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Research Article

Nonlinear Fractional Jaulent-Miodek and Whitham-Broer-Kaup Equations within Sumudu Transform

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We solve the system of nonlinear fractional Jaulent-Miodek and Whitham-Broer-Kaup equations via the Sumudu transform homotopy method (STHPM). The method is easy to apply, accurate, and reliable.

1. Introduction

Nonlinear partial differential equations arise in various areas of physics, mathematics, and engineering [1–4]. We notice that in fluid dynamics, the nonlinear evolution equations show up in the context of shallow water waves. Some of the commonly studied equations are the Korteweg-de Vries (KdV) equation, modified KdV equation, Boussinesq equation [5], Green-Naghdi equation, Gardeners equation, and Whitham-Broer-Kaup and Jaulent-Miodek (JM) equations. Analytical solutions of these equations are usually not available. Since only limited classes of equations are solved by analytical means, numerical solution of these nonlinear partial differential equations is of practical importance. Therefore, finding new methods and techniques to deal with these type of equations is still an open problem in this area. The purpose of this paper is to find an approximated solution for the system of fractional Jaulent-Miodek and Whitham-Broer-Kaup equations (FWBK) via the Sumudu transform method. The fractional systems of partial differential equations under investigation here are given below.

The nonlinear FWBK equation which will be considered in this paper has the following form:

$$\begin{gathered}\partial_t^{\eta} u + uu_x + u_x + \beta u_{xx} = 0, \quad 0 < \eta,\ \mu \le 1,\\ \partial_t^{\mu} v + (uv)_x + \alpha u_{xxx} - \beta v_{xx} = 0, (x,t) \in [a,b] \times [0,T],\end{gathered} \tag{1}$$

and the nonlinear FJM equation is

$$\begin{gathered}\partial_t^{\alpha} u + u_{xxx} + \frac{3}{2} v v_{xxx} + \frac{9}{2} v_x v_{xx} - 6uu_x + 6uvv_x - \frac{3}{2} u_x v_x^2 = 0,\\ \partial_t^{\mu} v + v_{xxx} - 6u_x v_x - \frac{15}{2} v_x v^2 = 0, \quad (x,t) \in [a,b] \times [0,T].\end{gathered} \tag{2}$$

The system of (1) and (2) is subjected to the following initial conditions:

$$\begin{gathered}u(x,0) = f(x),\\ v(x,0) = g(x).\end{gathered} \tag{3}$$

FWBK equation (1) describes the dispersive long wave in shallow water, where $u(x,t)$ is the field of horizontal velocity, $v(x,t)$ is the height which deviates from the equilibrium position of liquid, and α and β are constants that represent different powers. If $\alpha = 0$ and $\beta = 1$, (1) reduces to the classical long-wave equations which describe the shallow water wave with diffusion [6]. If $\alpha = 1$ and $\beta = 0$, (1) becomes the modified Boussinesq equations [7, 8]. FJM equation (2) appears in several areas of science such as condense matter physics [9], fluid mechanics [10], plasma physics [11], and optics [12] and associates with energy-dependent Schrödinger potential [13, 14].

The paper is organized as follows. In Section 2, we introduce briefly some of the basic tools of fractional order and of the Sumudu transform method. We show the numerical results in Section 4. The conclusions can be seen in Section 5.

2. Basic Tools

2.1. Properties and Definitions

Definition 1 (see [15–24]). A real function $f(x)$, $x > 0$, is said to be in the space C_μ, $\mu \in \mathbb{R}$ if there exists a real number $p > \mu$, such that $f(x) = x^p h(x)$, where $h(x) \in C[0,\infty)$, and it is said to be in space C_μ^m if $f^{(m)} \in C_\mu$, $m \in \mathbb{N}$.

Definition 2 (see [15–24]). The Riemann-Liouville fractional integral operator of order $\alpha \geq 0$, of a function $f \in C_\mu$, $\mu \geq -1$, is defined as

$$J^\alpha f(x) = \frac{1}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1} f(t)\,dt, \quad \alpha > 0,\ x > 0, \qquad J^0 f(x) = f(x). \tag{4}$$

Properties of the operator can be found in [15–23]; we mention only the following.

For $f \in C_\mu$, $\mu \geq -1$, $\alpha, \beta \geq 0$ and $\gamma > -1$

$$J^\alpha J^\beta f(x) = J^{\alpha+\beta} f(x), \qquad J^\alpha J^\beta f(x) = J^\beta J^\alpha f(x), \qquad J^\alpha x^\gamma = \frac{\Gamma(\gamma+1)}{\Gamma(\alpha+\gamma+1)} x^{\alpha+\gamma}. \tag{5}$$

Definition 3. The Caputo fractional order derivative is given as follows [15–18]:

$$ {}_0^C D_x^\alpha (f(x)) = \frac{1}{\Gamma(n-\alpha)} \int_0^x (x-t)^{n-\alpha-1} \frac{d^n f(t)}{dt^n} dt, \quad n-1 \leq \alpha \leq n. \tag{6}$$

Definition 4. The Riemann-Liouville fractional order derivative is given as follows [16–24]:

$$D_x^\alpha (f(x)) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dx^n} \int_0^x (x-t)^{n-\alpha-1} f(t)\,dt, \quad n-1 \leq \alpha \leq n. \tag{7}$$

Definition 5. The Jumarie Fractional order derivative is given as follows [24]:

$$D_x^\alpha (f(x)) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dx^n} \times \int_0^x (x-t)^{n-\alpha-1} \{f(t) - f(0)\}\,dt, \quad n-1 \leq \alpha \leq n. \tag{8}$$

Lemma 6. *If $m-1 < \alpha \leq m$, $m \in \mathbb{N}$ and $f \in C_\mu^m$, $\mu \geq -1$, then*

$$D^\alpha J^\alpha f(x) = f(x), \qquad J^\alpha D_0^\alpha f(x) = f(x) - \sum_{k=0}^{m-1} f^{(k)}(0^+) \frac{x^k}{k!}, \quad x > 0. \tag{9}$$

Definition 7 (partial derivatives of fractional order [15, 16, 19]). Assume now that $f(\mathbf{x})$ is a function of n variables x_i, $i = 1,\ldots,n$ also of class C on $D \in \mathbb{R}_n$. As an extension of Definition 3, we define partial derivative of order α for f with respect to x_i the function

$$a\partial_{\underline{\mathbf{x}}}^\alpha f = \frac{1}{\Gamma(m-\alpha)} \int_a^{x_i} (x_i - t)^{m-\alpha-1} \partial_{x_i}^m f(x_j) \Big|_{x_{j=t}} dt, \tag{10}$$

where $\partial_{x_i}^m$ is the usual partial derivative of integer order m.

3. Background of Sumudu Transform

Definition 8 (see [25]). The Sumudu transform of a function $f(t)$, defined for all real numbers $t \geq 0$, is the function $F_s(u)$, defined by

$$S(f(t)) = F_s(u) = \int_0^\infty \frac{1}{u} \exp\left[-\frac{t}{u}\right] f(t)\,dt. \tag{11}$$

Theorem 9 (see [26]). *Let $G(u)$ be the Sumudu transform of $f(t)$ such that*

(i) *$(G(1/s)/s)$ is a meromorphic function, with singularities having* $\mathrm{Re}[s] \leq \gamma$;

(ii) *there exist a circular region Γ with radius R and positive constants M and K with $|G(1/s)/s| < MR^{-K}$, then the function $f(t)$ is given by*

$$S^{-1}(G(s)) = \frac{1}{2\pi i} \int_{\gamma-i\infty}^{\gamma+i\infty} \exp[st]\, G\left(\frac{1}{s}\right) \frac{ds}{s} = \sum \text{residual}\left[\exp[st] \frac{G(1/s)}{s}\right]. \tag{12}$$

For the proof see [26].

3.1. Basics of the Sumudu Transform Homotopy Perturbation Method. We illustrate the basic idea of this method [27–32] by considering a general fractional nonlinear nonhomogeneous partial differential equation with the initial condition of the following form:

$$D_t^{\alpha} U(x,t) = L(U(x,t)) + N(U(x,t)) + f(x,t), \quad \alpha > 0, \tag{13}$$

subject to the initial condition

$$\begin{aligned} D_0^k U(x,0) &= g_k, \quad (k = 0, \dots, n-1), \\ D_0^n U(x,0) &= 0, \quad n = [\alpha], \end{aligned} \tag{14}$$

where D_t^{α} denotes without loss of generality the Caputo fraction derivative operator, f is a known function, N is the general nonlinear fractional differential operator, and L represents a linear fractional differential operator.

Applying the Sumudu transform on both sides of (10), we obtain

$$\begin{aligned} S[D_t^{\alpha} U(x,t]) &= S[L(U(x,t))] \\ &+ S[N(U(x,t))] + S[f(x,t)]. \end{aligned} \tag{15}$$

Using the property of the Sumudu transform, we have

$$\begin{aligned} S[U(x,t)] &= u^{\alpha} S[L(U(x,t))] + u^{\alpha} S[N(U(x,t))] \\ &+ u^{\alpha} S[f(x,t)] + g(x,t). \end{aligned} \tag{16}$$

Now applying the Sumudu inverse on both sides of (12) we obtain

$$\begin{aligned} U(x,t) &= S^{-1}[u^{\alpha} S[L(U(x,t))] + u^{\alpha} S[N(U(x,t))]] \\ &+ G(x,t), \end{aligned} \tag{17}$$

where $G(x,t)$ represents the term arising from the known function $f(x,t)$ and the initial conditions.

Now we apply the following HPM:

$$U(x,t) = \sum_{n=0}^{\infty} p^n U_n(x,t). \tag{18}$$

The nonlinear term can be decomposed to

$$NU(x,t) = \sum_{n=0}^{\infty} p^n \mathscr{H}_n(U), \tag{19}$$

using the He's polynomial $\mathscr{H}_n(U)$ given as

$$\mathscr{H}_n(U_0, \dots, U_n) = \frac{1}{n!} \frac{\partial^n}{\partial p^n} \left[N\left(\sum_{j=0}^{\infty} p^j U_j(x,t) \right) \right], \quad n = 0, 1, 2 \dots. \tag{20}$$

Substituting (15) and (16) gives

$$\begin{aligned} &\sum_{n=0}^{\infty} p^n U_n(x,t) \\ &= G(x,t) + p\left[S^{-1}\left[u^{\alpha} S\left[L\left(\sum_{n=0}^{\infty} p^n U_n(x,t) \right) \right] \right.\right. \\ &\left.\left. + u^{\alpha} S\left[N\left(\sum_{n=0}^{\infty} p^n U_n(x,t) \right) \right] \right] \right], \end{aligned} \tag{21}$$

which is the coupling of the Sumudu transform and the HPM using He's polynomials. Comparing the coefficients of like powers of p, the following approximations are obtained [29, 30]:

$$\begin{aligned} p^0 &: U_0(x,t) = G(x,t), \\ p^1 &: U_1(x,t) = S^{-1}[u^{\alpha} S[L(U_0(x,t)) + H_0(U)]], \\ p^2 &: U_2(x,t) = S^{-1}[u^{\alpha} S[L(U_1(x,t)) + H_1(U)]], \\ p^3 &: U_3(x,t) = S^{-1}[u^{\alpha} S[L(U_2(x,t)) + H_2(U)]], \\ p^n &: U_n(x,t) = S^{-1}[u^{\alpha} S[L(U_{n-1}(x,t)) + H_{n-1}(U)]]. \end{aligned} \tag{22}$$

Finally, we approximate the analytical solution $U(x,t)$ by truncated series:

$$U(x,t) = \lim_{N \to \infty} \sum_{n=0}^{N} U_n(x,t). \tag{23}$$

The above series solutions generally converge very rapidly [29, 30].

4. Applications

In this section, we apply this method for solving the system of the fractional differential equation. We will start with (1).

4.1. Approximate Solution of (1). Following carefully the steps involved in the STHPM, after comparing the terms of the same power of p and choosing the appropriate initials conditions, we arrive at the following series solutions:

$$\begin{aligned} u_0(x,t) &= G(x,t) = -\frac{c_1}{c_2} + 2c_1\sqrt{-\alpha-\beta^2}\,\operatorname{sech}(c_1 x), \\ v_0(x,t) &= G_1(x,t) \\ &= -c_1^2(\alpha+\beta^2) + 2c_1^2(\alpha+\beta^2)\operatorname{sech}(c_1 x)^2 \\ &\quad + 2c_1^2\beta\sqrt{-\alpha-\beta^2}\operatorname{sech}(c_1 x)\tanh(c_1 x), \\ u_1(x,t) &= S^{-1}[u^{\alpha} S[L(u_0(x,t)) + H_0(u)]] \\ &= \frac{c_1^2 t^{\eta}\operatorname{sech}(c_1 x)^3}{c_2\Gamma(\eta+1)} \\ &\quad \times \Big(c_1 c_2 \beta\sqrt{-\alpha-\beta^2}\cos(2c_1 x) \\ &\quad + 4c_1 c_2(\alpha+\beta^2)\sinh(c_1 x) + \sqrt{-\alpha-\beta^2} \\ &\quad \times \big(-3c_1 c_2 \beta \\ &\quad + (c_1 - c_2)\sinh(2c_1 x) \big) \Big), \end{aligned}$$

$$
\begin{aligned}
v_1(x,t) &= S^{-1}\left[u^{\alpha} S\left[L\left(v_0(x,t)\right)+H_0(v)\right]\right] \\
&= \frac{1}{c_2\Gamma(1+\mu)} \\
&\quad \times \Big(2c_1^2 t^{\mu}\Big(-2c_1 \operatorname{sech}(x)\left(\alpha+\beta^2\right) \\
&\qquad -2c_2\beta\left(\alpha+\beta^2\right)\operatorname{sech}(c_1x)^4 \\
&\qquad -\frac{1}{2}\sqrt{-\alpha-\beta^2}\operatorname{sech}(c_1x)^5 \\
&\qquad \times\Big(\beta-28c_1c_2\beta^2+\beta(1+18c_1c_2\beta) \\
&\qquad\quad \times\cosh(2c_1x)-5c_2\sinh(2c_1x)\Big) \\
&\qquad +2c_2\beta\left(\alpha+\beta^2\right)\operatorname{sech}(c_1x)^2\tanh(c_1x)^2 \\
&\qquad +\sqrt{-\alpha-\beta^2}\operatorname{sech}(c_1x) \\
&\qquad \times\Big(4c_1c_2\operatorname{sech}(x)\left(\alpha+\beta^2\right) \\
&\qquad\quad +\tanh(c_1x)^2 \\
&\qquad\quad \times\Big(\beta+c_2\tanh(c_1x) \\
&\qquad\qquad \times\left(-1+c_1\beta^2\tanh(c_1x)\right)\Big)\Big)\Big)\Big), \\
u_2(x,t) &= S^{-1}\left[u^{\alpha} S\left[L\left(u_1(x,t)\right)+H_1(v)\right]\right] \\
&= \frac{1}{c_2^2\Gamma(1+2\eta)} \\
&\quad \times\Big(4^{-1-\eta}c_1^3 t^{2\eta}\operatorname{sech}(c_1x)^5 \\
&\qquad \times\Big(-5\times4^{\eta}\sqrt{-\alpha-\beta^2} \\
&\qquad\quad \times\left(2c_1c_2-c_2^2+c_1^2\left(16\alpha+39\beta^2\right)\right)\Big) \\
&\qquad +3\times4^{2+\eta}c_1(c_1-c_2)c_2\left(\alpha+\beta^2\right)\cosh(c_1x) \\
&\qquad +4^{1+\eta}\sqrt{-\alpha-\beta^2}\Big(-2c_1c_2+c_2^2+c_1^2 \\
&\qquad\qquad \times\left(1+c_2^2\left(12\alpha+31\beta^2\right)\right)\Big) \\
&\qquad \times\cosh(2c_1x)+4^{2+\eta}c_1c_2(-c_1+c_2)\left(\alpha+\beta^2\right) \\
&\qquad \times\cosh(3c_1x)-4^{\eta}c_1^2\sqrt{-\alpha-\beta^2}\cosh(4c_1x)
\end{aligned}
$$

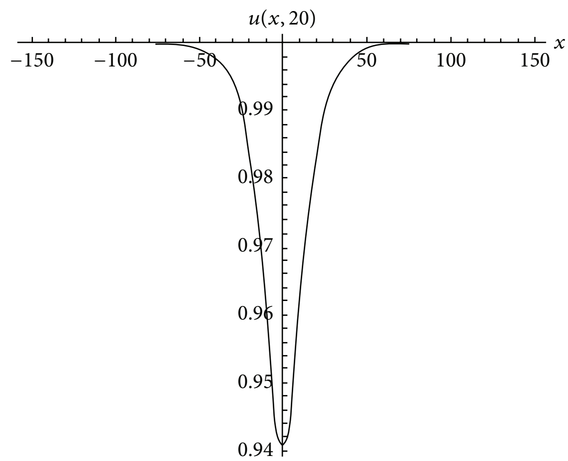

Figure 1: Approximate solution for FWBK equation.

$$
\begin{aligned}
&+2^{2\eta+1}c_1c_2\sqrt{-\alpha-\beta^2}\cosh(4c_1x) \\
&-4^{\eta}c_2^2\sqrt{-\alpha-\beta^2}\cosh(4c_1x) \\
&-4^{\eta}c_2^2c_1^2\beta^2\sqrt{-\alpha-\beta^2}\cosh(4c_1x) \\
&+11\times4^{1+\eta}c_2c_1^2\beta^2\sqrt{-\alpha-\beta^2}\sinh(2c_1x) \\
&-11\times4^{1+\eta}c_1c_2^2\beta^2\sqrt{-\alpha-\beta^2}\sinh(2c_1x) \\
&+2^{3+2\eta}c_1^2c_2^2\beta\left(\alpha+\beta^2\right) \\
&\times\left(37\sinh(c_1x)-3\sinh(3c_1x)\right) \\
&-4^{\eta+1}c_2c_1^2\beta\sqrt{-\alpha-\beta^2}\sinh(4c_1x) \\
&+4^{\eta+1}c_1c_2^2\beta\sqrt{-\alpha-\beta^2}\sinh(4c_1x)\Big).
\end{aligned}
\tag{24}
$$

And so on in the same manner one can obtain the rest of the components. However, here, few terms were computed and the asymptotic solution is given by

$$
\begin{aligned}
u(x,t) &= u_0(x,t)+u_1(x,t)+u_2(x,t)+u_3(x,t)+\cdots, \\
v(x,t) &= v_0(x,t)+v_1(x,t)+v_2(x,t)+v_3(x,t)+\cdots.
\end{aligned}
\tag{25}
$$

Figures 1, 2, 3, and 4 show the graphical representation of the approximated solution of the system of nonlinear fractional Whitham-Broer-Kaup equation for $\eta = 0.9$, $\mu = 0.98$, $c_1 = c_2 = 0.1$, and $\beta = \alpha = 0.1$.

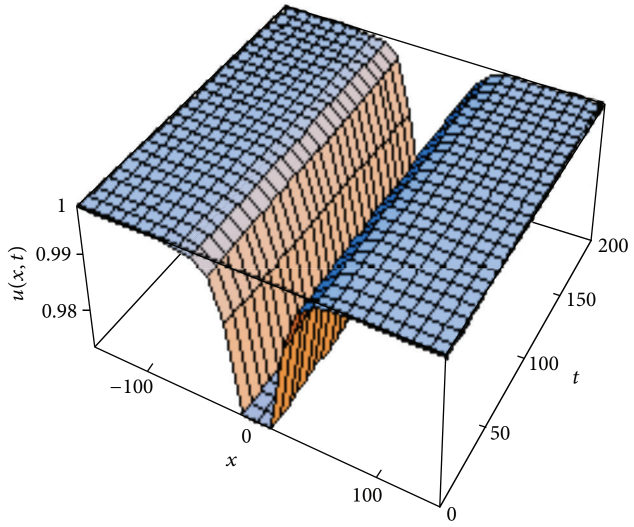

Figure 2: Approximate solution of FWBK equation.

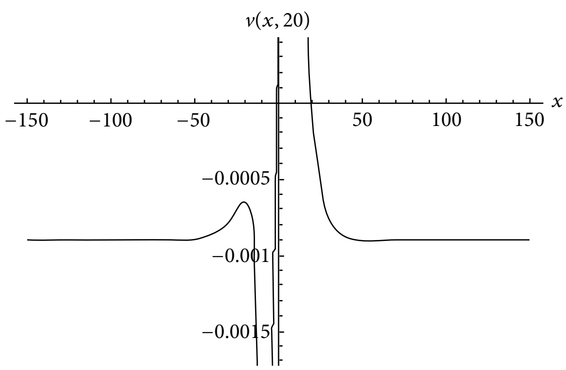

Figure 3: Approximate solution of FWBK equation.

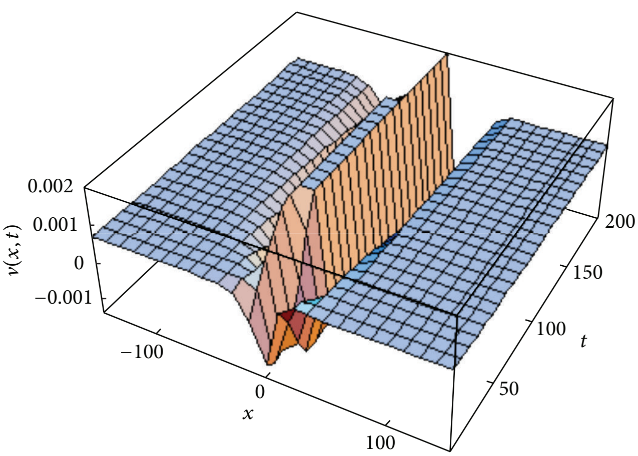

Figure 4: Approximate solution of FWBK equation.

4.2. Approximate Solution of (2). For (2), in the view of the Sumudu transform method, by choosing the appropriate initials conditions we are at the following series solutions:

$$u_0(x,t) = \frac{c^2}{8}\left(1 - \operatorname{sech}\left(\frac{cx}{2}\right)^2\right),$$

$$v_0(x,t) = c\operatorname{sech}\left(\frac{cx}{2}\right)^2,$$

$$\begin{aligned} u_1(x,t) = & -\frac{c^5 t^\eta \operatorname{sech}(cx/2)^5}{128\Gamma(\eta+1)} \\ & \times\left(192\cosh\left[\frac{cx}{2}\right] - 32\cosh\left[\frac{3cx}{2}\right]\right. \\ & \left. +3c\left(3\sinh\left[\frac{cx}{2}\right] + \sinh\left[\frac{3cx}{2}\right]\right)\right) \\ & \times\tanh\left[\frac{cx}{2}\right], \end{aligned}$$

$$\begin{aligned} v_1(x,t) = & -\frac{c^4 t^\mu \operatorname{sech}(cx/2)^3 \tanh[cx/2]}{16\Gamma(\mu+1)} \\ & \times\left(71 - \cosh[cx] + 6c\tanh\left[\frac{cx}{2}\right]\right), \end{aligned}$$

$$\begin{aligned} & u_2(x,t) \\ & = \left(\frac{4^{-10-\eta}c^5 t^\eta (cx/2)^{15}}{\Gamma(1+\mu)\Gamma(1+\eta)\Gamma(0.5+\eta)\Gamma(1+\mu+\eta)}\right. \\ & \times\left(-32c^3\sqrt{\pi}t^\eta \mu\cosh\left(\frac{cx}{2}\right)^4\Gamma(\mu)\right. \\ & \times\Gamma(1+\eta+\mu)\Gamma(1+2\eta+\mu) \\ & \times\left(221184 - 20532c^2\right)\cosh\left(\frac{cx}{2}\right) \\ & + 6\left(-11008 + 4813c^2\right)\cosh\left(\frac{3cx}{2}\right) \\ & - 69120\cosh\left(\frac{5cx}{2}\right) - 8622c^2\cosh\left(\frac{5cx}{2}\right) \\ & + 10368\cosh\left(\frac{7cx}{2}\right) + 267c^2\cosh\left(\frac{7cx}{2}\right) \\ & - 128\cosh\left(\frac{9cx}{2}\right) + 9c^2\cosh\left(\frac{9cx}{2}\right) \\ & + 61032c\sinh\left(\frac{cx}{2}\right) - 2772c^3\sinh\left(\frac{cx}{2}\right) \\ & + 29040c\sinh\left(\frac{3cx}{2}\right) + 828c^3\sinh\left(\frac{3cx}{2}\right) \\ & - 27312c\sinh\left(\frac{5cx}{2}\right) + 108c^3\sinh\left(\frac{5cx}{2}\right) \end{aligned}$$

$$+ 4596c\sinh\left(\frac{7cx}{2}\right)$$
$$-36c^3\sinh\left(\frac{7cx}{2}\right) - 84c\sinh\left(\frac{9cx}{2}\right)\Bigg)$$
$$+ 3\times 4^{\eta}\Gamma(0.5+\eta)$$
$$\times\Bigg(65536\mu\cosh\left(\frac{9cx}{2}\right)^9\Gamma(\mu)\Gamma(1+\eta+\mu)\Gamma$$
$$\times(1+2\eta+\mu)\sinh\left(\frac{cx}{2}\right)^2$$
$$\times(-2c+\sinh(cx))$$
$$+ 1024c^3 t^{\eta}\eta\Gamma(\eta)\Gamma(1+\mu)\Gamma$$
$$\times(1+2\eta+\mu)\sinh\left(\frac{cx}{2}\right)^2$$
$$\times\Bigg(-15745\cosh\left(\frac{cx}{2}\right) + 12951\cosh\left(\frac{3cx}{2}\right)$$
$$- 1175\cosh\left(\frac{3cx}{2}\right) + \cosh\left(\frac{7cx}{2}\right)$$
$$- 6240c\sinh\left(\frac{cx}{2}\right)$$
$$+ 1728c\sinh\left(\frac{3cx}{2}\right)$$
$$-96c\sinh\left(\frac{5cx}{2}\right)\Bigg)\Bigg)$$
$$+ 2c^6 t^{\eta+\mu}\Gamma(1+\eta+\mu)^2\sinh\left(\frac{cx}{2}\right)$$
$$\times\Big(-235648 - 1154128c^2 + 15804c^4$$
$$- 16\left(5584 - 7358c^2 + 1125c^4\right)$$
$$\times\cosh(cx)$$
$$+ 16\left(15904 - 60016c^2 + 99c^4\right)$$
$$\times\cosh(2cx)$$
$$+ 89216\cosh(3cx) - 296896c^2$$
$$\times\cosh(3cx)$$
$$+ 720c^4\cosh(3cx) - 18816\cosh(4cx)$$
$$+ 14672c^2\cosh(4cx) - 108c^4$$
$$\times\cosh(4cx)$$
$$+128\cosh(5cx) - 32c^2\cosh(5cx)\Big)$$
$$- 52680c\sinh(cx) - 391458c^3\sinh(cx)$$
$$- 240c\sinh(2cx) + 196824c^3\sinh(2cx)$$
$$+ 17580c\sinh(3cx) - 24207c^3\sinh(3cx)$$
$$+ 120c\sinh(4cx) - 156c^3\sinh(4cx)$$
$$- 12c\sinh(5cx) + 3c^3\sinh(4cx)\Big)\Big),$$

$$v_2(x,t)$$
$$= \Bigg(2^{-17-2\eta}c^4 t^{\mu}\operatorname{sech}\left(\frac{cx}{2}\right)^{13}$$
$$\times\Big(\Gamma(1+\mu)^2\Gamma(1+\eta)\Gamma(0.5+\mu)\Gamma$$
$$\times(1+\mu+\eta)\Gamma(1+3\mu)\Big)^{-1}$$
$$\times\Bigg(3\times 4^{4+\mu}c^4 t^{\eta}\cosh\left(\frac{cx}{2}\right)^4$$
$$\times\Gamma(1+\mu)^2\Gamma(1+\eta)\Gamma(0.5+\mu)$$
$$\times\Gamma(1+3\mu)\sinh\left(\frac{cx}{2}\right)$$
$$\times\Bigg(896\cosh\left(\frac{cx}{2}\right) - 608\cosh\left(\frac{3cx}{2}\right)$$
$$+ 32\cosh\left(\frac{5cx}{2}\right) + 78c\sinh\left(\frac{cx}{2}\right)$$
$$+3c\sinh\left(\frac{3cx}{2}\right) - 5c\sinh\left(\frac{5cx}{2}\right)\Bigg)\Bigg)$$
$$+ \Gamma(\eta+1)\Gamma(1+\eta+\mu)$$
$$\times(15\times 4^{\mu}\Gamma(0.5+\mu)$$
$$\times\Bigg(-65536\mu\cosh\left(\frac{cx}{2}\right)^9\Gamma(\mu)$$
$$\times\Gamma(1+3\mu)\sinh\left(\frac{3cx}{2}\right)\Bigg)\Bigg)$$
$$+ 2c^7 t^{2\mu}\Gamma(1+2\mu)$$
$$\times\Bigg(994\cosh\left(\frac{cx}{2}\right) - 435\cosh\left(\frac{3cx}{2}\right)$$
$$+ \cosh\left(\frac{5cx}{2}\right) + 204c\sinh\left(\frac{cx}{2}\right)$$
$$- 36c\sinh\left(\frac{3cx}{2}\right) -5c\sinh\left(\frac{3cx}{2}\right)\Bigg)^2\Bigg)$$
$$+ 16c^3\sqrt{\pi}t^{\mu}\mu\cosh\left(\frac{cx}{2}\right)^4$$

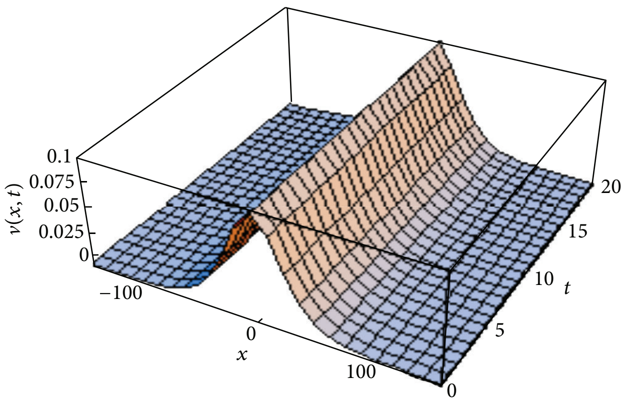

Figure 5: Approximate solution of FJM equation.

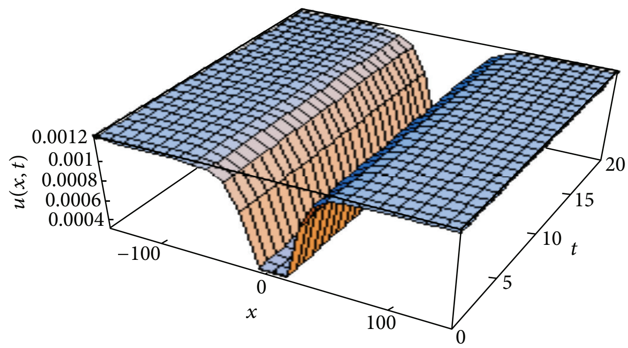

Figure 6: Approximate solution of FJM equation.

$$\times \Gamma(\mu)\Gamma(1+3\mu)$$
$$\times \left(-79965 - 22032c^2 + 8\left(-2633 + 3240c^2\right)\cosh(cx) + \left(54940 - 3888c^2\right)\cosh(2cx) - 3960\cosh(3cx) + \cosh(4cx) + 130152c\sinh(cx) - 39504c\sinh(2cx) - 216c\sinh(3cx)\right)\Bigg)\Bigg). \tag{26}$$

And so on in the same manner one can obtain the rest of the components. However, here, few terms were computed, and the asymptotic solution of the nonlinear fractional Jaulent-Miodek is given by

$$u(x,t) = u_0(x,t) + u_1(x,t) + u_2(x,t) + u_3(x,t) + \cdots,$$
$$v(x,t) = v_0(x,t) + v_1(x,t) + v_2(x,t) + v_3(x,t) + \cdots. \tag{27}$$

Figures 5 and 6 show the graphical representation of the approximated solution of the system of nonlinear fractional Jaulent-Miodek equation for $\eta = 0.98$, $\mu = 0.48$, and $c = 0.1$. Figures 5 and 6 show the approximate solution of the main problem.

5. Conclusion

We derived approximated solutions of nonlinear fractional Jaulent-Miodek and Whitham-Broer-Kaup equations using the relatively new analytical technique the STHPM. We presented the brief history and some properties of fractional derivative concept. It is demonstrated that STHPM is a powerful and efficient tool for the system of FPDEs. In addition, the calculations involved in STHPM are very simple and straightforward.

The STHPM is chosen to solve this nonlinear problem because of the following advantages that the method has over the existing methods. This method does not require the linearization or assumptions of weak nonlinearity. The solutions are not generated in the form of general solution as in the Adomian decomposition method (ADM) [33, 34]. No correction functional or Lagrange multiplier is required in the case of the variational iteration method [35, 36]. It is more realistic compared to the method of simplifying the physical problems. If the exact solution of the partial differential equation exists, the approximated solution via the method converges to the exact solution. STHPM provides us with a convenient way to control the convergence of approximation series without adapting h, as in the case of [37] which is a fundamental qualitative difference in the analysis between STHPM and other methods. And also there is nothing like solving a partial differential equation after comparing the terms of same power of p like in the case of homotopy perturbation method (HPM) [38].

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