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Research Article

Global Self-similar Solutions of a Class of Nonlinear Schrödinger Equations

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For a certain range of the value p in the nonlinear term $|u|^p u$, in this paper, we mainly study the global existence and uniqueness of global self-similar solutions to the Cauchy problem for some nonlinear Schrödinger equations using the method of harmonic analysis.

1. Introduction

This paper is devoted to study the initial value problem for the nonlinear Schrödinger equation

$$\begin{aligned} iu_t + (-\Delta)^m u &= \lambda |u|^p u, \quad x \in R^n,\ t \in R^+, \\ u(x,0) &= f(x), \quad x \in R^n, \end{aligned} \tag{1.1}$$

where $\lambda \in R$, $p > 0$ are constants, $m \geq 1$ is a positive integer, $u(t,x)$ is a complex-valued function defined in $R^+ \times R^n$, the initial value $f(x)$ is a complex-valued function defined in R^n.

When $m = 1$, (1.1) is a classical nonlinear Schrödinger equation of the second order:

$$iu_t - \Delta u = \lambda |u|^p u. \tag{1.2}$$

For the Cauchy problem of (1.2), the existence and the scattering theorem of solutions have been studied extensively by many authors with various methods and techniques [1–5], Cazenave and Weissler [6] (also Ribaud and Youssfi [7]) established existence of global self-similar solutions by introducing new function space. When $m \geq 1$, Pecher and von Wahl [8] established the existence of classical solution of the Cauchy problem (1.1) employing the related L^p estimate of the elliptic equation and the compact method. Sjölin and Sjögren in [9, 10] recently discussed the local smooth effect of solutions of the Cauchy problem (1.1) applying the Strichartz estimate in the nonhomogeneous Sobolev space. In [11], by constructing a time-weighted space and using the contractive mapping method, the author established global solutions of the problem (1.1) in the possible range of p, and further got the continuous dependence of the solution on the initial value together with its strong decay estimate. In addition, there are also much more efforts working for studying the scattering theorem and the existence of global strong solutions of the problem (1.1) [12, 13]. In this paper, we mainly investigate the existence of global self-similar solutions basing on the existence and uniqueness of global solution for the Cauchy problem (1.1).

In the following discussion, we suppose that p satisfies

$$p_0 < p < \frac{4m}{n-2m}, \quad n > 2m, \ p_0 < p < +\infty, \ n \leq 2m, \tag{1.3}$$

where p_0 is a positive solution of the equation $nx^2 + (n-2m)x - 4m = 0$, which also can be interpreted as a positive integer satisfying $(p+2)/(p+1) = np/2m$. In fact, condition (1.3) is equivalent to

$$\frac{p+2}{p+1} < \frac{np}{2m} < p+2. \tag{1.4}$$

For p which satisfies (1.3) or (1.4), let

$$\theta = \frac{4m-(n-2m)p}{2mp(p+2)}, \tag{1.5}$$

then we may introduce our work space X as follows. Let X be a space consisting of all Bochner measurable functions:

$$u(t) : (0,+\infty) \longrightarrow L^{p+2}(R^n), \tag{1.6}$$

such that

$$\|u\|_X = \sup_{t>0} t^{\theta}\|u(t)\|_{p+2} < +\infty. \tag{1.7}$$

In order to prove our main result, we should transform the Cauchy problem (1.1) into the following equivalent integral equation:

$$u(t) = S(t)f(x) - i\lambda\int_0^t S(t-s)\left(|u(s)|^p u(s)\right)ds, \tag{1.8}$$

where $S(t) = e^{i(-\Delta)^m t} = \mathcal{F}^{-1}(e^{i|\xi|^{2m}t}\mathcal{F}\cdot)$ is a free group produced by the free Schrödinger equation $iv_t + (-\Delta)^m v = 0$. Besides, we denote, respectively, by $\mathcal{F}$ and $\mathcal{F}^{-1}$ the Fourier transformation and the inverse Fourier transformation with respect to the space variables.

For convenience, we provide some useful symbols. $L^r(R^n)$ denotes the usual Lebesgue space on R^n with the norm $\|\cdot\|_r$, $1 \leq r \leq +\infty$. For any $q > 0$, q' stands for the dual to q, that is,

$(1/q)+(1/q')=1$. C which may be different when appeared every time is a constant depending on the dimension or any other constant.

In the end, we will review the definition of the homogeneous Besov space, the details on the properties, and the embedding theorems reference [1, 14].

Let $\widehat{\varphi}(\xi) \in \mathcal{S}$ be a symmetric Bump function with real values satisfying the conditions $\widehat{\varphi}(\xi)=1$, $|\xi| \le 1$, $\widehat{\varphi}(\xi)=0$, $|\xi|>2$, then

$$\widehat{\varphi}_j(\xi)=\widehat{\varphi}(2^{-j}\xi), \quad \widehat{\psi}_j(\xi)=\widehat{\psi}(2^{-j}\xi)=\widehat{\varphi}(2^{-j}\xi)-\widehat{\varphi}(2^{-j+1}\xi), \quad j \in Z \tag{1.9}$$

are also symmetric Bump functions. Denote by Δ_j and S_j the convolution operator of $\widehat{\psi}_j(\xi)$ and $\widehat{\varphi}_j(\xi)$, respectively, that is,

$$\Delta_j f=\mathcal{F}^{-1}\widehat{\psi}_j\mathcal{F}f=\psi_j * f, \quad S_j f=\mathcal{F}^{-1}\widehat{\varphi}_j\mathcal{F}f=\varphi_j * f \quad \forall j \in Z. \tag{1.10}$$

If $s \in R$, $1 \le p \le +\infty$, $1 \le q < +\infty$, then

$$\dot{B}_p^{s,q}=\left\{f \in \mathcal{S}' : \|f\|_{\dot{B}_p^{s,q}}=\left[\sum_{j \in Z} 2^{sjq}\|\Delta_j f\|_p^q\right]^{1/q}<+\infty\right\} \tag{1.11}$$

is called a homogeneous Besov space and

$$\dot{B}_p^{s,\infty}=\left\{f \in \mathcal{S}' : \|f\|_{\dot{B}_p^{s,\infty}}=\sup_{j \in Z} 2^{js}\left\|\Delta_j f\right\|_p<+\infty\right\}. \tag{1.12}$$

2. Lemmas and main results

The linear Schrödinger group $S(t)=e^{i(-\Delta)^m t}$ satisfies the following $L^{q'}-L^q$ estimate [14, 15]:

$$\left\|S(t)f(x)\right\|_q=\left\|\mathcal{F}^{-1}\left(e^{i|\xi|^{2m}t}\mathcal{F}f\right)\right\|_q \le C|t|^{-(n/m)(1/2-1/q)}\|\varphi\|_{q'}, \quad 2 \le q \le +\infty \ \forall t>0. \tag{2.1}$$

We first provide two lemmas that may be useful in in the following.

Lemma 2.1. *Let* $f(x)=\Omega(x/|x|)|x|^{-2m/p}$, $\theta=(4m-(n-2m)p)/2mp(p+2)$, *then* $\|u_0\|_X=\|S(t)f\|_X=\|S(1)f\|_{p+2}$.

Proof. According to the property of the Fourier transformation and $f(x)=\lambda^{2m/p}f(\lambda x)$, we get

$$S(t)f=\lambda^{2m/p}\left[S\left(\lambda^{2m}t\right)\right]f(\lambda x) \quad \forall \lambda>0. \tag{2.2}$$

Let $\lambda=1/\sqrt[2m]{t}$, then

$$S(t)f=t^{-1/p}[S(1)]f\left(\frac{x}{\sqrt[2m]{t}}\right). \tag{2.3}$$

Thus

$$t^{\theta}\left\|S(t)f\right\|_{p+2}=t^{\theta-1/p}\left\|[S(1)f]\left(\frac{x}{\sqrt[2m]{t}}\right)\right\|_{p+2}=t^{\theta-1/p+n/2m(p+2)}\left\|S(1)f\right\|_{p+2}. \tag{2.4}$$

Since

$$\theta - \frac{1}{p} + \frac{n}{2m(p+2)} = 0, \tag{2.5}$$

It is easy to see that from (2.4) and (2.5),

$$\sup_{t>0} t^{\theta} \left\| S(t)f \right\|_{p+2} = \left\| S(1)f \right\|_{p+2}, \tag{2.6}$$

namely,

$$\left\| u_0 \right\|_X = \left\| S(t)f \right\|_X = \left\| S(1)f \right\|_{p+2}. \tag{2.7}$$

□

Lemma 2.2. *Let* $\Omega \in C^k(S^{n-1})$, $k \geq 0$, $f(x) = \Omega(x/|x|)|x|^{-d}$, $0 < d < n$, *then*

$$\left| \Delta_0(f)(x) \right| \leq C \|\Omega\|_{C^k} (1 + |x|)^{-k-d}. \tag{2.8}$$

The detailed proof can be referred to [16].

In order to prove the main results, we need the following known theorems [11].

Theorem 2.3 (existence of global solutions). *Suppose that p satisfies* (1.3) *or* (1.4), $\theta = (4m - (n - 2m)p)/(2mp(p+2))$, $u_0(t,x) = [S(t)f](x)$ *if there is* $\varepsilon > 0$, *such that*

$$\left\| u_0 \right\|_X = \left\| S(t)f \right\|_X \leq \varepsilon, \tag{2.9}$$

then the Cauchy problem (1.1) *has a unique solution* $u(x,t) \in X$ *which satisfies* $\|u\|_X \leq 2\varepsilon$.

Theorem 2.4 (the continuous dependence of the solution on the initial value). *Suppose that* $f(x)$ *and* $g(x)$ *both satisfy the condition* (2.9), u, v *are two solutions of the Cauchy problem* (1.1) *corresponding to the initial value* $f(x)$ *and* $g(x)$, *then*

$$\|u - v\|_X \leq C \left\| S(t)(f-g) \right\|_X. \tag{2.10}$$

In addition, if

$$\sup_{t>0} t^{\theta}(1+t)^{\delta} \left\| S(t)(f-g) \right\|_{p+2} < +\infty, \tag{2.11}$$

then

$$\|u - v\|_{p+2} \leq C t^{-\theta}(1+t)^{-\delta}, \tag{2.12}$$

where $(p+1)\theta + \delta < 1$, $\delta > 0$.

In this paper, our object is to study the global self-similar solutions of the Cauchy problem (1.1). At first, we introduce the definition of the self-similar solution.

Definition 2.5. Suppose that $u(t,x)$ is a solution of the Cauchy problem (1.1), if

$$u(t,x) = u_\lambda(t,x) = \lambda^{2m/p} u\left(\lambda^{2m} t, \lambda x\right) \quad \forall \lambda > 0, \tag{2.13}$$

then $u(t,x)$ is called the self-similar solution of the problem (1.1).

One easily knows from the above definition that $u_\lambda(t,x) = \lambda^{2m/p} u(\lambda^{2m} t, \lambda x)$ $\forall \lambda > 0$ is a solution of the problem (1.1) which satisfies the initial value $\lambda^{2m/p} f(\lambda x)$, provide that $u(t,x)$ is just a solution of the Cauchy problem (1.1).

Now, we give our main result.

Theorem 2.6. *Let p satisfy* (1.3) *or* (1.4), $\theta = (4m - (n-2m)p)/2mp(p+2)$, $\Omega \in C^n(S^{n-1})$, *and*

$$f(x) = \frac{\Omega(x/|x|)}{|x|^{2m/p}}, \tag{2.14}$$

$u_0(t,x) = S(t)f(x)$, *then*

$$\left\| u_0 \right\|_X \leq C \|\Omega\|_{C^n}. \tag{2.15}$$

In particular, if existing $\varepsilon' = \varepsilon / C > 0$ such that $\|\Omega\|_{C^n} \leq \varepsilon'$, then there exists a unique self-similar solution of (1.1) *with the initial value* (2.14).

3. The proof of main result

To prove Theorem 2.6, we should provide the following two propositions.

Proposition 3.1. *Let*

$$f(x) = \Omega\left(\frac{x}{|x|}\right) |x|^{-2m/p}, \qquad \theta = \frac{4m - (n-2m)p}{2mp(p+2)}, \tag{3.1}$$

then

$$\left\| u_0 \right\|_X = \left\| S(t) f \right\|_X \leq C \left\| \Delta_0(f) \right\|_{(p+2)'}. \tag{3.2}$$

Proof. By Lemma 2.1, we only illustrate that the following inequality is valid:

$$\left\| S(1) f \right\|_{p+2} \leq C \left\| \Delta_0(f) \right\|_{(p+2)'}. \tag{3.3}$$

It follows that from the embedding $\dot{B}^{0,1}_{p+2} \hookrightarrow \dot{H}^0_{p+2} = L^{p+2}$, it is necessary to prove

$$\left\| S(1) f \right\|_{\dot{B}^{0,1}_{p+2}} \leq C \left\| \Delta_0(f) \right\|_{(p+2)'}. \tag{3.4}$$

Denote $F = S(1)f$, and then F can be decomposed as follows:

$$F = F_1 + F_2, \qquad F_1 = S(1)(\varphi * f), \qquad F_2 = S(1)\left(\mathcal{F}^{-1}(1 - \widehat{\varphi}) * f\right), \tag{3.5}$$

where φ is referred in the introduction.

Making use of the estimate (2.1) and noting that $\Delta_j \mathcal{F}^{-1}(1-\widehat{\varphi}) = 0$ for all $j \leq -1$, then we have

$$\begin{aligned}
\|F_2\|_{\dot{B}^{0,1}_{p+2}} &= \sum_{j\in Z}\|\Delta_j F_2\|_{p+2} = \sum_{j\in Z}\|S(1)\Delta_j(\mathcal{F}^{-1}((1-\widehat{\varphi})*f))\|_{p+2} \\
&\leq \sum_{j\in Z}\|\Delta_j(\mathcal{F}^{-1}((1-\widehat{\varphi})*f))\|_{(p+2)'} \\
&= \sum_{j\geq 0}\|\Delta_j(\mathcal{F}^{-1}((1-\widehat{\varphi})*f))\|_{(p+2)'} \\
&= \sum_{j\geq 0}\|\widetilde{\Delta}_j\Delta_j(\mathcal{F}^{-1}((1-\widehat{\varphi})*f))\|_{(p+2)'},
\end{aligned} \tag{3.6}$$

where $\widetilde{\Delta}_j = \sum_{l=-1}^{l=1}\Delta_{j+l}$.

For $l = \pm 1, 0$, we have

$$\|\Delta_{j+l}\mathcal{F}^{-1}(1-\widehat{\varphi})\|_1 = \|\mathcal{F}^{-1}(\widehat{\varphi}_{j+l}(1-\widehat{\varphi}))\|_1 \leq \|\varphi_{j+l}\|_1 + \|\varphi_{j+l}*\varphi\|_1. \tag{3.7}$$

Since $\varphi_{j+l}(x) = 2^{(j+l)n}\varphi_0(2^{j+l}x)$, then $\|\varphi_{j+l}\|_1 = \|\varphi_0\|_1$. Thus, it follows that from the Young inequality

$$\|\Delta_{j+l}\mathcal{F}^{-1}(1-\widehat{\varphi})\|_1 \leq C\|\varphi_{j+l}\|_1(1+\|\varphi\|_1) \leq C. \tag{3.8}$$

Besides, as $f(\lambda x) = \lambda^{-2m/p}f(x)$, so that

$$\begin{aligned}
\Delta_j f(x) = \varphi_j * f(x) &= \int_{R^n}\varphi_j(x-y)f(y)dy \\
&= 2^{jn}\int_{R^n}\varphi_0(2^j x - 2^j y)f(y)dy \\
&= \int_{R^n}\varphi_0(2^j x - z)f(2^{-j}z)dz \\
&= 2^{j(2m/p)}(\varphi_0 * f)(2^j x) = 2^{j(2m/p)}\Delta_0 f(2^j x).
\end{aligned} \tag{3.9}$$

Therefore,

$$\|\Delta_j f\|_{(p+2)'} = 2^{j(2m/p)}\|\Delta_0 f(2^j\,\cdot)\|_{(p+2)'} = 2^{j(2m/p-n/(p+2)')}\|\Delta_0 f\|_{(p+2)'}. \tag{3.10}$$

By (3.6) together with the Young inequality, we obtain

$$\|F_2\|_{\dot{B}^{0,1}_{p+2}} \leq \sum_{j\geq 0}\|\widetilde{\Delta}_j(\mathcal{F}^{-1}(1-\widehat{\varphi}))\|_1\|\Delta_j f\|_{(p+2)'} \leq C\|\Delta_0 f\|_{(p+2)'}\sum_{j\geq 0}2^{j(2m/p-n/(p+2)')}. \tag{3.11}$$

We know that from the left side of the inequality (1.4),

$$\frac{2m}{p} - \frac{n}{(p+2)'} < 0. \tag{3.12}$$

It yields from (3.11) that

$$\|F_2\|_{\dot{B}^{0,1}_{p+2}} \le C\|\Delta_0 f\|_{(p+2)'}. \tag{3.13}$$

On the other hand, $\Delta_j\varphi = 0$ for $j \ge 2$, thus

$$\|F_1\|_{\dot{B}^{0,1}_{p+2}} = \sum_{j\in Z}\|\Delta_j F_1\|_{p+2} = \sum_{j\le 1}\|S(1)\Delta_j(\varphi * f)\|_{p+2}. \tag{3.14}$$

It follows that by the Young inequality,

$$\|S(1)\Delta_j(\varphi * f)\|_{p+2} \le \|S(1)\varphi\|_1\|\Delta_j f\|_{p+2}. \tag{3.15}$$

We get that from (3.15) and $\|\Delta_j f\|_{p+2} = 2^{j(2m/p-n/(p+2))}\|\Delta_0 f\|_{p+2}$,

$$\|S(1)\Delta_j(\varphi * f)\|_{p+2} \le C2^{j(2m/p-n/(p+2))}\|\Delta_0 f\|_{p+2}. \tag{3.16}$$

Correspondingly,

$$\|F_1\|_{\dot{B}^{0,1}_{p+2}} \le C\|\Delta_0 f\|_{p+2}\sum_{j\le 1}2^{j(2m/p-n/(p+2))}. \tag{3.17}$$

The right side of (1.4) shows that $2m/p - n/(p+2) > 0$, consequently

$$\|F_1\|_{\dot{B}^{0,1}_{p+2}} \le C\|\Delta_0 f\|_{p+2}. \tag{3.18}$$

From $(p+2)' \le p+2$ and the Bernstein inequality, we get

$$\|F_1\|_{\dot{B}^{0,1}_{p+2}} \le C\|\Delta_0 f\|_{p+2} \le C\|\Delta_0 f\|_{(p+2)'}, \tag{3.19}$$

Combining (3.13) with (3.19), we have

$$\|F\|_{\dot{B}^{0,1}_{p+2}} \le C\|\Delta_0 f\|_{(p+2)'}. \tag{3.20}$$

The proof of Proposition 3.1 is finished. □

Proposition 3.2. *Let $\Omega \in C^n(S^{n-1})$, $f(x) = \Omega(x/|x|)|x|^{-2m/p}$, then*

$$\|\Delta_0 f\|_{(p+2)'} \le C\|\Omega\|_{C^n}. \tag{3.21}$$

Proof. Since $(p+2)' \ge 1$, then $(n+2m/p)(p+2)' > n$. Accordingly, we obtain by Lemma 2.2 that

$$\begin{aligned}
\|\Delta_0 f\|^{(p+2)'}_{(p+2)'} &= \int_{R^n}|\Delta_0 f(x)|^{(p+2)'}dx \\
&\le C\int_{R^n}\|\Omega\|^{(p+2)'}_{C^n}(1+|x|)^{-(n+2m/p)(p+2)'}dx \\
&\le C\|\Omega\|^{(p+2)'}_{C^n}\int_0^{+\infty}(1+r)^{-(n+2m/p)(p+2)'+n-1}dr \\
&\le C\|\Omega\|^{(p+2)'}_{C^n},
\end{aligned} \tag{3.22}$$

which implies that

$$\|\Delta_0 f\|_{(p+2)'} \leq C\|\Omega\|_{C^n}. \tag{3.23}$$

The proof is concluded. □

Now, we are ready to prove Theorem 2.6.

Proof. For

$$f(x) = \Omega\left(\frac{x}{|x|}\right)|x|^{-2m/p}, \tag{3.24}$$

we have from Proposition 3.1

$$\|S(t)f\|_X \leq C\|\Delta_0(f)\|_{(p+2)'}. \tag{3.25}$$

However, noting that $\Omega \in C^n(S^{n-1})$ as well as Proposition 3.2, we get

$$\|\Delta_0(f)\|_{(p+2)'} \leq C\|\Omega\|_{C^n}. \tag{3.26}$$

Then, it follows from (3.25) and (3.26) that

$$\|u_0\|_X = \|S(t)f\|_X \leq C\|\Omega\|_{C^n}. \tag{3.27}$$

Choosing $\varepsilon' = \varepsilon/C > 0$, then we have $\|u_0\|_X \leq \varepsilon$ for any $\|\Omega\|_{C^n} \leq \varepsilon'$. From Theorem 2.3, we conclude that there is a unique global solution $u(x,t)$ of the equation in (1.1) with the initial value (2.14). Besides,

$$\lambda^{2m/p} f(\lambda x) = \Omega\left(\frac{x}{|x|}\right)|x|^{-2m/p} = f(x), \tag{3.28}$$

which gives that by uniqueness

$$u(x,t) = \lambda^{2m/p} u(\lambda x, \lambda^{2m} t). \tag{3.29}$$

Thus, $u(x,t)$ is just a self-similar solution of the problem (1.1).

This completes the proof of Theorem 2.6. □

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