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Research Article

Periodic Solutions for a Semi-Ratio-Dependent Predator-Prey System with Delays on Time Scales

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This paper is devoted to the existence of periodic solutions for a semi-ratio-dependent predator-prey system with time delays on time scales. With the help of a continuation theorem based on coincidence degree theory, we establish necessary and sufficient conditions for the existence of periodic solutions. Our results show that for the most monotonic prey growth such as the logistic, the Gilpin, and the Smith growth, and the most celebrated functional responses such as the Holling type, the sigmoidal type, the Ivlev type, the Monod-Haldane type, and the Beddington-DeAngelis type, the system always has at least one periodic solution. Some known results are shown to be special cases of the present paper.

1. Introduction

In the past decades, many authors have investigated the existence of periodic solutions for population models governed by the differential and difference equations [1–7]. In particular, the existence of periodic solutions for semi-ratio-dependent predator-prey systems has been studied extensively in the literature and seen great progress [8–16].

Recently, in order to unify differential and difference equations, people have done a lot of research about dynamic equations on time scales. In fact, continuous and discrete systems are very important in implementing and applications. But it is troublesome to study the existence of periodic solutions for continuous and discrete systems, respectively. Therefore, it is meaningful to study that on time scales which can unify the continuous and discrete situations. For the theory of dynamic equations on time scales, we refer the reader to [17, 18]. For the research on periodic solutions of dynamic equations on time scales describing population dynamics, one may consult [19–26], and so forth.

In this paper, we consider the following periodic semi-ratio-dependent predator-prey system with time delays on a time scale $\mathbb{T}$:

$$\begin{aligned} u_1^{\Delta}(t) &= g\left(t, e^{u_1(t-\tau_1(t))}\right) - h\left(t, e^{u_1(t)}, e^{u_2(t)}\right) e^{u_2(t-\tau_2(t))-u_1(t)}, \\ u_2^{\Delta}(t) &= c(t) - d(t) e^{u_2(t)-u_1(t-\tau_3(t))}. \end{aligned} \tag{1.1}$$

Here $\mathbb{T}$ is a periodic time scale which has the subspace topology inherited from the standard topology on $\mathbb{R}$. The symbol Δ stands for the delta derivative which gives the ordinary derivative if $\mathbb{T} = \mathbb{R}$ and the forward difference operator if $\mathbb{T} = \mathbb{Z}$.

In system (1.1), set $x(t) = \exp[u_1(t)]$, $y(t) = \exp[u_2(t)]$. If $\mathbb{T} = \mathbb{R}$, then system (1.1) reduces to the standard semi-ratio-dependent predator-prey system governed by the ordinary differential equations:

$$\begin{aligned} x'(t) &= x(t) g(t, x(t-\tau_1(t))) - h(t, x(t), y(t)) y(t-\tau_2(t)), \\ y'(t) &= y(t)\left[c(t) - d(t)\frac{y(t)}{x(t-\tau_3(t))}\right], \end{aligned} \tag{1.2}$$

where $x(t)$ and $y(t)$ stand for the population of the prey and the predator, respectively. The function $g(t,x)$ is the growth rate of the prey in the absence of the predator. The predator consumes the prey according to the functional response $h(t,x,y)$ and grows logistically with growth rate $c(t)$ and carrying capacity $x(t)/d(t)$ proportional to the population size of the prey. The function $d(t)$ is a measure of the food quality that the prey provides for conversion into the predator birth. If $\mathbb{T} = \mathbb{Z}$, then system (1.1) is reformulated as

$$\begin{aligned} x(k+1) &= x(k)\exp\left[g(k, x(k-\tau_1(k))) - h(k, x(k), y(k))\frac{y(k-\tau_2(k))}{x(k)}\right], \\ y(k+1) &= y(k)\exp\left[c(k) - d(k)\frac{y(k)}{x(k-\tau_3(k))}\right], \end{aligned} \tag{1.3}$$

which is the discrete time semi-ratio-dependent predator-prey system and is a discrete analogue of (1.2).

We note that Ding and Jiang [8, 9], Ding et al. [10], Liu [11], Liu and Huang [12], and Wang et al. [13] studied some special cases of system (1.2). Fan and Wang [14], Fazly and Hesaaraki [15], and Liu [16] discussed some special cases of system (1.3). Bohner et al. [19], Fazly and Hesaaraki [21], and Zhuang [26] investigated some special cases of system (1.1). So far as we know, there is no published paper concerned system (1.1).

The main purpose of this paper is, by using the coincidence degree theory developed by Gaines and Mawhin [27], to derive necessary and sufficient conditions for the existence of periodic solutions of system (1.1). Furthermore, we will see that our result for the above system can be easily extended to the one with distributed or state-dependent delays. Our result generalizes some theorems in [8, 9, 11, 12, 15, 16, 21], improves and generalizes some theorems in [10, 13, 14, 19, 26].

2. Preliminaries

In this section, we briefly give some elements of the time scale calculus, recall the continuation theorem from coincidence degree theory, and state an auxiliary result that will be used in this paper.

First, let us present some foundational definitions and results from the calculus on time scales so that the paper is self-contained. For more details, we refer the reader to [17, 18].

A time scale $\mathbb{T}$ is an arbitrary nonempty closed subset $\mathbb{T}$ of the real numbers $\mathbb{R}$, which inherits the standard topology of $\mathbb{R}$. Thus, the real numbers $\mathbb{R}$, the integers $\mathbb{Z}$, and the natural numbers $\mathbb{N}$ are examples of time scales, while the rational numbers $\mathbb{Q}$ and the open interval $(1,2)$ are no time scales.

Let $\omega > 0$. Throughout this paper, the time scale $\mathbb{T}$ is assumed to be ω-periodic; that is, $t \in \mathbb{T}$ implies $t+\omega \in \mathbb{T}$. In particular, the time scale $\mathbb{T}$ under consideration is unbounded above and below.

For $t \in \mathbb{T}$, the forward and backward jump operators $\sigma, \rho : \mathbb{T} \to \mathbb{T}$ are defined by

$$\sigma(t) = \inf\{s \in \mathbb{T} : s > t\}, \qquad \rho(t) = \sup\{s \in \mathbb{T} : s < t\} \tag{2.1}$$

respectively.

If $\sigma(t) = t$, t is called right-dense (otherwise: right-scattered), and if $\rho(t) = t$, then t is called left-dense (otherwise left-scattered).

A function $f : \mathbb{T} \to \mathbb{R}$ is said to be rd-continuous if it is continuous at right-dense points in $\mathbb{T}$ and its left-sided limits exist (finite) at left-dense points in $\mathbb{T}$. The set of rd-continuous functions is denoted by $C_{\mathrm{rd}}(\mathbb{T})$.

For $f : \mathbb{T} \to \mathbb{R}$ and $t \in \mathbb{T}$ we define $f^{\Delta}(t)$, the delta-derivative of f at t, to be the number (provided it exists) with the property that, given any $\varepsilon > 0$, there is a neighborhood U of t (i.e., $U = (t-\delta, t+\delta) \cap \mathbb{T}$ for some $\delta > 0$) in $\mathbb{T}$ such that

$$\left|[f(\sigma(t)) - f(s)] - f^{\Delta}(t)[\sigma(t) - s]\right| \le \varepsilon|\sigma(t) - s|, \quad \forall s \in U. \tag{2.2}$$

f is said to be delta-differentiable if its delta-derivative exists for all $t \in \mathbb{T}$. The set of functions $f : \mathbb{T} \to \mathbb{R}$ that are delta-differentiable and whose delta-derivative is rd-continuous functions is denoted by $C^1_{\mathrm{rd}}(\mathbb{T})$.

A function $F : \mathbb{T} \to \mathbb{R}$ is called a delta-antiderivative of $f : \mathbb{T} \to \mathbb{R}$ provided $F^{\Delta}(t) = f(t)$, for all $t \in \mathbb{T}$. Then, we define the delta integral by

$$\int_a^b f(t)\Delta t = F(b) - F(a), \quad \forall a, b \in \mathbb{T}. \tag{2.3}$$

Lemma 2.1. *Every delta differentiable function is continuous.*

Lemma 2.2. *Every rd-continuous function has a delta-antiderivative.*

Lemma 2.3. *If a, b, $c \in \mathbb{T}$, α, $\beta \in \mathbb{R}$ and f, $g \in C_{\mathrm{rd}}(\mathbb{T})$, then*

(a) $\int_a^b [\alpha f(t) + \beta g(t)]\ \Delta t = \alpha \int_a^b f(t)\ \Delta t + \beta \int_a^b \beta g(t)\ \Delta t$,

(b) $\int_a^b f(t)\ \Delta t = \int_a^c f(t)\ \Delta t + \int_c^b f(t)\ \Delta t$,

(c) *if* $f(t) \geq 0$ *for all* $a \leq t < b$, *then* $\int_a^b f(t)\ \Delta t \geq 0$,

(d) *if* $|f(t)| \leq g(t)$ *on* $[a,b) := \{t \in \mathbb{T} : a \leq t < b\}$, *then* $|\int_a^b f(t)\ \Delta t| \leq \int_a^b g(t)\ \Delta t$.

Next, let us recall the continuation theorem in coincidence degree theory. To do so, we need to introduce the following notation.

Let X, Y be real Banach spaces, let $L : \operatorname{Dom} L \subset X \to Y$ be a linear mapping, and let $N : X \to Y$ be a continuous mapping.

The mapping L is said to be a Fredholm mapping of index zero, if $\dim \operatorname{Ker} L = \operatorname{codim} \operatorname{Im} L < +\infty$ and $\operatorname{Im} L$ is closed in Y.

If L is a Fredholm mapping of index zero, then there exist continuous projectors $P : X \to X$ and $Q : Y \to Y$, such that $\operatorname{Im} P = \operatorname{Ker} L$, $\operatorname{Ker} Q = \operatorname{Im} L = \operatorname{Im}(I - Q)$. It follows that the restriction L_P of L to $\operatorname{Dom} L \cap \operatorname{Ker} P : (I - P)X \to \operatorname{Im} L$ is invertible. Denote the inverse of L_P by K_P.

The mapping N is said to be L-compact on $\overline{\Omega}$, if Ω is an open bounded subset of X, $QN(\overline{\Omega})$ is bounded, and $K_P(I - Q)N : \overline{\Omega} \to X$ is compact.

Since $\operatorname{Im} Q$ is isomorphic to $\operatorname{Ker} L$, there exists an isomorphism $J : \operatorname{Im} Q \to \operatorname{Ker} L$.

Here we state the Gaines-Mawhin theorem, which is a main tool in the proof of our main result.

Lemma 2.4 (continuation theorem [27, page 40]). *Let* $\Omega \subset X$ *be an open bounded set, let* L *be a Fredholm mapping of index zero and let* N *be* L*-compact on* $\overline{\Omega}$. *Assume*

(a) *for each* $\lambda \in (0,1)$, $x \in \partial\Omega \cap \operatorname{Dom} L, Lx \neq \lambda Nx$;

(b) *for each* $x \in \partial\Omega \cap \operatorname{Ker} L, QNx \neq 0$;

(c) $\deg(JQN, \Omega \cap \operatorname{Ker} L, 0) \neq 0$.

Then $Lx = Nx$ *has at least one solution in* $\overline{\Omega} \cap \operatorname{Dom} L$.

For convenience and simplicity in the following discussion, we always use the following notation:

$$\kappa = \min\{[0,+\infty) \cap \mathbb{T}\}, \qquad I_\omega = [\kappa, \kappa+\omega] \cap \mathbb{T}, \qquad \widehat{a} = \frac{1}{\omega}\int_\kappa^{\kappa+\omega} a(t)\Delta t, \qquad \widehat{A} = \frac{1}{\omega}\int_\kappa^{\kappa+\omega} |a(t)|\Delta t,$$
$$\widetilde{b}(x) = \frac{1}{\omega}\int_\kappa^{\kappa+\omega} b(t,x)\Delta t, \qquad \widetilde{B}(x) = \frac{1}{\omega}\int_\kappa^{\kappa+\omega} |b(t,x)|\Delta t, \qquad \overline{\varphi}(x,y) = \frac{1}{\omega}\int_\kappa^{\kappa+\omega} \varphi(t,x,y)\Delta t, \tag{2.4}$$

where $a \in C_{\mathrm{rd}}(\mathbb{T})$ is an ω-periodic function, $b : \mathbb{T} \times \mathbb{R} \to \mathbb{R}$ and $\varphi : \mathbb{T} \times \mathbb{R}^2 \to \mathbb{R}$ are rd-continuous and ω-periodic in their first variable.

In order to achieve the priori estimation in the case of dynamic equations on a time scale $\mathbb{T}$, we now give the following inequality which is proved in [19, Lemma 2.4].

Lemma 2.5. *Let* t_1, $t_2 \in I_\omega$ *and* $t \in \mathbb{T}$. *If* $\varphi \in C^1_{\mathrm{rd}}(\mathbb{T})$ *is an* ω*-periodic real function, then*

$$\varphi(t) \leq \varphi(t_1) + \int_\kappa^{\kappa+\omega} \left|\varphi^\Delta(t)\right|\Delta t, \qquad \varphi(t) \geq \varphi(t_2) - \int_\kappa^{\kappa+\omega} \left|\varphi^\Delta(t)\right|\Delta t. \tag{2.5}$$

3. Existence of Periodic Solutions

In this section, we study the existence of periodic solutions of system (1.1). For the sake of generality, we make the following fundamental assumptions for system (1.1).

(H_1) $\tau_i : \mathbb{T} \to \mathbb{R}^+$ is rd-continuous and ω-periodic such that $t - \tau_i(t) \in \mathbb{T}$ for $i = 1, 2, 3$, and $t \in \mathbb{T}$.

(H_2) $c : \mathbb{T} \to \mathbb{R}$ and $d : \mathbb{T} \to (0, +\infty)$ are rd-continuous and ω-periodic.

(H_3) $g : \mathbb{T} \times \mathbb{R} \to \mathbb{R}$ is rd-continuous and ω-periodic in the first variable and is continuously differentiable in the second variable and $(\partial g/\partial x)(t, x) < 0$, $\lim_{x\to+\infty} g(t, x) < 0$ for all $t \in \mathbb{T}$, $x > 0$.

(H_4) $h : \mathbb{T} \times \mathbb{R}^2 \to \mathbb{R}^+$ is rd-continuous and ω-periodic in the first variable and is continuously differentiable in the last two variables. In addition, there exist a positive integer m and ω-periodic rd-continuous functions $a_i : \mathbb{T} \to \mathbb{R}^+$, $i = 0, 1, \ldots, m-1$, such that

$$h(t, x, y) \leq a_0(t)x^m + a_1(t)x^{m-1} + \cdots + a_{m-1}(t)x, \quad \forall t \in \mathbb{T},\ x > 0,\ y > 0. \tag{3.1}$$

Readers familiar with predator-prey models may notice that the above assumptions are reasonable for population models. Under the above assumptions, system (1.1) covers many models that have appeared in the literature. For instance, $g(t, x)$ can be taken as the logistic growth $a - bx$, the Gilpin growth $a - bx^\theta$, and the Smith growth $(a - bx)/(D + x)$. $h(t, x, y)$ can be taken as functional responses of the Lotka-Volterra type mx, the Holling type $mx^n/(A + x^n)(n \geq 1)$, the Ivlev type $m(1 - e^{-Ax})$, the sigmoidal type $mx^2/[(A + x)(B + x)]$, the Monod-Haldane type $mx/(A + Bx + x^2)$, and the Beddington-DeAngelis type $mx/(A + Bx + Cy)$, and so forth.

By (H_3), we have

$$\widetilde{g}'(x) = \frac{1}{\omega}\int_\kappa^{\kappa+\omega} \frac{\partial g}{\partial x}(t, x)\ \Delta t < 0, \qquad \lim_{x\to+\infty} \widetilde{g}(x) = \frac{1}{\omega}\int_\kappa^{\kappa+\omega} \lim_{x\to+\infty} g(t, x)\ \Delta t < 0. \tag{3.2}$$

Thus $\widetilde{g}(x)$ is strictly decreasing on $[0, +\infty)$.

We are now in a position to state and prove our main result.

Theorem 3.1. *Under the assumptions (H_1)–(H_4), system* (1.1) *has at least one ω-periodic solution if and only if*

(H_5) $\widetilde{g}(0) > 0$,

(H_6) $\widehat{c} > 0$

hold.

Proof. "Only if" part: Suppose that $(u_1(t), u_2(t))^T$ is an ω-periodic solution of system (1.1). Then by integrating (1.1) on both side from κ to $\kappa+\omega$, we have

$$\int_{\kappa}^{\kappa+\omega}\left[g\left(t, e^{u_1(t-\tau_1(t))}\right) - h\left(t, e^{u_1(t)}, e^{u_2(t)}\right)e^{u_2(t-\tau_2(t))-u_1(t)}\right]\Delta t = \int_{\kappa}^{\kappa+\omega} u_1^{\Delta}(t)\ \Delta t = 0, \tag{3.3}$$

$$\int_{\kappa}^{\kappa+\omega}\left[c(t) - d(t)e^{u_2(t)-u_1(t-\tau_3(t))}\right]\ \Delta t = \int_{\kappa}^{\kappa+\omega} u_2^{\Delta}(t)\ \Delta t = 0. \tag{3.4}$$

By (H_4) and the monotonicity of function $\widetilde{g}(x)$, we obtain from (3.3) that

$$\widetilde{g}(0) > \frac{1}{\omega}\int_{\kappa}^{\kappa+\omega} g\left(t, e^{u_1(t-\tau_1(t))}\right)\Delta t = \frac{1}{\omega}\int_{\kappa}^{\kappa+\omega} h\left(t, e^{u_1(t)}, e^{u_2(t)}\right)e^{u_2(t-\tau_2(t))-u_1(t)}\ \Delta t \geq 0, \tag{3.5}$$

which is (H_5).
By (H_2) and (3.4), we have

$$\widehat{c} = \frac{1}{\omega}\int_{\kappa}^{\kappa+\omega} d(t)e^{u_2(t)-u_1(t-\tau_3(t))}\ \Delta t > 0, \tag{3.6}$$

which gives (H_6).

"If" part: Take

$$\begin{gathered} X = Y = \left\{u = (u_1(t), u_2(t))^T \mid u_i \in C_{\mathrm{rd}}(\mathbb{T}),\ u_i(t+\omega) = u_i(t),\quad i = 1,2\right\}, \\ \|u\| = \left\|(u_1(t), u_2(t))^T\right\| = \max_{t\in I_\omega}|u_1(t)| + \max_{t\in I_\omega}|u_2(t)| \end{gathered} \tag{3.7}$$

Then X and Y are Banach spaces with the norm $\|\cdot\|$. Set

$$L:\ \operatorname{Dom} L \subset X \longrightarrow Y, \qquad L\begin{pmatrix} u_1(t) \\ u_2(t)\end{pmatrix} = \begin{pmatrix} u_1^{\Delta}(t) \\ u_2^{\Delta}(t)\end{pmatrix}, \tag{3.8}$$

where $\operatorname{Dom} L = \{u = (u_1(t), u_2(t))^T \in X \mid u_i \in C_{\mathrm{rd}}^1(\mathbb{T}),\ i = 1,2\}$ and

$$N : X \longrightarrow Y, \quad N\begin{pmatrix} u_1(t) \\ u_2(t)\end{pmatrix} = \begin{pmatrix} g\left(t, e^{u_1(t-\tau_1(t))}\right) - h\left(t, e^{u_1(t)}, e^{u_2(t)}\right)e^{u_2(t-\tau_2(t))-u_1(t)} \\ c(t) - d(t)e^{u_2(t)-u_1(t-\tau_3(t))}\end{pmatrix}. \tag{3.9}$$

With these notations system (1.1) can be written in the form

$$Lu = Nu, \quad u \in X. \tag{3.10}$$

Obviously, $\operatorname{Ker} L = \mathbb{R}^2$, $\operatorname{Im} L = \{(u_1(t), u_2(t))^{\mathrm{T}} \in Y : \int_\kappa^{\kappa+\omega} u_i(t)\ \Delta t = 0,\ i = 1, 2\}$ is closed in Y, and $\dim \operatorname{Ker} L = \operatorname{codim} \operatorname{Im} L = 2$. Therefore L is a Fredholm mapping of index zero. Now define two projectors $P : X \to X$ and $Q : Y \to Y$ as

$$P\begin{pmatrix} u_1(t) \\ u_2(t) \end{pmatrix} = Q\begin{pmatrix} u_1(t) \\ u_2(t) \end{pmatrix} = \begin{pmatrix} \widehat{u}_1 \\ \widehat{u}_2 \end{pmatrix}, \quad \begin{pmatrix} u_1(t) \\ u_2(t) \end{pmatrix} \in X = Y. \tag{3.11}$$

Then P and Q are continuous projectors such that

$$\operatorname{Im} P = \operatorname{Ker} L, \quad \operatorname{Ker} Q = \operatorname{Im} L = \operatorname{Im}(I - Q). \tag{3.12}$$

Furthermore, through an easy computation we find that the generalized inverse K_P of L_P has the form

$$\begin{gathered} K_P : \operatorname{Im} L \longrightarrow \operatorname{Dom} L \cap \operatorname{Ker} P, \\ K_P(u) = \int_\kappa^t u(s)\Delta s - \frac{1}{\omega}\int_\kappa^{\kappa+\omega}\int_\kappa^t u(s)\Delta s \Delta t. \end{gathered} \tag{3.13}$$

Then $QN : X \to Y$ and $K_P(I - Q)N : X \to X$ read as

$$QNu = \frac{1}{\omega}\begin{pmatrix} \int_\kappa^{\kappa+\omega}\left[g\left(t, \mathrm{e}^{u_1(t-\tau_1(t))}\right) - h\left(t, \mathrm{e}^{u_1(t)}, \mathrm{e}^{u_2(t)}\right)\mathrm{e}^{u_2(t-\tau_2(t))-u_1(t)}\right]\ \Delta t \\ \int_\kappa^{\kappa+\omega}\left[c(t) - d(t)\mathrm{e}^{u_2(t)-u_1(t-\tau_3(t))}\right]\Delta t \end{pmatrix}, \tag{3.14}$$

$$\begin{aligned} K_P(I - Q)Nu = & \begin{pmatrix} \int_\kappa^t \left[g\left(s, \mathrm{e}^{u_1(s-\tau_1(s))}\right) - h\left(s, \mathrm{e}^{u_1(s)}, \mathrm{e}^{u_2(s)}\right)\mathrm{e}^{u_2(s-\tau_2(s))-u_1(s)}\right]\Delta s \\ \int_\kappa^t \left[c(s) - d(s)\mathrm{e}^{u_2(s)-u_1(s-\tau_3(s))}\right]\Delta s \end{pmatrix} \\ & - \frac{1}{\omega}\left(t - \kappa - \frac{1}{\omega}\int_\kappa^{\kappa+\omega}(s - \kappa)\ \Delta s\right) \\ & \times \begin{pmatrix} \int_\kappa^{\kappa+\omega}\left[g\left(s, \mathrm{e}^{u_1(s-\tau_1(s))}\right) - h\left(s, \mathrm{e}^{u_1(s)}, \mathrm{e}^{u_2(s)}\right)\mathrm{e}^{u_2(s-\tau_2(s))-u_1(s)}\right]\Delta s \\ \int_\kappa^{\kappa+\omega}\left[c(s) - d(s)\mathrm{e}^{u_2(s)-u_1(s-\tau_3(s))}\right]\Delta s \end{pmatrix} \\ & - \frac{1}{\omega}\begin{pmatrix} \int_\kappa^{\kappa+\omega}\int_\kappa^t \left[g\left(s, \mathrm{e}^{u_1(s-\tau_1(s))}\right) - h\left(s, \mathrm{e}^{u_1(s)}, \mathrm{e}^{u_2(s)}\right)\mathrm{e}^{u_2(s-\tau_2(s))-u_1(s)}\right]\Delta s \Delta t \\ \int_\kappa^{\kappa+\omega}\int_\kappa^t \left[c(s) - d(s)\mathrm{e}^{u_2(s)-u_1(s-\tau_3(s))}\right]\Delta s \Delta t \end{pmatrix}. \end{aligned} \tag{3.15}$$

Clearly, QN and $K_P(I - Q)N$ are continuous. By using the Arzela-Ascoli theorem, it is not difficult to prove that $\overline{K_P(I - Q)N(\overline{\Omega})}$ is compact for any open bounded set $\Omega \subset X$. Moreover, $QN(\overline{\Omega})$ is bounded. Therefore N is L-compact on $\overline{\Omega}$ with any open bounded set $\Omega \subset X$.

In order to apply Lemma 2.4, we need to find appropriate open, bounded subsets in X. Corresponding to the operator equation $Lu = \lambda Nu$, $\lambda \in (0, 1)$, we have

$$\begin{aligned} u_1^\Delta(t) &= \lambda\left[g\left(t, \mathrm{e}^{u_1(t-\tau_1(t))}\right) - h\left(t, \mathrm{e}^{u_1(t)}, \mathrm{e}^{u_2(t)}\right)\mathrm{e}^{u_2(t-\tau_2(t))-u_1(t)}\right], \\ u_2^\Delta(t) &= \lambda\left[c(t) - d(t)\mathrm{e}^{u_2(t)-u_1(t-\tau_3(t))}\right]. \end{aligned} \tag{3.16}$$

Suppose that $(u_1(t), u_2(t))^{\mathrm{T}} \in X$ is a solution of (3.16) for a certain $\lambda \in (0,1)$. Integrating (3.16) on both side from κ to $\kappa+\omega$ leads to

$$\begin{aligned}\int_{\kappa}^{\kappa+\omega} \lambda\Big[g\Big(t, \mathrm{e}^{u_1(t-\tau_1(t))}\Big) - h\Big(t, \mathrm{e}^{u_1(t)}, \mathrm{e}^{u_2(t)}\Big)\mathrm{e}^{u_2(t-\tau_2(t))-u_1(t)}\Big]\Delta t &= \int_{\kappa}^{\kappa+\omega} u_1^{\Delta}(t)\ \Delta t = 0, \\ \int_{\kappa}^{\kappa+\omega} \lambda\Big[c(t) - d(t)\mathrm{e}^{u_2(t)-u_1(t-\tau_3(t))}\Big]\ \Delta t &= \int_{\kappa}^{\kappa+\omega} u_2^{\Delta}(t)\ \Delta t = 0.\end{aligned} \tag{3.17}$$

That is

$$\int_{\kappa}^{\kappa+\omega} g\Big(t, \mathrm{e}^{u_1(t-\tau_1(t))}\Big)\Delta t = \int_{\kappa}^{\kappa+\omega} h\Big(t, \mathrm{e}^{u_1(t)}, \mathrm{e}^{u_2(t)}\Big)\mathrm{e}^{u_2(t-\tau_2(t))-u_1(t)}\ \Delta t \tag{3.18}$$

$$\int_{\kappa}^{\kappa+\omega} d(t)\mathrm{e}^{u_2(t)-u_1(t-\tau_3(t))}\Delta t\ = \int_{\kappa}^{\kappa+\omega} c(t)\Delta t = \hat{c}\omega. \tag{3.19}$$

From (3.18), we have

$$\tilde{g}(0)\omega = \int_{\kappa}^{\kappa+\omega}\Big\{\Big[g(t,0) - g\Big(t, \mathrm{e}^{u_1(t-\tau_1(t))}\Big)\Big] + h\Big(t, \mathrm{e}^{u_1(t)}, \mathrm{e}^{u_2(t)}\Big)\mathrm{e}^{u_2(t-\tau_2(t))-u_1(t)}\Big\}\Delta t\ . \tag{3.20}$$

It follows from (3.16), (3.18), (3.19), (3.20), and (H_2)–(H_4) that

$$\begin{aligned}\int_{\kappa}^{\kappa+\omega}\Big|u_1^{\Delta}(t)\Big|\ \Delta t &\le \lambda\int_{\kappa}^{\kappa+\omega}\Big|g\Big(t, \mathrm{e}^{u_1(t-\tau_1(t))}\Big) - h\Big(t, \mathrm{e}^{u_1(t)}, \mathrm{e}^{u_2(t)}\Big)\mathrm{e}^{u_2(t-\tau_2(t))-u_1(t)}\Big|\Delta t \\ &< \int_{\kappa}^{\kappa+\omega}\Big\{\Big[g(t,0) - g\Big(t, \mathrm{e}^{u_1(t-\tau_1(t))}\Big)\Big] + h\Big(t, \mathrm{e}^{u_1(t)}, \mathrm{e}^{u_2(t)}\Big)\mathrm{e}^{u_2(t-\tau_2(t))-u_1(t)}\Big\}\Delta t \\ &\quad + \int_{\kappa}^{\kappa+\omega}\big|g(t,0)\big|\Delta t \\ &= \Big[\tilde{G}(0) + \tilde{g}(0)\Big]\omega,\end{aligned} \tag{3.21}$$

$$\begin{aligned}\int_{\kappa}^{\kappa+\omega}\Big|u_2^{\Delta}(t)\Big|\ \Delta t &\le \lambda\int_{\kappa}^{\kappa+\omega}\Big|c(t) - d(t)\mathrm{e}^{u_2(t)-u_1(t-\tau_3(t))}\Big|\ \Delta t \\ &< \int_{\kappa}^{\kappa+\omega}|c(t)|\ \Delta t + \int_{\kappa}^{\kappa+\omega} d(t)\mathrm{e}^{u_2(t)-u_1(t-\tau_3(t))}\ \Delta t \\ &= \Big[\hat{C} + \hat{c}\Big]\omega.\end{aligned} \tag{3.22}$$

Since $(u_1(t), u_2(t))^{\mathrm{T}} \in X$, there exist $\xi_i, \eta_i \in I_\omega$ such that

$$u_i(\xi_i) = \min_{t\in I_\omega} u_i(t), \quad u_i(\eta_i) = \max_{t\in I_\omega} u_i(t), \quad i = 1, 2. \tag{3.23}$$

Then from (3.19) and (H_2), we have

$$\widehat{c} \leq \frac{1}{\omega}\int_{\kappa}^{\kappa+\omega} d(t)e^{u_2(\eta_2)-u_1(\xi_1)}\ \Delta t = \widehat{d}e^{u_2(\eta_2)-u_1(\xi_1)},$$
$$\widehat{c} \geq \frac{1}{\omega}\int_{\kappa}^{\kappa+\omega} d(t)e^{u_2(\xi_2)-u_1(\eta_1)}\ \Delta t = \widehat{d}e^{u_2(\xi_2)-u_1(\eta_1)}. \tag{3.24}$$

These, together with (H_6), yield

$$u_2(\eta_2) \geq \ln\left[\frac{\widehat{c}}{\widehat{d}}\right] + u_1(\xi_1), \tag{3.25}$$

$$u_2(\xi_2) \leq \ln\left[\frac{\widehat{c}}{\widehat{d}}\right] + u_1(\eta_1). \tag{3.26}$$

From (3.18), (H_4), and the monotonicity of function $\widetilde{g}(x)$, we have

$$\widetilde{g}\left(e^{u_1(\xi_1)}\right) \geq \frac{1}{\omega}\int_{\kappa}^{\kappa+\omega} g\left(t, e^{u_1(t-\tau_1(t))}\right)\ \Delta t \geq 0. \tag{3.27}$$

In view of (H_3), (H_5), and the continuity of function $\widetilde{g}(x)$, it is easy to see that there exists a positive constant α_1 such that

$$\widetilde{g}(\alpha_1) = 0. \tag{3.28}$$

Then, from (3.27), (3.28), and the monotonicity of function $\widetilde{g}(x)$, we have

$$u_1(\xi_1) \leq \ln \alpha_1. \tag{3.29}$$

By Lemma 2.5, we obtain from (3.21) and (3.29) that for all $t \in I_\omega$

$$u_1(t) \leq u_1(\xi_1) + \int_{\kappa}^{\kappa+\omega}\left|u_1^{\Delta}(t)\right|\Delta t \leq \ln \alpha_1 + \left[\widetilde{G}(0) + \widetilde{g}(0)\right]\omega := \beta_1. \tag{3.30}$$

By Lemma 2.5, we also obtain from (3.22), (3.26), and (3.30) that for all $t \in I_\omega$

$$u_2(t) \leq u_2(\xi_2) + \int_{\kappa}^{\kappa+\omega}\left|u_2^{\Delta}(t)\right|\Delta t \leq \beta_1 + \ln\left[\frac{\widehat{c}}{\widehat{d}}\right] + \left[\widehat{C} + \widehat{c}\right]\omega := \beta_2. \tag{3.31}$$

It follows from $(H_4)$and (3.30) that

$$0 \leq \frac{1}{\omega}\int_{\kappa}^{\kappa+\omega} h\left(t, e^{u_1(t)}, e^{u_2(t)}\right)e^{-u_1(t)}\ \Delta t \leq \widehat{a}_0 e^{(m-1)\beta_1} + \widehat{a}_1 e^{(m-2)\beta_1} + \cdots + \widehat{a}_{m-1} := c_0. \tag{3.32}$$

In order to obtain β_3 and β_4 such that $u_1(t) \geq \beta_3$ and $u_2(t) \geq \beta_4$ for all $t \in I_\omega$, we consider the following two cases.

Case 1. If $u_2(\eta_2) \geq u_1(\eta_1)$, then from (3.18), (3.23), (3.32), (H_4), and monotonicity of function $\widetilde{g}(x)$, we have

$$\begin{aligned}\widetilde{g}\left(e^{u_2(\eta_2)}\right) &\leq \widetilde{g}\left(e^{u_1(\eta_1)}\right) \leq \frac{1}{\omega}\int_\kappa^{\kappa+\omega} g\left(t, e^{u_1(t-\tau_1(t))}\right)\,\Delta t \\ &\leq \frac{e^{u_2(\eta_2)}}{\omega}\int_\kappa^{\kappa+\omega} h\left(t, e^{u_1(t)}, e^{u_2(t)}\right)e^{-u_1(t)}\,\Delta t \leq c_0 e^{u_2(\eta_2)}.\end{aligned} \tag{3.33}$$

From (H_3), (H_5), and the continuity of function $\widetilde{g}(x)$, one can easily see that there exists a positive constant α_2 such that

$$\widetilde{g}(\alpha_2) - c_0\alpha_2 = 0. \tag{3.34}$$

Then, from (3.33), (3.34), and the monotonicity of function $\widetilde{g}(x) - c_0 x$, we have

$$u_2(\eta_2) \geq \ln\alpha_2. \tag{3.35}$$

By Lemma 2.5, we obtain from (3.22) and (3.35) that for all $t \in I_\omega$

$$u_2(t) \geq u_2(\eta_2) - \int_\kappa^{\kappa+\omega}\left|u_2^\Delta(t)\right|\Delta t \geq \ln\alpha_2 - \left[\widehat{C} + \widehat{c}\,\right]\omega := \beta_4^1. \tag{3.36}$$

By Lemma 2.5, we also obtain from (3.21), (3.26) that for all $t \in I_\omega$

$$u_1(t) \geq u_1(\eta_1) - \int_\kappa^{\kappa+\omega}\left|u_1^\Delta(t)\right|\Delta t \geq \beta_4^1 - \ln\left[\frac{\widehat{c}}{\widehat{d}}\right] - \left[\widetilde{G}(0) + \widetilde{g}(0)\right]\omega := \beta_3^1. \tag{3.37}$$

Case 2. If $u_2(\eta_2) < u_1(\eta_1)$, then from (3.18), (3.23), (3.32), (H_4), and monotonicity of function $\widetilde{g}(x)$, we have

$$\begin{aligned}\widetilde{g}\left(e^{u_1(\eta_1)}\right) &\leq \frac{1}{\omega}\int_\kappa^{\kappa+\omega} g\left(t, e^{u_1(t-\tau_1(t))}\right)\,\Delta t \leq \frac{e^{u_2(\eta_2)}}{\omega}\int_\kappa^{\kappa+\omega} h\left(t, e^{u_1(t)}, e^{u_2(t)}\right)e^{-u_1(t)}\,\Delta t \\ &\leq c_0 e^{u_2(\eta_2)} \leq c_0 e^{u_1(\eta_1)}.\end{aligned} \tag{3.38}$$

Then, from (3.34) and the monotonicity of function $\widetilde{g}(x) - c_0 x$, we have

$$u_1(\eta_1) \geq \ln\alpha_2. \tag{3.39}$$

By Lemma 2.5, we obtain from (3.21), (3.39) that for all $t \in I_\omega$

$$u_1(t) \geq u_1(\eta_1) - \int_\kappa^{\kappa+\omega}\left|u_1^\Delta(t)\right|\Delta t \geq \ln\alpha_2 - \left[\widetilde{G}(0) + \widetilde{g}(0)\right]\omega := \beta_3^2. \tag{3.40}$$

By Lemma 2.5, we also obtain from (3.22), (3.25), and (3.40) that for all $t \in I_\omega$

$$u_2(t) \geq u_2(\eta_2) - \int_\kappa^{\kappa+\omega} \left| u_2^\Delta(t) \right| \Delta t \geq \beta_3^2 + \ln\left[\frac{\widehat{c}}{\widehat{d}}\right] - \left[\widehat{C} + \widehat{c}\right]\omega := \beta_4^2. \tag{3.41}$$

Now, we take $\beta_3 =: \min\{\beta_3^1, \beta_3^2\}$ and $\beta_4 =: \min\{\beta_4^1, \beta_4^2\}$. Then it follows from (3.36), (3.37), (3.40), and (3.41) that for all $t \in I_\omega$, $u_1(t) \geq \beta_3$ and $u_2(t) \geq \beta_4$. Hence from these, (3.30), and (3.31), we have

$$\sup_{t\in I_\omega} |u_1(t)| \leq \max\{|\beta_1|, |\beta_3|\} := \beta_5, \qquad \sup_{t\in I_\omega} |u_1(t)| \leq \max\{|\beta_2|, |\beta_4|\} := \beta_6. \tag{3.42}$$

Clearly, β_5 and β_6 are independent of λ.

On the other hand, for $\mu \in [0,1]$, we consider the following algebraic system:

$$\begin{gathered} \widetilde{g}(\mathrm{e}^{u_1}) - \mu\overline{h}(\mathrm{e}^{u_1}, \mathrm{e}^{u_2})\mathrm{e}^{u_2-u_1} = 0, \\ \widehat{c} - \widehat{d}\mathrm{e}^{u_2-u_1} = 0, \end{gathered} \tag{3.43}$$

where $(u_1, u_2)^T \in \mathbb{R}^2$. From the second equation of (3.43) and (H_6), we have

$$u_2 = u_1 + \ln\left[\frac{\widehat{c}}{\widehat{d}}\right]. \tag{3.44}$$

From the first equation of (3.43) and (H_4), we also have

$$\widetilde{g}(\mathrm{e}^{u_1}) = \mu\overline{h}(\mathrm{e}^{u_1}, \mathrm{e}^{u_2})\mathrm{e}^{u_2-u_1} \geq 0. \tag{3.45}$$

Then, from (3.44) and the monotonicity of function $\widetilde{g}(x)$, we obtain

$$u_1 \leq \ln \alpha_1. \tag{3.46}$$

Substituting (3.44) into the first equation of (3.43), we can get from (H_4), (3.30), (3.32), and (3.46) that

$$\begin{aligned} \widetilde{g}(\mathrm{e}^{u_1}) &= \mu\frac{\widehat{c}}{\widehat{d}}\overline{h}(\mathrm{e}^{u_1}, \mathrm{e}^{u_2}) \leq \frac{\widehat{c}}{\widehat{d}}\overline{h}(\mathrm{e}^{u_1}, \mathrm{e}^{u_2}) \leq \frac{\widehat{c}}{\widehat{d}}\left[\widehat{a}_0\mathrm{e}^{(m-1)u_1} + \widehat{a}_1\mathrm{e}^{(m-2)u_1} + \cdots + \widehat{a}_{m-1}\right]\mathrm{e}^{u_1} \\ &\leq \frac{\widehat{c}}{\widehat{d}}\left[\widehat{a}_0\mathrm{e}^{(m-1)\beta_1} + \widehat{a}_1\mathrm{e}^{(m-2)\beta_1} + \cdots + \widehat{a}_{m-1}\right]\mathrm{e}^{u_1} = c_0\frac{\widehat{c}}{\widehat{d}}\mathrm{e}^{u_1}. \end{aligned} \tag{3.47}$$

In view of (H_3), (H_5), and the continuity of function $\widetilde{g}(x)$, it is easy to see that there exists a positive constant α_3 such that

$$\widetilde{g}(\alpha_3) - c_0\frac{\widehat{c}}{\widehat{d}}\alpha_3 = 0. \tag{3.48}$$

Then, from (3.47), (3.48), and the monotonicity of function $\widetilde{g}(x)$, we obtain

$$u_1 \geq \ln \alpha_3. \tag{3.49}$$

It follows from (3.44), (3.46), and (3.49) that

$$|u_1| \leq \max\{|\ln \alpha_1|, |\ln \alpha_3|\} := \beta_7, \qquad |u_2| \leq \beta_7 + \left|\ln \widehat{c} - \ln \widehat{d}\right| := \beta_8. \tag{3.50}$$

Clearly, β_7 and β_8 are also independent of μ.

We take $\Omega = \{u = (u_1(t), u_2(t))^{\mathrm{T}} \in X \mid \|u\| < \beta\}$, here $\beta = \beta_5 + \beta_6 + \beta_7 + \beta_8$. Now we check the conditions of Lemma 2.4.

(a) By (3.42), one can conclude that for each $\lambda \in (0,1)$, $u \in \partial\Omega \cap \operatorname{Dom} L$, $Lu \neq \lambda N u$.

(b) When $(u_1(t), u_2(t))^{\mathrm{T}} \in \partial\Omega \cap \operatorname{Ker} L$, $(u_1(t), u_2(t))^{\mathrm{T}}$ is a constant vector in $\mathbb{R}^2$, we denote it by $(u_1, u_2)^{\mathrm{T}}$ and $|u_1| + |u_2| = \beta$. If

$$QNu = \begin{pmatrix} \widetilde{g}(e^{u_1}) - \overline{h}(e^{u_1}, e^{u_2})e^{u_2-u_1} \\ \widehat{c} - \widehat{d}e^{u_2-u_1} \end{pmatrix} = 0, \tag{3.51}$$

then $(u_1, u_2)^{\mathrm{T}}$ is a constant solution of system (3.43) with $\mu = 1$. By (3.50), we have $|u_1| + |u_2| \leq \beta_7 + \beta_8 < \beta$. This contradiction implies that for each $u \in \partial\Omega \cap \operatorname{Ker} L, QNu \neq 0$.

(c) In order to verify the condition (c) in Lemma 2.4, we define $\phi : \mathbb{R}^2 \times [0,1] \to \mathbb{R}^2$ by

$$\phi(u_1, u_2, \mu) = \begin{pmatrix} \widetilde{g}(e^{u_1}) \\ \widehat{c} - \widehat{d}e^{u_2-u_1} \end{pmatrix} + \mu \begin{pmatrix} -\overline{h}(e^{u_1}, e^{u_2})e^{u_2-u_1} \\ 0 \end{pmatrix}, \tag{3.52}$$

where $\mu \in [0,1]$ is a parameter. When $(u_1, u_2)^{\mathrm{T}} \in \partial\Omega \cap \operatorname{Ker} L$, $(u_1, u_2)^{\mathrm{T}}$ is a constant vector in $\mathbb{R}^2$ with $|u_1| + |u_2| = \beta$. By (3.50) we know $\phi(u_1, u_2, \mu) \neq (0,0)^{\mathrm{T}}$ on $\partial\Omega \cap \operatorname{Ker} L$. Thus, ϕ is a homotopy mapping. Moreover, it is not difficult to see that the following algebraic system:

$$\widetilde{g}(e^{u_1}) = 0, \qquad \widehat{c} - \widehat{d}e^{u_2-u_1} = 0, \tag{3.53}$$

has a unique solution $(u_1^{\dagger}, u_2^{\dagger})^{\mathrm{T}} \in \partial\Omega \cap \operatorname{Ker} L$. So, due to homotopy invariance theorem of topology degree and taking $J = I : \operatorname{Im} Q \to \operatorname{Ker} L, (u_1, u_2)^{\mathrm{T}} \to (u_1, u_2)^{\mathrm{T}}$, we obtain

$$\begin{aligned} \deg\left(JQN, \Omega \cap \operatorname{Ker} L, (0,0)^{\mathrm{T}}\right) &= \deg\left(\phi(u_1, u_2, 1), \Omega \cap \operatorname{Ker} L, (0,0)^{\mathrm{T}}\right) \\ &= \deg\left(\phi(u_1, u_2, 0), \Omega \cap \operatorname{Ker} L, (0,0)^{\mathrm{T}}\right) = \operatorname{sign}\left\{-\widetilde{g}\left(e^{u_1^{\dagger}}\right)e^{u_2^{\dagger}}\right\} \neq 0. \end{aligned} \tag{3.54}$$

By now we have proved that Ω satisfies all the requirements in Lemma 2.4. Hence, system (1.1) has at least one ω-periodic solution. This completes the proof. □

Noticing that both systems (1.2) and (1.3) are special cases of system (1.1), by Theorem 3.1, we can obtain the following results.

Theorem 3.2. *Under the assumptions* (H_1)–(H_4), *system* (1.2) *has at least one positive* ω*-periodic solution if and only if* (H_5) *and* (H_6) *hold.*

Theorem 3.3. *Under the assumptions* (H_1)–(H_4), *system* (1.3) *has at least one positive* ω*-periodic solution if and only if* (H_5) *and* (H_6) *hold.*

The proof of Theorem 3.1 shows that it remains valid for the following periodic semi-ratio-dependent predator-prey system on a time scale:

$$\begin{aligned} u_1^{\Delta}(t) &= g\left(t, e^{u_1(t-\tau_1(t))}\right) - h\left(t, e^{u_1(t)}, e^{u_2(t)}\right) e^{u_2(t-\tau_2(t))-u_1(t)}, \\ u_2^{\Delta}(t) &= c(t) - d(t) e^{u_2(t-\tau_3(t))-u_1(t-\tau_3(t))}. \end{aligned} \tag{3.55}$$

$$\begin{aligned} x'(t) &= x(t) g(t, x(t-\tau_1(t))) - h(t, x(t), y(t)) y(t-\tau_2(t)), \\ y'(t) &= y(t)\left[c(t) - d(t)\frac{y(t-\tau_3(t))}{x(t-\tau_3(t))}\right], \end{aligned} \tag{3.56}$$

$$\begin{aligned} x(k+1) &= x(k)\exp\left[g(k, x(k-\tau_1(k))) - h(k, x(k), y(k))\frac{y(k-\tau_2(k))}{x(k)}\right], \\ y(k+1) &= y(k)\exp\left[c(k) - d(k)\frac{y(k-\tau_3(k))}{x(k-\tau_3(k))}\right], \end{aligned} \tag{3.57}$$

Remark 3.4. One can easily verify that if their parameters are positive ω-periodic functions, all the prey growth types and the functional responses mentioned previously satisfy the assumptions of Theorem 3.1. Therefore, by Theorem 3.1, the system (1.1) with the logistic, the Gilpin, or the Smith prey growth and with the Lotka-Volterra, the Holling, the sigmoidal, the Ivlev, the Monod-Haldane, or the Beddington-DeAngelis functional responses always has at least one ω-periodic solution.

Remark 3.5. Similarly, by Theorems 3.2 and 3.3, the systems (1.2) and (1.3) with the logistic, the Gilpin, or the Smith prey growth and with the Lotka-Volterra, the Holling, the sigmoidal, the Ivlev, the Monod-Haldane, or the Beddington-DeAngelis functional responses, always have at least one positive ω-periodic solution, respectively.

Remark 3.6. Bohner et al. [19], Fazly and Hesaaraki [21] studied the special cases of system (1.1) for $\tau_1(t) = \tau_2(t) = \tau_3(t) = 0$, $g(t,x) = a(t) - b(t)x$, and $h(t,x,y) = p(t,x)$. Zhuang [26] studied the special case of system (1.1) for $\tau_2(t) = 0$, $g(t,x) = a(t) - b(t)x$, and $h(t,x,y) = k(t)x/(m^2 + x^2)$. Therefore, our Theorem 3.1 generalizes and improves Theorem 3.4 in [19] and Theorem 3.1 in [26] and generalizes Theorem 1 in [21].

Remark 3.7. Wang et al. [13] studied the special case of system (1.2) for $\tau_1(t) = \tau_2(t) = \tau_3(t) = 0$, $g(t,x) = a(t) - b(t)x$, and $h(t,x,y) = p(t,x)$. Ding et al. [10] studied the special case of system (1.2) for $\tau_2(t) = \tau_3(t) = 0$, $g(t,x) = a(t) - b(t)x$, and $h(t,x,y) = k(t)x/(m^2 + x^2)$. Ding and Jiang [8] studied the special case of system (1.2) for $h(t,x,y) = p(t,x)$. Liu [11] studied the special case of system (1.2) for $\tau_1(t) = \tau_2(t) = \tau_3(t) = 0$ and $g(t,x) = a(t) - b(t)x$. Liu and Huang [12] studied the special case of system (3.56) for $\tau_1(t) = \tau_2(t) = 0$ and $g(t,x) = a(t)-b(t)x$. Ding and Jiang [9] studied the special case of system (3.56) for $\tau_1(t) = 0$. Therefore, our Theorem 3.2 generalizes and improves Theorem 3.3 in [13] and Theorem 2.1 in [10] and generalizes Theorem 2.2 in [8], Theorem 2.2 in [9], Theorem 2.1 in [11], and Corollary 3.1 in [12].

Remark 3.8. Fan and Wang [14], Fazly and Hesaaraki [15] studied the special cases of system (1.3) for $\tau_1(k) = \tau_2(k) = \tau_3(k) = 0$, $g(k,x) = a(k) - b(k)x$, and $h(k,x,y) = p(k,x)$. Liu [16] studied the special case of system (1.3) for $\tau_1(k) = \tau_2(k) = \tau_3(k) = 0$ and $g(k,x) = a(k)-b(k)x$. Therefore, our Theorem 3.3 generalizes and improves Theorem 2.1 in [14] and generalizes Theorem 1 in [15] and Theorem 2.1 in [16].

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References

[1] X. Ding and J. Jiang, "Multiple periodic solutions in generalized Gause-type predator-prey systems with non-monotonic numerical responses," *Nonlinear Analysis: Real World Applications*, vol. 10, no. 5, pp. 2819–2827, 2009.

[2] Y. Li and Y. Kuang, "Periodic solutions in periodic state-dependent delay equations and population models," *Proceedings of the American Mathematical Society*, vol. 130, no. 5, pp. 1345–1353, 2002.

[3] G. Liu and J. Yan, "Existence of positive periodic solutions for neutral delay Gause-type predator-prey system," *Applied Mathematical Modelling*, vol. 35, no. 12, pp. 5741–5750, 2011.

[4] X. Tang and X. Zou, "On positive periodic solutions of Lotka-Volterra competition systems with deviating arguments," *Proceedings of the American Mathematical Society*, vol. 134, no. 10, pp. 2967–2974, 2006.

[5] Z. Zhang and J. Luo, "Multiple periodic solutions of a delayed predator-prey system with stage structure for the predator," *Nonlinear Analysis: Real World Applications*, vol. 11, no. 5, pp. 4109–4120, 2010.

[6] X. Chen, "Periodicity in a nonlinear discrete predator-prey system with state dependent delays," *Nonlinear Analysis: Real World Applications*, vol. 8, no. 2, pp. 435–446, 2007.

[7] W.-T. Li and H.-F. Huo, "Positive periodic solutions of delay difference equations and applications in population dynamics," *Journal of Computational and Applied Mathematics*, vol. 176, no. 2, pp. 357–369, 2005.

[8] X. Ding and J. Jiang, "Periodicity in a generalized semi-ratio-dependent predator-prey system with time delays and impulses," *Journal of Mathematical Analysis and Applications*, vol. 360, no. 1, pp. 223–234, 2009.

[9] X. Ding and J. Jiang, "Positive periodic solutions for a generalized two-species semi-ratio-dependent predator-prey system in a two-path environment," *Mathematical and Computer Modelling*, vol. 52, no. 1-2, pp. 361–369, 2010.

[10] X. Ding, C. Lu, and M. Liu, "Periodic solutions for a semi-ratio-dependent predator-prey system with nonmonotonic functional response and time delay," *Nonlinear Analysis: Real World Applications*, vol. 9, no. 3, pp. 762–775, 2008.

[11] X. Liu, "A note on periodic solutions for semi-ratio-dependent predator-prey systems," *Applied Mathematics-A Journal of Chinese Universities Series B*, vol. 25, no. 1, pp. 1–8, 2010.
[12] X. Liu and L. Huang, "Periodic solutions for impulsive semi-ratio-dependent predator-prey systems," *Nonlinear Analysis: Real World Applications*, vol. 10, no. 5, pp. 3266–3274, 2009.
[13] Q. Wang, M. Fan, and K. Wang, "Dynamics of a class of nonautonomous semi-ratio-dependent predator-prey systems with functional responses," *Journal of Mathematical Analysis and Applications*, vol. 278, no. 2, pp. 443–471, 2003.
[14] M. Fan and Q. Wang, "Periodic solutions of a class of nonautonomous discrete time semi-ratio-dependent predator-prey systems," *Discrete and Continuous Dynamical Systems B*, vol. 4, no. 3, pp. 563–574, 2004.
[15] M. Fazly and M. Hesaaraki, "Periodic solutions for a discrete time predator-prey system with monotone functional responses," *Comptes Rendus Mathématique*, vol. 345, no. 4, pp. 199–202, 2007.
[16] X. Liu, "A note on the existence of periodic solutions in discrete predator-prey models," *Applied Mathematical Modelling*, vol. 34, no. 9, pp. 2477–2483, 2010.
[17] M. Bohner and A. Peterson, *Dynamic Equations on Time Scales: An Introduction with Applications*, Birkhäuser, Boston, Mass, USA, 2001.
[18] M. Bohner and A. Peterson, *Advances in Dynamic Equations on Time Scales*, Birkhäuser, Boston, Mass, USA, 2003.
[19] M. Bohner, M. Fan, and J. Zhang, "Existence of periodic solutions in predator-prey and competition dynamic systems," *Nonlinear Analysis: Real World Applications*, vol. 7, no. 5, pp. 1193–1204, 2006.
[20] X. Ding, J. Hao, and C. Liu, "Multiple periodic solutions for a delayed predator-prey system on time scales," *World Academy of Science, Engineering and Technology*, vol. 60, pp. 1532–1537, 2011.
[21] M. Fazly and M. Hesaaraki, "Periodic solutions for a semi-ratio-dependent predator-prey dynamical system with a class of functional responses on time scales," *Discrete and Continuous Dynamical Systems B*, vol. 9, no. 2, pp. 267–279, 2008.
[22] Y. Li and H. Zhang, "Existence of periodic solutions for a periodic mutualism model on time scales," *Journal of Mathematical Analysis and Applications*, vol. 343, no. 2, pp. 818–825, 2008.
[23] Z. Zeng, "Periodic solutions for a delayed predator-prey system with stage-structured predator on time scales," *Computers & Mathematics with Applications*, vol. 61, no. 11, pp. 3298–3311, 2011.
[24] J. Zhang, M. Fan, and H. Zhu, "Periodic solution of single population models on time scales," *Mathematical and Computer Modelling*, vol. 52, no. 3-4, pp. 515–521, 2010.
[25] L. Zhang, H.-X. Li, and X.-B. Zhang, "Periodic solutions of competition Lotka-Volterra dynamic system on time scales," *Computers & Mathematics with Applications*, vol. 57, no. 7, pp. 1204–1211, 2009.
[26] K. Zhuang, "Periodicity for a semi-ratio-dependent predator-prey system with delays on time scales," *International Journal of Difference Equations*, vol. 4, no. 2, pp. 273–281, 2009.
[27] R. E. Gaines and J. L. Mawhin, *Coincidence Degree and Nonlinear Differential Equations*, Springer, Berlin, Germany, 1977.