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Research Article

On Pexider Differences in Topological Vector Spaces

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Let X be a normed space and Y a sequentially complete Hausdorff topological vector space over the field $\mathbb{Q}$ of rational numbers. Let $D_1 = \{(x, y) \in X \times X : \|x\| + \|y\| \geq d\}$, and $D_2 = \{(x, y) \in X \times X : \|x\| + \|y\| < d\}$ where $d > 0$. We prove that the Pexiderized Jensen functional equation is stable for functions defined on $D_1(D_2)$, and taking values in Y. We consider also the Pexiderized Cauchy functional equation.

1. Introduction

The functional equation (ξ) is stable if any function g satisfying the equation (ξ) *approximately* is near to true solution of (ξ). The stability of functional equations was first introduced by Ulam [1] in 1940. More precisely, Ulam proposed the following problem: given a group G_1, a metric group (G_2, d), and a positive number ϵ, does there exist a $\delta > 0$ such that if a function $f : G_1 \to G_2$ satisfies the inequality $d(f(xy), f(x)f(y)) < \delta$ for all $x, y \in G_1$, then there exists a homomorphism $T : G_1 \to G_2$ such that $d(f(x), T(x)) < \epsilon$ for all $x \in G_1$? As it is mentioned above, when this problem has a solution, we say that the homomorphisms from G_1 to G_2 are stable. In 1941, Hyers [2] gave a partial solution of Ulam's problem for the case of approximate additive mappings under the assumption that G_1 and G_2 are Banach spaces. Aoki [3] and Rassias [4] provided a generalization of Hyers' theorem for additive and linear mappings, respectively, by allowing the Cauchy difference to be unbounded. During the last decades several stability problems of functional equations have been investigated by several mathematicians. A large list of references concerning the stability of functional equations can be found in [5–8].

2. Hyers-Ulam Stability of Jensen's Functional Equation

Jung investigated the Hyers-Ulam stability for Jensen's equation on a restricted domain [9]. In this section, we prove a local Hyers-Ulam stability of the Pexiderized Jensen functional equation in topological vector spaces. In this section X is a normed space and Y is a sequentially complete Hausdorff topological vector space over the field $\mathbb{Q}$ of rational numbers.

Theorem 2.1. *Let V be a nonempty bounded convex subset of Y containing the origin. Suppose that $f, g, h : X \to Y$ satisfies*

$$2f\left(\frac{x+y}{2}\right) - g(x) - h(y) \in V \tag{2.1}$$

for all $x, y \in X$ with $\|x\| + \|y\| \geqslant d$, where $d > 0$. Then there exists a unique additive mapping $A : X \to Y$ such that

$$A(x) - f(x) + f(0) \in \overline{W - W}, \tag{2.2}$$

$$A(x) - g(x) + g(0) \in \overline{2(W - W)} + W - W, \tag{2.3}$$

$$A(x) - h(x) + h(0) \in \overline{2(W - W)} + W - W \tag{2.4}$$

for all $x \in \mathbb{R}$, where $W = 3V - 2V$ and $\overline{2(W - W)}$ denotes the sequential closure of $2(W - W)$.

Proof. Suppose $\|x\| + \|y\| < d$. If $\|x\| + \|y\| = 0$, let $z \in X$ with $\|z\| = d$, otherwise

$$z := \begin{cases} (d + \|x\|)\dfrac{x}{\|x\|}, & \text{if } \|x\| \geqslant \|y\|; \\ (d + \|y\|)\dfrac{y}{\|y\|}, & \text{if } \|y\| \geqslant \|x\|. \end{cases} \tag{2.5}$$

It is easy to verify that

$$\begin{gathered} \|x - z\| + \|y + z\| \geqslant d; \|2z\| + \|x - z\| \geqslant d; \|y\| + \|2z\| \geqslant d; \\ \|y + z\| + \|z\| \geqslant d; \|x\| + \|z\| \geqslant d. \end{gathered} \tag{2.6}$$

It follows from (2.1) and (2.6) that

$$\begin{aligned}2f\left(\frac{x+y}{2}\right)-g(x)-h(y)&=2f\left(\frac{x+y}{2}\right)-g(y+z)-h(x-z)\\&\quad-\left[2f\left(\frac{x+z}{2}\right)-g(2z)-h(x-z)\right]\\&\quad+\left[2f\left(\frac{y+2z}{2}\right)-g(2z)-h(y)\right]\\&\quad-\left[2f\left(\frac{y+2z}{2}\right)-g(y+z)-h(z)\right]\\&\quad+\left[2f\left(\frac{x+z}{2}\right)-g(x)-h(z)\right]\\&\in 3V-2V\end{aligned}\tag{2.7}$$

for all $x, y \in X$ with $\|x\| + \|y\| < d$. Hence, by (2.1) and (2.7), we have

$$2f\left(\frac{x+y}{2}\right)-g(x)-h(y)\in 3V-2V\tag{2.8}$$

for all $x, y \in X$. Letting $x = 0$ $(y = 0)$ in (2.8), we get

$$\begin{aligned}&2f\left(\frac{y}{2}\right)-g(0)-h(y)\in 3V-2V,\\&2f\left(\frac{x}{2}\right)-g(x)-h(0)\in 3V-2V\end{aligned}\tag{2.9}$$

for all $x, y \in X$. It follows from (2.8) and (2.9) that

$$\begin{aligned}2f\left(\frac{x+y}{2}\right)-2f\left(\frac{x}{2}\right)-2f\left(\frac{y}{2}\right)+2f(0)&=2f\left(\frac{x+y}{2}\right)-g(x)-h(y)-\left[2f\left(\frac{x}{2}\right)-g(x)-h(0)\right]\\&\quad-\left[2f\left(\frac{y}{2}\right)-g(0)-h(y)\right]+\left[2f(0)-g(0)-h(0)\right]\\&\in 2(W-W)\end{aligned}\tag{2.10}$$

for all $x, y \in X$, where $W = 3V - 2V$. So we get from (2.10) that

$$f(x+y)-f(x)-f(y)+f(0)\in W-W\tag{2.11}$$

for all $x, y \in X$. Setting $y = x$ in (2.10), we infer that

$$f(x)-2f\left(\frac{x}{2}\right)+f(0)\in W-W\tag{2.12}$$

for all $x \in X$. It is easy to prove that

$$\frac{f(2^{n+1}x)}{2^{n+1}} - \frac{f(2^n x)}{2^n} + \frac{f(0)}{2^{n+1}} \in \frac{1}{2^{n+1}}(W - W) \subseteq W - W, \tag{2.13}$$

$$\frac{f(2^n x)}{2^n} - f(x) + \sum_{k=1}^{n} \frac{1}{2^k} f(0) \in \sum_{k=1}^{n} \frac{1}{2^k}(W - W) \subseteq W - W \tag{2.14}$$

for all $x \in X$ and all integers $n \geqslant 1$. Since V is a nonempty bounded convex subset of X containing the origin, $W - W$ is a nonempty bounded convex subset of X containing the origin. It follows from (2.13) that

$$\begin{aligned}\frac{f(2^n x)}{2^n} - \frac{f(2^m x)}{2^m} + \sum_{k=m}^{n-1} \frac{1}{2^{k+1}} f(0) &= \sum_{k=m}^{n-1} \left[\frac{f(2^{k+1}x)}{2^{k+1}} - \frac{f(2^k x)}{2^k} + \frac{1}{2^{k+1}} f(0) \right] \\ &\in \sum_{k=m}^{n-1} \frac{1}{2^{k+1}}(W - W) \subseteq \frac{1}{2^m}(W - W)\end{aligned} \tag{2.15}$$

for all $x \in X$ and all integers $n > m \geqslant 0$. Let U be an arbitrary neighborhood of the origin in Y. Since $W - W$ is bounded, there exists a rational number $t > 0$ such that $t(W - W) \subseteq U$. Choose $n_0 \in \mathbb{N}$ such that $2^{n_0} t > 1$. Let $x \in X$ and $m, n \in \mathbb{N}$ with $n \geqslant m \geqslant n_0$. Then (2.15) implies that

$$\frac{f(2^n x)}{2^n} - \frac{f(2^m x)}{2^m} + (2^{-m} - 2^{-n}) f(0) \in U. \tag{2.16}$$

Thus, the sequence $\{2^{-n} f(2^n x)\}$ forms a Cauchy sequence in Y. By the sequential completeness of Y, the limit $A(x) = \lim_{n\to\infty} 2^{-n} f(2^n x)$ exists for each $x \in X$. So (2.2) follows from (2.14).

To show that $A : X \to Y$ is additive, replace x and y by $2^n x$ and $2^n y$, respectively, in (2.11) and then divide by 2^n to obtain

$$\frac{f(2^n(x + y))}{2^n} - \frac{f(2^n x)}{2^n} - \frac{f(2^n y)}{2^n} + \frac{f(0)}{2^n} \in \frac{1}{2^n}(W - W) \tag{2.17}$$

for all $x, y \in X$ and all integers $n \geqslant 0$. Since $\overline{W - W}$ is bounded, on taking the limit as $n \to \infty$, we get that A is additive. It follows from (2.2) and (2.9) that

$$\begin{aligned}A(2x) - g(2x) + g(0) &= 2A(x) - 2f(x) + 2f(0) + 2f(x) \\ &\quad - g(2x) - h(0) - 2f(0) + g(0) + h(0) \\ &\in \overline{2(W - W)} + W - W\end{aligned} \tag{2.18}$$

for all $x \in X$. So we obtain (2.3). Similarly, we get (2.4).

To prove the uniqueness of A, assume on the contrary that there is another additive mapping $T : X \to Y$ satisfying (2.2) and there is an $a \in X$ such that $y = T(a) - A(a) \neq 0$. So there is a neighborhood U of the origin in Y such that $y \notin U$, since Y is Hausdorff. Since A and T satisfy (2.2), we get $T(x) - A(x) \in \overline{2(W - W)}$ for all $x \in X$. Since $\overline{W - W}$ is bounded, there exists a positive integer m such that $\overline{2(W - W)} \subseteq mU$. Therefore, $my = T(ma) - A(ma) \in mU$ which is a contradiction with $y \notin U$. This completes the proof. □

We apply the result of Theorem 2.1 to study the asymptotic behavior of additive mappings.

Theorem 2.2. *Suppose that Y has a bounded convex neighborhood of 0. Let $f, g, h : X \to Y$ be functions satisfying*

$$2f\left(\frac{x+y}{2}\right) - g(x) - h(y) \longrightarrow 0 \quad \text{as } \|x\| + \|y\| \longrightarrow \infty. \tag{2.19}$$

Then $f - f(0)$, $g - g(0)$, $h - h(0)$ are additive and $f - f(0) = g - g(0) = h - h(0)$.

Proof. Let V be a bounded convex neighborhood of 0 in Y. It follows from (2.19) that there exists an increasing sequence $\{d_n\}_n$ such that

$$2f\left(\frac{x+y}{2}\right) - g(x) - h(y) \in \frac{1}{n}V \tag{2.20}$$

for all $x, y \in X$ with $\|x\| + \|y\| \geq d_n$. Applying (2.20) and Theorem 2.1, we obtain a sequence $\{A_n : X \to Y\}$ of unique additive mappings satisfying

$$\begin{gathered} A_n(x) - f(x) + f(0) \in \frac{1}{n}\overline{(W - W)}, \\ A_n(x) - g(x) + g(0) \in \frac{3}{n}\overline{(W - W)}, \\ A_n(x) - h(x) + h(0) \in \frac{3}{n}\overline{(W - W)} \end{gathered} \tag{2.21}$$

for all $x \in X$, where $W = 3V - 2V$. Since $\overline{(W - W)}$ is convex and $0 \in \overline{(W - W)}$, we have

$$\begin{gathered} A_m(x) - f(x) + f(0) \in \frac{1}{m}\overline{(W - W)} \subseteq \frac{1}{n}\overline{(W - W)}, \\ A_m(x) - g(x) + g(0) \in \frac{3}{m}\overline{(W - W)} \subseteq \frac{3}{n}\overline{(W - W)}, \\ A_m(x) - h(x) + h(0) \in \frac{3}{m}\overline{(W - W)} \subseteq \frac{3}{n}\overline{(W - W)} \end{gathered} \tag{2.22}$$

for all $x \in X$ and all $m \geqslant n$. The uniqueness of A_n implies $A_m = A_n$ for all $m \geqslant n$. Hence, letting $n \to \infty$ in (2.21), we obtain that $f - f(0)$, $g - g(0)$, $h - h(0)$ are additive and $f - f(0) = g - g(0) = h - h(0)$. □

Theorem 2.3. *Let V be a nonempty bounded convex subset of Y containing the origin. Suppose that $f, g, h : X \to Y$ with $g(0) = h(0) = 0$ satisfies*

$$2f\left(\frac{x+y}{2}\right) - g(x) - h(y) \in \left(\|x\|^p + \|y\|^p\right)V \tag{2.23}$$

for all $x, y \in X$ with $\|x\| + \|y\| \leqslant d$, where $d > 0$ and $p > 1$. Then there exists a unique additive mapping $\varphi : X \to Y$ such that

$$\varphi(x) - f(x) \in \frac{2^p}{2^p - 2}\|x\|^p\overline{(V - V)}, \tag{2.24}$$

$$\varphi(x) - g(x) \in \frac{2^p}{2^p - 2}\|x\|^p\left(\overline{V - V} + V\right), \tag{2.25}$$

$$\varphi(x) - h(x) \in \frac{2^p}{2^p - 2}\|x\|^p\left(\overline{V - V} + V\right) \tag{2.26}$$

for all $x \in X$ with $\|x\| \leqslant d/2$. Moreover, $\varphi(x) = \lim_{n\to\infty} 2^n f(2^{-n}x)$ for all $x \in X$.

Proof. Letting $y = x = 0$ in (2.23), we get $f(0) = 0$. Letting $y = x$ in (2.23), we get

$$2f(x) - g(x) - h(x) \in 2\|x\|^p V \tag{2.27}$$

for all $x \in X$ with $\|x\| \leqslant d/2$. If we put $y = 0$ $(x = 0)$ in (2.23), we have

$$\begin{aligned} 2f\left(\frac{x}{2}\right) - g(x) &\in \|x\|^p V, \quad \|x\| \leqslant \frac{d}{2}, \\ 2f\left(\frac{y}{2}\right) - h(y) &\in \|y\|^p V, \quad \|y\| \leqslant \frac{d}{2}. \end{aligned} \tag{2.28}$$

Hence it follows from (2.27) and (2.28) that

$$2f\left(\frac{x}{2}\right) - f(x) \in \|x\|^p(V - V) \tag{2.29}$$

for all $x \in X$ with $\|x\| \leqslant d/2$. We can replace x by $x/2^n$ in (2.29) for all nonnegative integers n. So we have

$$2^{n+1}f\left(2^{-n-1}x\right) - 2^n f\left(2^{-n}x\right) \in \left(\frac{2}{2^p}\right)^n \|x\|^p(V - V) \tag{2.30}$$

for all $x \in X$ with $\|x\| \leqslant d/2$. Therefore,

$$
\begin{aligned}
2^{n+1} f\left(2^{-n-1} x\right) - 2^m f\left(2^{-m} x\right) &= \sum_{k=m}^{n} \left[2^{k+1} f\left(2^{-k-1} x\right) - 2^k f\left(2^{-k} x\right)\right] \\
&\in \sum_{k=m}^{n} \left(\frac{2}{2^p}\right)^k \|x\|^p (V - V) \\
&\subseteq \frac{2^{m+p}}{2^{mp}(2^p - 2)} \|x\|^p (V - V)
\end{aligned}
\tag{2.31}
$$

for all $x \in X$ with $\|x\| \leqslant d/2$ and all integers $n \geqslant m \geqslant 0$. Let $x \in X$ with $\|x\| \leqslant d/2$, and let U be an arbitrary neighborhood of the origin in Y. Since $V - V$ is bounded, there exists a rational number $t > 0$ such that $t(V - V) \subseteq U$. Choose $k \in \mathbb{N}$ such that $(2^{kp}(2^p - 2)/2^{k+p})t > \|x\|^p$. Let $m, n \in \mathbb{N}$ with $n \geqslant m \geqslant k$. Then (2.31) implies that

$$
2^{n+1} f\left(2^{-n-1} x\right) - 2^m f\left(2^{-m} x\right) \in U. \tag{2.32}
$$

Thus, the sequence $\{2^n f(2^{-n} x)\}$ forms a Cauchy sequence in Y for all $x \in X$ with $\|x\| \leqslant d/2$. By the sequential completeness of Y, the limit $A(x) = \lim_{n \to \infty} 2^n f(2^{-n} x)$ exists for each $\|x\| \leqslant d/2$. It follows from (2.28) that

$$
A(x) = \lim_{n \to \infty} 2^n f(2^{-n} x) = \lim_{n \to \infty} 2^n g(2^{-n} x) = \lim_{n \to \infty} 2^n h(2^{-n} x) \tag{2.33}
$$

for all $x \in X$ with $\|x\| \leqslant d/2$. Letting $m = 0$ and $n \to \infty$ in (2.31), we obtain that A satisfies

$$
A(x) - f(x) \in \frac{2^p}{2^p - 2} \|x\|^p \overline{(V - V)} \tag{2.34}
$$

for all $x \in X$ with $\|x\| \leqslant d/2$. It follows from the definition of A that $A(0) = 0$, and we conclude from (2.23) that $A(x + y) = A(x) + A(y)$ for all $x, y \in X$ with $\|x\|, \|y\|, \|x + y\| \leqslant d/2$. Using an extension method of Skof [10], we extend the additivity of A to the whole space X (see also [11]). Let φ be the extension of A in which $\varphi(x) = A(x)$ for all $x \in X$ with $\|x\| \leqslant d/2$. It follows from (2.34) that φ satisfies (2.24). To prove (2.25), we have from (2.24) and (2.28) that

$$
\begin{aligned}
\phi(x) - g(x) &\in \phi(x) - 2f\left(\frac{x}{2}\right) + 2f\left(\frac{x}{2}\right) - g(x) \\
&\in \frac{2}{2^p - 2} \|x\|^p \overline{(V - V)} + \|x\|^p V \\
&= \frac{2^p}{2^p - 2} \|x\|^p \left(\overline{V - V} + V\right)
\end{aligned}
\tag{2.35}
$$

for all $x \in X$ with $\|x\| \leqslant d/2$. Similarly, we obtain (2.26). □

3. Hyers-Ulam Stability of Cauchy's Functional Equation

The following theorems are alternative results for the Pexiderized Cauchy functional equation.

Theorem 3.1. *Let V be a nonempty bounded convex subset of Y containing the origin. Suppose that $f, g, h : X \to Y$ satisfies*

$$f(x+y) - g(x) - h(y) \in V \tag{3.1}$$

for all $x, y \in X$ with $\|x\| + \|y\| \geqslant d$, where $d > 0$. Then there exists a unique additive mapping $A : X \to Y$ such that

$$\begin{aligned} A(x) - f(x) + f(0) &\in \overline{2(W-W)}, \\ A(x) - g(x) + g(0) &\in \overline{2(W-W)} + W - W, \\ A(x) - h(x) + h(0) &\in \overline{2(W-W)} + W - W \end{aligned} \tag{3.2}$$

for all $x \in \mathbb{R}$, where $W = 3V - 2V$.

Proof. Using the method in the proof of Theorem 2.1, we get

$$f(x+y) - g(x) - h(y) \in W \tag{3.3}$$

for all $x, y \in X$. Letting $x = 0$ $(y = 0)$ in (3.3), we get

$$\begin{aligned} f(y) - g(0) - h(y) &\in W, \\ f(x) - g(x) - h(0) &\in W \end{aligned} \tag{3.4}$$

for all $x, y \in X$. It follows from (3.3) and (3.4) that

$$\begin{aligned} f(x+y) - f(x) - f(y) + f(0) &= f(x+y) - g(x) - h(y) - [f(x) - g(x) - h(0)] \\ &\quad - [f(y) - g(0) - h(y)] + [f(0) - g(0) - h(0)] \\ &\in 2(W-W) \end{aligned} \tag{3.5}$$

for all $x, y \in X$. The rest of the proof is similar to the proof of Theorem 2.1, and we omit the details. □

Corollary 3.2. *Suppose that Y has a bounded convex neighborhood of 0. Let $f, g, h : X \to Y$ be functions satisfying*

$$f(x+y) - g(x) - h(y) \longrightarrow 0 \quad \text{as } \|x\| + \|y\| \longrightarrow \infty. \tag{3.6}$$

Then $f - f(0)$, $g - g(0)$, $h - h(0)$ are additive and $f - f(0) = g - g(0) = h - h(0)$.

Theorem 3.3. *Let V be a nonempty bounded convex subset of Y containing the origin. Suppose that $f, g, h : X \to Y$ with $g(0) = h(0) = 0$ satisfies*

$$f(x+y) - g(x) - h(y) \in \left(\|x\|^p + \|y\|^p\right)V \tag{3.7}$$

for all $x, y \in X$ with $\|x\| + \|y\| \leqslant d$, where $d > 0$ and $p > 1$. Then there exists a unique additive mapping $\varphi : X \to Y$ such that

$$\begin{aligned} \varphi(x) - f(x) &\in \frac{2}{2^p - 2}\|x\|^p \overline{(V - V)}, \\ \varphi(x) - g(x) &\in \frac{2^p}{2^p - 2}\|x\|^p \left(\overline{V - V} + V\right), \\ \varphi(x) - h(x) &\in \frac{2^p}{2^p - 2}\|x\|^p \left(\overline{V - V} + V\right) \end{aligned} \tag{3.8}$$

for all $x \in X$ with $\|x\| \leqslant d/2$. Moreover, $\varphi(x) = \lim_{n\to\infty} 2^n f(2^{-n}x)$ for all $x \in X$.

Proof. Using the same method in the proof of Theorem 2.3, we get from (3.7) that $2f(x) - f(2x) \in 2(V - V)\|x\|^p$ for all $x \in X$ with $\|x\| \leqslant d/2$. Therefore,

$$2f\left(\frac{x}{2}\right) - f(x) \in \frac{2}{2^p}\|x\|^p (V - V) \tag{3.9}$$

for all $x \in X$ with $\|x\| \leqslant d/2$. The rest of the proof is similar to the proof of Theorem 2.3, and we omit the details. □

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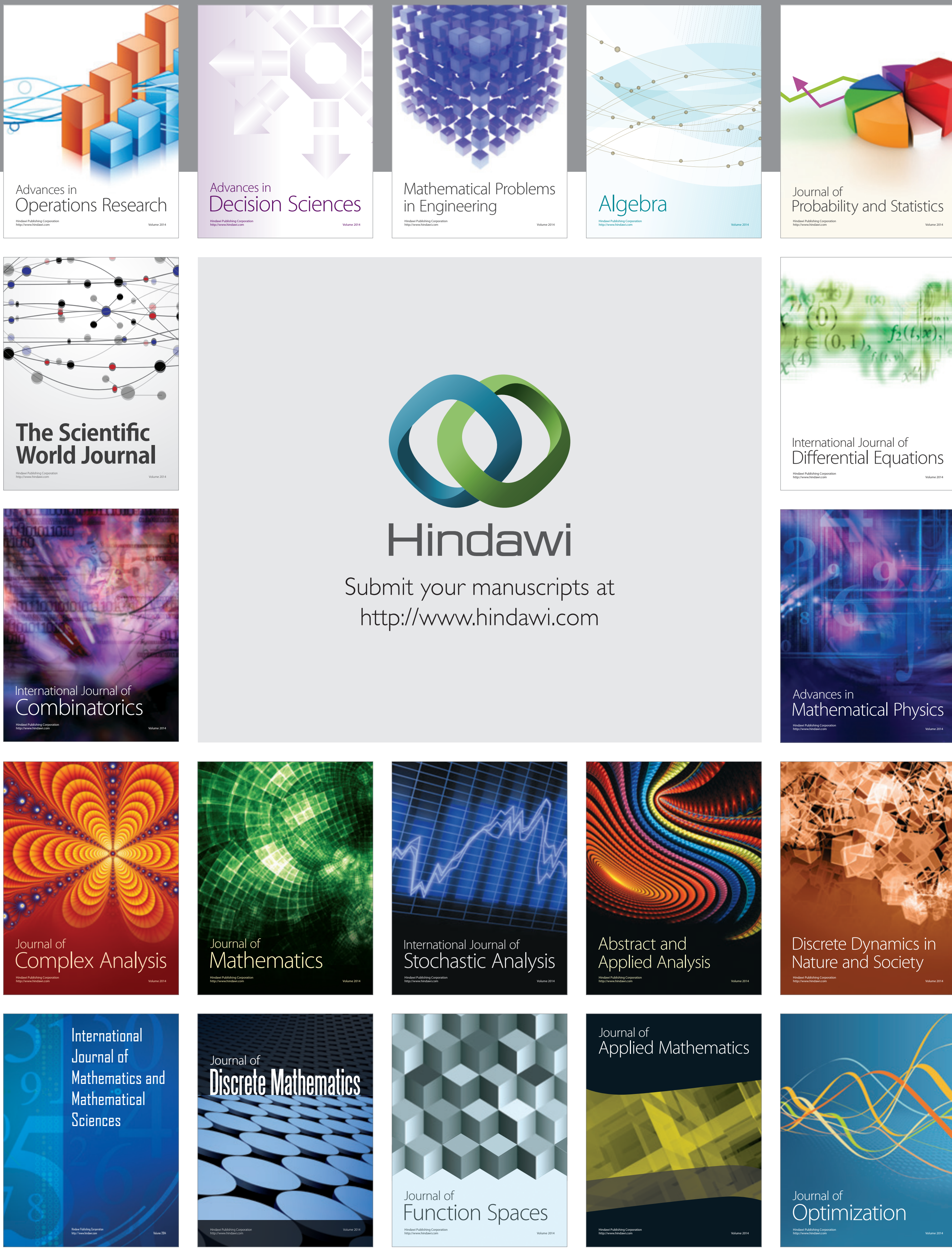
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