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Research Article

A Discrete Two-Sector Economic Growth Model

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This paper studies a key model in economic theory—the two-sector growth model—with an alternative utility function. We show that the system has a unique stable equilibrium when the production functions take on the Cobb-Douglas form. We also simulate the model and demonstrate effects of changes in some parameters.

1. Introduction

Solow's one-sector growth model and Uzawa's two-sector growth model have played the role of the key models in the neoclassical growth theory (Uzawa [1], Solow [2], Burmeister and Dobell [3]). These two models and their various extensions and generalizations are fundamental for the development of new economic growth theories as well (e.g., Barro and Sala-i-Martin [4], Aghion and Howitt [5]). Since Uzawa proposed the model in [1], many works have been published to extend and generalize the model in from the 1960s till today (e.g., Drandakis [6], Diamond [7], Weizsäcker, [8], Corden [9], Stiglitz [10], Gram [11], Mino [12], Drugeon and Venditti [13]). But all these studies follow the Solow or Ramsey approach to consumer behavior. This study proposes another approach to consumer behavior to reexamine the basic issues addressed by the two-sector growth model in discrete time. The paper is organized as follows. Section 2 defines the two-sector growth with an alternative approach to consumer behavior with saving and consumption. Section 3 examines dynamic properties of the model when the production functions are specified with the Cobb-Douglas form and simulates the model. Section 4 carries out comparative dynamic analysis with regard to technological and preference changes. Section 5 concludes the study.

2. The two-sector model

This paper reexamines dynamics of the two-sector economic model proposed by Uzawa [1]. The Uzawa model extends the Solow model by a break down of the productive system into two sectors using capital and labor, one of which produces capital goods, the other consumption goods (Solow [2]). This paper introduces an alternative approach to consumer decision to examine structural change for a two-sector economy with capital accumulation. Like in the traditional two-sector growth model, it is assumed that consumption and capital goods are different commodities, which are produced in two distinct sectors. We develop the model with endogenous saving in discrete time. The economy has an infinite future. We represent the passage of time in a sequence of periods, numbered from zero and indexed by $t = 0,1,2,\ldots$. The end of period $t-1$ coincides with the beginning of period t; it can also be called period t. We assume that transactions are made in each period. The population, N, is constant. Most aspects of our model are similar to the Solow one-sector growth model and the Uzawa two-sector model. The discrete version of the Solow-model is referred to Diamond [7] and Zhang [14]. It is assumed that there is only one (durable) good in the economy under consideration. Households own assets of the economy and distribute their incomes to consume and save. Exchanges take place in perfectly competitive markets. Production sectors sell their product to households or to other sectors and households sell their labor and assets to production sectors. Factor markets work well; the available factors are fully utilized. Saving is undertaken only by households, which implies that all earnings of firms are distributed in the form of payments to factors of production, labor, managerial skill, and capital ownership.

We assume perfect competition in all markets and select commodity to serve as numeraire, with all the other prices being measured relative to its price. Let $K(t)$ denote the capital existing in period t and N the flow of labor services used in period t for production. Capital depreciates at a constant exponential rate δ_k, which is independent of the manner of use. The two inputs are smoothly substitutable for each other in each sector and are freely transferable from one sector to the other. Both exogenously determined labor supply and irrevocably existing capital stock are inelastically offered for employment. Both sectors use neoclassical technology with the standard Inada conditions. The production functions are given by $F_j(K_j(t),N_j(t))$, $j = i,s$, where the subscripts i and s denote the capital good sector and the consumption good sector and F_j are the output of sector j; $K_j(t)$ and $N_j(t)$ are, respectively, the capital and labor used in sector j. For simplicity, we assume that F_j takes on the Cobb-Douglas form as follows:

$$F_j(t) = A_j K_j^{\alpha_j}(t) N_j^{\beta_j}(t), \quad \alpha_j,\beta_j > 0,\ \alpha_j + \beta_j = 1,\ j = i,s. \tag{2.1}$$

We express the above equations by

$$f_j(t) = A_j k_j^{\alpha_j}(t), \quad f_j(t) \equiv \frac{F_j(t)}{N_j(t)}, \quad k_j(t) \equiv \frac{K_j(t)}{N_j(t)}, \quad j = i,s. \tag{2.2}$$

Markets are competitive; thus labor and capital earn their marginal products, and firms earn zero profits. We assume that the capital good serves as a medium of exchange and is taken as numeraire. The price of consumption good is denoted by $p(t)$. The rate of interest, $r(t)$, and wage rate, $w(t)$, are determined by markets. Hence, for any individual firm, $r(t)$ and $w(t)$ are given in each period. The production sector chooses the two variables, $K_j(t)$ and $N_j(t)$, to maximize its profit. The marginal conditions are given by

$$r(t)+\delta_k=\alpha_i A_i k_i^{-\beta_i}(t)=p(t)\alpha_s A_s k_s^{-\beta_s}(t), \qquad w(t)=\beta_i A_i k_i^{\alpha_i}(t)=\alpha_s A_s p(t) k_s^{\alpha_s}(t). \tag{2.3}$$

The total capital stock, $K(t)$, is allocated to the two sectors. As full employment of labor and capital is assumed, we have

$$K_i(t)+K_s(t)=K(t), \qquad N_i(t)+N_s(t)=N, \tag{2.4}$$

where $N(=1)$ is the fixed population. We rewrite the above equations as

$$n_i(t)k_i(t)+n_s(t)k_s(t)=k(t), \quad n_i(t)+n_s(t)=1, \tag{2.5}$$

where

$$k(t)\equiv\frac{K(t)}{N}, \qquad n_j(t)\equiv\frac{N_j(t)}{N}, \quad j=i,s. \tag{2.6}$$

A representative household's current income, $y(t)$, from the interest payment, $r(t)k(t)$, and the wage payment, $w(t)$, in period t are given as follows:

$$y(t)=r(t)k(t)+w(t). \tag{2.7}$$

The sum of money that consumers are using for consuming, saving, or transferring are not necessarily equal to the current income because consumers can sell wealth to pay, for instance, current consumption if the temporary income is not sufficient for purchasing goods and services. Retired people may live not only on the interest payment but also have to spend some of their wealth. The total value of wealth that a representative household can sell to purchase goods and to save is equal to $k(t)$. Here, we do not allow borrowing for current consumption. We assume that selling and buying wealth can be conducted instantaneously without any transaction cost. This is evidently a strict consumption as it may take time to draw savings from bank or to sell one's properties. The per capita disposable income of the household is defined as the sum of the current income and the wealth available for purchasing consumption goods and saving:

$$\hat{y}(t)=y(t)+k(t)=\left(1+r(t)\right)k(t)+w(t). \tag{2.8}$$

The disposable income is used for saving and consumption. In each period, a consumer would distribute the total available budget between savings, $s(t)$, and consumption of goods, $c(t)$. The budget constraint is given by

$$p(t)c(t)+s(t)=\hat{y}(t). \tag{2.9}$$

Equation (2.9) means that consumption and savings exhaust the consumers' disposable income.

In our model, in each period, consumers have two variables, the level of consumption and saving, to decide. We assume that utility level, $U(t)$, that the consumers obtain is dependent on the consumption level of commodity, $c(t)$, and the savings, $s(t)$, as follows:

$$U(t) = c^{\xi}(t)s^{\lambda}(t), \quad \xi,\lambda > 0,\ \xi + \lambda = 1, \tag{2.10}$$

where ξ is called the propensity to consume, and λ the propensity to own wealth. This model was proposed by Zhang in the early 1990s. A comprehensive explanation of the model is referred to Zhang [15]. In particular, the economic growth theory based on Zhang's approach is systematically compared with traditional economic growth theories. This type of utility functions has also been applied to different economic problems formed with difference equations [14].

For consumers, wage rate, $w(t)$, and rate of interest, $r(t)$, are given in markets and wealth, $k(t)$, is predetermined before decision. Maximizing $U(t)$ in (2.10) subject to (2.9) yields

$$p(t)c(t) = \xi\hat{y}(t), \qquad s(t) = \lambda\hat{y}(t). \tag{2.11}$$

According to the definitions, the household's wealth in period $t+1$ is equal to the savings made in period t:

$$k(t+1) = s(t) = \lambda\hat{y}(t). \tag{2.12}$$

This equation describes the accumulation of the households' wealth. The output of the consumption good sector is consumed by the households. That is,

$$c(t)N = F_s(t). \tag{2.13}$$

As output of the capital good sector is equal to the depreciation of capital stock and the net savings, we have

$$S(t) - K(t) + \delta_k K(t) = F_i(t), \tag{2.14}$$

where $S(t) - K(t) + \delta_k K(t)$ is the sum of the net saving and depreciation. It is straightforward to show that this equation can be derived from the other equations in the system. We have thus built the dynamic model.

3. The dynamics, equilibrium, and stability

The dynamic system consists of many equations. Before analyzing dynamic properties of the system, we first show that the motion of the entire system is actually controlled by a single difference equation. First, from (2.3), we obtain

$$k_s(t) = \alpha k_i(t), \tag{3.1}$$

where $\alpha \equiv \beta_i\alpha_s/\beta_s\alpha_i$. The capital intensity of the consumption good sector is proportional to that of the capital good sector. By $k_s = \alpha k_i$ and $\beta_i f_i = \beta_s p f_s$, we solve

$$p(t) = \frac{\beta_i A_i}{\beta_s \alpha^{\alpha_s} A_s} k_i(t)^{\alpha_i - \alpha_s}. \tag{3.2}$$

The price of consumption goods is positively related to the technological level of the capital good sector but negatively related to that of the consumption good sector. The price is positively or negatively related to the capital intensity of the capital good sector, depending on the sign of $\alpha_i - \alpha_s$. If $\alpha_i = \alpha_s$, then the price is constant, $p = A_i/A_s$. In the remainder of this section, we require $\alpha_i \neq \alpha_s$. The analysis of case $\alpha_i = \alpha_s$ is straightforward. From (2.5) and $k_s = \alpha k_i(t)$, we solve the labor distribution as follows:

$$n_i(t) = \frac{\alpha k_i(t) - k(t)}{(\alpha - 1) k_i(t)}, \qquad n_s(t) = \frac{k(t) - k_i(t)}{(\alpha - 1) k_i(t)}. \tag{3.3}$$

The labor distribution between the two sectors is uniquely determined by $k(t)$ and $k_i(t)$. According to the definitions of $S(t)$, $K(t)$, $s(t)$, and $k(t)$, we have

$$S(t) - \delta K(t) = \big(s(t) - \delta k(t)\big) N, \tag{3.4}$$

where $\delta \equiv 1 - \delta_k$. From the above equation, (2.14), and $s = \lambda \hat{y}$, we obtain

$$\hat{y}(t) = \frac{n_i(t) f_i(t) + \delta k(t)}{\lambda}. \tag{3.5}$$

From $pc = \xi \hat{y}$, $c = n_s f_s$, and $p = f_i'/f_s'$, we get $\hat{y} = n_s f_s f_i'/\xi f_s'$. From this equation and (3.5), we have

$$n_s(t) f_i'(t) f^*(t) = n_i(t) f_i(t) + \delta k(t), \tag{3.6}$$

where $f^*(k_s(t)) \equiv \lambda k_s(t)/\xi\alpha_s$. Substituting $n_i(t) = 1 - n_s(t)$ and $n_s(t)$ in (3.3) into (3.6) yields

$$k(t) = \Phi\big(k_i(t)\big) k_i(t), \tag{3.7}$$

where

$$\Phi\big(k_i(t)\big) \equiv \frac{1}{A_0\big(1 + A k_i^{\beta_i}(t)\big)}, \qquad A_0 \equiv \frac{1 + \lambda_0}{\alpha + \lambda_0} > 0, \qquad A \equiv \frac{(1-\alpha)\delta/A_i}{1 + \lambda_0}, \quad \lambda_0 \equiv \frac{\beta_i \lambda}{\beta_s \xi}. \tag{3.8}$$

By (3.6) and (3.7) and according to the definitions of A and A_0, we solve

$$n_i(t) = \Phi\big(k_i(t)\big)\big(\alpha_1 - \alpha_2 k_i^{\beta_i}(t)\big), \tag{3.9}$$

where $\alpha_1 \equiv \lambda_0/(\alpha + \lambda_0)$ and $\alpha_2 \equiv \alpha\delta/A_i(\alpha + \lambda_0)$. Hence, for $n_i(t)$ to satisfy $1 > n_i(t) > 0$, it is sufficient to have

$$0 < k_i(t) < \left(\frac{\alpha_1}{\alpha_2}\right)^{1/\beta_i}. \tag{3.10}$$

It is straightforward to check that under inequalities (3.10), from (3.9), we have $k(t) > 0$ for all $k_i(t) > 0$. Insert (3.7) and (3.9) into (3.5),

$$\widehat{y}(t) = \frac{[(\alpha_1 - \alpha_2 k_i^{\beta_i}(t)) f_i(t) + \delta k_i(t)]\Phi(k_i(t))}{\lambda}. \tag{3.11}$$

Substituting (3.7) and (3.11) into (2.12),

$$\Phi(k_i(t+1))k_i(t+1) = [(\alpha_1 - \alpha_2 k_i^{\beta_i}(t)) f_i(t) + \delta k_i(t)]\Phi(k_i(t)). \tag{3.12}$$

It is straightforward to show

$$\frac{d[\Phi(k_i(t+1))k_i(t+1)]}{dk_i(t+1)} = \left[\frac{1 + \alpha_i A k_i^{\beta_i}(t+1)}{1 + A k_i^{\beta_i}(t+1)}\right]\Phi > 0. \tag{3.13}$$

According to the implicit function theorem, for any positive $k_i(t) > 0$, we can solve the difference equation (3.12) as follows:

$$k_i(t+1) = \overline{\Phi}(k_i(t)), \tag{3.14}$$

where $\overline{\Phi}(k_i)$ is a unique function of $k_i(t)$. The one-dimensional difference equation (3.14) contains a single variable $k_i(t)$. For a proper initial condition satisfying inequalities (3.10), the evolution of $k_i(t)$ is determined by the difference equation (3.14).

Lemma 3.1. *For a solution $k_i(t)$ of the difference equation (3.14), all the other variables are uniquely determined in any period by the following procedure: $k_s(t) = \alpha k_i(t) \to r(t)$ and $w(t)$ by (2.3) $\to k(t)$ by (3.7) $\to n_i(t)$ and $n_s(t)$ by (3.3) $\to f_j(k_j)$, $j = i, s \to p(t)$ by (3.2) $\to N_j(t) = n_j(t)N \to K_j(t) = k_j(t)N_j(t) \to F_j(t) = f_j(t)N_j(t) \to \widehat{y}(t)$ by (3.5) $\to c(t)$ and $s(t)$ by (2.11).*

The above lemma guarantees that it is sufficient to analyze dynamic properties of difference equation (3.14) for describing the whole system. By difference equation (3.12), an equilibrium point is determined by

$$k_i = (\alpha_1 - \alpha_2 k_i^{\beta_i}) f_i + \delta k_i. \tag{3.15}$$

Solve (3.15),

$$k_i^* = \left(\frac{A_i \alpha_1}{\delta_k + A_i \alpha_2}\right)^{1/\beta_i}. \tag{3.16}$$

To determine stability of the equilibrium point, we take derivatives of the two sides of (3.12) with respect to $k_i(t)$ and then evaluate the resulted equation at $k_i(t) = k_i^*$:

$$\begin{aligned} &\frac{d[\Phi(k_i(t+1))k_i(t+1)]}{dk_i(t+1)} \frac{dk_i(t+1)}{dk_i(t)}\bigg|_{k_i(t+1)=k_i^*} \\ &\quad = \frac{d\{[(\alpha_1 - \alpha_2 k_i^{\beta_i}(t)) f_i(t) + \delta k_i(t)]\Phi(k_i(t))\}}{dk_i(t)}\bigg|_{k_i(t)=k_i^*}. \end{aligned} \tag{3.17}$$

From (3.13) and (3.17), we calculate

$$\left.\frac{dk_i(t+1)}{dk_i(t)}\right|_{k_i(t+1)=k_i^*} = 1 - \frac{\beta_i\alpha_1 f_i}{k_i}\left(\frac{1+Ak_i^{\beta_i}}{1+\alpha_i Ak_i^{\beta_i}}\right)$$
$$= \left(\frac{\alpha_i\alpha+\lambda_0-\delta_k\beta_i\lambda_0}{\alpha+\lambda_0}\right) - \left(\frac{\beta_i^2\alpha_1 AA_i}{\delta_k+A_i\alpha_2+\alpha_1\alpha_i AA_i}\right)\left(\frac{\delta_k\lambda_0+\alpha}{\alpha+\lambda_0}\right). \tag{3.18}$$

From the terms between the two equalities, we see that the derivative is less than 1 under (3.10). As

$$1 > \frac{\alpha_i\alpha+\lambda_0-\delta_k\beta_i\lambda_0}{\alpha+\lambda_0}, \frac{\delta_k\lambda_0+\alpha}{\alpha+\lambda_0} > 0, \tag{3.19}$$

we see that the derivative is greater than -1 if the following positive term is less than 1, that is,

$$\frac{\beta_i^2\alpha_1 AA_i}{\delta_k+A_i\alpha_2+\alpha_1\alpha_i AA_i} \le 1. \tag{3.20}$$

The numerator is positive as shown below:

$$\delta_k+A_i\alpha_2+\alpha_1\alpha_i AA_i = \frac{\delta_k\lambda_0}{(\alpha+\lambda_0)} + \frac{\alpha+\alpha_i\lambda_0\delta+(\beta_i+\alpha_i\delta_i)\alpha\lambda_0}{(\alpha+\lambda_0)(1+\lambda_0)} > 0. \tag{3.21}$$

Hence, inequality (3.20) is guaranteed if $\beta_i^2\alpha_1 A \le \alpha_2+\alpha_1\alpha_i A$, that is,

$$(\beta_i^2-\alpha_i)\lambda_0\frac{(1-\alpha)}{1+\lambda_0} \le \alpha, \tag{3.22}$$

where we use the definitions of α_1, α_2, and A.

THEOREM 3.2. *The dynamic system has a unique equilibrium point. The equilibrium values of all the other variables are given by the procedure in Lemma 3.1. Moreover, if (3.22) holds, the unique equilibrium point is stable.*

As it is difficult to explicitly interpret the stability properties, we illustrate the analytical results by simulation.

4. Equilibrium and motion by simulation

This section demonstrates properties of the dynamic system by simulation. First, we specify the parameters as follows:

$$N=1, \qquad A_i=1.3, \qquad A_s=1.2, \qquad \alpha_i=\frac{1}{3},$$
$$\alpha_s=0.30, \qquad \lambda=0.85, \qquad \delta_k=0.05. \tag{4.1}$$

The labor force is normalized to unit. As the dynamic system is characterized of constant returns to scale, the normalization will not affect our analytical results. Some empirical studies on the US economy demonstrate that the value of the parameter, α, in the

Cobb-Douglas production is approximately equal to 0.3 (e.g., Abel and Bernanke [16]). As shown below, the specification, $\alpha_i = 1/3$, makes it possible to explicitly express $k_i(t+1)$ as a unique function of $k_i(t)$. This is important for simulating the motion of the system. The propensity to save is 0.85, which implies that the household consumes 15 per cent of the available income.

Following Lemma 3.1, we solve the equilibrium values of the variables as follows:

$$\begin{gathered} r = 0.033, \qquad w = 2.274, \qquad p = 2.777, \qquad k_i = 18.065, \\ k_s = 15.484, \qquad f_i = 3.411, \qquad f_s = 2.730, \\ k = 15.844, \qquad n_i = 0.139, \qquad \hat{y} = 18.640, \qquad c = 1.007, \qquad s = 15.844. \end{gathered} \tag{4.2}$$

The per-worker output level of the capital good sector is higher than the consumption good sector. The capital intensity of the capital good sector is higher than that of the consumption good sector. Most of the labor force is employed by the consumption good sector. We introduce the national product as

$$F = n_i f_i + p n_s f_s. \tag{4.3}$$

The total product is equal to 7.001. The shares of the capital good sector and consumption good sector are, respectively, 6.2 per cent and 93.8 per cent. The consumption good sector plays the most important role in the economic system. To determine stability, we calculate

$$\left.\frac{dk_i(t+1)}{dk_i(t)}\right|_{k_i^*} = 0.883. \tag{4.4}$$

The unique equilibrium point is stable.

Although we have found out the equilibrium point and stability conditions, it is important to study motion of the system when it deviates from its equilibrium point. Nevertheless, the motion is controlled by (3.12) and the system is expressed in implicit form. To simulate the motion of the system, we need to find out the explicit expression of (3.12) in $k_i(t+1)$. With $\alpha_i = 1/3$, (3.12) can be rewritten as

$$k_i(t+1) - A\overline{f}(k_i(t))k_i^{2/3}(t+1) - \overline{f}(k_i(t)) = 0, \tag{4.5}$$

in which $A > 0$ under (4.1) and

$$\overline{f}(k_i(t)) \equiv \lfloor (\alpha_1 - \alpha_2 k_i^{\beta_i}(t)) f_i(t) + \delta k_i(t) \rfloor A_0 \Phi(k_i(t)). \tag{4.6}$$

From (3.11), we see that it is necessary to require $\overline{f}(k_i(t)) > 0$. Introducing $x = k_i^{1/3}(t+1)$, (4.5) becomes

$$x^3 - A\overline{f}(k_i(t))x^2 - \overline{f}(k_i(t)) = 0. \tag{4.7}$$

The cubic equation has a unique positive (real) solution in x. We solve (4.5) as follows:

$$k_i(t+1) = \left(\frac{A\overline{f}(k_i(t))}{3} + \frac{\sqrt[3]{2}A^2\overline{f}^2(k_i(t))}{3\widetilde{F}(k_i(t))} + \frac{\widetilde{F}(k_i(t))}{3\sqrt[3]{2}}\right)^3, \tag{4.8}$$

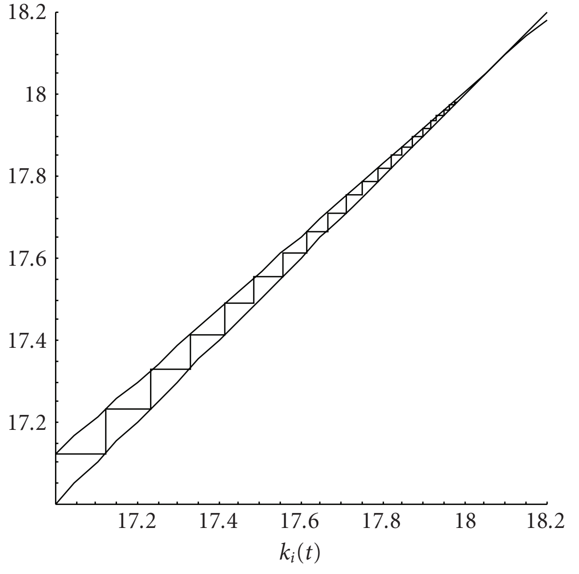


Figure 4.1. The convergence to the equilibrium point.

where

$$\tilde{F}(k_i(t)) \equiv \left[2A^3\bar{f}^3(k_i(t)) + 27\bar{f}(k_i(t)) + \sqrt{27\bar{f}(k_i(t))\left(4A^3\bar{f}^3(k_i(t)) + 27\bar{f}(k_i(t))\right)}\right]^{1/3}. \tag{4.9}$$

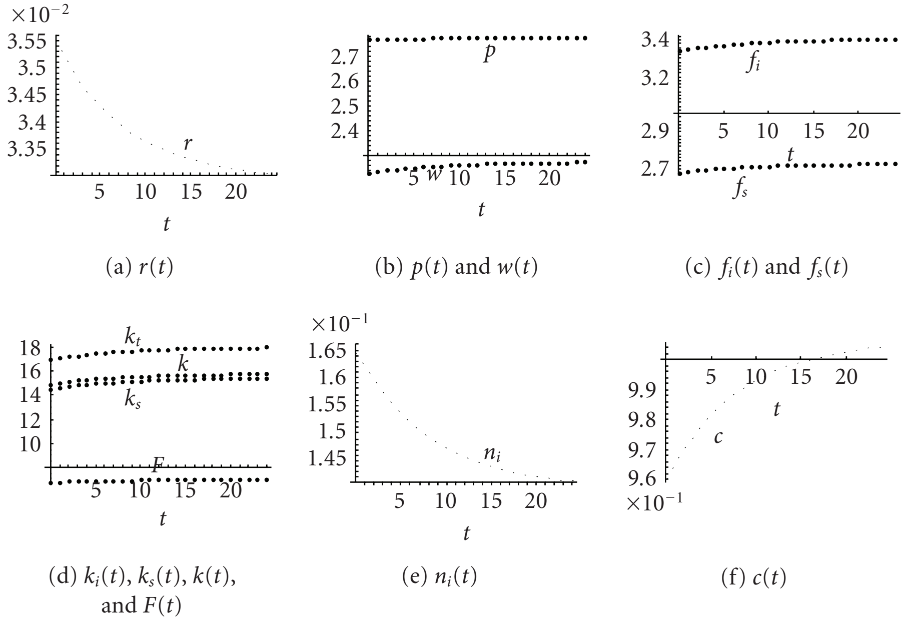
Equation (4.8) describes the motion of the variable, $k_i(t)$.

With (4.8) and Lemma 3.1, it is straightforward to describe the motion of the dynamic system. We start with $k_i(0) = 17$. The convergence from the initial state to the long-run equilibrium point is illustrated in Figure 4.1.

The convergence of all the variables are illustrated in Figure 4.2. The rate of interest falls over time as the initial level of the capital stocks is lower than its equilibrium value. The price of consumption goods rises slightly and the wage rate rises over time. The per-worker output levels of the two sectors are increased. The national output, total capital stocks, and capital intensities of the two sector rise. The share of the capital good sector in the national labor force falls over time. The per-capita consumption level rises.

5. Comparative dynamic analysis

It is important to ask questions such as how a change in technology or preference affects the national economy. This section examines impact of changes in some parameters on the national economy and economic structures. As we have explicitly provided the procedure to simulate the motion, it is straightforward to make comparative dynamic analysis. First, we examine the case that all the parameters, except the capital good sector productivity, A_i, are the same as in (4.1). We increase the productivity level, A_i, from 1.3 to 1.5.



(a) $r(t)$ (b) $p(t)$ and $w(t)$ (c) $f_i(t)$ and $f_s(t)$

(d) $k_i(t)$, $k_s(t)$, $k(t)$, and $F(t)$ (e) $n_i(t)$ (f) $c(t)$

Figure 4.2. The motion of the variables.

The simulation results are summarized in Figure 5.1, in which a variable, $\bar{\Delta}x(t)$, stands for the change rate of the value of the variable, $x(t)$, in period t in percentage due to the change in the parameter value from A_{i0} (= 1.3 in this case) to A_i (= 1.5). That is,

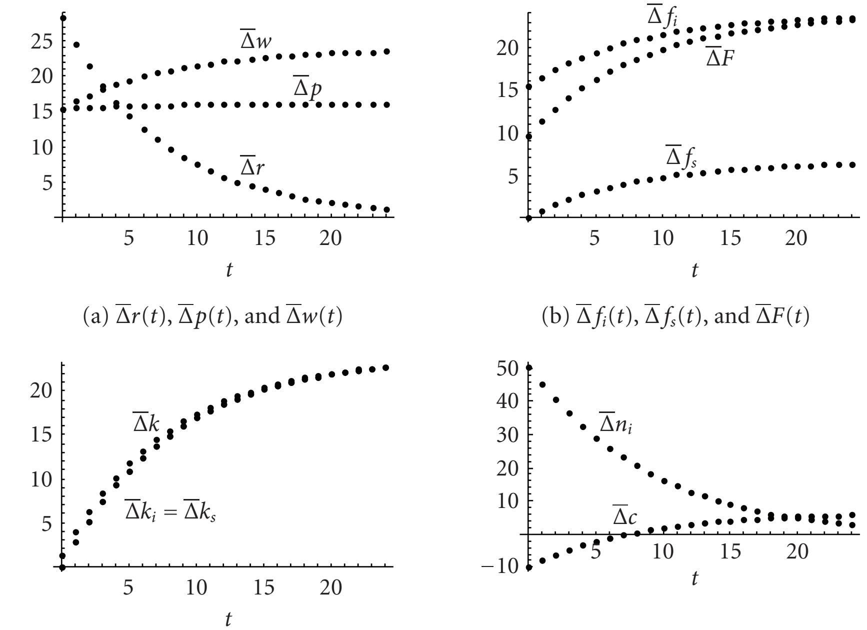
$$\bar{\Delta}x(t) \equiv \frac{x(t, A_i) - x(t, A_{i0})}{x(t, A_{i0})} \times 100. \tag{5.1}$$

We will use the symbol $\bar{\Delta}$ with the same meaning when we analyze other parameters.

As shown in Figure 5.1, as the productivity is improved, the rate of interest falls and the price of consumption goods slightly rises. The wage rate rises over time. The per-worker output levels of the two sectors, the national output, the national wealth, the capital intensities of the two sectors, and per-capita consumption level are all increased. The share of the labor force of the capital good sector falls over time.

Figure 5.2 summarizes the effects of a fall in the propensity to save, from 0.85 to 0.83. The rate of interest rises and the price of consumption goods and the wage rate are reduced. The per-worker output levels of the two sectors, the national output, the national wealth, the capital intensities of the two sectors, and per-capita consumption level are all reduced. The share of the labor force of the capital good sector rises over time.

It can be seen that it is also important to examine what happen to the system when the economy is experiencing preference and technological changes at the same time. This is straightforward for our model as we provided a procedure to determine all the variables in any period with any parameter values.



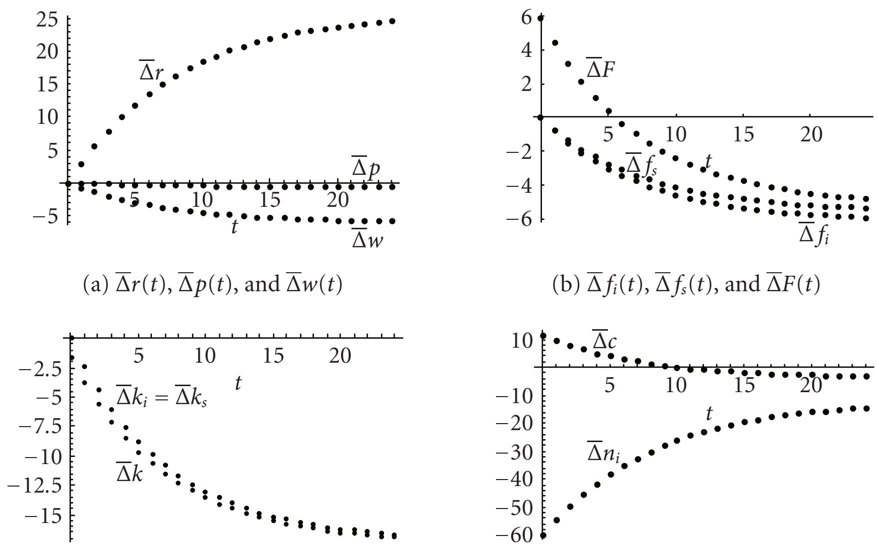
(a) $\overline{\Delta}r(t)$, $\overline{\Delta}p(t)$, and $\overline{\Delta}w(t)$

(b) $\overline{\Delta}f_i(t)$, $\overline{\Delta}f_s(t)$, and $\overline{\Delta}F(t)$

(c) $\overline{\Delta}k_i(t)$, $\overline{\Delta}k_s(t)$, and $\overline{\Delta}k(t)$

(d) $\overline{\Delta}n_i(t)$ and $\overline{\Delta}c(t)$

Figure 5.1. An improvement in the capital good sector's productivity.



(a) $\overline{\Delta}r(t)$, $\overline{\Delta}p(t)$, and $\overline{\Delta}w(t)$

(b) $\overline{\Delta}f_i(t)$, $\overline{\Delta}f_s(t)$, and $\overline{\Delta}F(t)$

(c) $\overline{\Delta}k_i(t)$, $\overline{\Delta}k_s(t)$, and $\overline{\Delta}k(t)$

(d) $\overline{\Delta}n_i(t)$ and $\overline{\Delta}c(t)$

Figure 5.2. A fall in the propensity to save.

6. Conclusions

This paper reexamined the Uzawa two-sector growth model with an alternative utility function. Different from the two-sector growth models with the Ramsey approach in the literature, we used a utility function, which determines saving and consumption with utility optimization without leading to a higher-dimensional dynamic system like the traditional approach. The dynamics is controlled by a one-dimensional difference equation. Although we can express explicitly the dynamics in one-dimensional difference equation, it is difficult to explicitly interpret analytical results because the expressions are tedious. For illustration, we simulate the model.

We may extend the model in different ways. It is important to study the economy when both the production and utility functions take on general forms. As mentioned in the introduction, Uzawa's two-sector model has been generalized and extended in many directions. It is not difficult to generalize our model along these lines. Our model may exhibit nonlinear behavior if possible externalities are considered as in the approach taken by, for instance, Drugeon and Venditti [13] and Boldrin et al.[17], within the two-sector model framework.

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