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Research Article

The Volatility of the Index of Shanghai Stock Market Research Based on ARCH and Its Extended Forms

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The proposed ARCH and its extension model have brought a powerful tool for the study of stock market volatility as well as verify that a "high risk brings high-yield" and the "leverage effect" of stock market. This paper gives modeling analysis by using the ARCH group models; in the last ten years Shanghai's index returns, concluded that there are significant "high-yield associated with high-risk" phenomenon and the "leverage effect" in the domestic securities market. The previous studies in fitting return series of ARMA models, mostly with low accuracy have a very subjective "observation autocorrelation and partial autocorrelation function method," and even directly use "random walk" model. That will inevitably have some impact on the accuracy of the model. While this paper adopts the Pandit-Wu formulaic modeling method, the ARMA model is built on a strong theoretical foundation.

1. ARCH Model and Its Extended Forms

Autoregressive conditional Heteroscedasticity Mode1 was raised by Engle in 1982 [1]. The model sets up yield obedience to the conditional expectation of the error term to be zero. The conditional variance obedience to the numbers of previous period yields square error function of the conditions of normal distribution. Its nature coincides with characteristics such as volatility clustering and heteroscedasticity of financial market. Bollerslev (1986) extended ARCH models, introduced an infinite period of entry error term in the variance explained, and got the generalized ARCH model (GARCH) [2]; Engle, Lilien, and Robbins explained the expected return in the introduction of ARCH models residual variance items in 1987 [3] and obtained ARCH-M model. Black (1976) [4] discovered that the volatility of the leverage effect first, that is, the unanticipated price decreases (bad news) and the unexpected price increases (good news) on the impact of the extent of fluctuations is nonsymmetrical. In response to this phenomenon, Glosten et al. (1993) [5], Zakoian (1990) [6], and Nelson (1991) [7] revised the traditional ARCH model proposed two nonsymmetrical models: TARCH and the EGARCH [8].

ARCH

The research process of ARCH model considers of σ_t^2 to be the residual variance ε_t of the regression equation that meets $\sigma_t^2 = \omega + \alpha_1\varepsilon_{t-1}^2$. It consists of two parts: a constant and the former moment of residuals squared. Usually ε_{t-1}^2 is called ARCH item. In general, the variance can be dependent on any number of lagged error term, that is, $\sigma_t^2 = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \cdots + \alpha_p\varepsilon_{t-p}^2$, recorded as ARCH ($p$) model.

GARCH

The most commonly used GARCH model is GARCH (1,1) model that meets $\sigma_t^2 = \omega + \alpha_1\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2$. Given conditional variance equation has three components: the constant term, using the mean equation, the lagged squared residuals to measure the volatility obtained from the previous information ε_{t-1}^2 (ARCH items), and the last forecast variance σ_{t-1}^2 (GARCH items).

GARCH-M

Using conditional variance denotes the expected risk model which is known as the ARCH mean regression model (ARCH-M). The expression $Y_t = X_t\gamma + \rho\sigma_t^2 + \varepsilon_t$, $\sigma_t^2 = \omega + \alpha_1\varepsilon_{t-1}^2 + \cdots + \alpha_p\varepsilon_{t-p}^2$ where the parameter ρ is measured in terms of variance of σ_t^2 can be observed in the risk of fluctuations in the expected degree of influence on Y_t.

TARCH

The conditional variance in this model is set as follows: $\sigma_t^2 = \omega + \alpha_1\varepsilon_{t-1}^2 + \gamma_1\varepsilon_{t-1}^2 I_{t-1}^- + \beta_1\sigma_{t-1}^2$, where I_{t-1}^- is a dummy variable, when $\varepsilon_{t-1} < 0$, $I_{t-1}^- = 1$; otherwise, $I_{t-1}^- = 0$. As long as $\gamma_1 \neq 0$, there exists an asymmetric effect.

EGARCH

The conditional variance equation in the EGARCH model is set as follows: $\ln(\sigma_t^2) = \omega + \beta\ln(\sigma_{t-1}^2) + \alpha|(\varepsilon_{t-1}/\sigma_{t-1}) - \sqrt{2/\pi}| + \gamma(\varepsilon_{t-1}/\sigma_{t-1})$. The left is the logarithm of conditional variance which means that the lever effect is exponential, rather than secondary; so the predictive value of conditional variance certain is nonnegative. The existence of leverage effect is tested through the hypothesis $\gamma < 0$. As long as $\gamma \neq 0$, the effect of shocks exist is nonsymmetries.

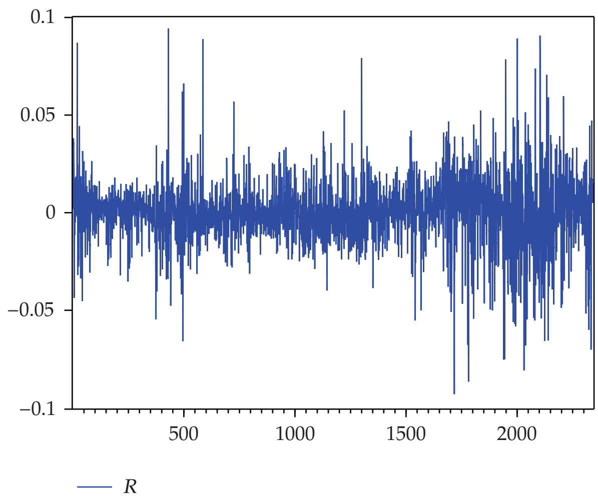

Figure 1

2. The Empirical Analysis

2.1. Data Acquisition and Finishing

The paper used data from the Shanghai Securities each day at Shanghai Composite Index closing. (The Shanghai Composite Index, since July 15, 1991, with a sample of all stocks listed on the Shanghai Stock Exchange stocks, in general, reflects the stock price movements of the Shanghai Stock Exchange. It has gradually become a "barometer" of China's economy.) Data time spans from January 4, 2000 to September 11, 2009, a total of 2341 observations. At the same time, the definition of day yield on closing price of the first-order difference of the natural logarithm is expressed as $r_i = \ln p_i - \ln p_{i-1}$. where r_i denotes the day's rate of return, and p_i denotes the day's closing price.

2.2. The Test Data

2.2.1. Normality Tests

Figure 1 shows the daily rate of return of the Shanghai Index, the Fluctuations Show time-varying volatility, and sudden and clustering characteristics. Figure 2 indicated its frequency chart and statistics characteristics. We can see that the partial degrees −0.073892, sample distribution is left skewed peak degrees are 6.982480, significantly higher than peak 3 of the normal distribution, and therefore has a clear "pike apex and thick tail" phenomenon, and JB value is 1548.493, indicating that the distribution of return series shows the nonnormality [9].

2.2.2. Smooth Test

Do the ADF test to return series $\{r_i\}$, assuming that yields fluctuate up-down on 0; so to calculate the ADF statistic on the assumption that the regression equation does not contain

Table 1

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		−47.69910	0.0001
Test critical values	1% level	−2.565951	
	5% level	−1.940959	
	10% level	−1.616608	

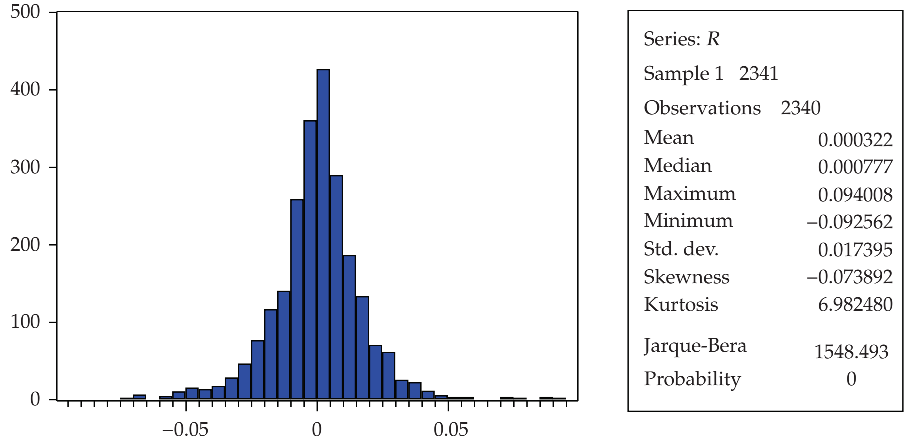

Figure 2

the constant term and time trend items, calculated by the ADF statistic which is less than 1% significance level under the critical value, it rejected the hypothesis of existing the unit root, indicating that the sequence is stationary series [10]; see Table 1.

2.3. *ARMA Model Fitting of Return Series*

Based on the fact that $\{r_i\}$ is a stationary series, we use Pandit-Wu model to fit the ARMA $(2n, 2n-1)$ model: Pandit-Wu modeling approach is based on Box-Jenkins method; proven and further development in 1977 proposed a new method of system modeling; this approach is not a function identification counted as sample (partial) autocorrelation function. It is based on the following understanding: any sequence can always use an ARMA $(n, n-1)$ model to represent, while the AR (n), MA (m), and ARMA (m, n) are a special case. The modeling idea can be summarized as follows: increasing the order of the model gradually, fitting the higher-order ARMA $(n, n-1)$ model, and a further increasing the order of the model and the remaining sum of squares that no longer significantly decrease.

Main steps are as follows:

(1) on the model of zero-mean,

(2) from $n = 1$, start and gradually increase the model order, fitting ARMA $(2n, 2n-1)$ model, until the F test showed that the model order to increase the number of remaining squares is no longer significantly reduced.

(3) model of the adaptive test,

(4) find the optimal model [11].

Table 2

F-statistic	0.042488	Probability	0.958402
Obs* R-squared	0.085434	Probability	0.958182

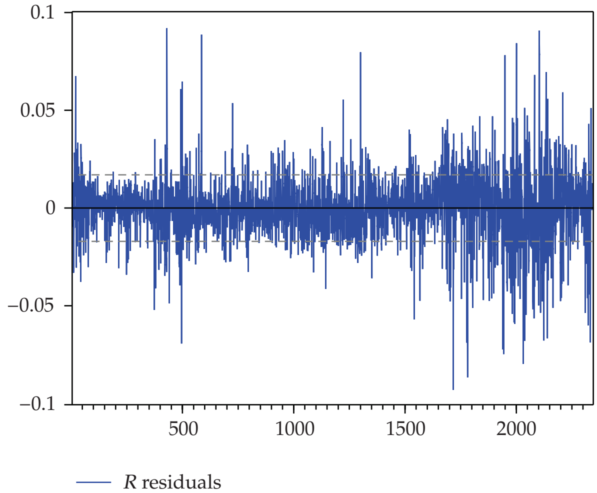

Figure 3

Through the fitting, ARMA (8,7) model and ARMA (6,5) model have no significant differences:

$$F = \frac{(0.689486 - 0.689920)/4}{0.689920/(2430 - 8 - (8+7))} = -0.3785 < F_{0.01}(4, \infty) = 3.32. \tag{2.1}$$

So choose ARMA (6,5) model.

Again ARMA, (6,5) $p = 2$, the residual autocorrelation test, see Table 2.

Clearly, there is no significant residual autocorrelation, another model of the coefficient is significant. So this model is appropriate.

The use of (6,5) model regression to $\{r_i\}$ is

$$\begin{aligned} r_t = {} & 0.000309 - 0.264845 r_{t-1} + 0.047856 r_{t-2} + 0.243321 r_{t-3} - 0.758743 r_{t-4} \\ & - 0.379365 r_{t-5} - 0.028096 r_{t-6} + \varepsilon_t + 0.274071 \varepsilon_{t-1} - 0.061145 \varepsilon_{t-2} - 0.229173 \varepsilon_{t-3} \\ & + 0.810033 \varepsilon_{t-4} + 0.410553 \varepsilon_{t-5}. \end{aligned} \tag{2.2}$$

2.4. The ARCH Group Model-Building of Return Series

Analysis residuals graphs of the regression result Figure 3.

Note the phenomenon of fluctuations in these clusters: fluctuations in some of the longer period of time is very small and in some other longer period of time is very large, indicating the error term may have a condition of heteroscedasticity. Therefore, its ARCH LM test of conditional heteroscedasticity has been got in the lag order of $p = 3$ Table 3.

Table 3

F-statistic	35.22275	Probability	0.000000
Obs* R-squared	101.2521	Probability	0.000000

Table 4

	Variance equation			
C	3.79E–06	7.15E–07	5.291596	0.0000
RESID $(-1)^2$	0.110112	0.009052	12.16377	0.0000
GARCH (–1)	0.884263	0.008639	102.3603	0.0000
R-squared	0.014862	Mean dependent var		0.000314
Adjusted R-squared	0.008915	S.D. dependent var		0.017347
S.E. of regression	0.017269	Akaike info criterion		–5.526109
Sum squared resid	0.691589	Schwarz criterion		–5.489121
Log likelihood	6463.969	F-statistic		2.498929
Durbin-Watson stat	2.015616	Prob(F-statistic)		0.001566

P-value is 0, so reject the original hypothesis, indicating the residual sequence existing ARCH effect.

2.4.1. GARCH (1,1) Model

As can be seen in Table 4, the variance equation in the ARCH and GARCH is significant, while AIC value and the SC values are smaller, indicating that GARCH (1,1) model can better fit the data. Then make the ARCH LM test to this equation heteroscedasticity. That can get the results of the lagging order of the residual sequence when $p = 3$ see Table 5.

At this time the accompanied probability is 0.82, accepting the null hypothesis that there is no ARCH effect in the series that shows the use of GARCH (1,1) model eliminating the conditional heteroscedasticity of residual sequence.

In addition, the variance equation in the ARCH and GARCH coefficient entries equal to 0.994375 is less than 1, to meet the parameters of constraints; as the coefficient is very close to 1, indicating that the impact on conditional variance is persistent. It means that all future projections have an important role.

2.4.2. GARCH-M Model

In Table 6, the return rate equation including the terms of the standard deviation σ_t is in order to integrate the risk measurement in the process of revenue generation, which is the basis of many capital pricing theories—the meaning of “Mean-variance assumptions”. In this assumption, the coefficient ρ of conditional standard deviation should be positive. The result is exactly the case, the conditional standard deviation which has larger expected value associated with high rates of return. Estimated coefficient of the equation is less than 1, to meet stable condition. The conditional standard deviation coefficient in the equation is 0.083511, indicating that market is expected to increase the risk of a percentage point; that will lead to a corresponding increase in yield of 0.083511 percent.

Table 5

F-statistic	0.311432	Probability	0.817141
Obs* R-squared	0.935528	Probability	0.816847

Table 6

	Coefficient	Std. Error	z-Statistic	Prob.
@SQRT(GARCH)	0.083511	0.052562	1.588815	0.1121
C	−0.000625	0.000701	−0.891278	0.3728
AR(1)	−0.199086	0.351060	−0.567099	0.5706
AR(2)	0.124659	0.050907	2.448740	0.0143
AR(3)	0.174735	0.062016	2.817584	0.0048
AR(4)	−0.785433	0.052558	−14.94409	0.0000
AR(5)	−0.199962	0.293003	−0.682457	0.4949
AR(6)	−0.046151	0.023580	−1.957159	0.0503
MA(1)	0.216611	0.349176	0.620347	0.5350
MA(2)	−0.146851	0.043255	−3.394974	0.0007
MA(3)	−0.159767	0.056709	−2.817310	0.0048
MA(4)	0.815284	0.052174	15.62612	0.0000
MA(5)	0.231638	0.299560	0.773261	0.4394
		Variance equation		
C	3.97E–06	7.56E–07	5.251923	0.0000
RESID $(-1)^2$	0.114202	0.009615	11.87704	0.0000
GARCH (−1)	0.879996	0.009193	95.72147	0.0000

2.4.3. TARCH and EARCH Model

In the TARCH model (see Table 7), the coefficient of leverage effect $\gamma_1 = 0.055381$, indicating the stock price, has "leverage" effect: the same amount of bad news generate greater volatility than good news. When appears the "good news", $\varepsilon_{t-1} > 0$, then $I_{t-1}^{-} = 0$, so the impact will only bring about a stock price index of 0.076231 times, while a "bad news", $\varepsilon_{t-1} < 0$, $I_{t-1}^{-} = 1$, then the "bad news" will bring $0.055381 + 0.076231 = 0.131612$ times impact. The bad news generates greater volatility than the same amount of good news. The results also can be confirmed in EARCH models. In the EARCH model (see Table 8), the estimated value of α is 0.218522; the estimated value of nonsymmetric key γ is −0.040285. When $\varepsilon_{t-1} > 0$, the information on the logarithm of conditional variance will bring $0.218522 + (-0.040285) = 0.178237$ times impact; when $\varepsilon_{t-1} < 0$, it will bring $0.218522 + (-0.040285) \times (-1) = 0.258807$ times impact to logarithm of conditional variance.

3. Conclusion

3.1. Model of Comparative Analysis

From the test results, rates of return series do have a heteroscedastic phenomenon. In the GARCH (1,1) model, the ARCH item and GARCH item of variance equation are significant, while the AIC value and the SC value are smaller, indicating it can fit data better. GARCH-M

Table 7: TARCH.

Variance equation				
C	3.74E–06	7.01E–07	5.332952	0.0000
RESID $(-1)^2$	0.076231	0.010432	7.307341	0.0000
RESID $(-1)^2$ *(RESID $(-1) < 0$)	0.055381	0.012945	4.278287	0.0000
GARCH (−1)	0.889139	0.008770	101.3873	0.0000

Table 8: EARCH.

Variance equation				
$C(13)$	−0.348419	0.039287	−8.868469	0.0000
$C(14)$	0.218522	0.017791	12.28297	0.0000
$C(15)$	−0.040285	0.008560	−4.706245	0.0000
$C(16)$	0.977814	0.003808	256.8123	0.0000

model and TARCH, EARCH models measure market from the "high-risk brings high-yield" and "leverage effect" of the stock market. All of them have achieved good results, indicating that the use of ARCH group models to market research is appropriate [12].

3.2. *Empirical Results*

This paper uses time series analysis method on the Shanghai index; last decade, the daily rate of return was analyzed and found showing the left side and the distribution form of pike apex and the thick trail, not subject to normal, and there is a self-related phenomena, can be used (6,5) model fitting. When fitting ARCH group model, we found that its variance has a strong volatility clustering and continuity. Rates of return and the risk of changes in the same direction; high-risk for high returns; high-yield associated with high-risk, which indicate investors concern on marketing a higher degree. The fast transmission of information, with the risk of change, will have an impact on yields, reflecting investor a certain preference for the risk; the domestic securities market exists significant leverage effect and "bad news" roles were clearly stronger than "good news" effect showing that our investors are often more sensitive to the decline of stocks as a result of avoiding risk.

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