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Research Article

Oscillation of Even-Order Neutral Delay Differential Equations

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By using Riccati transformation technique, we will establish some new oscillation criteria for the even order neutral delay differential equations $[z(t)]^{(n)}+q(t)f(x(\sigma(t)))=0$, $t\geq t_0$, where n is even, $z(t)=x(t)+p(t)x(\tau(t))$, $0\leq p(t)\leq p_0<\infty$, and $q(t)\geq 0$. These oscillation criteria, at least in some sense, complement and improve those of Zafer (1998) and Zhang et al. (2010). An example is considered to illustrate the main results.

1. Introduction

This paper is concerned with the oscillatory behavior of the even-order neutral delay differential equations

$$[z(t)]^{(n)}+q(t)f(x(\sigma(t)))=0, \quad t\geq t_0, \tag{1.1}$$

where n is even $z(t)=x(t)+p(t)x(\tau(t))$.

In what follows we assume that

(I_1) $p,q\in C([t_0,\infty),R)$, $0\leq p(t)\leq p_0<\infty$, $q(t)\geq 0$,

(I_2) $\tau\in C^1([t_0,\infty),R)$, $\sigma\in C([t_0,\infty),R)$, $\tau(t)\leq t$, $\tau'(t)=\tau_0>0$, $\lim_{t\to\infty}\sigma(t)=\infty$, $\tau\circ\sigma=\sigma\circ\tau$, where τ_0 is a constant,

(I_3) $f\in C(R,R)$ and $f(y)/y\geq\alpha>0$, for $y\neq 0$, α is a constant.

Neutral differential equations find numerous applications in natural science and technology. For instance, they are frequently used for the study of distributed networks containing lossless transmission lines; see Hale [1].

In the last decades, there are many studies that have been made on the oscillatory behavior of solutions of differential equations [2–6] and neutral delay differential equations [7–23].

For instance, Grammatikopoulos et al. [10] examined the oscillation of second-order neutral delay differential equations

$$\left[x(t)+p(t)x(t-\tau)\right]''+q(t)x(t-\sigma)=0,\quad t\geq t_0, \tag{1.2}$$

where $0\leq p(t)<1$.

Liu and Bai [13] investigated the second-order neutral differential equations

$$\left(r(t)\left|Z'(t)\right|^{\alpha-1}Z'(t)\right)'+q(t)\left|y(\sigma(t))\right|^{\alpha-1}y(\sigma(t))=0,\quad t\geq t_0, \tag{1.3}$$

where $Z(t)=x(t)+p(t)x(\tau(t)),\ 0\leq p(t)<1$.

Meng and Xu [14] studied the oscillation of even-order neutral differential equations

$$\left[r(t)\left|\left(x(t)+p(t)x(t-\tau)\right)^{(n-1)}\right|^{\alpha-1}\left(x(t)+p(t)x(t-\tau)\right)^{(n-1)}\right]'+q(t)f(x(\sigma(t)))=0,\quad t\geq t_0, \tag{1.4}$$

where $0\leq p(t)<1$.

Ye and Xu [21] considered the second-order quasilinear neutral delay differential equations

$$\left(r(t)\psi(x(t))\left|Z'(t)\right|^{\alpha-1}Z'(t)\right)'+q(t)f(x(\sigma(t)))=0,\quad t\geq t_0, \tag{1.5}$$

where $Z(t)=x(t)+p(t)x(\tau(t)),\ 0\leq p(t)<1$.

Zafer [22] discussed oscillation criteria for the equations

$$\left[x(t)+p(t)x(\tau(t))\right]^{(n)}+f(t,x(t),x(\sigma(t)))=0,\quad t\geq t_0, \tag{1.6}$$

where $0\leq p(t)<1$.

In 2009, Zhang et al. [23] considered the oscillation of the even-order nonlinear neutral differential equation (1.1) when $0\leq p(t)<1$.

To the best of our knowledge, the above oscillation results cannot be applied when $p(t)>1$, and it seems to have few oscillation results for (1.1) when $p(t)>1$.

Xu and Xia [17] established a new oscillation criteria for the second-order neutral differential equations

$$\left[x(t)+p(t)x(t-\tau)\right]''+q(t)f(x(t-\sigma))=0,\quad t\geq t_0. \tag{1.7}$$

Motivated by Liu and Bai [13], we will further the investigation and offer some more general new oscillation criteria for (1.1), by employing a class of function Y and operator T. The method used in this paper is different from [17].

Following [13], we say that a function $\phi=\phi(t,s,l)$ belongs to the function class Y if $\phi\in C(E,R)$, where $E=\{(t,s,l):t_0\leq l\leq s\leq t<\infty\}$, which satisfies

$$\phi(t,t,l)=0,\quad \phi(t,l,l)=0,\quad \phi(t,s,l)>0, \tag{1.8}$$

for $l<s<t$ and has the partial derivative $\partial\phi/\partial s$ on E such that $\partial\phi/\partial s$ is locally integrable with respect to s in E.

By choosing the special function ϕ, it is possible to derive several oscillation criteria for a wide range of differential equations.

Define the operator $T[\cdot;l,t]$ by

$$T[g;l,t]=\int_l^t\phi(t,s,l)g(s)\mathrm{d}s, \tag{1.9}$$

for $t\geq s\geq l\geq t_0$ and $g\in C([t_0,\infty),R)$. The function $\varphi=\varphi(t,s,l)$ is defined by

$$\frac{\partial\phi(t,s,l)}{\partial s}=\varphi(t,s,l)\phi(t,s,l). \tag{1.10}$$

It is easy to verify that $T[\cdot;l,t]$ is a linear operator and that it satisfies

$$T[g';l,t]=-T[g\varphi;l,t],\quad \text{for } g\in C^1([t_0,\infty),R). \tag{1.11}$$

2. Main Results

In this section, we give some new oscillation criteria for (1.1). In order to prove our theorems we will need the following lemmas.

Lemma 2.1 (see [5]). *Let $u\in C^n([t_0,\infty),R^+)$. If $u^{(n)}(t)$ is eventually of one sign for all large t, say $t_1>t_0$, then there exist a $t_x>t_0$ and an integer $l, 0\leq l\leq n$, with $n+l$ even for $u^{(n)}(t)\geq 0$ or $n+l$ odd for $u^{(n)}(t)\leq 0$ such that $l>0$ implies that $u^{(k)}(t)>0$ for $t>t_x$, $k=0,1,2,\ldots,l-1$, and $l\leq n-1$ implies that $(-1)^{l+k}u^k(t)>0$ for $t>t_x$, $k=l,l+1,\ldots,n-1$.*

Lemma 2.2 (see [5]). *If the function u is as in Lemma 2.1 and $u^{(n-1)}(t)u^{(n)}(t)\leq 0$ for $t>t_x$, then for every $\lambda, 0<\lambda<1$, there exists a constant $M>0$ such that*

$$u(\lambda t)\geq Mt^{n-1}\left|u^{(n-1)}(t)\right|,\quad \textit{for all large } t. \tag{2.1}$$

Lemma 2.3 (see [14]). *Suppose that x is an eventually positive solution of* (1.1). *Then there exists a number $t_1 \geq t_0$ such that for $t \geq t_1$,*

$$z(t) > 0, \quad z'(t) > 0, \quad z^{(n-1)}(t) > 0, \quad z^{(n)}(t) \leq 0. \tag{2.2}$$

Theorem 2.4. *Assume that $\sigma(t) \geq \tau(t)$.*

Further, there exist functions $\phi \in Y$ and $k \in C^1([t_0,\infty), R^+)$, such that for some λ, $0 < \lambda < 1$ and for every $M > 0$,

$$\limsup_{t\to\infty} T\left[k(s)Q(s) - \frac{(1+p_0/\tau_0)(\varphi + k'(s)/k(s))^2}{4\lambda M}\frac{k(s)}{\tau^{n-2}(s)\tau_0}; l, t\right] > 0, \tag{2.3}$$

where $Q(t) = \min\{\alpha q(t), \alpha q(\tau(t))\}$, the operator T is defined by (1.9), *and $\varphi = \varphi(t,s,l)$ is defined by* (1.10). *Then every solution x of* (1.1) *is oscillatory.*

Proof. Let x be a nonoscillatory solution of (1.1). Then there exists $t_1 \geq t_0$ such that $x(t) \neq 0$, for all $t \geq t_1$. Without loss of generality, we assume that $x(t) > 0$, $x(\tau(t)) > 0$, and $x(\sigma(t)) > 0$, for all $t \geq t_1$.

By Lemma 2.3, there exists $t_2 \geq t_1$ such like that (2.2) for $t \geq t_2$. Using definition of z and applying (1.1), we get for sufficiently large t

$$z^{(n)}(t) + \alpha q(t)x(\sigma(t)) + \alpha p_0 q(\tau(t))x(\sigma(\tau(t))) + p_0 z^{(n)}(\tau(t)) \leq 0, \tag{2.4}$$

thus

$$z^{(n)}(t) + Q(t)z(\sigma(t)) + p_0 z^{(n)}(\tau(t)) \leq 0, \tag{2.5}$$

where $Q(t) = \min\{\alpha q(t), \alpha q(\tau(t))\}$.

It is easy to check that we can apply Lemma 2.2 for $u = z'$ and conclude that there exist $M > 0$ and $t_3 \geq t_2$ such that

$$z'(\lambda\tau(t)) \geq M\tau^{n-2}(t)z^{(n-1)}(\tau(t)) \geq M\tau^{n-2}(t)z^{(n-1)}(t), \quad t \geq t_3. \tag{2.6}$$

Next, define

$$\omega(t) = k(t)\frac{z^{(n-1)}(t)}{z(\lambda\tau(t))}, \quad t \geq t_3. \tag{2.7}$$

Then, $\omega(t) > 0$ and

$$\omega'(t) = k'(t)\frac{z^{(n-1)}(t)}{z(\lambda\tau(t))} + k(t)\frac{z^{(n)}(t)z(\lambda\tau(t)) - \lambda z^{(n-1)}(t)z'(\lambda\tau(t))\tau'(t)}{z^2(\lambda\tau(t))}. \tag{2.8}$$

From (2.6), (2.7), and (2.8), we have

$$\omega'(t) \le k(t)\frac{z^{(n)}(t)}{z(\lambda\tau(t))} + \frac{k'(t)}{k(t)}\omega(t) - \lambda M\frac{\tau^{n-2}(t)\tau'(t)}{k(t)}\omega^2(t). \tag{2.9}$$

Similarly, define

$$\nu(t) = k(t)\frac{z^{(n-1)}(\tau(t))}{z(\lambda\tau(t))}, \quad t \ge t_3. \tag{2.10}$$

Then, $\nu(t) > 0$ and

$$\nu'(t) = k'(t)\frac{z^{(n-1)}(\tau(t))}{z(\lambda\tau(t))} + k(t)\frac{z^{(n)}(\tau(t))\tau'(t)z(\lambda\tau(t)) - \lambda z^{(n-1)}(\tau(t))z'(\lambda\tau(t))\tau'(t)}{z^2(\lambda\tau(t))}. \tag{2.11}$$

From (2.6), (2.10), and (2.11), we have

$$\nu'(t) \le k(t)\tau'(t)\frac{z^{(n)}(\tau(t))}{z(\lambda\tau(t))} + \frac{k'(t)}{k(t)}\nu(t) - \lambda M\frac{\tau^{n-2}(t)\tau'(t)}{k(t)}\nu^2(t). \tag{2.12}$$

Therefore, from (2.9) and (2.12), we get

$$\begin{aligned}\omega'(t) + \frac{p_0}{\tau'(t)}\nu'(t) \le k(t)\frac{z^{(n)}(t)}{z(\lambda\tau(t))} + p_0k(t)\frac{z^{(n)}(\tau(t))}{z(\lambda\tau(t))} + \frac{k'(t)}{k(t)}\omega(t)\\ - \lambda M\frac{\tau^{n-2}(t)\tau'(t)}{k(t)}\omega^2(t) + \frac{p_0}{\tau'(t)}\frac{k'(t)}{k(t)}\nu(t) - \frac{p_0}{\tau'(t)}\lambda M\frac{\tau^{n-2}(t)\tau'(t)}{k(t)}\nu^2(t).\end{aligned} \tag{2.13}$$

From (2.5), note that $\sigma(t) \ge \tau(t)$, $z'(t) > 0$, and $\tau'(t) = \tau_0$, then we obtain

$$\begin{aligned}\omega'(t) + \frac{p_0}{\tau_0}\nu'(t) \le -k(t)Q(t) + \frac{k'(t)}{k(t)}\omega(t) - \lambda M\frac{\tau^{n-2}(t)\tau_0}{k(t)}\omega^2(t)\\ + \frac{p_0}{\tau_0}\frac{k'(t)}{k(t)}\nu(t) - \frac{p_0}{\tau_0}\lambda M\frac{\tau^{n-2}(t)\tau_0}{k(t)}\nu^2(t).\end{aligned} \tag{2.14}$$

Applying $T[\cdot;l,t]$ to (2.14), we get

$$\begin{aligned}T\left[\omega'(s) + \frac{p_0}{\tau_0}\nu'(s);l,t\right] \le T\Bigg[-k(s)Q(s) + \frac{k'(s)}{k(s)}\omega(s) - \lambda M\frac{\tau^{n-2}(s)\tau_0}{k(s)}\omega^2(s)\\ + \frac{p_0}{\tau_0}\frac{k'(s)}{k(s)}\nu(s) - \frac{p_0}{\tau_0}\lambda M\frac{\tau^{n-2}(s)\tau_0}{k(s)}\nu^2(s);l,t\Bigg].\end{aligned} \tag{2.15}$$

By (1.11) and the above inequality, we obtain

$$T[k(s)Q(s);l,t] \le T\left[\left(\varphi + \frac{k'(s)}{k(s)}\right)\omega(s) - \lambda M \frac{\tau^{n-2}(s)\tau_0}{k(s)}\omega^2(s) + \frac{p_0}{\tau_0}\left(\varphi + \frac{k'(s)}{k(s)}\right)\nu(s) - \frac{p_0}{\tau_0}\lambda M \frac{\tau^{n-2}(s)\tau_0}{k(s)}\nu^2(s);l,t\right]. \tag{2.16}$$

Hence, from (2.16), we have

$$T[k(s)Q(s);l,t] \le T\left[\left(\frac{(\varphi + k'(s)/k(s))^2}{4\lambda M} + \frac{(p_0/\tau_0)(\varphi + k'(s)/k(s))^2}{4\lambda M}\right)\frac{k(s)}{\tau^{n-2}(s)\tau_0};l,t\right], \tag{2.17}$$

that is,

$$T\left[k(s)Q(s) - \frac{(1+p_0/\tau_0)(\varphi + k'(s)/k(s))^2}{4\lambda M}\frac{k(s)}{\tau^{n-2}(s)\tau_0};l,t\right] \le 0. \tag{2.18}$$

Taking the super limit in the above inequality, we get

$$\limsup_{t\to\infty} T\left[k(s)Q(s) - \frac{(1+p_0/\tau_0)(\varphi + k'(s)/k(s))^2}{4\lambda M}\frac{k(s)}{\tau^{n-2}(s)\tau_0};l,t\right] \le 0, \tag{2.19}$$

which contradicts (2.3). This completes the proof. □

We can apply Theorem 2.4 to the second-order neutral delay differential equations

$$z''(t) + q(t)f(x(\sigma(t))) = 0, \quad t \ge t_0. \tag{2.20}$$

We get the following new result.

Theorem 2.5. *Assume that $\sigma(t) \ge \tau(t)$. Further, there exist functions $\phi \in Y$ and $k \in C^1([t_0,\infty), R^+)$, such that*

$$\limsup_{t\to\infty} T\left[k(s)Q(s) - \frac{k(s)(1+p_0/\tau_0)(\varphi + k'(s)/k(s))^2}{4\tau_0};l,t\right] > 0, \tag{2.21}$$

where Q is defined as in Theorem 2.4, the operator T is defined by (1.9)*, and $\varphi = \varphi(t,s,l)$ is defined by* (1.10)*. Then every solution x of* (2.20) *is oscillatory.*

Proof. Let x be a nonoscillatory solution of (2.20). Then there exists $t_1 \ge t_0$ such that $x(t) \ne 0$, for all $t \ge t_1$.

Without loss of generality, we assume that $x(t) > 0$, and $x(\tau(t)) > 0$, $x(\sigma(t)) > 0$, for all $t \geq t_1$. Proceeding as in the proof of Theorem 2.4, we have (2.2) and (2.5) ($n = 2$). Next, define

$$\omega(t) = k(t)\frac{z'(t)}{z(\tau(t))}, \quad t \geq t_3. \tag{2.22}$$

Then $\omega(t) > 0$ and

$$\omega'(t) = k'(t)\frac{z'(t)}{z(\tau(t))} + k(t)\frac{z''(t)z(\tau(t)) - z'(t)z'(\tau(t))\tau'(t)}{z^2(\tau(t))}. \tag{2.23}$$

From (2.22) and (2.23), we have

$$\omega'(t) \leq k(t)\frac{z''(t)}{z(\tau(t))} + \frac{k'(t)}{k(t)}\omega(t) - \frac{\tau'(t)}{k(t)}\omega^2(t). \tag{2.24}$$

Similarly, define

$$\nu(t) = k(t)\frac{z'(\tau(t))}{z(\tau(t))}, \quad t \geq t_3. \tag{2.25}$$

Then $\nu(t) > 0$. The rest of the proof is similar to that of the proof of Theorem 2.4, hence we omit the details. □

Remark 2.6. With the different choice of k and ϕ, Theorem 2.4 (or Theorem 2.5) can be stated with different conditions for oscillation of (1.1) (or (2.20)). For example, if we choose $\phi(t,s,l) = \rho(s)(t-s)^{v}(s-l)^{\mu}$ for $v > 1/2$, $\mu > 1/2$, $\rho \in C^1([t_0,\infty),(0,\infty))$, then

$$\varphi(t,s,l) = \frac{\rho'(s)}{\rho(s)} + \frac{\mu t - (v+\mu)s + vl}{(t-s)(s-l)}. \tag{2.26}$$

By Theorem 2.4 (or Theorem 2.5) we can obtain the oscillation criterion for (1.1) (or (2.20)); the details are left to the reader.

For an application, we give the following example to illustrate the main results.

Example 2.7. Consider the following equations:

$$[x(t) + 2x(t-5\pi)]'' + x(t-\pi) = 0, \quad t \geq t_0. \tag{2.27}$$

Let $p(t) = 2$, $q(t) = 1$, and $\tau(t) = t - 5\pi$, $\sigma(t) = t - \pi$. Take $\phi(t,s,l) = (t-s)(s-l)$, it is easy to verify that

$$\varphi(t,s,l) = \frac{(t-s)-(s-l)}{(t-s)(s-l)}. \tag{2.28}$$

By Theorem 2.5, let $k(t) = 1$, $\alpha = 1$, and $p_0 = 2$, $Q(t) = 1$, one has (2.21). Hence, every solution of (2.27) oscillates. For example, $x(t) = \sin t$ is an oscillatory solution of (2.27).

Remark 2.8. The recent results cannot be applied in (1.1) and (2.20) when $p(t) > 1$ for $t \geq t_0$. Therefore, our results are new.

Remark 2.9. It would be interesting to find another method to study (1.1) and (2.20) when $\tau'(t) \not\equiv \tau_0$, $\tau(\sigma(t)) \not\equiv \sigma(\tau(t))$, or $\sigma(t) \leq \tau(t)$ for $t \geq t_0$.

Remark 2.10. It would be more interesting to find another method to study (1.1) when n is odd.

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