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Research Article

Self-Commutators of Composition Operators with Monomial Symbols on the Dirichlet Space

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Let $\varphi(z) = z^n, z \in \mathbb{U}$, for some positive integer n and C_φ the composition operator on the Dirichlet space $\mathfrak{D}$ induced by φ. In this paper, we completely determine the point spectrum, spectrum, essential spectrum, and essential norm of the operators $C_\varphi^* C_\varphi$, $C_\varphi C_\varphi^*$ and self-commutators of C_φ, which expose that the spectrum and point spectrum coincide. We also find the eigenfunctions of the operators.

1. Introduction

Let φ be a holomorphic self-map of the unit disk $\mathbb{U} := \{z \in \mathbb{C} : |z| < 1\}$. The function φ induces the *composition operator* C_φ, defined on the space of holomorphic functions on $\mathbb{U}$ by $C_\varphi f = f \circ \varphi$. The restriction of C_φ to various Banach spaces of holomorphic functions on $\mathbb{U}$ has been an active subject of research for more than three decades, and it will continue to be for decades to come (see [1–3]). Let $\mathfrak{D}$ denote the *Dirichlet space* of analytic functions on the unit disk with derivatives that are square integrable with respect to the area measure on the disk. In recent years, the study of composition operators on the the Dirichlet space has received considerable attention (see [4–9] and references cited therein).

Let $\varphi(z) = z^n$, $z \in \mathbb{U}$, for some positive integer n, and $C_\varphi : \mathfrak{D} \to \mathfrak{D}$ the composition operator on the Dirichlet space $\mathfrak{D}$ induced by φ. The main aim here is to find the spectrum, point spectrum, essential spectrum, and essential norm of $C_\varphi^* C_\varphi, C_\varphi C_\varphi^*$, self-commutator $[C_\varphi^*, C_\varphi] = C_\varphi^* C_\varphi - C_\varphi C_\varphi^*$ and anti-self-commutator $\{C_\varphi^*, C_\varphi\} = C_\varphi^* C_\varphi + C_\varphi C_\varphi^*$, for composition operators C_φ on the Dirichlet space.

In [10], by using Cowen's formula for the adjoint of C_φ on $H^2(\mathbb{U})$, the authors have completely determined the spectrum, essential spectrum, and point spectrum for selfcommutators of automorphic composition operators acting on the Hardy space of unit disk. In [4], the first author, has extended these results from the Hardy space to the Dirichlet space.

The other problem which is important to the study of composition operators is finding the relationships between the properties of the symbol φ and essential normality of the composition operator C_φ. Recall that an operator T on a Hilbert space $\mathscr{H}$ is called *essentially normal* if its image in the Calkin algebra is normal or equivalently if the *self-commutator* $[T^*, T] = T^*T - TT^*$ is compact on $\mathscr{H}$.

In [11], the authors have determined which composition operators with automorphism symbol are essentially normal on $A^2(B_N)$ and $H^2(B_N)$ for $N \geq 1$. They have shown that the only essential normal automorphic composition operators are actually normal. This was first shown in the setting $H^2(\mathbb{U})$ by Zorboska in [12]. The related works and some historical remarks can be found in [10–13].

In [5], the authors consider composition operators C_φ, where φ is a linear-fractional self-map of the unit disk $\mathbb{U}$, acting on the Dirichlet space $\mathfrak{D}$. By using the E. Gallardo and A. Montes' adjoint formula given in [6], they show that the essentially normal linear fractional composition operators on $\mathfrak{D}$ are precisely those whose symbol is not a hyperbolic nonautomorphism with a boundary fixed point. They also obtained conditions for the linear fractional symbols φ and ψ of the unit disk for which $C_\psi^* C_\varphi$ or $C_\varphi C_\psi^*$ is compact.

In the next section, after giving some background material and presenting formula for the adjoint of C_φ on $\mathfrak{D}$, we give useful formula for the operators $C_\varphi^* C_\varphi$, $C_\varphi C_\varphi^*$, $[C_\varphi^*, C_\varphi]$, and $\{C_\varphi^*, C_\varphi\}$, when φ is an arbitrary monomial symbol $\varphi(z) = z^n$. In Section 3, we completely determine the point spectrum, spectrum, and essential spectrum of $C_\varphi^* C_\varphi$ and $C_\varphi C_\varphi^*$. Finally, in Section 4, we determine the same for $[C_\varphi^*, C_\varphi]$ and $\{C_\varphi^*, C_\varphi\}$.

2. Preliminaries

Throughout the paper, for a Hilbert space $\mathscr{H}$, $\mathcal{B}(\mathscr{H})$ denotes the set of bounded operators on $\mathscr{H}$ and $\mathcal{B}_0(\mathscr{H})$ denotes the closed ideal of all compact operators in $\mathcal{B}(\mathscr{H})$. The natural homomorphism of $\mathcal{B}(\mathscr{H})$ onto the quotient Banach algebra $\mathcal{B}(\mathscr{H})/\mathcal{B}_0(\mathscr{H}) = \mathcal{B}/\mathcal{B}_0$—the *Calkin algebra*—is denoted by $T \mapsto \widetilde{T} = T + \mathcal{B}_0(\mathscr{H})$.

For an operator $T \in \mathcal{B}(\mathscr{H})$, the essential norm of T is defined by

$$\|T\|_e := \inf\{\|T + K\| : K \in \mathcal{B}_0(\mathscr{H})\}, \tag{2.1}$$

and the *essential spectrum* $\sigma_e(T)$ is defined as the spectrum of the image $\widetilde{T}$ of T in the Calkin algebra $\mathcal{B}(\mathscr{H})/\mathcal{B}_0(\mathscr{H})$. It is well known that the essential spectrum of a normal operator consists of all points in the spectrum of the operator except the isolated eigenvalues of finite multiplicity (see [14]).

As we mentioned in the Introduction, an operator T on a Hilbert space $\mathscr{H}$ is called *essentially normal* if its image in the Calkin algebra is normal or equivalently if the self-commutator $[T^*, T] = T^*T - TT^*$ is compact on $\mathscr{H}$.

The *Dirichlet space*, which we denote by $\mathfrak{D}$, is the set of all analytic functions f on the unit disk $\mathbb{U}$ for which

$$\int_{\mathbb{U}} |f'(z)|^2 dA(z) < \infty, \tag{2.2}$$

where dA denote the normalized area measure, and equivalently an analytic function f is in $\mathfrak{D}$ if $\sum_{n=1}^{\infty} n|\widehat{f}(n)|^2 < \infty$, where $\widehat{f}(n)$ denotes the n-th Taylor coefficient of f at 0. Background on the Dirichlet space can be found in [15] and the references cited therein.

For each holomorphic self-map φ of $\mathbb{U}$, we define the *composition operator* C_φ by $C_\varphi f = f \circ \varphi (f \in \mathfrak{D})$.

Martín and Vukotić in [9] express and prove formulas for the adjoint of C_φ on the Hardy space, when φ is finite Blaschke product and also is rational self-map of the unit disk $\mathbb{U}$. By using the same arguments as in [9] for the Hardy space, one can prove the following theorem for the Dirichlet space case.

Theorem 2.1. *Let $\varphi(z) = z^n$. For an arbitrary point $w = re^{i\theta}$ in $\mathbb{U}$, writing its nth roots as $r_{k,w} = r^{1/n}e^{i(\theta+2k\pi)/n}$, $k = 0, 1, \ldots, n-1$. The adjoint of C_φ (viewed as an operator on the Dirichlet space $\mathfrak{D}$) is given by the formula*

$$C_\varphi^* f(w) = \sum_{k=0}^{n-1} f(r_{k,w}) - (n-1)f(0). \tag{2.3}$$

Throughout this paper, we denote by $\mathcal{M}$ the closed subspace of $\mathfrak{D}$ spanned by the monomials $\{z^{nk} : k = 0, 1, \ldots\}$ and by $P_{\mathcal{M}} : \mathfrak{D} \to \mathcal{M}$ the corresponding orthogonal projection onto $\mathcal{M}$.

Remark 2.2. Let $\varphi(z) = z^n$ and $r_{k,\varphi(w)} = r^{1/n}e^{i(\theta+2k\pi)/n}$, $k = 0, 1, \ldots, n-1$, be the nth roots of $\varphi(z) = \varphi(w)$. For $f \in \mathfrak{D}$ with $f(z) = \sum_{m=0}^{\infty} c_m z^m$, we have

$$\begin{aligned} \sum_{k=0}^{n-1} f\left(r_{k,\varphi(w)}\right) &= \sum_{k=0}^{n-1}\sum_{m=0}^{\infty} c_m r_{k,\varphi(w)}^m \\ &= \sum_{m=0}^{\infty} c_m \sum_{k=0}^{n-1} r_{k,\varphi(w)}^m \\ &= n\sum_{j=0}^{\infty} c_{nj} w^{nj} = n\left(P_{\mathcal{M}} f\right)(w). \end{aligned} \tag{2.4}$$

Before stating our main results, we also need the next results.

Theorem 2.3. *Let $\varphi(z) = z^n$. Then,*

$$C_\varphi^* C_\varphi = nI \bmod \mathcal{B}_0(\mathfrak{D}), \tag{2.5}$$

$$\left(C_\varphi C_\varphi^* f\right)(w) = n\left(P_{\mathcal{M}} f\right)(w) - (n-1)f(0), \tag{2.6}$$

$\mathcal{B}_0(\mathfrak{D})$ is the ideal of compact operators on $\mathfrak{D}$.

Proof. By a simple computation and using formula (2.3), it follows that

$$\left(C_\varphi^* C_\varphi f\right)(w) = nf(w) - (n-1)f(0), \tag{2.7}$$

$$\left(C_\varphi C_\varphi^* f\right)(w) = \sum_{k=0}^{n-1} f\left(r_{k,\varphi(w)}\right) - (n-1)f(0), \tag{2.8}$$

for each $w \in \mathbb{U}$, $f \in \mathfrak{D}$. Thus,

$$\begin{gathered} C_\varphi^* C_\varphi = nI \bmod \mathcal{B}_0(\mathfrak{D}), \\ \left(C_\varphi C_\varphi^* f\right)(w) = n\left(P_{\mathcal{M}} f\right)(w) - (n-1)f(0). \end{gathered} \tag{2.9}$$

□

3. Spectrum of $C_\varphi^* C_\varphi$ and $C_\varphi C_\varphi^*$

Let $\varphi(z) = z^n$. In this section, we are going to find the point spectrum, spectrum, essential spectrum, and the eigenfunctions of the operators $C_\varphi^* C_\varphi$ and $C_\varphi C_\varphi^*$.

Theorem 3.1. *Let $\varphi(z) = z^n$. Then,*

$$\sigma_e\left(C_\varphi^* C_\varphi\right) = \{n\}, \qquad \sigma_p\left(C_\varphi^* C_\varphi\right) = \sigma\left(C_\varphi^* C_\varphi\right) = \{1, n\}, \tag{3.1}$$

and, for $n \geq 2$,

$$\sigma_e\left(C_\varphi C_\varphi^*\right) = \{0, n\}, \qquad \sigma_p\left(C_\varphi C_\varphi^*\right) = \sigma\left(C_\varphi C_\varphi^*\right) = \{1, 0, n\}, \tag{3.2}$$

and, in the case that $n = 1$,

$$\sigma_p\left(C_\varphi C_\varphi^*\right) = \sigma_e\left(C_\varphi C_\varphi^*\right) = \sigma\left(C_\varphi C_\varphi^*\right) = \{1\}. \tag{3.3}$$

Proof. Since the operator $C_\varphi^* C_\varphi$ is a finite rank perturbation of nI, the essential spectrum of this operator is $\{n\}$. Since any points in the spectrum of a normal operator which are not in the essential spectrum are isolated eigenvalues of finite multiplicity, it is enough to find the eigenvalues. We first do this for the operator $C_\varphi^* C_\varphi$. Let $\lambda \in \mathbb{C}$ be an eigenvalue of the operator $C_\varphi^* C_\varphi$ with corresponding eigenvector $f \in \mathfrak{D}$. Then, $C_\varphi^* C_\varphi f = \lambda f$. By using formula (2.7) for $C_\varphi^* C_\varphi$, we have

$$nf(w) - (n-1)f(0) = \lambda f(w), \quad w \in \mathbb{U}. \tag{3.4}$$

By putting $w = 0$, it follows that $nf(0) - (n-1)f(0) = \lambda f(0)$. If $f(0) \neq 0$, then $\lambda = 1$. Thus for the case $\lambda = 1$, the function $f(w) = f(0)$ is a nonzero function in $\mathfrak{D}$ that satisfies the equation, and, hence, $\lambda = 1$ is an eigenvalue of the operator $C_\varphi^* C_\varphi$. If $f(0) = 0$, then $\lambda = n$, and, in this case, the function $w^k, k \geq 1$ is a nonzero function in $\mathfrak{D}$ that satisfies (3.4). Hence, $\lambda = n$ is an eigenvalue of the operator $C_\varphi^* C_\varphi$ with infinite multiplicity. So

$$\sigma_p\left(C_\varphi^* C_\varphi\right) = \sigma\left(C_\varphi^* C_\varphi\right) = \{1, n\}. \tag{3.5}$$

Now, let $\lambda \in \mathbb{C}$ be an eigenvalue of the operator $C_\varphi C_\varphi^*$ with corresponding eigenvector $f \in \mathfrak{D}$. Then, $C_\varphi C_\varphi^* f = \lambda f$. By using formula (2.6) for $C_\varphi C_\varphi^*$, we have

$$n\left(P_{\mathcal{M}} f\right)(w) - (n-1) f(0) = \lambda f(w). \tag{3.6}$$

By putting $w = 0$, it follows that

$$n f(0) - (n-1) f(0) = \lambda f(0). \tag{3.7}$$

If $f(0) \neq 0$, then $\lambda = 1$. Thus,

$$n\left(P_{\mathcal{M}} f\right)(w) - (n-1) f(0) = f(w). \tag{3.8}$$

Let $f(z) = \sum_{m=0}^{\infty} c_m z^m$. Then,

$$n \sum_{j=0}^{\infty} c_{nj} w^{nj} - (n-1) c_0 = \sum_{m=0}^{\infty} c_m w^m, \quad w \in \mathbb{U}. \tag{3.9}$$

For $n \geq 2$, it follows that $c_m = 0$, whenever $m \geq 1$. So $f \equiv 1$ is an eigenfunction corresponding to $\lambda = 1$. Thus, $\lambda = 1$ is an eigenvalue of the operator $C_\varphi C_\varphi^*$.

Now suppose that $f(0) = 0$. Then, $nP_{\mathcal{M}} f = \lambda f$ and so λ/n is an eigenvalue of $P_{\mathcal{M}}$. Hence, $\lambda = 0$ or $\lambda = n$.

So, for the case $\lambda = 0$ and $n \geq 2$, the function w is a nonzero function in $\mathfrak{D}$ that satisfies (3.6), and, hence, $\lambda = 0$ is an eigenvalue of the operator $C_\varphi C_\varphi^*$.

So when $n \geq 2$, the eigenvalues of $C_\varphi C_\varphi^*$ are $\{1, 0, n\}$. In the case that $\lambda = n$, for each natural number k, w^{nk} is a nonzero function in $\mathfrak{D}$ that satisfies (3.6), and, hence, the essential spectrum $C_\varphi C_\varphi^*$ contains n. If $\lambda = 0$ and $n \geq 2$, then we conclude that for each natural number m which is not a multiple of n, w^m is a nonzero function in $\mathfrak{D}$ that satisfies (3.6), and, hence, $\lambda = 0$ is an eigenvalue of the operator $C_\varphi C_\varphi^*$ with infinite multiplicity. So the essential spectrum $C_\varphi C_\varphi^*$ contains 0. Since

$$\sigma\left(C_\varphi C_\varphi^* + \mathcal{B}_0(\mathfrak{D})\right) \cup \{0\} = \sigma\left(C_\varphi^* C_\varphi + \mathcal{B}_0(\mathfrak{D})\right) \cup \{0\}, \tag{3.10}$$

and $\sigma_e(C_\varphi^* C_\varphi) = \{n\}$, for $n \geq 2$, we conclude that

$$\sigma_e\left(C_\varphi C_\varphi^*\right) = \{0, n\}. \tag{3.11}$$

So when $\varphi(z) = z^n$ and $n \geq 2$,

$$\sigma_p\left(C_\varphi C_\varphi^*\right) = \sigma\left(C_\varphi C_\varphi^*\right) = \{1, 0, n\}, \tag{3.12}$$

and, for $n = 1$,

$$\sigma_p\left(C_\varphi C_\varphi^*\right) = \sigma_e\left(C_\varphi C_\varphi^*\right) = \sigma\left(C_\varphi C_\varphi^*\right) = \{1\}. \tag{3.13}$$

□

4. The Spectrum of $[C_\varphi^*, C_\varphi]$ and $\{C_\varphi^*, C_\varphi\}$

Theorem 4.1. *Let $\varphi(z) = z^n$. Then, for $n = 1$,*

$$\sigma_p\left(\left[C_\varphi^*, C_\varphi\right]\right) = \sigma_e\left(\left[C_\varphi^*, C_\varphi\right]\right) = \sigma\left(\left[C_\varphi^*, C_\varphi\right]\right) = \{0\}, \tag{4.1}$$

and, for $n \geq 2$,

$$\sigma_p\left(\left[C_\varphi^*, C_\varphi\right]\right) = \sigma_e\left(\left[C_\varphi^*, C_\varphi\right]\right) = \sigma\left(\left[C_\varphi^*, C_\varphi\right]\right) = \{0, n\}. \tag{4.2}$$

Proof. Let $T = C_\varphi^* C_\varphi - C_\varphi C_\varphi^*$. Then,

$$T = n(I - P_{\mathcal{M}}). \tag{4.3}$$

Since any points in the spectrum of a normal operator which are not in the essential spectrum are isolated eigenvalues of finite multiplicity, we first find the eigenvalues.

If $n \geq 2$, then $I - P_{\mathcal{M}}$ is a nontrivial projection and so $\sigma(T) = \{0, n\}$.

In the case that $\lambda = n$, for each natural number m which is not a multiple of n, the function w^m is an eigenfunction of T, and, hence, $\lambda = n$ is an eigenvalue of the operator T with infinite multiplicity. For the case $\lambda = 0$, for each natural number k, w^{kn} is an eigenfunction of T, and, hence, $\lambda = 0$ is an eigenvalue of the operator T with infinite multiplicity.

The essential spectrum of $[C_\varphi^*, C_\varphi]$ can be computed directly by using the following:

$$\begin{aligned}\sigma_e\left(\left[C_\varphi^*, C_\varphi\right]\right) &= \sigma\left(C_\varphi^* C_\varphi - C_\varphi C_\varphi^* + \mathcal{B}_0(\mathfrak{D})\right) \\ &= \sigma\left(nI - C_\varphi C_\varphi^* + \mathcal{B}_0(\mathfrak{D})\right).\end{aligned} \tag{4.4}$$

So if $\varphi(z) = z$, then

$$\sigma_p\left(\left[C_\varphi^*, C_\varphi\right]\right) = \sigma_e\left(\left[C_\varphi^*, C_\varphi\right]\right) = \sigma\left(\left[C_\varphi^*, C_\varphi\right]\right) = \{0\}, \tag{4.5}$$

and, if $\varphi(z) = z^n$ and $n \geq 2$, then

$$\sigma_p\left(\left[C_\varphi^*, C_\varphi\right]\right) = \sigma_e\left(\left[C_\varphi^*, C_\varphi\right]\right) = \sigma\left(\left[C_\varphi^*, C_\varphi\right]\right) = \{0, n\}. \tag{4.6}$$

□

Theorem 4.2. *Let $\varphi(z) = z^n$. Then, for $n = 1$,*

$$\sigma_p\left(\left\{C_\varphi^*, C_\varphi\right\}\right) = \sigma_e\left(\left\{C_\varphi^*, C_\varphi\right\}\right) = \sigma\left(\left\{C_\varphi^*, C_\varphi\right\}\right) = \{2\}, \tag{4.7}$$

for $n \geq 2$,

$$\begin{gathered}\sigma_e\left(\left\{C_\varphi^*, C_\varphi\right\}\right) = \{2n, n\}, \\ \sigma_p\left(\left\{C_\varphi^*, C_\varphi\right\}\right) = \sigma\left(\left\{C_\varphi^*, C_\varphi\right\}\right) = \{2, n, 2n\}.\end{gathered} \tag{4.8}$$

Proof. Let $S = C_\varphi^* C_\varphi + C_\varphi C_\varphi^*$. Then for each $f \in \mathfrak{D}$ and $w \in \mathbb{U}$,

$$(Sf)(w) = nf(w) + n(P_{\mathcal{M}} f)(w) - 2(n-1)f(0). \tag{4.9}$$

Since S is self-adjoint, any points in the spectrum of S which are not in the essential spectrum are eigenvalues of finite multiplicity. So we first find such points.

Let $\lambda \in \mathbb{C}$ be an eigenvalue of the operator S with corresponding eigenvector $f \in \mathfrak{D}$. Then, $Sf = \lambda f$. So we have

$$nf(w) + n(P_{\mathcal{M}} f)(w) - 2(n-1)f(0) = \lambda f(w). \tag{4.10}$$

By putting $w = 0$, it follows that $2nf(0) - 2nf(0) + 2f(0) = \lambda f(0)$. If $f(0) \neq 0$, then $\lambda = 2$. Thus,

$$nf(w) + n(P_{\mathcal{M}} f)(w) - 2(n-1)f(0) = 2f(w). \tag{4.11}$$

The function $f(w) = 1$ is a nonzero function in $\mathfrak{D}$ that satisfies the equation, and, hence, $\lambda = 2$ is an eigenvalue of the operator S. If $f(0) = 0$, then

$$n(P_{\mathcal{M}} f)(w) = (\lambda - n)f(w), \tag{4.12}$$

and it follows that $\lambda = n$ or $\lambda = 2n$. For the case $\lambda = n$ with $n \geq 2$, for each natural number m which is not a multiple of n, the function w^m is a nonzero function in $\mathfrak{D}$ that satisfies (4.12), and, hence, $\lambda = n$ is an eigenvalue of the operator S with infinite multiplicity. In the case that $\lambda = 2n$, for each natural number k, w^{kn} is a nonzero function in $\mathfrak{D}$ that satisfies (4.12), and, hence, $\lambda = 2n$ is an eigenvalues of the operator S with infinite multiplicity. The essential spectrum of $\{C_\varphi^*, C_\varphi\}$ can be computed directly by using the following:

$$\begin{aligned}\sigma_e\left(\left\{C_\varphi^*, C_\varphi\right\}\right) &= \sigma\left(C_\varphi^* C_\varphi + C_\varphi C_\varphi^* + \mathcal{B}_0(\mathfrak{D})\right)\\ &= \sigma\left(nI + C_\varphi C_\varphi^* + \mathcal{B}_0(\mathfrak{D})\right).\end{aligned} \tag{4.13}$$

Hence, we conclude that when $\varphi(z) = z^n$ and $n \geq 2$,

$$\sigma_e\left(\left\{C_\varphi^*, C_\varphi\right\}\right) = \{n, 2n\}. \tag{4.14}$$

Also, if $\varphi(z) = z$, then

$$\sigma_p\left(\left\{C_\varphi^*, C_\varphi\right\}\right) = \sigma_e\left(\left\{C_\varphi^*, C_\varphi\right\}\right) = \sigma\left(\left\{C_\varphi^*, C_\varphi\right\}\right) = \{2\}, \tag{4.15}$$

and, for $n \geq 2$,

$$\sigma_p\left(\left\{C_\varphi^*, C_\varphi\right\}\right) = \sigma\left(\left\{C_\varphi^*, C_\varphi\right\}\right) = \{2, n, 2n\}. \tag{4.16}$$

□

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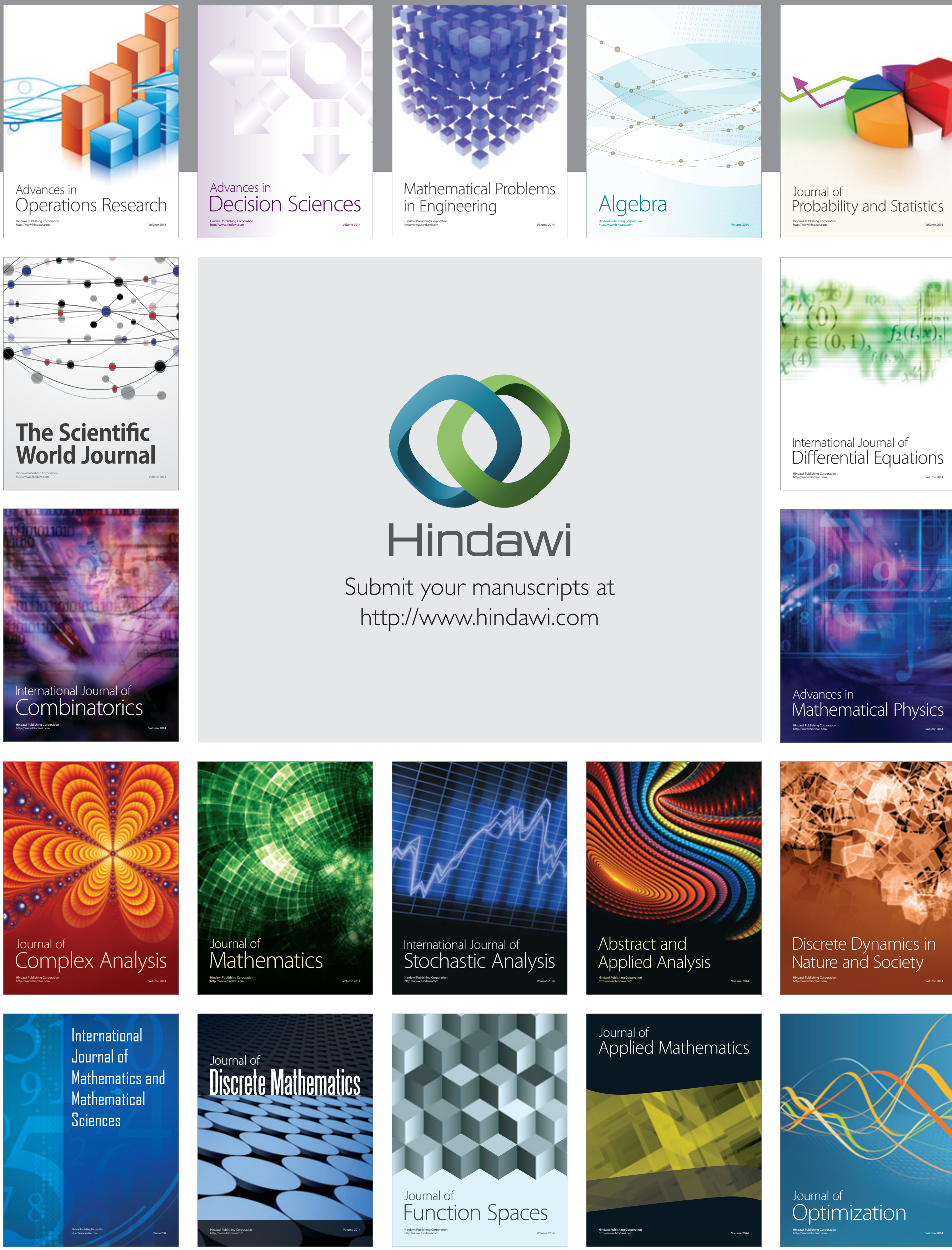
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