Hindawi Publishing Corporation
Discrete Dynamics in Nature and Society
Volume 2012, Article ID 181752, 21 pages
doi:10.1155/2012/181752

Research Article

Multi-State Dependent Impulsive Control for Holling I Predator-Prey Model

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Received 18 February 2012; Accepted 10 April 2012

Academic Editor: Recai Kilic

According to the different effects of biological and chemical control, we propose a model for Holling I functional response predator-prey system concerning pest control which adopts different control methods at different thresholds. By using differential equation geometry theory and the method of successor functions, we prove that the existence of order one periodic solution of such system and the attractiveness of the order one periodic solution by sequence convergence rules and qualitative analysis. Numerical simulations are carried out to illustrate the feasibility of our main results which show that our method used in this paper is more efficient and easier than the existing ones for proving the existence of order one periodic solution.

1. Introduction

More and more scholars have paid close attention, and studied impulsive differential equation since the 1980s. Impulsive differential equation theory, especially the one in a fixed time, has been deeply developed and widely applied in various fields through years of research [1–5]. In population dynamical system, Tang et al. [6] discussed the stage-structure system for single population with birth pulse and got the existence and stability of periodic solution; Liu et al. [7–9] studied the impulse control strategy of Lotka-Volterra system; he also set up and discussed the Holling type II predator-prey model with impulse control strategy. Ballinger and Liu [10] discussed the persistence of population model with impulse effect. Tang and Cheke [11] first proposed the “Volterra” model with state-dependence, and they applied this model to pest management and proved existence and stability of periodic solution of first and second orders. Then, Liu et al. [12] also proposed bait-dependent digestive model with state pulse, and the model had the existence of positive periodic solution and stability of orbit. Recently, Jiang and Liu et al. [12–14] have proposed pest management model with state pulse and phase structure and several predator-prey models with state pulse and had the existence of semitrivial periodic solution and positive periodic solution and stability of orbit.

In consideration of predator-prey capacity, Holling [15] proposes three different predations with functional response based on experiments; the average predator-prey system with Holling response is as follows:

$$\begin{aligned} x'(t) &= xg(x) - y\phi(x), \\ y'(t) &= -dy + ey\phi(x), \end{aligned} \tag{1.1}$$

where x represents the prey's density, while y is the predator's; $g(x)$ is the unit rate of prey density in lack of predators; $\phi(x)$ is the Holling functional response, among which Holling type I functional response is

$$\phi(x) = \begin{cases} cx, & x \leq x_0, \\ cx_0, & x > x_0, \end{cases} \tag{1.2}$$

where c is a constant; when the amount of prey is greater than certain threshold value x_0, predatory rate is a constant. Referring to [15] for details.

As the Lotka-Volterra predator-prey system with Holling functional response is more practical, many authors have studied it [12, 14, 15]. The researches mostly focus on Lotka-Volterra predator-prey model with Holling type II or Holling type III functional response in contrast to the model with Holling type I. This paper sets up a state-dependent impulsive mathematical model concerning pest control which adopts different control methods at different thresholds and adopts new mathematic method to study existence and attractiveness of order one periodic solution of such system; thus the following pest-control model with Holling type-I functional response is set up:

$$\begin{aligned}
&\left.\begin{aligned} &\left.\begin{aligned} x'(t) &= rx(t) - cx(t)y(t), \\ y'(t) &= -dy(t) + ecx(t)y(t), \end{aligned}\right\} x \leq x_0, \\ &\left.\begin{aligned} x'(t) &= rx(t) - cx_0y(t), \\ y'(t) &= -dy(t) + ecx_0y(t), \end{aligned}\right\} x > x_0, \end{aligned}\right\} x \neq h_1, h_2 \text{ or } x = h_1,\ y > y^*, \\
&\begin{aligned} \Delta x(t) &= 0, \\ \Delta y(t) &= \delta, \end{aligned} \quad x = h_1,\ y \leq y^*, \\
&\begin{aligned} \Delta x(t) &= -\alpha x(t), \\ \Delta y(t) &= -\beta y(t) + q, \end{aligned} \quad x = h_2,
\end{aligned} \tag{1.3}$$

where $r, c, d, e,\ h_1$, and h_2 are all positive constants, $x(t)$ and $y(t)$ represent the densities of prey (pest) and predator (natural enemy), respectively; r is the intrinsic growth rate of the prey; d denotes the death rate of the predator; $(\alpha, \beta) \in (0, 1)$ represent the proportion of killed prey and predator by spraying pesticides respectively, $\delta > 0$ is the number of natural enemies released at this time t_{h_1}, when the amount of the prey reaches the threshold h_1 at time t_{h_1}, control measures are taken (releasing natural enemies) and the amount of predator abruptly turns to $y(t_{h_1}) + \delta$. When the amount of prey reaches the threshold h_2 at time t_{h_2}, control measures are taken and the amount of prey and predator abruptly turns to $(1 - \alpha)h_2$ and

$(1-\beta)y(t_{h_2})$, respectively. $\Delta x = x(t^+) - x(t)$, $\Delta y = y(t^+) - y(t)$, $x(t^+) = \lim_{w\to 0} x(t+w)$, $y(t^+) = \lim_{w\to 0} y(t+w)$. Referring to [12] for details.

2. Preliminaries

We first consider the model (1.3) without impulse effects:

$$\begin{aligned} x'(t) &= rx(t) - cx(t)y(t), \\ y'(t) &= -dy(t) + ecx(t)y(t), \end{aligned} \quad x \le x_0, \qquad \begin{aligned} x'(t) &= rx(t) - cx_0 y(t), \\ y'(t) &= -dy(t) + ecx_0 y(t). \end{aligned} \quad x > x_0. \tag{2.1}$$

We consider the following function:

$$V(x,y) = \int_{x^*}^{x} \frac{-d + e\phi(s)}{\phi(s)} ds + \int_{y^*}^{y} \frac{s - y^*}{s} ds, \tag{2.2}$$

we can easily know that $V(x,y)$ is positive definite in the first quartile and fits for all conditions of Lyapunov function.

We can get that

$$V'(x,y) = \frac{exy^*}{\phi(x)}(\phi(x) - \phi(x^*))\left(\frac{\phi(x^*)}{x^*} - \frac{\phi(x)}{x}\right). \tag{2.3}$$

It is easily proved that $V'(x,y) \equiv 0$ on condition that $x \le x_0$; so all solutions of model (1.3) form a set $\{(x,y)/V(x,y) \le V(x_0,y^*)\}$ are closed trajectory $V(x,y) = C$, where $0 < C < V(x_0,y^*)$.

Since $V'(x,y) > 0$ on condition that $x > x_0$; so the trajectory of system (2.1) passes through closed curve $V(x,y) = C$ when it is out of the curve $V(x,y) = V(x_0,y^*)$.

Therefore, we observe the straight line:

$$L(x,y) = y + x - n, \quad n > 0, \ x_0 < x \le h. \tag{2.4}$$

The derivative of $L(x,y)$ along (2.1) is that

$$\begin{aligned} L'(x,y)/_{L=0} &= x' + y' = -dy + ecx_0 y + rx - ecx_0 \\ &= -(dn - ecx_0 n + cx_0 n + cx_0 h - dx_0) - (ecx_0 - r - cx_0) \\ &\le dh - ecx_0^2 + rh + cx_0 h - (d - ecx_0 + cx_0)n. \end{aligned} \tag{2.5}$$

We have that $L'/_{L=0} < 0$ on condition that $n > ((dh - ecx_0^2 + rh + cx_0 h)/(d - ecx_0 + cx_0))$. Therefore, we can get the following Lemma.

Lemma 2.1. *The system* (2.1) *possesses:*

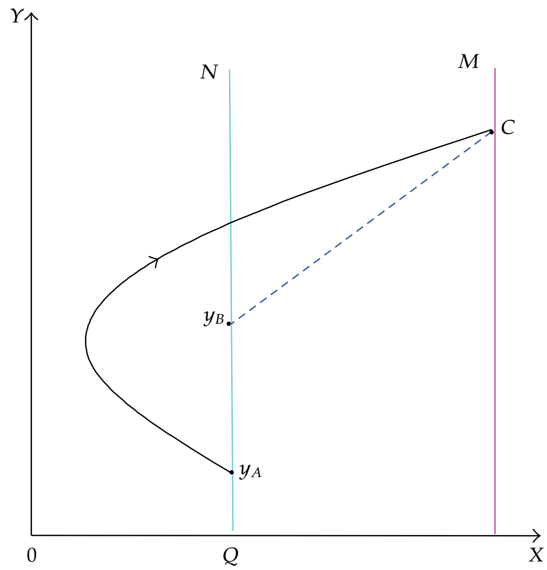

Figure 1

(I) *two steady states* $0(0,0)$—*saddle point, and* $R(d/ec, r/c) = R(x^*, y^*)$—*stable centre on the condition that* $x \le x_0$ *and that* $d \le ecx_0$*;*

(II) *the trajectory of system* (2.1) *goes across the straight line* $y+x-n=0$ *from the right to the left on condition that* $x_0 \le x \le h$ *and that* $n > ((dh - ecx_0^2 + rh + cx_0h)/(d - ecx_0 + cx_0))$ *and intersects with the straight line* $x = x_0$.

Definition 2.2. Suppose that the impulse set M and the phase set N are both lines, as shown in Figure 1. Define the coordinate in the phase set N as follows: denote the point of intersection Q between N and x-axis as O, then the coordinate of any point A in N is defined as the distance between A and Q and is denoted by y_A. Let C denote the point of intersection between the trajectory starting from A and the impulse set M, and let B denote the phase point of C after impulse with coordinate y_B. Then, we define B as the successor point of A, and then the successor function [16] of point A is that $f(A) = y_B - y_A$.

Lemma 2.3. *In system* (1.3)*, if there exist* $A \in N$*,* $B \in N$ *satisfying successor function* $f(A)f(B) < 0$*, then there must exist a point* P $(P \in N)$ *satisfying* $f(P) = 0$ *the function between the point of A and the point of B, thus there is an order one periodic solution in system* (1.3).

In this paper, we assume that the condition $d \le ecx_0$ holds. By the biological background of system (1.3), we only consider $D = \{(x, y) : x \ge 0, y \ge 0\}$.

This paper is organized as follows. In the next section, we present some basic definitions and important lemmas as preliminaries. In Section 3, we prove the existence for an order one periodic solution of system (1.3). The sufficient conditions for the attractiveness of order one periodic solutions of system (1.3) are obtained in Section 4. At last, we state conclusion, and the main results are carried out to illustrate the feasibility by numerical simulations.

3. Existence of Order One Periodic Solution

In this section, we shall investigate the existence of an order one periodic solution of system (1.3) by using the successor function defined in this paper. For this goal, we denote

$$M_1 = \left\{ \frac{(x, y)}{x} = h_1,\ 0 \le y \le \frac{r}{c} + \delta \right\},$$
$$M_2 = \{(x, y) \mid x = h_2,\ y \ge 0\},$$
$$N_1 = I(M_1) = \left\{ (x, y) \mid x = h_1,\ \frac{r}{c} < y \le \frac{r}{c} + \delta \right\},$$
$$N_2 = I(M_2) = \{(x, y) \mid x = (1 - \alpha)h_2,\ y \ge 0\}. \tag{3.1}$$

Isoclinic line is denoted respectively by lines:

$$L_1 = \left\{ (x, y) \mid y = \frac{r}{c},\ 0 \le x \le x_0 \right\},$$
$$L_2 = \left\{ (x, y) \mid x = \frac{d}{ec},\ 0 \le x \le x_0,\ y \ge 0 \right\},$$
$$L_3 = \left\{ (x, y) \mid y = \frac{r}{cx_0}x,\ x > x_0,\ y \ge \frac{r}{c} \right\}. \tag{3.2}$$

For the convenience, if $P \in \Omega - M$, $F(P)$ is defined as the first point of intersection of $C^+(P)$ and M, that is, there exists a $t_1 \in R_+$ such that $F(P) = \Pi(P, t_1) \in M$, and for $0 < t < t_1, \Pi(P, t) \notin M$; if $B \in N, R(B)$ is defined as the first point of intersection of $C^-(P)$ and N, that is, there exists a $t_2 \in R_+$ such that $R(B) = \Pi(B, -t_2) \in N$, and for $-t < t < 0$, $\Pi(B, t) \notin N$.

For any point P, denote y_P as its ordinate. If the point $P(h, y_P) \in M$, pulse occurs at the point P, the impulsive function transfers the point P into $P^+ \in N$. Without loss of generality, we assume the initial point of the trajectory lies in phase set N unless otherwise specified.

According to the practical significance, in this paper we assume that the set N always lies in the left side of stable centre R, that is, $h_1 < r/c$, $(1 - \alpha)h_2 < r/c$.

In the light of the different position of the set N_1 and the set N_2, we consider the following three cases.

Case 1 ($0 < h_1 < d/ec$). In this case, set M_1 and N_1 are both in the left side of stable center R (as shown in Figure 2). Take a point $B(h_1, r/c + \varepsilon) \in N_1$ above A, where $\varepsilon > 0$ is small enough, then there must exist a trajectory passing through B which intersects with the set M_1 at point $P_1(h_1, y_{p_1})$, we have $y_{p_1} < r/c$. Since $p_1 \in M_1$, pulse occurs at the point P_1, the impulsive function transfers the point P_1 into $P_1^+(h_1, y_{p_1} + \delta)$ and P_1^+ must lie above B; therefore, inequation $a/b + \varepsilon < y_{p_1} + \delta$ holds, thus the successor function of B is that $f(B) = y_{p_1} + \delta - (r/c + \varepsilon) > 0$.

On the other hand, the trajectory with the initial point P_1^+ intersects with M_1 at point $P_2(h_1, y_{p_2})$, in view of vector field and disjointness of any two trajectories, we know

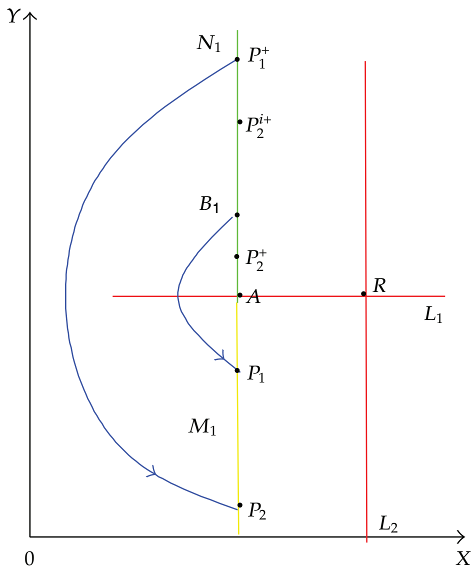

Figure 2

$y_{p_2} < y_{p_1} < r/c$. Supposing that the point P_2 is subject to impulsive effects to point $P_2^+(h_1, y_{p_2^+})$, where $y_{p_2^+} = y_{p_2} + \delta$, the position of P_2^+ has the following two cases.

Subcase 1.1 ($r/c < y_{p_2} + \delta < y_{p_1} + \delta$). In this case, the point P_2^+ lies above the point A and under P_1^+, we have $f(P_1^+) = y_{p_2} + \delta - (y_{p_1} + \delta) < 0$.

By Lemma 2.3, there exists an order one periodic solution of system (1.3), whose initial point is between B and P_1^+ in set N_1.

Subcase 1.2 ($r/c \geq y_{p_2} + \delta$ (as shown in Figure 2)). The point P_2^+ lies below the point A, that is, $P_2^+ \in M_1$, then pulse occurs at the point P_2^+, the impulsive function transfers the point P_2^+ into $P_2^{++}(h_1, y_{p_2} + 2\delta)$.

If $r/c < y_{p_2} + 2\delta < y_{p_1} + \delta$, like the analysis of Subcase 1.1, there exists an order one periodic solution of system (1.3).

If $r/c > y_{p_2} + 2\delta$, that is, $P_2^{++} \in M_1$; we repeat the above process until there exists $k \in Z_+$ such that P_2^{++} jumps to $P_2^{i+}((h_1, y_{p_2} + (k+2)\delta)$ after k times' impulsive effects which satisfies $r/c < y_{p_2} + (k+2)\delta < y_{p_1} + \delta$. Like the analysis of Subcase 1.1, there exists an order one periodic solution of system (1.3).

Now we can summarize the above results in the following theorem.

Theorem 3.1. *If $d < ec$, $0 < h_1 < d/ec$, then there exists an order one periodic solution of the system* (1.3).

Remark 3.2. It shows from the proved process of Theorem 3.1 that the number of natural enemies should be selected appropriately, which aims to reduce releasing impulsive times to save manpower and resources.

Case 2 ($h_2 < d/ec$). In this case, sets M_2 and N_2 are both in the left side of stable center R, in the light of the different position of the set N_2, we consider the following two cases.

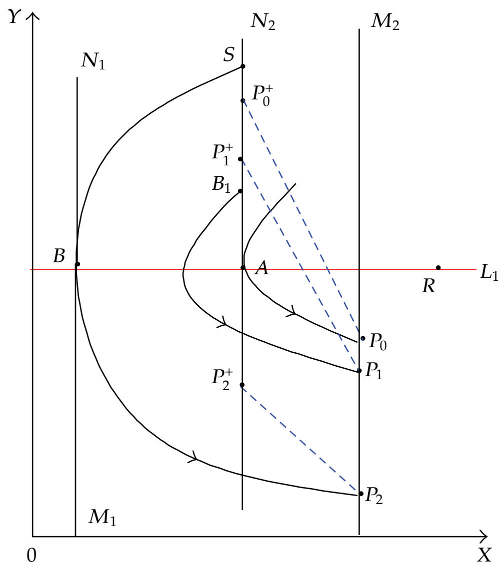

Figure 3

Subcase 2.1 ($0 < h_1 < (1-\alpha)h_2 < h_2$). In this case, the set N_2 is in the right side of M_1 (as shown in Figure 3). The trajectory passing through point A which tangents to N_2 at point A intersects with the set M_2 at point $P_0(h_2, y_{p_0})$. Since the point $P_0 \in M_2$, then impulse occurs at point P_0, supposing the point P_0 is subject to impulsive effects to point $P_0^+((1-\alpha)h_2, y_{P_0^+})$, where $y_{P_0^+} = (1-\beta)y_{P_0} + q$, the position of P_0^+ has the following three cases.

(1) $(1-\beta)y_{P_0} + q > r/c$: Take a point $B_1((1-\alpha)h_2, \varepsilon + r/c) \in N_2$ above A, where $\varepsilon > 0$ is small enough. Then there must exist a trajectory passing through the point B_1 which intersects with M_2 at point $P_1(h_2, y_{P_1})$. In view of continuous dependence of the solution on initial value and time, we know that $y_{P_1} < y_{P_0}$, and the point P_1 is close to P_0 enough, so we have the point P_1^+ close to P_0^+ enough and $y_{P_1^+} < y_{P_0^+}$, then we obtain $f(B_1) = y_{P_1^+} - y_{B_1} > 0$. On the other hand, the trajectory passing through point B which tangents to N_1 at point B intersect with N_2 at point S. Set $F(S) = P_2(h_2, y_{P_2}) \in M_2$. Denote the coordinates of impulsive point $P_2^+((1-\alpha)h_2, y_{P_2^+})$ corresponding to the point $P_2(h_2, y_{P_2})$. If $y_S \geq y_{P_0^+}$, then $y_{P_2^+} < y_{P_0^+}$. So we obtain $f(S) = y_{P_2^+} - y_S < 0$. There exists an order one periodic solution of system (1.3), whose initial point is between the point B_1 and the point S in set N_2. If $y_S < y_{P_0^+}$ and $y_{P_2^+} > y_S$, from the vector field of system (1.3), we know that the trajectory of system (1.3) with any initiating point on the N_2 will ultimately stay in Γ_1 after one impulsive effect (as shown in Figure 4). Therefore, there is no an order one periodic solution of system (1.3);

(2) $(1-\beta)y_{P_0} + q < r/c$ (as shown in Figure 5): In this case, the point P_0^+ lies below the point A, that is, $(1-\beta)y_{P_0} + q < r/c$, thus the successor function of the point A is $f(A) = (1-\beta)y_{P_0} + q - r/c < 0$. Take another point $B_1((1-\alpha)h_2, \varepsilon) \in N_2$, where $\varepsilon > 0$ is small enough. Then there must exist a trajectory passing through the point B_1 which intersects with M_2 at a point $P_1(h_2, y_{P_1}) \in M_2$. Supposing the point

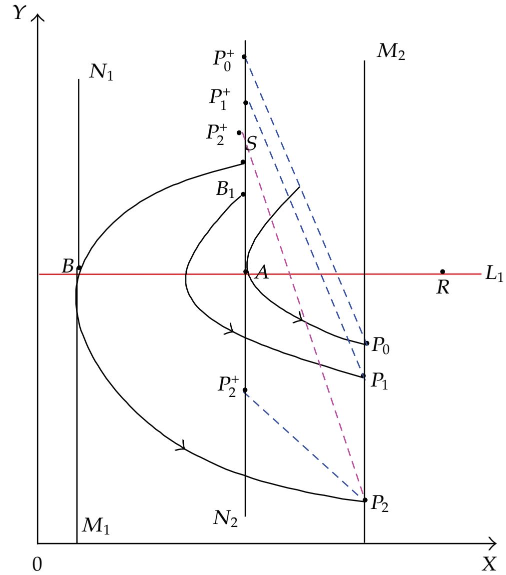

Figure 4

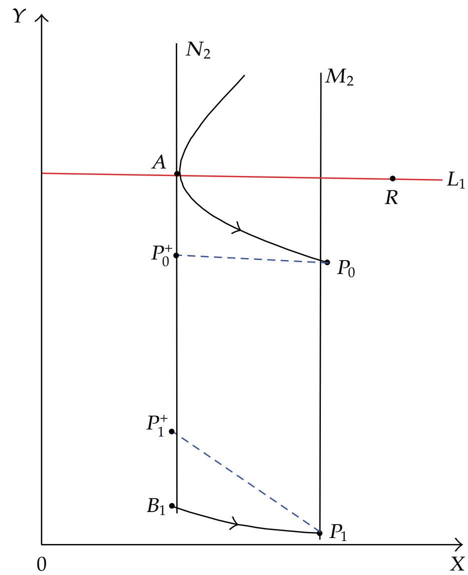

Figure 5

$P_1(h_2, y_{P_1})$ is subject to impulsive effects to point $P_1^+((1-\alpha)h_2, y_{P_1^+})$, then we have $y_{P_1^+} > \varepsilon$, so we have $f(C_1) = y_{P_1^+} - \varepsilon > 0$. From Lemma 2.3, there exists an order one periodic solution of system(1.3), whose initial point is between B_1 and A in set N_2;

(3) $(1-\beta)y_{P_0} + q = r/c$: P_0^+ coincides with A, and the successor function of A is that $f(A) = 0$, so there exists an order one periodic solution of system (1.3) which is just a part of the trajectory passing through the A.

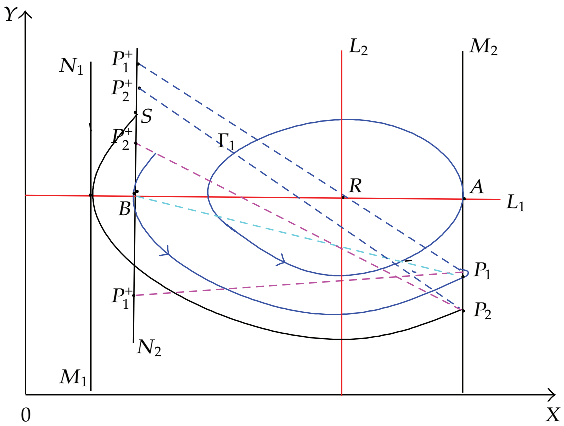

Figure 6

Now we can summarize the above results in the following theorem.

Theorem 3.3. *Assuming that* $d < ecx_0$, $0 < h_1 < (1-\alpha)h_2 < h_2 < d/ec$.

If $(1-\beta)y_{P_0} + q \le r/c$, *there exists an order one periodic solutions of the system* (1.3).

If $(1-\beta)y_{P_0} + q > r/c$ *and* $y_S \ge y_{P_0^+}$ *or* $y_S < y_{P_0^+}$ *and* $y_{P_2^+} \le y_S$, *there exists an order one periodic solutions of the system* (1.3).

Subcase 2.2 $(0 < (1-\alpha)h_2 < h_1 < h_2)$. In this case, the set N_2 is in the left side of N_1. Any trajectory from initial point $(x_0^+, y_0^+) \in N_2$ will intersect with M_1 at some point with time increasing. Like the analysis of Case 1, the trajectory from initial point $(x_0^+, y_0^+) \in N_2$ on the set N_2 will stay in the region $\Omega_1 = \{(x,y) \mid x \ge 0,\ y \ge 0,\ x \le h_1\}$. Similarly, any trajectory from initial point $(x_0^+, y_0^+) \in \Omega_0 = \{(x,y) \mid x \ge 0,\ y \ge 0,\ x \le h_2\}$ will stay in the region Ω_1 after one impulsive effect or free from impulsive effect.

Theorem 3.4. *If* $d < ecx_0$ *and* $0 < (1-\alpha)h_2 < h_1 < h_2 < d/ec$, *there is no order one periodic solutions to the system* (1.3), *and the trajectory with initial point* $(x_0^+, y_0^+) \in \Omega_0 = \{(x,y) \mid x \ge 0, y \ge 0,\ x \le h_2\}$ *will stay in the region* $\Omega_1 = \{(x,y) \mid x \ge 0,\ y \ge 0, x \le h_1\}$.

Case 3 $(d/ec < h_2 \le x_0)$. In this case, the set M_2 is in the right side of stable center R. In the light of the different position of N_2, we consider the following two subject cases.

Subcase 3.1 $(h_1 < (1-\alpha)h_2)$. In this case, the set M_2 is in the right side of R. Then there exists a unique closed trajectory Γ_1 of system (1.3) which contains the point R and is tangent to M_2 at the point A.

Since Γ_1 is a closed trajectory, we take the minimal value $\delta_{\min}$ of abscissas at the trajectory Γ_1, namely, $\delta_{\min} \le x$ holds for any abscissas of Γ_1.

(1) $h_1 < (1-\alpha)h_2 < \delta_{\min}$: In this case, there is a trajectory, which contains the point R and is tangent to the N_2 at the point B intersecting with M_2 at a point $P_1(h_2, y_{P_1}) \in M_2$. Suppose point P_1 is subject to impulsive effects to point $P_1^+((1-\alpha)h_2, y_{P_1^+})$, here $y_{P_1^+} = (1-\beta)y_{P_1} + q$. Like the analysis of Subcase 2.1, we can prove that there exists an order one periodic solution to the system (1.3) in this case (as shown in Figure 6);

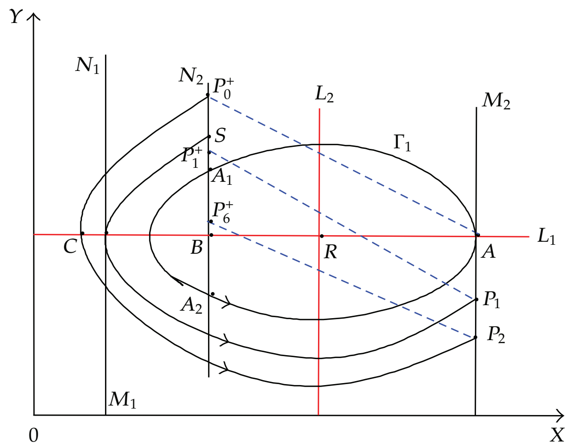

Figure 7

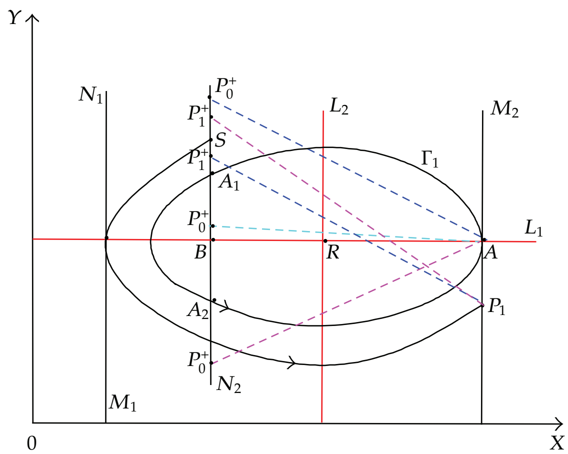

Figure 8

(2) $h_1 < \delta_{\min} < (1-\alpha)h_2$: In this case, denote the closed trajectory Γ_1 of system (1.3) intersecting with the set N_2 two point $A_1((1-\alpha)h_2, y_{A_1})$ and $A_2((1-\alpha)h_2, y_{A_2})$ (as shown in Figure 7). Since $A \in M_2$, impulse occurs at the point A. Suppose point A is subject to impulsive effects to point $P_0^+((1-\alpha)h_2, y_{P_0^+})$, here $y_{P_0^+} = (1-\beta)(r/c) + q$. If $(1-\beta)(r/c) + q < y_{A_2}$, the point P_0^+ lies below the point A_2. Like the analysis of (2) of Subcase 2.1, we can prove that there exists an order one periodic solution to the system (1.3) in this case. If $(1-\beta)(r/c) + q > y_{A_1}$, the point P_0^+ is above the point A_1. Suppose the trajectory passing through point B which tangents to N_1 at point B intersects with N_2 at a point S. Like the analysis of (1) of Subcase 2.1, we obtain sufficient conditions of existence of order one periodic solution to the system (1.3);

(3) $y_{A_2} < (1-\beta)(r/c) + q < y_{A_1}$: In this case, we note that the point P_0^+ must lie between the point A_1 and the point A_2 (as shown in Figure 8). Take a point $B_1 \in M_2$ such that B_1 jumps to A_2 after the impulsive effect and denote $A_2 = B_1^+$. Since $y_{P_0^+} > y_{B_1^+}$, we have $y_A > y_{B_1}$. Let $R(B_1) = B_2^+ \in N_2$, take a point $B_2 \in M_2$ such that B_2 jumps

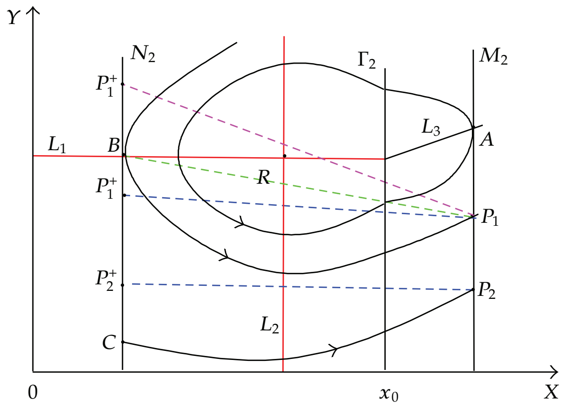

Figure 9

to B_2^+ after the impulsive effects, then we have $y_{B_1^+} > y_{B_2^+}, y_{B_1} > y_{B_2}$. This process continues until there exists a $B_K^+ \in N_2 (K \in Z_+)$ satisfying $y_{B_k^+} < q$. So we obtain a sequence $\{B_k^+\}_{k=1,2,\dots,K}$ of set M_2 and a sequence $\{B_k\}_{k=1,2,\dots,K}$ of set N_2 satisfying $R(B_{k-1}) = B_k^+ \in N_2, y_{B_{k-1}^+} > y_{B_k^+}$. In the following, we will prove that the trajectory of system (1.3) with any initiating point of set N_2 will ultimately stay in Γ_1. From the vector field of system (1.3), we know the trajectory of system (1.3) with any initiating point between the point A_1 and A_2 will be free from impulsive effect and ultimately will stay in Γ_1. For any point below A_2, it must lie between B_k^+ and B_{k-1}^+, here $k = 2, 3 \dots, K+1$ and $A_2 = B_1^+$. After k times' impulsive effects, the trajectory with this initiating point will arrive at some point of the set N_2 which must be between A_1 and A_2, and then ultimately stay in Γ_1. Denote the intersection of the trajectory passing through the point B which tangents to N_1 at point B with the set N_2 at point $S((1-\alpha)h_2, y_S)$ (Figure 7). The trajectory of system (1.3) with any initiating point on segment $\overline{A_1 S}$ intersects with the set N_2 at some point below A_2 with time increasing, so just like the analysis above we obtain it will ultimately stay in Γ_1. Therefore, for any point below S will ultimately stay in region Γ_1 with time increasing.

Now we can summarize the above results as the following theorem.

Theorem 3.5. *Assuming that* $d < ecx_0, h_1 < \delta_{\min} < (1-\alpha)h_2 < d/ec < h_2 \le x_0$ *and* $y_{A_2} < (1-\beta)(r/c) + q < y_{A_1}$*, there is no periodic solution in system* (1.3) *and the trajectory with any initiating point below S will stay in* Γ_1 *or in the region* $\Omega_1 = \{(x, y) \mid x \ge 0, y \ge 0, x \le h_1\}$.

Case 4 ($x_0 < h_2$). In this case, denote the intersection of the line L_1 with the set N at point $B((1-\alpha)h_2, r/c)$, and the intersection between the line L_3 and the set M_2 at point $A(h_2, rh_2/cx_0)$ (as shown in Figure 9). By Lemma 2.3 and means of qualitative analysis, there exists a unique closed trajectory Γ_2 of system (1.3) which is tangent to the set M_2 at the point A and has minimal value $\lambda_{\min}$ at the line L_1. In the light of the different position of the set N_2, we consider the following two cases.

Subcase 4.1 ($0 < h_1 < (1-\alpha)h_2 < \lambda_{\min}$). In this case, there exists a unique trajectory of system (1.3) which is tangent to the set N_2 at the point B. Set $F(B) = P_1 \in M_2$, then pulse occurs at point P_1, the impulsive function transfers the point P_1 into P_1^+. Like the analysis of (1) of Subcase 2.1, we can prove that there exists an order one periodic solution in system (1.3) in this case.

Subcase 4.2 ($h_1 < \lambda_{\min} < (1-\alpha)h < x_0 < h_2$). In this case, let the closed trajectory Γ_2 of system (1.3) intersects N_2 at two point $A_1((1-\alpha)h, y_{A_1})$ and $A_2((1-\alpha)h, y_{A_2})$. Like the analysis of (2) of Subcase 3.1, we can prove that there exists an order one periodic solution in system (1.3) in this case; like the analysis of (3) of Subcase 3.1 we can prove that there is no periodic solution in system (1.3).

4. Attractiveness of the Order One Periodic Solution

In this section, under the condition of existence of order one periodic solution to system (1.3) and the initial value of pest population $x(0) \le h_2$, we discuss its attractiveness. We focus on Cases 1 and 2; by similar method we can obtain similar results about Cases 3 and 4.

Theorem 4.1. *Assuming that $d < cex_0$, $h_1 < h_2 < r/c$ and $\delta \ge r/c$. If $y_{P_0^+} > y_{P_2^+} > y_{P^+}$ or $y_{P_0^+} < y_{P_2^+} < y_{P^+}$, then*

(I) *there exists a unique order one periodic solution of system* (1.3)*;*

(II) *If $(1-\alpha)h_2 < h_1$, order one periodic solution of system* (1.3) *is attractive in the region $\Omega_0 = \{(x,y) \mid x \ge 0, y \ge 0, x \le h_2\}$.*

Proof. By the derivation of Theorem 3.1, we know that there exists an order one periodic solution of system (1.3). We assume that trajectory $\widehat{PP^+}$ and segment $\overline{PP^+}$ formulate an order one periodic solution of system (1.3), that is, there exists a $P^+ \in N_2$ such that the successor function of P^+ satisfies $f(P^+) = 0$. First, we will prove the uniqueness of the order one periodic solution.

We take any two points $C_1(h_1, y_{C_1}) \in N_1, C_2(h_1, y_{C_2}) \in N_1$ satisfying $y_{C_2} > y_{C_1} > y_A$, then we obtain two trajectories whose initiate points are C_1 and C_2 intersect with the set M_1 at two points $D_1(h_1, y_{D_1})$ and $D_2(h_1, y_{D_2})$, respectively (Figure 10). In view of the vector field of system (1.3) and the disjointness of any two trajectories without impulse, we know that $y_{D_1} > y_{D_2}$. Suppose the points D_1 and D_2 are subject to impulsive effect to points $D_1^+(h_1, y_{D_1^+})$ and $D_2^+(h_2, y_{D_2^+})$, respectively, then we have $y_{D_1^+} > y_{D_2^+}$ and $f(C_1) = y_{D_1^+} - y_{C_1}, f(C_2) = y_{D_2^+} - y_{C_2}$, so we get $f(C_1) - f(C_2) < 0$, thus we obtain that the successor function $f(x)$ is decreasing monotonously in N_1; therefore there is a unique point $P^+ \in N_1$ satisfying $f(P^+) = 0$, and the trajectory $\widehat{P^+PP^+}$ is a unique order one periodic solution of system (1.3).

Next we prove the attractiveness of the order one periodic solution $\widehat{P^+PP^+}$ in the region $\Omega_0 = \{(x,y) \mid x \ge 0, y \ge 0, x \le h_2\}$. We focus on the case $y_{P_0^+} > y_{P_2^+} > y_{P^+}$; by similar method we can obtain similar results about case $y_{P_0^+} < y_{P_2^+} < y_{P^+}$ (Figure 11).

Take any point $P_0^+(h_1, y_{P_0^+}) \in N_1$ above P^+. Denote the first intersection point of the trajectory from initiating point $P_0^+(h_1, y_{P_0^+})$ with the set M_1 at $P_1(h_1, y_{P_1})$, and the corresponding consecutive points are $P_2(h_1, y_{P_2}), P_3(h_1, y_{P_3}), P_4(h_1, y_{P_4}), \ldots$, respectively. Consequently, under the effect of impulsive function I, the corresponding points after pulse are $P_1^+(h_1, y_{P_1^+}), P_2^+(h_1, y_{P_2^+}), P_3^+(h_1, y_{P_3^+}), \ldots$

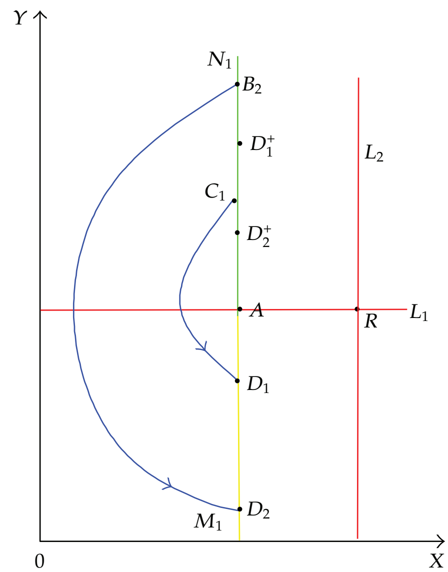

Figure 10

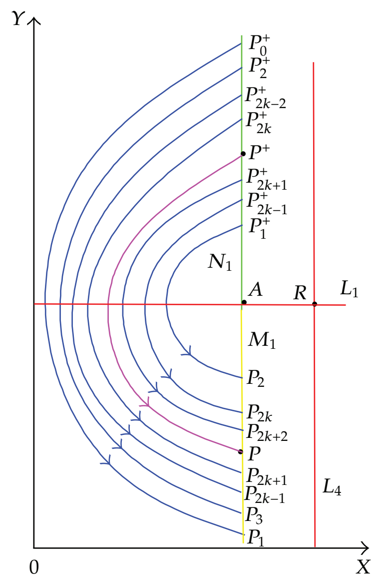

Figure 11

Due to conditions $y_{P_0^+} > y_{P_2^+} > y_{P^+}$, $y_{P_k^+} = y_{P_k} + \delta$, $\delta \geq a/b$ and disjointness of any two trajectories, we get a sequence $\{P_k^+\}_{k=1,2,\ldots}$ of the set N_1 satisfying

$$y_{P_1^+} < y_{P_3^+} < \cdots < y_{P_{2k-1}^+} < y_{P_{2k+1}^+} < \cdots < y_{P^+} < \cdots < y_{P_{2k}^+} < y_{P_{2k-2}^+} < \cdots < y_{P_2^+} < y_{P_0^+}. \tag{4.1}$$

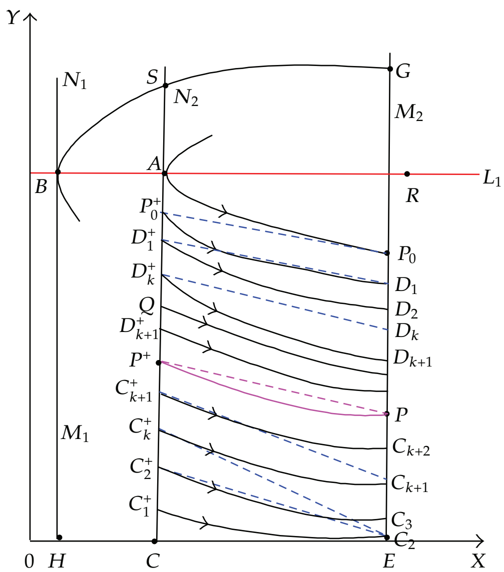

Figure 12

So the successor function $f(P^+_{2k-1}) = y_{P^+_{2k}} - y_{P^+_{2k-1}} > 0$ and $f(P^+_{2k}) = y_{P^+_{2k+1}} - y_{P^+_{2k}} < 0$ hold. Series $\{y_{P_{2k-1}}\}_{k=1,2,\ldots}$ increases monotonously and has upper bound, so $\lim_{k\to\infty} y_{P^+_{2k-1}}$ exists. Next we will prove $\lim_{k\to\infty} y_{P^+_{2k-1}} = y_{P^+}$. Set $\lim_{k\to\infty} P_{2k-1} = C^+$, we will prove $P^+ = C^+$. Otherwise $P^+ \neq C^+$, then there is a trajectory passing through the point C^+ which intersects the set M_1 at point $\widetilde{C}$, then we have $y_{\widetilde{C}} > y_P$, $y_{\widetilde{C}^+} > y_{P^+}$. Since $f(C^+) \geq 0$ and $P^+ \neq C^+$, according to the uniqueness of the periodic solution, then we have $f(C^+) = y_{\widetilde{C}^+} - y_{C^+} > 0$, thus $y_{C^+} < y_{P^+} < y_{\widetilde{C}^+}$ hold. Analogously, let trajectory passing through the point C^+ which intersects the set M_1 at point $\widetilde{\widetilde{C}}$, and the corresponding consecutive points is $\widetilde{\widetilde{C}}$, then $y_{\widetilde{C}} > y_{\widetilde{\widetilde{C}}} > y_p > y_{\widetilde{\widetilde{C}}}$, $y_{\widetilde{C}^+} > y_{\widetilde{\widetilde{C}}^+} > y_{\widetilde{\widetilde{C}}^+} > y_{p^+} > y_{C^+}$, then we have $f(\widetilde{\widetilde{C}}^+) = y_{\widetilde{\widetilde{C}}^+} - y_{\widetilde{\widetilde{C}}^+} > 0$, this contradicts to the fact that C^+ is a limit of sequence $\{P^+_{2k-1}\}_{k=1,2,\ldots}$, so we obtain $P^+ = C^+$. Therefore, we have $\lim_{k\to\infty} y_{P^+_{2k-1}} = y_{P^+}$. Similarly, we can prove $\lim_{k\to\infty} y_{P^+_{2k}} = y_{P^+}$.

From above analysis, we know that there exists a unique order one periodic solution in system (1.3), and the trajectory from initiating any point of the N_1 will ultimately tend to be order one periodic solution $\widehat{P^+PP^+}$.

Any trajectory from initial point $(x_0^+, y_0^+) \in \Omega_0 = \{(x, y) \mid x \geq 0, y \geq 0, x \leq h_2\}$ will intersect with N_1 at some point with time increasing on the condition that $(1-\alpha)h_2 < h_1 < h_2 < d/b(\lambda - dh)$; therefore, the trajectory from initial point on N_1 ultimately tends to be order one periodic solution $\widehat{P^+PP^+}$. Therefore, order one periodic solution $\widehat{P^+PP^+}$ is attractive in the region Ω_0. This completes the proof. □

Remark 4.2. Assuming that $d < ecx_0, h_1 < h_2 < d/ec$ and $\delta \geq r/c$, if $y_{P^+} < y_{P_0^+} < y_{P_2^+}$ or $y_{P^+} > y_{P_0^+} > y_{P_2^+}$, then the order one periodic solution is unattractive.

Theorem 4.3. *Assuming that $d < ecx_0, h_1 < (1-\alpha)h_2 < h_2 < d/ec$ and $y_{P_0^+} < y_A$ (as shown in Figure 12), then*

(I) *there exists an odd number of order one periodic solutions of system* (1.3) *with initial value between* C_1^+ *and* A *in set* N_2;

(II) *if* $(1-\alpha)h_2 < h_1$ *and the periodic solution is unique, then the periodic solution is attractive in region* Ω_2, *here* Ω_2 *is open region which is constituted by trajectory* $\widehat{GB}$, *segment* $\overline{BH}$, *segment* $\overline{HE}$, *and segment* $\overline{EG}$.

Proof. (I) due to the (2) of Subcase 2.1, $f(A) < 0$ and $f(C_1^+) > 0$ and the continuous successor function $f(x)$, there exists an odd number of root satisfying $f(x) = 0$, then we can get that there exists an odd number of order one periodic solutions of system (1.3) with initial value between C_1^+ and A in set N_2;

(II) by the derivation of Theorem 3.3, we know that there exists an order one periodic solution of system (1.3) whose initial point is between C_1^+ and P_0^+ in the set N_2. Assume trajectory $\widehat{P^+P}$ and segment $\overline{PP^+}$ formulate the unique order one periodic solution of system (1.3) with initial point $P^+ \in N_2$.

On the one hand, take a point $C_1^+((1-\alpha)h_2, y_{C_1^+}) \in N_2$ satisfying $y_{C_1^+} = \varepsilon < q$ and $y_{C_1^+} < y_{P^+}$. The trajectory passing through the point $C_1^+((1-\alpha)h_2, \varepsilon)$ which intersects with set M_2 at point $C_2(h_2, y_{C_2})$, that is, $F(C_1^+) = C_2 \in M_2$, then we have $y_{C_2} < y_P$, thus $y_{C_2^+} < y_{P^+}$. Since $y_{C_2^+} = (1-\beta)y_{C_2} + q > \varepsilon$, so we obtain $f(C_1^+) = y_{C_2^+} - y_{C_1^+} = y_{C_2^+} - \varepsilon > 0$; set $F(C_2^+) = C_3 \in M_2$, because $y_{C_1^+} < y_{C_2^+} < y_{P^+}$, we know that $y_{C_2} < y_{C_3} < y_P$, then we have $y_{C_2^+} < y_{C_3^+} < y_{P^+}$ and $f(C_2^+) = y_{C_3^+} - y_{C_2^+} > 0$. This process is continuing, then we get a sequence $\{C_k^+\}_{k=1,2,\dots}$ of the set N_2 satisfying

$$y_{C_1^+} < y_{C_2^+} < \cdots < y_{C_k^+} < \cdots < y_{P^+} \tag{4.2}$$

and $f(C_k^+) = y_{C_{k+1}^+} - y_{C_k^+} > 0$. Series $\{y_{C_k^+}\}_{k=1,2,\dots}$ increases monotonously and has upper bound, so $\lim_{k\to\infty} y_{C_k^+}$ exists. Like the proof of Theorem 4.1, we can prove $\lim_{k\to\infty} y_{C_k^+} = y_{P^+}$.

On the other hand, set $F(P_0^+) = D_1 \in M_2$, then D_1 jumps to $D_1^+ \in N_2$ under the impulsive effects. Since $y_{P^+} < y_{P_0^+} < y_A$, we have $y_P < y_{D_1} < y_{P_0}$, thus we obtain $y_{P^+} < y_{D_1^+} < y_{P_0^+}$, $f(P_0^+) = y_{D_1^+} - y_{P_0^+} < 0$. Set $F(D_1^+) = D_2 \in M_2$, then D_2 jumps to $D_2^+ \in N_2$ under the impulsive effects. We have $y_{P^+} < y_{D_2^+} < y_{D_1^+}$. This process is continuing, we can obtain a sequence $\{D_k^+\}_{k=1,2\dots}$ of the set N_2 satisfying

$$y_{P_0^+} > y_{D_1^+} > y_{D_2^+} > \cdots > y_{D_k^+} > \cdots > y_{P^+} \tag{4.3}$$

and $f(D_k^+) = y_{D_{k+1}^+} - y_{D_k^+} < 0$. Series $\{y_{D_k^+}\}_{k=1,2,\dots}$ decreases monotonously and has lower bound, so $\lim_{k\to\infty} y_{D_k^+}$ exists. Similarly, we can prove $\lim_{k\to\infty} y_{D_k^+} = y_{P^+}$.

Any point $Q \in N_2$ below A must be in some interval $[y_{D_{k+1}^+}, y_{D_k^+})_{k=1,2,\dots}$, $[y_{D_1^+}, y_{P_0^+})$, $[y_{P_0^+}, y_A)$, $[y_{C_k^+}, y_{C_{k+1}^+})_{k=1,2,\dots}$. Without loss of generality, we assume that the point $Q \in [y_{D_{k+1}^+}, y_{D_k^+})$. The trajectory with initiating point Q moves between trajectory $\widehat{D_k^+ D_{k+1}}$ and $\widehat{D_{k+1}^+ D_{k+2}}$ and intersects with M_2 at some point between D_{k+2} and D_{k+1}, under the impulsive effects it jumps to the point of N_2 which is between $[y_{D_{k+2}^+}, y_{D_{k+1}^+})$, then trajectory $\widetilde{\Pi}(Q,t)$ continues to move between trajectory $\widehat{D_{k+1}^+ D_{k+2}}$ and $\widehat{D_{k+2}^+ D_{k+3}}$. This process can be continued unlimitedly. Since $\lim_{k\to\infty} y_{D_k^+} = y_{P^+}$, the intersection sequence of trajectory $\widetilde{\Pi}(Q,t)$ with the set N_2 will ultimately tend to be the point P^+. Similarly, if $Q \in [y_{C_k^+}, y_{C_{k+1}^+})$, we also can get the intersection sequence of trajectory $\widetilde{\Pi}(Q,t)$, and the set N_2 will ultimately tend to be point P^+. Thus the trajectory from initiating any point below A ultimately tends to be the unique order one periodic solution $\widehat{P^+PP^+}$.

Denote the intersection of the trajectory passing through the point B which tangents to N_1 at the point B, and the set N_2 by a point $S((1-\alpha)h_2, y_S)$. The trajectory from any initiating point on segment $\overline{AS}$ will intersect with the set N_2 at some point below A with time increasing, so like the analysis above we obtain that the trajectory from any initiating point on segment $\overline{AS}$ will ultimately tend to be the unique order one periodic solution $\widehat{P^+PP^+}$.

Since the trajectory with any initiating point of the Ω_2 will definitely intersect with set N_2. From the above analysis, we know that the trajectory with any initiating point on segment $\overline{AS}$ will ultimately tend to be order one periodic solution $\widehat{P^+PP^+}$. Therefore, the unique order one periodic solution $\widehat{P^+PP^+}$ is attractive in the region Ω_2. This completes the proof. □

Remark 4.4. Assuming that $d < ecx_0, h_1 < (1-\alpha)h_2 < h_2 < d/ec$ and $y_{C_1^+} < y_A < y_{P_0^+}$, the order one periodic solution with initial point between A and P_0^+ is unattractive.

5. Conclusion

In this paper, a state-dependent impulsive dynamical model with Holling I functional response predator-prey concerning different control methods at different thresholds is proposed; we find a new method to study existence and attractiveness of order one periodic solution of such system. We define semicontinuous dynamical system and successor function and demonstrate the sufficient conditions that system (1.3) exists order one periodic solution with differential geometry theory and successor function. Besides, we successfully prove the attractiveness of the order one periodic solution by sequence convergence rules and qualitative analysis. In order to testify the validity of our results, we consider the following example:

$$
\begin{aligned}
&\left.\begin{aligned}
&\left.\begin{aligned} x'(t) &= 0.4x(t) - 0.6x(t)y(t), \\ y'(t) &= -0.2y(t) + 0.3x(t)y(t), \end{aligned}\right\} \quad x \le 0.8, \\
&\left.\begin{aligned} x'(t) &= 0.4x(t) - 0.48y(t), \\ y'(t) &= -0.6y(t) + 0.24y(t), \end{aligned}\right\} \quad x > 0.8,
\end{aligned}\right\} \quad x \neq h_1, h_2 \text{ or } x = h_1, y > y^*, \\
&\qquad \left.\begin{aligned} \Delta x(t) &= 0, \\ \Delta y(t) &= 0.8, \end{aligned}\right\} \quad x = h_1,\ y \le y^*, \\
&\qquad \left.\begin{aligned} \Delta x(t) &= -0.5x(t), \\ \Delta y(t) &= -0.2y(t) + 0.5, \end{aligned}\right\} \quad x = h_2,
\end{aligned}
\tag{5.1}
$$

where $0 < h_1 < h_2 < x^*$. Now, we consider the impulsive effects influences on the dynamics of system (5.1).

Example 5.1. Existence and attractiveness of order one periodic solution.

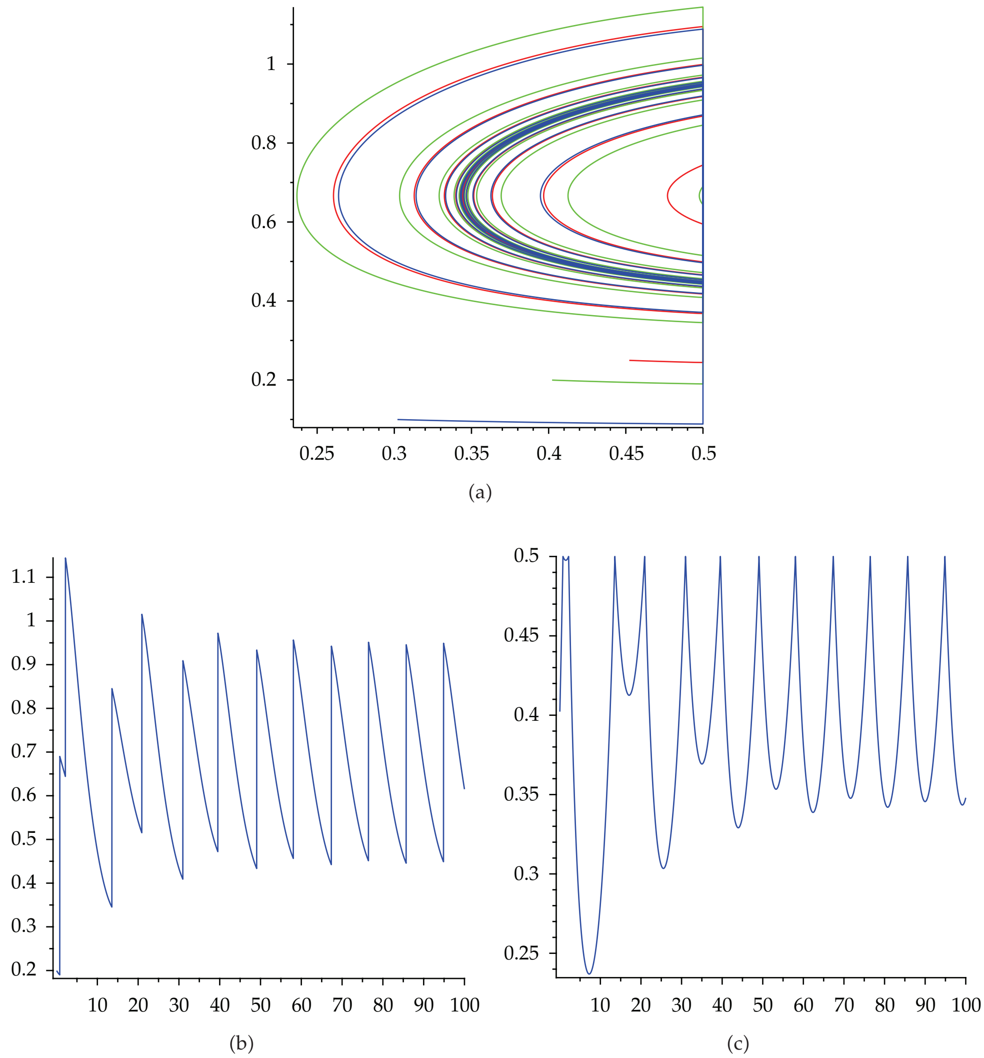

Figure 13: The time series and phase diagram for system (1.3) starting from initial value (0.45, 0.25) (red), (0.4, 0.2) (green), and (0.3, 0.1) (blue) $0 < h_1 < h_2 < x^*$.

We set $h_2 = 0.6, (1-\alpha)h_2 < h_2 < x^*$, initiating points are (0.45, 0.25) (red), (0.4, 0.2) (green), and (0.3, 0.1) (blue), respectively. Figure 13 shows that the conditions of Theorems 3.1 and 4.1 hold, system (1.3) exists order one periodic solution, and the trajectory from different initiating must ultimately tend to be the order one periodic solution. Therefore, order one periodic solution is attractive.

Example 5.2. Existence and attractiveness of positive periodic solution.

We set $h_1 = 0.3, h_2 = 0.6, h_1 < (1-\alpha)h_2 < x^* < h_2 < x_0$, initiating points are (0.45, 0.25) (red), (0.4, 0.2) (green), and (0.3, 0.1) (blue), respectively. Figure 14 shows that the conditions of Theorems 3.3 and 4.3 hold, system (1.3) exists order one periodic solution,

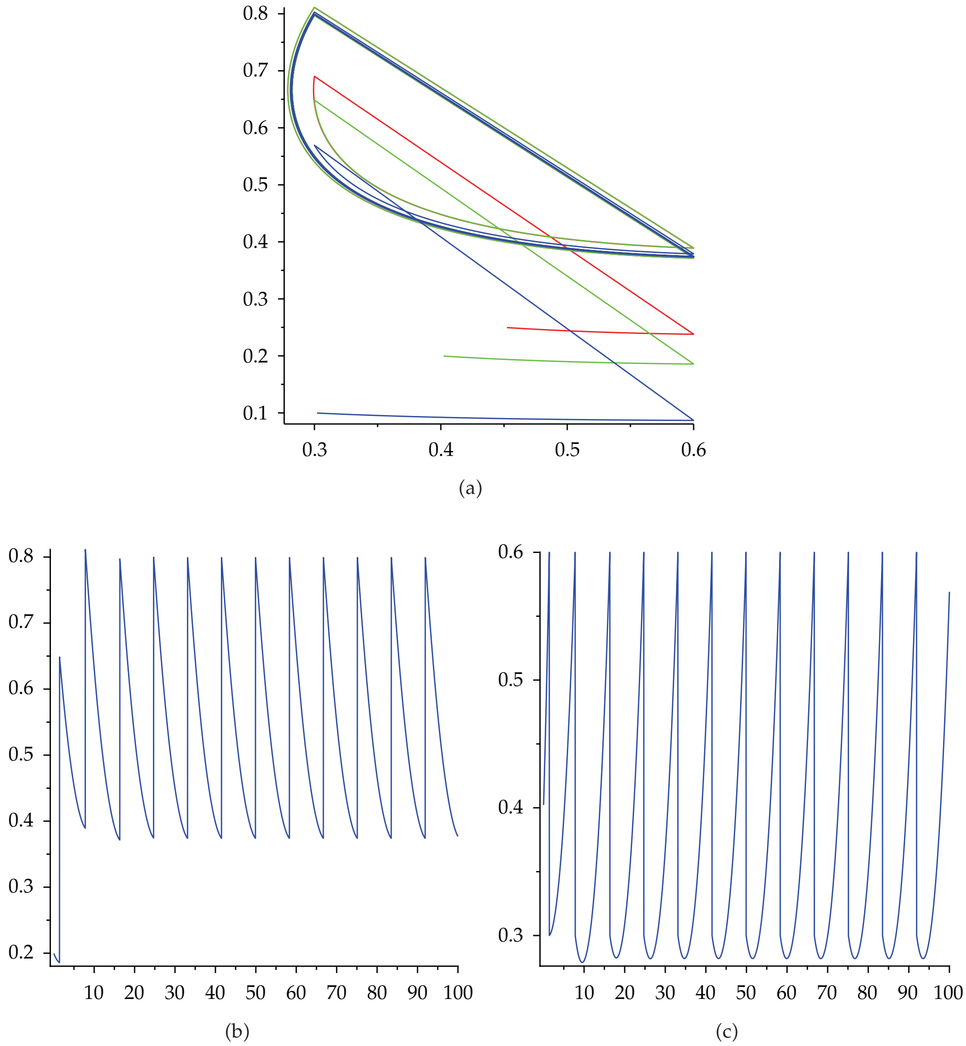

Figure 14: The time series and phase diagram for system (1.3) starting from initial value $(0.45, 0.25)$ (red), $(0.4, 0.2)$ (green), and $(0.3, 0.1)$ (blue) $h_1 < (1-\alpha)h_2 < x^* < h_2 < x_0$.

and the trajectory from different initiating must ultimately tend to be the order one periodic solution. Therefore, order one periodic solution is attractive.

Example 5.3. Existence and attractive of positive periodic solutions.

We set $h_1 = 0.3, h_2 = 0.8, h_1 < (1-\alpha)h_2 < x^* < h_2$, initiating points are $(0.4, 0.1)$ (red), $(0.4, 0.2)$ (green), and $(0.4, 0.15)$ (blue) as shown in Figure 15. Therefore, the conditions of Theorem 3.3 and Theorem 4.3 hold, so system (1.3) exists order one periodic solution, and it is attractive.

Example 5.4. Existence and attractive of positive periodic solutions.

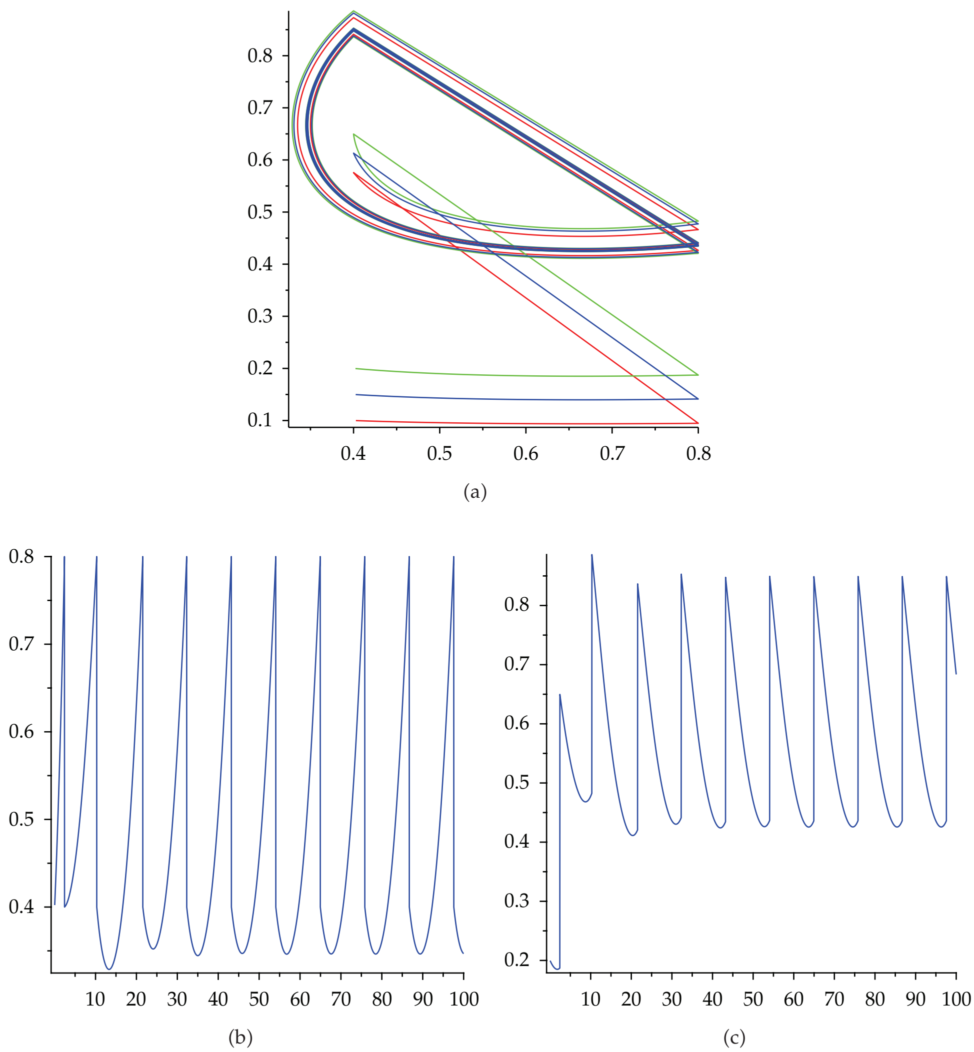

Figure 15: The time series and phase diagram for system (1.3) starting from initial value $(0.4, 0.1)$ (red), $(0.4, 0.2)$ (green), and $(0.4, 0.15)$ (blue) $h_1 = 0.3$, $h_2 = 0.8$, $h_1 < (1-\alpha)h_2 < x^* < h_2$.

We set $h_1 = 0.3, h_2 = 1.5, h_1 < (1-\alpha)h_2 < x^* < x_0 < h_2$, initiating points are $(0.9, 0.2)$ Figure 16 shows that results of Case 4 are valid.

These results show that the state-dependent impulsive effects contribute significantly to the richness of the dynamics of the model. Our results show that, in theory, a pest can be controlled such that its population size is no larger than its ET by applying effects impulsively once, twice, or at most, a finite number of times, or according to a periodic regime. The methods of the theorems are proved to be new in this paper, and these methods are more efficient and easier to operate than the existing research methods which have been applied to the models with impulsive state feedback control [12–15], so they are deserved further promotion.

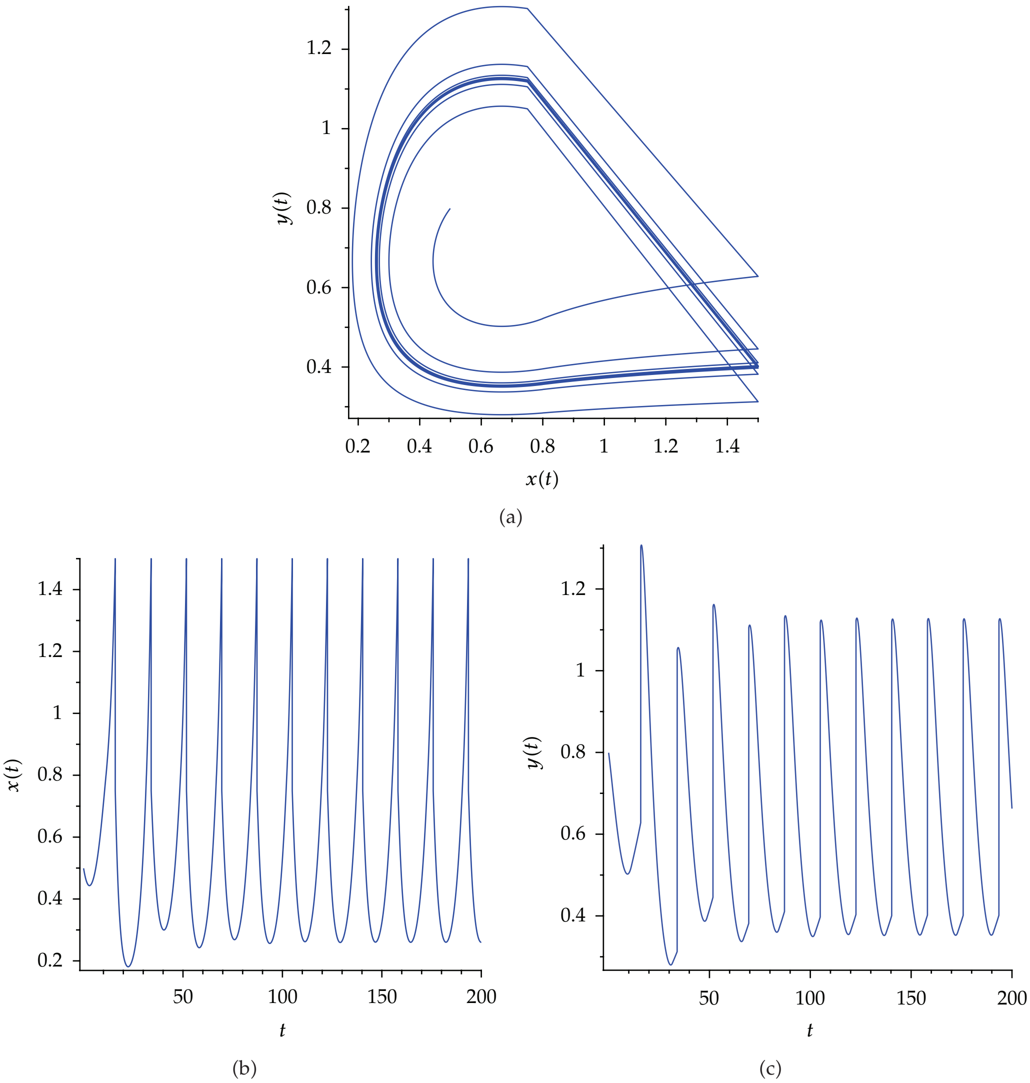

Figure 16: The time series and phase diagram for system (1.3) starting from initial value $(0.9, 0.2) h_1 = 0.3, h_2 = 1.5, h_1 < (1 - \alpha) h_2 < x^* < x_0 < h_2$.

Acknowledgment

This project was supported by the National Natural Science Foundation of China (no. 10872118).

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