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Research Article

Approximately Ternary Homomorphisms and Derivations on C^*-Ternary Algebras

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We investigate the stability and superstability of ternary homomorphisms between C^*-ternary algebras and derivations on C^*-ternary algebras, associated with the following functional equation $f((x_2 - x_1)/3) + f((x_1 - 3x_3)/3) + f((3x_1 + 3x_3 - x_2)/3) = f(x_1)$.

1. Introduction

A C^*-ternary algebra is a complex Banach space A, equipped with a ternary product $(x, y, z) \rightarrowtail [x, y, z]$ of A^3 into A, which is $\mathbb{C}$-linear in the outer variables, conjugate $\mathbb{C}$-linear in the middle variable, and associative in the sense that $[x, y, [z, w, v]] = [x, [w, z, y], v] = [[x, y, z], w, v]$, and satisfies $\|[x, y, z]\| \leq \|x\| \cdot \|y\| \cdot \|z\|$ and $\|[x, x, x]\| = \|x\|^3$. If a C^*-ternary algebra $(A, [\cdot, \cdot, \cdot])$ has an identity, that is, an element $e \in A$ such that $x = [x, e, e] = [e, e, x]$ for all $x \in A$, then it is routine to verify that A, endowed with $xoy := [x, e, y]$ and $x^* := [e, x, e]$, is a unital C^*-algebra. Conversely, if (A, o) is a unital C^*-algebra, then $[x, y, z] := xoy^*oz$ makes A into a C^*-ternary algebra. A $\mathbb{C}$-linear mapping $H : A \to B$ is called a C^*-ternary algebra homomorphism if

$$H([x, y, z]) = [H(x), H(y), H(z)], \tag{1.1}$$

for all $x, y, z \in A$. A $\mathbb{C}$-linear mapping $\delta : A \to A$ is called a C^*-ternary algebra derivation if

$$\delta([x,y,z]) = [\delta(x),y,z] + [x,\delta(y),z] + [x,y,\delta(z)], \tag{1.2}$$

for all $x, y, z \in A$.

Ternary structures and their generalization the so-called n-ary structures raise certain hopes in view of their applications in physics (see [1–8]).

We say a functional equation ζ is stable if any function g satisfying the equation ζ approximately is near to true solution of ζ. Moreover, ζ is superstable if every approximately solution of ζ is an exact solution of it.

The study of stability problems originated from a famous talk given by Ulam [9] in 1940: "Under what condition does there exist a homomorphism near an approximate homomorphism?" In the next year 1941, Hyers [10] answered affirmatively the question of Ulam for additive mappings between Banach spaces.

A generalized version of the theorem of Hyers for approximately additive maps was given by Rassias [11] in 1978 as follows.

Theorem 1.1. *Let $f : E_1 \to E_2$ be a mapping from a normed vector space E_1 into a Banach space E_2 subject to the inequality:*

$$\|f(x+y) - f(x) - f(y)\| \le \epsilon(\|x\|^p + \|y\|^p), \tag{1.3}$$

for all $x, y \in E_1$, where ϵ and p are constants with $\epsilon > 0$ and $p < 1$. Then, there exists a unique additive mapping $T : E_1 \to E_2$ such that

$$\|f(x) - T(x)\| \le \frac{2\epsilon}{2 - 2^p}\|x\|^p, \tag{1.4}$$

for all $x \in E_1$.

The stability phenomenon that was introduced and proved by Rassias is called Hyers-Ulam-Rassias stability. And then the stability problems of several functional equations have been extensively investigated by a number of authors, and there are many interesting results concerning this problem (see [12–27]).

Throughout this paper, we assume that A is a C^*-ternary algebra with norm $\|\cdot\|_A$ and that B is a C^*-ternary algebra with norm $\|\cdot\|_B$. Moreover, we assume that $n_0 \in \mathbb{N}$ is a positive integer and suppose that $\mathbb{T}^1_{1/n_o} := \{e^{i\theta};\ 0 \le \theta \le 2\pi/n_o\}$.

2. Superstability

In this section, first we investigate homomorphisms between C^*-ternary algebras. We need the following Lemma in the main results of the paper.

Lemma 2.1. *Let $f : A \to B$ be a mapping such that*

$$\left\| f\left(\frac{x_2 - x_1}{3}\right) + f\left(\frac{x_1 - 3x_3}{3}\right) + f\left(\frac{3x_1 + 3x_3 - x_2}{3}\right)\right\|_B \le \|f(x_1)\|_B, \tag{2.1}$$

for all $x_1, x_2, x_3 \in A$. Then f is additive.

Proof. Letting $x_1 = x_2 = x_3 = 0$ in (2.1), we get

$$\|3f(0)\|_B \le \|f(0)\|_B. \tag{2.2}$$

So $f(0) = 0$. Letting $x_1 = x_2 = 0$ in (2.1), we get

$$\|f(-x_3) + f(x_3)\|_B \le \|f(0)\|_B = 0, \tag{2.3}$$

for all $x_3 \in A$. Hence $f(-x_3) = -f(x_3)$ for all $x_3 \in A$. Letting $x_1 = 0$ and $x_2 = 6x_3$ in (2.1), we get

$$\|f(2x_3) - 2f(x_3)\|_B \le \|f(0)\|_B = 0, \tag{2.4}$$

for all $x_3 \in A$. Hence

$$f(2x_3) = 2f(x_3), \tag{2.5}$$

for all $x_3 \in A$. Letting $x_1 = 0$ and $x_2 = 9x_3$ in (2.1), we get

$$\|f(3x_3) - f(x_3) - 2f(x_3)\|_B \le \|f(0)\|_B = 0, \tag{2.6}$$

for all $x_3 \in A$. Hence

$$f(3x_3) = 3f(x_3), \tag{2.7}$$

for all $x_3 \in A$. Letting $x_1 = 0$ in (2.1), we get

$$\left\|f\left(\frac{x_2}{3}\right) + f(-x_3) + f\left(x_3 - \frac{x_2}{3}\right)\right\|_B \le \|f(0)\|_B = 0, \tag{2.8}$$

for all $x_2, x_3 \in A$. So

$$f\left(\frac{x_2}{3}\right) + f(-x_3) + f\left(x_3 - \frac{x_2}{3}\right) = 0, \tag{2.9}$$

for all $x_2, x_3 \in A$. Let $t_1 = x_3 - (x_2/3)$ and $t_2 = x_2/3$ in (2.9). Then

$$f(t_2) - f(t_1 + t_2) + f(t_1) = 0, \tag{2.10}$$

for all $t_1, t_2 \in A$, this means that f is additive. □

Now, we prove the first result in superstability as follows.

Theorem 2.2. *Let $p \neq 1$ and θ be nonnegative real numbers, and let $f : A \to B$ be a mapping such that*

$$\left\| f\left(\frac{x_2 - x_1}{3}\right) + f\left(\frac{x_1 - 3\mu x_3}{3}\right) + \mu f\left(\frac{3x_1 + 3x_3 - x_2}{3}\right) \right\|_B \leq \|f(x_1)\|_B, \tag{2.11}$$

$$\|f([x_1, x_2, x_3]) - [f(x_1), f(x_2), f(x_3)]\|_B \leq \theta\left(\|x_1\|_A^{3p} + \|x_2\|_A^{3p} + \|x_3\|_A^{3p}\right), \tag{2.12}$$

for all $\mu \in \mathbb{T}^1_{1/n_o}$ and all x_1, x_2, $x_3 \in A$. Then, the mapping $f : A \to B$ is a C^*-ternary algebra homomorphism.

Proof. Assume $p > 1$.

Let $\mu = 1$ in (2.11). By Lemma 2.1, the mapping $f : A \to B$ is additive. Letting $x_1 = x_2 = 0$ in (2.11), we get

$$\|f(-\mu x_3) + \mu f(x_3)\|_B \leq \|f(0)\|_B = 0, \tag{2.13}$$

for all $x_3 \in A$ and $\mu \in \mathbb{T}^1$. So

$$-f(\mu x_3) + \mu f(x_3) = f(-\mu x_3) + \mu f(x_3) = 0, \tag{2.14}$$

for all $x_3 \in A$ and all $\mu \in \mathbb{T}^1$. Hence $f(\mu x_3) = \mu f(x_3)$ for all $x_3 \in A$ and all $\mu \in \mathbb{T}^1_{1/n_o}$. By same reasoning as proof of Theorem 2.2 of [28], the mapping $f : A \to B$ is $\mathbb{C}$-linear. It follows from (2.12) that

$$\begin{aligned}
&\|f([x_1, x_2, x_3]) - [f(x_1), f(x_2), f(x_3)]\|_B \\
&\quad = \lim_{n\to\infty} 8^n \left\| f\left(\frac{[x_1, x_2, x_3]}{2^n \cdot 2^n \cdot 2^n}\right) - \left[f\left(\frac{x_1}{2^n}\right), f\left(\frac{x_2}{2^n}\right), f\left(\frac{x_3}{2^n}\right)\right] \right\|_B \\
&\quad \leq \lim_{n\to\infty} \frac{8^n\theta}{8^{np}} \left(\|x_1\|_A^{3p} + \|x_2\|_A^{3p} + \|x_3\|_A^{3p}\right) = 0,
\end{aligned} \tag{2.15}$$

for all x_1, x_2, $x_3 \in A$. Thus,

$$f([x_1, x_2, x_3]) = [f(x_1), f(x_2), f(x_3)], \tag{2.16}$$

for all x_1, x_2, $x_3 \in A$. Hence, the mapping $f : A \to B$ is a C^*-ternary algebra homomorphism. Similarly, one obtains the result for the case $p < 1$. □

Now, we establish the superstability of derivations on C^*-ternary algebras as follows.

Theorem 2.3. *Let $p \neq 1$ and θ be nonnegative real numbers, and let $f : A \to A$ be a mapping satisfying* (2.11) *such that*

$$\begin{aligned}
&\left\| f([x_1, x_2, x_3]) - [f(x_1), x_2, x_3] - [x_1, f(x_2), x_3] - [x_1, x_2, f(x_3)] \right\|_A \\
&\quad \leq \theta\left(\|x_1\|_A^{3p} + \|x_2\|_A^{3p} + \|x_3\|_A^{3p} \right),
\end{aligned} \tag{2.17}$$

for all $x_1, x_2, x_3 \in A$. Then the mapping $f : A \to A$ is a C^*-ternary derivation.

Proof. Assume $p > 1$.

By the Theorem 2.2, the mapping $f : A \to A$ is $\mathbb{C}$-linear. It follows from (2.17) that

$$\begin{aligned}
&\left\| f([x_1, x_2, x_3]) - [f(x_1), x_2, x_3] - [x_1, f(x_2), x_3] - [x_1, x_2, f(x_3)] \right\|_A \\
&\quad = \lim_{n \to \infty} 8^n \left\| f\left(\frac{[x_1, x_2, x_3]}{8^n} \right) - \left[f\left(\frac{x_1}{2^n} \right), \frac{x_2}{2^n}, \frac{x_3}{2^n} \right] - \left[\frac{x_1}{2^n}, f\left(\frac{x_2}{2^n} \right), \frac{x_3}{2^n} \right] \right. \\
&\qquad\qquad \left. - \left[\frac{x_1}{2^n}, \frac{x_2}{2^n}, f\left(\frac{x_3}{2^n} \right) \right] \right\|_A \\
&\quad \leq \lim_{n \to \infty} \frac{8^n \theta}{8^{np}} \left(\|x_1\|_A^{3p} + \|x_2\|_A^{3p} + \|x_3\|_A^{3p} \right) = 0,
\end{aligned} \tag{2.18}$$

for all $x_1, x_2, x_3 \in A$. So

$$f([x_1, x_2, x_3]) = [f(x_1), x_2, x_3] + [x_1, f(x_2), x_3] + [x_1, x_2, f(x_3)] \tag{2.19}$$

for all $x_1, x_2, x_3 \in A$. Thus, the mapping $f : A \to A$ is a C^*-ternary derivation. Similarly, one obtains the result for the case $p < 1$. □

3. Stability

First we prove the generalized Hyers-Ulam-Rassias stability of homomorphisms in C^*-ternary algebras.

Theorem 3.1. *Let $p > 1$ and θ be nonnegative real numbers, and let $f : A \to B$ be a mapping such that*

$$\begin{aligned}
&\left\| f\left(\frac{x_2 - x_1}{3} \right) + f\left(\frac{x_1 - 3\mu x_3}{3} \right) + \mu f\left(\frac{3x_1 + 3x_3 - x_2}{3} \right) - f(x_1) \right\|_B \\
&\quad \leq \theta\left(\|x_1\|_A^p + \|x_2\|_A^p + \|x_3\|_A^p \right),
\end{aligned} \tag{3.1}$$

$$\left\| f([x_1, x_2, x_3]) - [f(x_1), f(x_2), f(x_3)] \right\|_B \leq \theta\left(\|x_1\|_A^{3p} + \|x_2\|_A^{3p} + \|x_3\|_A^{3p} \right), \tag{3.2}$$

for all $\mu \in \mathbb{T}^1_{1/n_o}$, *and all* $x_1, x_2, x_3 \in A$. *Then there exists a unique* C^**-ternary homomorphism* $H : A \to B$ *such that*

$$\|H(x_1) - f(x_1)\|_B \leq \frac{\theta(1+2^p)\|x_1\|_A^p}{1-3^{1-p}}, \tag{3.3}$$

for all $x_1 \in A$.

Proof. Let us assume $\mu = 1$, $x_2 = 2x_1$ and $x_3 = 0$ in (3.1). Then we get

$$\left\|3f\left(\frac{x_1}{3}\right) - f(x_1)\right\|_B \leq \theta(1+2^p)\|x_1\|_A^p, \tag{3.4}$$

for all $x_1 \in A$. So by induction, we have

$$\left\|3^n f\left(\frac{x_1}{3^n}\right) - f(x_1)\right\|_B \leq \theta(1+2^p)\|x_1\|_A^p \sum_{i=0}^{n-1} 3^{i(1-p)}, \tag{3.5}$$

for all $x_1 \in A$. Hence

$$\begin{aligned}\left\|3^{n+m} f\left(\frac{x_1}{3^{n+m}}\right) - 3^m f\left(\frac{x_1}{3^m}\right)\right\|_B &\leq \theta(1+2^p)\|x_1\|_A^p \sum_{i=0}^{n-1} 3^{(i+m)(1-p)} \\ &\leq \theta(1+2^p)\|x_1\|_A^p \sum_{i=m}^{n+m-1} 3^{i(1-p)},\end{aligned} \tag{3.6}$$

for all nonnegative integers m and n with $n \geq m$, and all $x_1 \in A$. It follows that the sequence $\{3^n f(x_1/3^n)\}$ is a Cauchy sequence for all $x_1 \in A$. Since B is complete, the sequence $\{3^n f(x_1/3^n)\}$ converges. Thus, one can define the mapping $H : A \to B$ by

$$H(x_1) := \lim_{n\to\infty} 3^n f\left(\frac{x_1}{3^n}\right), \tag{3.7}$$

for all $x_1 \in A$. Moreover, letting $m = 0$ and passing the limit $n \to \infty$ in (3.6), we get (3.3). It follows from (3.1) that

$$\begin{aligned}&\left\|H\left(\frac{x_2 - x_1}{3}\right) + H\left(\frac{x_1 - 3\mu x_3}{3}\right) + \mu H\left(\frac{3x_1 + 3x_3 - x_2}{3}\right) - H(x_1)\right\|_B \\ &\quad = \lim_{n\to\infty} 3^n \left\|f\left(\frac{x_2 - x_1}{3^{n+1}}\right) + f\left(\frac{x_1 - 3\mu x_3}{3^{n+1}}\right) + f\left(\frac{3x_1 + 3x_3 - x_2}{3^{n+1}}\right) - f\left(\frac{x_1}{3^n}\right)\right\|_B \\ &\quad \leq \lim_{n\to\infty} \frac{3^n\theta}{3^{np}}\left(\|x_1\|_A^p + \|x_2\|_A^p + \|x_3\|_A^p\right) = 0,\end{aligned} \tag{3.8}$$

for all $\mu \in \mathbb{T}^1_{1/n_o}$, and all $x_1, x_2, x_3 \in A$. So

$$H\left(\frac{x_2 - x_1}{3}\right) + H\left(\frac{x_1 - 3\mu x_3}{3}\right) + \mu H\left(\frac{3x_1 + 3x_3 - x_2}{3}\right) = H(x_1), \tag{3.9}$$

for all $\mu \in \mathbb{T}^1_{1/n_o}$, and all x_1, x_2, $x_3 \in A$. By the same reasoning as proof of Theorem 2.2 of [28], the mapping $H : A \to B$ is $\mathbb{C}$-linear.

Now, let $H' : A \to B$ be another additive mapping satisfying (3.3). Then, we have

$$\begin{aligned} \|H(x_1) - H'(x_1)\|_B &= 3^n \left\| H\left(\frac{x_1}{3^n}\right) - H'\left(\frac{x_1}{3^n}\right) \right\|_B \\ &\le 3^n \left(\left\| H\left(\frac{x_1}{3^n}\right) - f\left(\frac{x_1}{3^n}\right) \right\|_B + \left\| H'\left(\frac{x_1}{3^n}\right) - f\left(\frac{x_1}{3^n}\right) \right\|_B \right) \\ &\le \frac{2 \cdot 3^n \theta (1 + 2^p)}{3^{np}(1 - 3^{1-p})} \|x\|_A^p, \end{aligned} \tag{3.10}$$

which tends to zero as $n \to \infty$ for all $x_1 \in A$. So we can conclude that $H(x_1) = H'(x_1)$ for all $x_1 \in A$. This proves the uniqueness of H.

It follows from (3.2) that

$$\begin{aligned} &\|H([x_1, x_2, x_3]) - [H(x_1), H(x_2), H(x_3)]\|_B \\ &\quad = \lim_{n \to \infty} 27^n \left\| f\left(\frac{[x_1, x_2, x_3]}{3^n \cdot 3^n \cdot 3^n}\right) - \left[f\left(\frac{x_1}{3^n}\right), f\left(\frac{x_2}{3^n}\right), f\left(\frac{x_3}{3^n}\right) \right] \right\|_B \\ &\quad \le \lim_{n \to \infty} \frac{27^n \theta}{27^{np}} \left(\|x_1\|_A^{3p} + \|x_2\|_A^{3p} + \|x_3\|_A^{3p} \right) = 0, \end{aligned} \tag{3.11}$$

for all x_1, x_2, $x_3 \in A$.

Thus, the mapping $H : A \to B$ is a unique C^*-ternary homomorphism satisfying (3.3). □

Theorem 3.2. *Let $p < 1$ and θ be nonnegative real numbers, and let $f : A \to B$ be a mapping satisfying* (3.1) *and* (3.2). *Then, there exists a unique C^*-ternary homomorphism $H : A \to B$ such that*

$$\|H(x_1) - f(x_1)\|_B \le \frac{\theta(1 + 2^p)\|x_1\|_A^p}{3^{1-p} - 1}, \tag{3.12}$$

for all $x_1 \in A$.

Proof. The proof is similar to the proof of Theorem 3.1. □

Now, we prove the generalized Hyers-Ulam-Rassias stability of derivations on C^*-ternary algebras.

Theorem 3.3. *Let $p > 1$ and θ be nonnegative real numbers, and let $f : A \to A$ be a mapping such that*

$$\left\| f\left(\frac{x_2 - x_1}{3}\right) + f\left(\frac{x_1 - 3\mu x_3}{3}\right) + \mu f\left(\frac{3x_1 + 3x_3 - x_2}{3}\right) - f(x_1) \right\|_A \leq \theta\left(\|x_1\|_A^p + \|x_2\|_A^p + \|x_3\|_A^p\right), \tag{3.13}$$

$$\left\| f([x_1, x_2, x_3]) - [f(x_1), x_2, x_3] - [x_1, f(x_2), x_3] - [x_1, x_2, f(x_3)] \right\|_A \leq \theta\left(\|x_1\|_A^{3p} + \|x_2\|_A^{3p} + \|x_3\|_A^{3p}\right), \tag{3.14}$$

for all $\mu \in \mathbb{T}^1_{1/n_o}$, and all x_1, x_2, $x_3 \in A$. Then, there exists a unique C^*-ternary derivation $D : A \to A$ such that

$$\left\| D(x_1) - f(x_1) \right\|_A \leq \frac{\theta(1 + 2^p)\|x_1\|_A^p}{1 - 3^{1-p}}, \tag{3.15}$$

for all $x_1 \in A$.

Proof. By the same reasoning as in the proof of the Theorem 3.1, there exists a unique $\mathbb{C}$-linear mapping $D : A \to A$ satisfying (3.15). The mapping $D : A \to A$ is defined by

$$D(x_1) := \lim_{n\to\infty} 3^n f\left(\frac{x_1}{3^n}\right), \tag{3.16}$$

for all $x_1 \in A$. It follows from (3.14) that

$$\begin{aligned} &\|D([x_1, x_2, x_3]) - [D(x_1), x_2, x_3] - [x_1, D(x_2), x_3] - [x_1, x_2, D(x_3)]\|_A \\ &\quad = \lim_{n\to\infty} 27^n \left\| \frac{[x_1, x_2, x_3]}{3^n \cdot 3^n \cdot 3^n} - \left[f\left(\frac{x_1}{3^n}\right), \frac{x_2}{3^n}, \frac{x_3}{3^n}\right] - \left[\frac{x_1}{3^n}, f\left(\frac{x_2}{3^n}\right), \frac{x_3}{3^n}\right] - \left[\frac{x_1}{3^n}, \frac{x_2}{3^n}, f\left(\frac{x_3}{3^n}\right)\right] \right\|_A \\ &\quad \leq \lim_{n\to\infty} \frac{27^n \theta}{27^{np}} \left(\|x_1\|_A^{3p} + \|x_2\|_A^{3p} + \|x_3\|_A^{3p}\right) = 0, \end{aligned} \tag{3.17}$$

for all x_1, x_2, $x_3 \in A$. So

$$D([x_1, x_2, x_3]) = [D(x_1), x_2, x_3] + [x_1, D(x_2), x_3] + [x_1, x_2, D(x_3)] \tag{3.18}$$

for all x_1, x_2, $x_3 \in A$.

Thus, the mapping $D : A \to A$ is a unique C^*-ternary derivation satisfying (3.15). □

Theorem 3.4. *Let $p < 1$ and θ be nonnegative real numbers, and let $f : A \to A$ be a mapping satisfying* (3.13)*and* (3.14)*. Then, there exists a unique C^*-ternary derivation $D : A \to A$ such that*

$$\left\| D(x_1) - f(x_1) \right\|_A \leq \frac{\theta(1+2^p)\|x_1\|_A^p}{3^{1-p}-1}, \tag{3.19}$$

for all $x_1 \in A$.

Proof. The proof is similar to the proof of Theorems 3.1 and 3.3. □

4. Conclusions

In this paper, we have analyzed some detail C^*-ternary algebras and derivations on C^*-ternary algebras, associated with the following functional equation:

$$f\left(\frac{x_2 - x_1}{3}\right) + f\left(\frac{x_1 - 3x_3}{3}\right) + f\left(\frac{3x_1 + 3x_3 - x_2}{3}\right) = f(x_1). \tag{4.1}$$

A detailed study of how we can have the generalized Hyers-Ulam-Rassias stability of homomorphisms and derivations on C^*-ternary algebras is given.

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