Hindawi Publishing Corporation
Discrete Dynamics in Nature and Society
Volume 2012, Article ID 961642, 10 pages
doi:10.1155/2012/961642

Research Article

Nearly Quadratic *n*-Derivations on Non-Archimedean Banach Algebras

Madjid Eshaghi Gordji,[1] Badrkhan Alizadeh,[2] Young Whan Lee,[3] and Gwang Hui Kim[4]

[1] *Department of Mathematics, Semnan University, P.O. Box 35195-363, Semnan, Iran*
[2] *Technical and Vocational University of Iran, Technical and Vocational Faculty of Tabriz, P.O. Box 51745-135, Tabriz, Iran*
[3] *Department of Computer Hacking and Information Security, Daejeon University, Dong-gu, Daejeon 300-716, Republic of Korea*
[4] *Department of Mathematics, Kangnam University, Yongin, Gyeonggi 446-702, Republic of Korea*

Correspondence should be addressed to Young Whan Lee, ywlee@dju.kr

Received 27 January 2012; Revised 18 March 2012; Accepted 19 March 2012

Academic Editor: John Rassias

Let $n > 1$ be an integer, let A be an algebra, and X be an A-module. A quadratic function $D : A \to X$ is called a quadratic n-derivation if $D(\prod_{i=1}^{n} a_i) = D(a_1)a_2^2 \cdots a_n^2 + a_1^2 D(a_2)a_3^2 \cdots a_n^2 + \cdots + a_1^2 a_2^2 \cdots a_{n-1}^2 D(a_n)$ for all $a_1,\ldots,a_n \in A$. We investigate the Hyers-Ulam stability of quadratic n-derivations from non-Archimedean Banach algebras into non-Archimedean Banach modules by using the Banach fixed point theorem.

1. Introduction

A functional equation (ξ) is stable if any function g satisfying the equation (ξ) approximately is near to a true solution of (ξ).

The stability of functional equations was first introduced by Ulam [1] in 1964. In 1941, Hyers [2] gave a first affirmative answer to the question of Ulam for Banach spaces. In 1978, Th. M. Rassias [3] generalized the theorem of Hyers by considering the stability problem with unbounded Cauchy differences $\|f(x+y) - f(x) - f(y)\| \le \epsilon(\|x\|^p + \|y\|^p)$, $(\epsilon > 0, p \in [0,1))$. In 1994, a generalization of Th. M. Rassias theorem was obtained by Găvruţa [4], who replaced the bound $\epsilon(\|x\|^p + \|y\|^p)$ by a general control function $\varphi(x,y)$ (see also [5–7]).

Every solution of the following functional equation

$$f(x+y) + f(x-y) = 2f(x) + 2f(y) \tag{1.1}$$

is said to be a quadratic function [8]. It is well known that a mapping f between real vector spaces is quadratic mapping if and only if there exists a unique symmetric biadditive mapping B_1 such that $f(x) = B_1(x,x)$ for all x. The biadditive mapping B_1 is given by $B_1(x,y) = (1/4)(f(x+y) - f(x-y))$.

The stability problem of the quadratic functional equation was proved by Skof [9] for mappings $f : A \to B$, where A is a normed space and B is a Banach space (see also [10, 11]). Let A be an algebra and let X be a A-bimodule. A quadratic function $D : A \to X$ is called a quadratic n-derivation if

$$D\left(\prod_{i=1}^{n} a_i\right) = D(a_1)a_2^2 \cdots a_n^2 + a_1^2 D(a_2)a_3^2 \cdots a_n^2 + \cdots + a_1^2 a_2^2 \cdots a_{n-1}^2 D(a_n) \tag{1.2}$$

for all $a_1, \ldots, a_n \in A$. Recently, Gordji and Ghobadipour [12] introduced the quadratic derivations on Banach algebras. Indeed, they investigated the Hyers-Ulam-Aoki-Rassias stability and Ulam-Gavruta-Rassias type stability of quadratic derivations on Banach algebras.

More recently, Gordji et al. [13] investigated the Hyers-Ulam stability and the superstability of higher ring derivations on non-Archimedean Banach algebras (see also [12–32]). In this paper we investigate the Hyers-Ulam stability of quadratic n-derivations from non-Archimedean Banach algebras into non-Archimedean Banach modules by using the weighted space method (see [33]).

2. Preliminaries

Let us recall that a non-Archimedean field is a field $\mathbb{K}$ equipped with a function (valuation) $|\cdot|$ from $\mathbb{K}$ into $[0,\infty)$ such that $|r| = 0$ if and only if $r = 0$, $|rs| = |r||s|$, and $|r+s| \leq \max\{|r|, |s|\}$ for all $r, s \in \mathbb{K}$. An example of a non-Archimedean valuation is the mapping $|\cdot|$ taking everything but 0 into 1 and $|0| = 0$. This valuation is called trivial (see [34]).

Definition 2.1. Let X be a vector space over a scalar field $\mathbb{K}$ with a non-Archimedean non-trivial valuation $|\cdot|$. A function $\|\cdot\| : X \to \mathbb{R}$ is a non-Archimedean norm (valuation) if it satisfies the following conditions:

(NA_1) $\|x\| = 0$ if and only if $x = 0$;

(NA_2) $\|rx\| = |r|\|x\|$ for all $r \in \mathbb{K}$ and $x \in X$;

(NA_3) $\|x+y\| \leq \max\{\|x\|, \|y\|\}$ for all $x, y \in X$ (the strong triangle inequality).

In 1897, Hensel [35] introduced a normed space which does not have the Archimedean property. It turned out that non-Archimedean spaces have many nice applications. The most important examples of non-Archimedean spaces are p-adic numbers. Let p be a prime number. For any nonzero rational number $x = (a/b)p^{n_x}$ such that a and b are integers not divisible by p, define the p-adic absolute value $|x|_p := p^{-n_x}$. Then $|\cdot|_p$ is a non-Archimedean norm on $\mathbb{Q}$. The completion of $\mathbb{Q}$ with respect to $|\cdot|_p$ is denoted by $\mathbb{Q}_p$ which is called the p-adic number field.

Definition 2.2. Let X be a nonempty set and let $d : X \times X \to [0,\infty)$ satisfy the following properties:

(D_1) $d(x,y) = 0$ if and only if $x = y$,

(D_2) $d(x,y) = d(y,x)$ (symmetry),

(D_3) $d(x,z) \le \max\{d(x,y), d(y,z)\}$ (strong triangle inequality),

for all $x,y,z \in X$. Then (X,d) is called a non-Archimedean metric space. (X,d) is called a non-Archimedean complete metric space if every d-Cauchy sequence in X is d-convergent.

Theorem 2.3 (Non-Archimedean Banach Contraction Principle). *Let (X,d) be a non-Archimedean complete metric space and let $T : X \to X$ be a contraction; that is, there exists $\alpha \in [0,1)$ such that*

$$d(Tx,Ty) \le \alpha d(x,y), \quad \forall x,y \in X. \tag{2.1}$$

Then there exists a unique element $a \in X$ such that $Ta = a$. Moreover, $a = \lim_{n\to\infty} T^n x$, and

$$d(a,x) \le d(x,Tx), \quad \forall x \in X. \tag{2.2}$$

Proof. A similar argument as Archimedean case can be applied to show that T has a unique element $a \in X$ such that $Ta = a$ and $a = \lim_{n\to\infty} T^n x$. It follows from strong triangle inequality that for all $x \in X$ and for each $n \in \mathbb{N}$, we have

$$\begin{aligned} d(T^n x, x) &\le \max\left\{d(T(x),x),\dots,d\left(T^n(x),T^{n-1}(x)\right)\right\} \\ &\le \max\left\{d(T(x),x),\dots,\alpha^{n-1}d(T(x),(x))\right\} \\ &= d(T(x),x). \end{aligned} \tag{2.3}$$

□

3. Main Results

In this section A denotes a non-Archimedean Banach algebra over a non-Archimedean field $\mathbb{K}$ and X is a non-Archimedean Banach A-module.

Theorem 3.1. *Let $\varphi : A \times A \to [0,\infty)$, $\psi : A \times \cdots \times A \to [0,\infty)$ be functions. Let $f : A \to X$ be a given mapping such that $f(0) = 0$,*

$$\|f(x+y) + f(x-y) - 2f(x) - 2f(y)\| \le \varphi(x,y) \tag{3.1}$$

and that

$$\left\| f\left(\prod_{i=1}^{n} x_i\right) - f(x_1)x_2^2\cdots x_n^2 - x_1^2 f(x_2)x_3^2\cdots x_n^2 - \cdots - x_1^2\cdots x_{n-1}^2 f(x_n)\right\| \le \psi(x_1,\dots,x_n) \tag{3.2}$$

for all $x_1,\ldots,x_n,x,y \in A$. Suppose that there exist a natural number $k \in \mathbb{K}$ and $L,K \in (0,1)$, such that

$$|k|^2\varphi\left(k^{-1}x,k^{-1}y\right) \le L\varphi(x,y), \qquad |k|^2\psi\left(k^{-1}x_1,\ldots,k^{-1}x_n\right) \le K\psi(x_1,\ldots,x_n) \tag{3.3}$$

for all $x_1,\ldots,x_n,x,y \in A$. Then there exists a unique quadratic n-derivation h from A into X such that

$$\|f(x)-h(x)\| \le \frac{L\Phi(x)}{|k|^2} \tag{3.4}$$

for all $x \in A$, where

$$\Phi(x) = \max\{\varphi(0,0),\varphi(x,x),\varphi(2x,x),\ldots,\varphi((k-1)x,x)\} \quad (x \in A). \tag{3.5}$$

Proof. By induction on i, one can show that for all $x \in A$ and $i \ge 2$,

$$\left\|f(ix)-i^2f(x)\right\| \le \max\{\varphi(0,0),\varphi(x,x),\varphi(2x,x),\ldots,\varphi((i-1)x,x)\}. \tag{3.6}$$

Let $x=y$ in (3.1). Then

$$\left\|f(2x)-2^2f(x)\right\| \le \max\{\varphi(0,0),\varphi(x,x)\} \quad (x \in A). \tag{3.7}$$

This proves (3.6) for $i=2$. Let (3.6) hold for $i=1,2,\ldots,j$. Replacing x by jx and y by x in (3.1) for all $x \in A$, we get

$$\|f((j+1)x)+f((j-1)x)-2f(jx)-2f(x)\| \le \max\{\varphi(0,0),\varphi(jx,x)\} \tag{3.8}$$

for all $x \in A$. Since

$$\begin{aligned} f((j+1)x)+f((j-1)x)-2f(jx)-2f(x) &= f((j+1)x)-(j+1)^2f(x) \\ &\quad + f((j-1)x)-(j-1)^2f(x)-2\left[f(jx)-j^2f(x)\right] \end{aligned} \tag{3.9}$$

for all $x \in A$, it follows from induction hypothesis and (3.8) that for all $x \in A$,

$$\begin{aligned} \left\|f((j+1)x)-(j+1)^2f(x)\right\| &\le \max\Big\{\|f((j+1)x)+f((j-1)x)-2f(jx)-2f(x)\|, \\ &\qquad \left\|f((j-1)x)-(j-1)^2f(x)\right\|, |2|\left\|j^2f(x)-f(jx)\right\|\Big\} \\ &\le \max\{\varphi(0,0),\varphi(x,x),\varphi(2x,x),\ldots,\varphi((j)x,x)\}. \end{aligned} \tag{3.10}$$

This proves (3.6) for all $i \geq 2$. In particular

$$\left\| f(kx) - k^2 f(x) \right\| \leq \Phi(x) \quad (x \in A). \tag{3.11}$$

Replacing x by $k^{-1}x$ in (3.11), we get

$$\left\| f(x) - k^2 f\left(k^{-1}x\right) \right\| \leq \Phi\left(k^{-1}x\right) \leq \frac{L}{|k|^2}\Phi(x) \tag{3.12}$$

for all $x \in A$. Let Ω be the set of all functions $u : A \to X$. We define the metric d on Ω as follows:

$$d(u,v) = \sup_{x \in A} D(x), \tag{3.13}$$

where $D(x) = (\|u(x) - v(x)\|)/\Phi(x)$ if $\Phi(x) \neq 0$ and $D(x) = \|u(x) - v(x)\|$ if $\Phi(x) = 0$. One has the operator $J : \Omega \to \Omega$ by $J(u)(x) = k^2 u(k^{-1}x)$. Then J is strictly contractive on Ω; in fact, if

$$\|u(x) - v(x)\| \leq \alpha\Phi(x) \quad (x \in A), \tag{3.14}$$

then by (3.3),

$$\begin{aligned} \|J(u)(x) - J(v)(x)\| &= |k|^2 \left\| u\left(k^{-1}x\right) - v\left(k^{-1}x\right) \right\| \\ &\leq \alpha |k|^2 \Phi\left(k^{-1}x\right) \leq L\alpha\Phi(x), \quad (x \in A). \end{aligned} \tag{3.15}$$

It follows that

$$d(J(u), J(v)) \leq L d(u,v) \quad (u, v \in \Omega). \tag{3.16}$$

Hence J is a contractive with Lipschitz constant L. By Theorem 2.3, J has a unique fixed point $h : A \to X$ and

$$h(x) = \lim_{m \to \infty} J^m(f(x)) = \lim k^{2m} f(k^{-m}x) \tag{3.17}$$

for all $x \in A$.

Therefore

$$\begin{aligned} &\|h(x+y) + h(x-y) - 2h(x) - 2h(y)\| \\ &\quad = \lim_{m \to \infty} |k|^{2m} \|f(k^{-m}(x+y)) + f(k^{-m}(x-y)) - 2f(k^{-m}x) - 2f(k^{-m}y)\| \\ &\quad \leq \lim_{m \to \infty} |k|^{2m} \varphi(k^{-m}x, k^{-m}y) \\ &\quad \leq \lim_{m \to \infty} L^m \varphi(x,y) = 0 \end{aligned} \tag{3.18}$$

for all $x, y \in A$. This shows that h is quadratic. It follows from Theorem 2.3 that

$$d(f, h) \leq d(J(f), f), \tag{3.19}$$

that is,

$$\|f(x) - h(x)\| \leq \frac{L\Phi(x)}{|k|^2} \quad (x \in A). \tag{3.20}$$

Replacing x_i by $k^{-m}x_i$, $i = 1, \dots, n$ in (3.2), we get

$$\begin{aligned}
&\Bigg\| f\left(\prod_{i=1}^{n} k^{-mn} x_i\right) - f\left(k^{-m}x_1\right) k^{-2m(n-1)} x_2^2 \cdots x_n^2 \\
&-k^{-2m(n-1)} x_1^2 f\left(k^{-m}x_2\right) x_3^2 \cdots k^{-2m(n-1)} x_n^2 - \cdots - x_1^2 \cdots x_{n-1}^2 f\left(k^{-m}x_n\right) \Bigg\| \\
&\quad \leq \varphi\left(k^{-m}x_1, \dots, k^{-m}x_n\right),
\end{aligned} \tag{3.21}$$

and so

$$\begin{aligned}
&|k|^{2mn} \Bigg\| f\left(\prod_{i=1}^{n} k^{-mn} x_i\right) - f\left(k^{-m}x_1\right) k^{-2m(n-1)} x_2^2 \cdots x_n^2 \\
&\qquad -k^{-2m(n-1)} x_1^2 f\left(k^{-m}x_2\right) x_3^2 \cdots x_n^2 - \cdots - k^{-2m(n-1)} x_1^2 \cdots x_{n-1}^2 f\left(k^{-m}x_n\right) \Bigg\| \\
&\quad = \Bigg\| 2^{2mn} f\left(\prod_{i=1}^{n} k^{-mn} x_i\right) - k^{2m} f\left(k^{-m}x_1\right) x_2^2 \cdots x_n^2 \\
&\qquad -x_1^2 k^{2m} f\left(k^{-m}x_2\right) x_3^2 \cdots x_n^2 - \cdots - x_1^2 \cdots x_{n-1}^2 k^{2m} f\left(k^{-m}x_n\right) \Bigg\| \\
&\quad \leq |k|^{2mn} \varphi\left(k^{-m}x_1, \dots, k^{-m}x_n\right) \leq |k|^{2mn} \frac{K^m}{|k|^{2m}} \varphi(x_1, \dots, x_n)
\end{aligned} \tag{3.22}$$

for all $x_1, \dots, x_n \in A$ and each $m \in \mathbb{N}$. By taking $m \to \infty$, we have

$$h\left(\prod_{i=1}^{n} x_i\right) = h(x_1) x_2^2 \cdots x_n^2 + x_1^2 h(x_2) x_3^2 \cdots x_n^2 + \cdots - x_1^2 \cdots x_{n-1}^2 h(x_n) \tag{3.23}$$

for all $x_1, \dots, x_n \in A$. □

In the following corollaries we will assume that A is a non-Archimedean Banach algebra over $\mathbb{K} = \mathbb{Q}_p$ the field of p-adic numbers, where $p > 2$ is a prime number.

Corollary 3.2. *Let $r < 1$ and let ε be δ be positive real numbers. Suppose that $f : A \to X$ is a mapping such that*

$$\begin{aligned}&\|f(x+y)+f(x-y)-2f(x)-2(y)\| \leq \varepsilon\|x\|^r\|y\|^r,\\&\left\|f\left(\prod_{i=1}^{n}x_i\right)-f(x_1)x_2^2\cdots x_n^2-x_1^2f(x_2)x_3^2\cdots x_n^2-\cdots-x_1^2\cdots x_{n-1}^2f(x_n)\right\|\\&\quad\leq \delta\max\{\|x_1\|^r,\ldots,\|x_n\|^r\}\end{aligned} \tag{3.24}$$

for all $x_1,\ldots,x_n,x,y \in A$. Then there exists a unique quadratic n-derivation h from A into X such that

$$\|f(x)-h(x)\| \leq \varepsilon p^{2r}\|x\|^{2r} \tag{3.25}$$

for all $x \in A$.

Proof. By (3.24), $f(0)=0$. Let $\varphi(x,y)=\varepsilon\|x\|^r\|y\|^r$ and $\psi(x_1,\ldots,x_n\}=\delta\max\{\|x_1\|^r,\ldots,\|x_n\|^r\}$ for all $x_1,\ldots,x_n,x,y \in A$. Then

$$|p|^2\varphi\left(p^{-1}x,p^{-1}y\right)=p^{2r-2}\varphi(x,y), \qquad |p|^2\psi\left(p^{-1}x_1,\ldots,p^{-1}x_n\right\}=p^{r-2}\psi(x_1,\ldots,x_n\} \tag{3.26}$$

for all $x_1,\ldots,x_n,x,y \in A$.

Moreover,

$$\Phi(x)=\max\{\varphi(0,0),\varphi(x,x),\varphi(2x,x),\ldots,\varphi((p-1)x,x)\}=\epsilon\|x\|^{2r} \quad (x \in A). \tag{3.27}$$

Put $L=p^{2r-2}$ and $K=p^{r-2}$ in Theorem 3.1. Then there exists a unique quadratic n-derivation h from A into X such that

$$\|f(x)-h(x)\| \leq \varepsilon p^{2r}\|x\|^{2r} \tag{3.28}$$

for all $x \in A$. □

Similarly, we can prove the following result.

Corollary 3.3. *Let $r < 2$ and let ε be δ be positive real numbers. Suppose that $f : A \to X$ is a mapping such that*

$$\begin{aligned}&\|f(x+y)+f(x-y)-2f(x)-2(y)\| \leq \varepsilon\max\{\|x\|^r,\|y\|^r\},\\&\left\|f\left(\prod_{i=1}^{n}x_i\right)-f(x_1)x_2^2\cdots x_n^2-x_1^2f(x_2)x_3^2\cdots x_n^2-\cdots-x_1^2\cdots x_{n-1}^2f(x_n)\right\|\\&\quad\leq \delta\max\{\|x_1\|^r,\ldots,\|x_n\|^r\}\end{aligned} \tag{3.29}$$

for all $x_1, \ldots, x_n, x, y \in A$*. Then there exists a unique quadratic* n*-derivation* h *from* A *into* X *such that*

$$\|f(x) - h(x)\| \leq \varepsilon p^r \|x\|^r \tag{3.30}$$

for all $x \in A$*.*

Remark 3.4. We can use similar arguments to obtain corollaries like Corollaries 3.2 and 3.3, when $r > 1$ and $r > 2$.

By using the same technique of proving Theorem 3.1, we can prove the following result.

Remark 3.5. Let $\varphi : A \times A \to [0, \infty), \psi : A \times \cdots \times A \to [0, \infty)$ be functions. Let $f : A \to X$ be a given mapping such that $f(0) = 0$,

$$\|f(x+y) + f(x-y) - 2f(x) - 2f(y)\| \leq \varphi(x, y) \tag{3.31}$$

and that

$$\left\| f\left(\prod_{i=1}^{n} x_i\right) - f(x_1)x_2^2 \cdots x_n^2 - x_1^2 f(x_2)x_3^2 \cdots x_n^2 - \cdots - x_1^2 \cdots x_{n-1}^2 f(x_n) \right\| \leq \psi(x_1, \ldots, x_n) \tag{3.32}$$

for all $x_1, \ldots, x_n, x, y \in A$. Suppose that there exist a natural number $k \in \mathbb{K}$ and $L, K \in (0, 1)$, such that

$$\varphi(kx, y) \leq |k|^2 L \varphi(x, y), \qquad \psi(kx_1, \ldots, kx_n) \leq |k|^2 K \psi(x_1, \ldots, x_n) \tag{3.33}$$

for all $x_1, \ldots, x_n, x, y \in A$. Then there exists a unique quadratic n-derivation d from A into X such that

$$\|f(x) - d(x)\| \leq |k|^2 L \Phi(x) \tag{3.34}$$

for all $x \in A$, where

$$\Phi(x) = \max\left\{ \varphi(0, 0), \varphi(x, x), \varphi\left(\frac{x}{2}, x\right), \ldots, \varphi\left(\frac{x}{(k-1)}, x\right) \right\} \quad (x \in A). \tag{3.35}$$

Acknowledgment

The third author of this work was partially supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology (Grant no. 2011-0021253).

References

[1] S. M. Ulam, *Problems in Modern Mathematics*, chapter 6, John Wiley & Sonsc, New York, NY, USA, 2nd edition, 1964.

[2] D. H. Hyers, "On the stability of the linear functional equation," *Proceedings of the National Academy of Sciences of the United States of America*, vol. 27, pp. 222–224, 1941.

[3] T. M. Rassias, "On the stability of the linear mapping in Banach spaces," *Proceedings of the American Mathematical Society*, vol. 72, no. 2, pp. 297–300, 1978.

[4] P. Găvruţa, "A generalization of the Hyers-Ulam-Rassias stability of approximately additive mappings," *Journal of Mathematical Analysis and Applications*, vol. 184, no. 3, pp. 431–436, 1994.

[5] R. Badora, "Report of Meeting: The Thirty-fourth International Symposium on Functional Equations, June 10 to 19, 1996, Wisła-Jawornik, Poland," *Aequationes Mathematicae*, vol. 53, no. 1-2, pp. 162–205, 1997.

[6] H. Khodaei and T. M. Rassias, "Approximately generalized additive functions in several variables," *International Journal of Nonlinear Analysis and Applications*, vol. 1, no. 1, pp. 22–41, 2010.

[7] J. Tabor, "Remark 20, In Report on the 34th ISFE," *Aequationes Mathematicae*, vol. 53, pp. 194–196, 1997.

[8] J. Aczél and J. Dhombres, *Functional Equations in Several Variables*, vol. 31 of *Encyclopedia of Mathematics and its Applications*, Cambridge University Press, Cambridge, UK, 1989.

[9] F. Skof, "Local properties and approximation of operators," *Rendiconti del Seminario Matematico e Fisico di Milano*, vol. 53, pp. 113–129, 1983.

[10] P. W. Cholewa, "Remarks on the stability of functional equations," *Aequationes Mathematicae*, vol. 27, no. 1-2, pp. 76–86, 1984.

[11] St. Czerwik, "On the stability of the quadratic mapping in normed spaces," *Abhandlungen aus dem Mathematischen Seminar der Universität Hamburg*, vol. 62, pp. 59–64, 1992.

[12] M. Eshaghi Gordji and N. Ghobadipour, "Hyers–Ulam–Aoki–Rassias stability and Ulam–Gavruta–Rassias stability of quadratic homomorphisms and quadratic derivations on Banach Algebras," in *Functional Equations, Difference Inequalities*, vol. and Ulam Stability Notions (F.U.N.) of *Mathematics Research Developments*, NOVA Publishers, 2010.

[13] M. Eshaghi Gordji, M. B. Ghaemi, and B. Alizadeh, "A fixed point method for perturbation of higher ring derivations in non–Archimedean Banach algebras," *International Journal of Geometric Methods in Modern Physics*, vol. 8, no. 7, pp. 1611–1625, 2011.

[14] E. H. Lee, I.-S. Chang, and Y.-S. Jung, "On stability of the functional equations having relation with a multiplicative derivation," *Bulletin of the Korean Mathematical Society*, vol. 44, no. 1, pp. 185–194, 2007.

[15] H.-M. Kim and I.-S. Chang, "Stability of the functional equations related to a multiplicative derivation," *Journal of Applied Mathematics & Computing A*, vol. 11, no. 1-2, pp. 413–421, 2003.

[16] M. Eshaghi Gordji, "Nearly ring homomorphisms and nearly ring derivations on non-Archimedean Banach algebras," *Abstract and Applied Analysis*, vol. 2010, Article ID 393247, 12 pages, 2010.

[17] M. Eshaghi Gordji and Z. Alizadeh, "Stability and superstability of ring homomorphisms on non-Archimedean Banach algebras," *Abstract and Applied Analysis*, vol. 2011, Article ID 123656, 10 pages, 2011.

[18] M. Eshaghi Gordji, M. B. Ghaemi, and B. Alizadeh, "A fixed point approach to superstability of generalized derivations on non-Archimedean Banach algebras," *Abstract and Applied Analysis*, vol. 2011, Article ID 587097, 9 pages, 2011.

[19] M. E. Gordji and H. Khodaei, *Stability of Functional Equations*, LAP LAMBERT Academic Publishing, 2010.

[20] M. Eshaghi Gordji, H. Khodaei, and R. Khodabakhsh, "General quartic-cubic-quadratic functional equation in non-Archimedean normed spaces," *University of Bucharest: Scientific Bulletin A*, vol. 72, no. 3, pp. 69–84, 2010.

[21] M. Eshaghi Gordji and M. B. Savadkouhi, "Stability of cubic and quartic functional equations in non-Archimedean spaces," *Acta Applicandae Mathematicae*, vol. 110, no. 3, pp. 1321–1329, 2010.

[22] M. Eshaghi Gordji and M. B. Savadkouhi, "Stability of a mixed type cubic-quartic functional equation in non-Archimedean spaces," *Applied Mathematics Letters*, vol. 23, no. 10, pp. 1198–1202, 2010.

[23] M. Eshaghi Gordji, M. B. Savadkouhi, and M. Bidkham, "Stability of a mixed type additive and quadratic functional equation in non-Archimedean spaces," *Journal of Computational Analysis and Applications*, vol. 12, no. 2, pp. 454–462, 2010.

[24] J. M. Rassias, "On approximation of approximately linear mappings by linear mappings," *Journal of Functional Analysis*, vol. 46, no. 1, pp. 126–130, 1982.

[25] J. M. Rassias, "On approximation of approximately linear mappings by linear mappings," *Bulletin des Sciences Mathématiques*, vol. 108, no. 4, pp. 445–446, 1984.

[26] J. M. Rassias, "Solution of a problem of Ulam," *Journal of Approximation Theory*, vol. 57, no. 3, pp. 268–273, 1989.

[27] G. Z. Eskandani, H. Vaezi, and Y. N. Dehghan, "Stability of a mixed additive and quadratic functional equation in non-Archimedean Banach modules," *Taiwanese Journal of Mathematics*, vol. 14, no. 4, pp. 1309–1324, 2010.

[28] T. Z. Xu, J. M. Rassias, and W. X. Xu, "Stability of a general mixed additive-cubic functional equation in non-Archimedean fuzzy normed spaces," *Journal of Mathematical Physics*, vol. 51, no. 9, Article ID 093508, 19 pages, 2010.

[29] T. Z. Xu, J. M. Rassias, and W. X. Xu, "Intuitionistic fuzzy stability of a general mixed additive-cubic equation," *Journal of Mathematical Physics*, vol. 51, no. 6, Article ID 063519, 21 pages, 2010.

[30] T. Z. Xu, J. M. Rassias, and W. X. Xu, "On the stability of a general mixed additive-cubic functional equation in random normed spaces," *Journal of Inequalities and Applications*, vol. 2010, Article ID 328473, 16 pages, 2010.

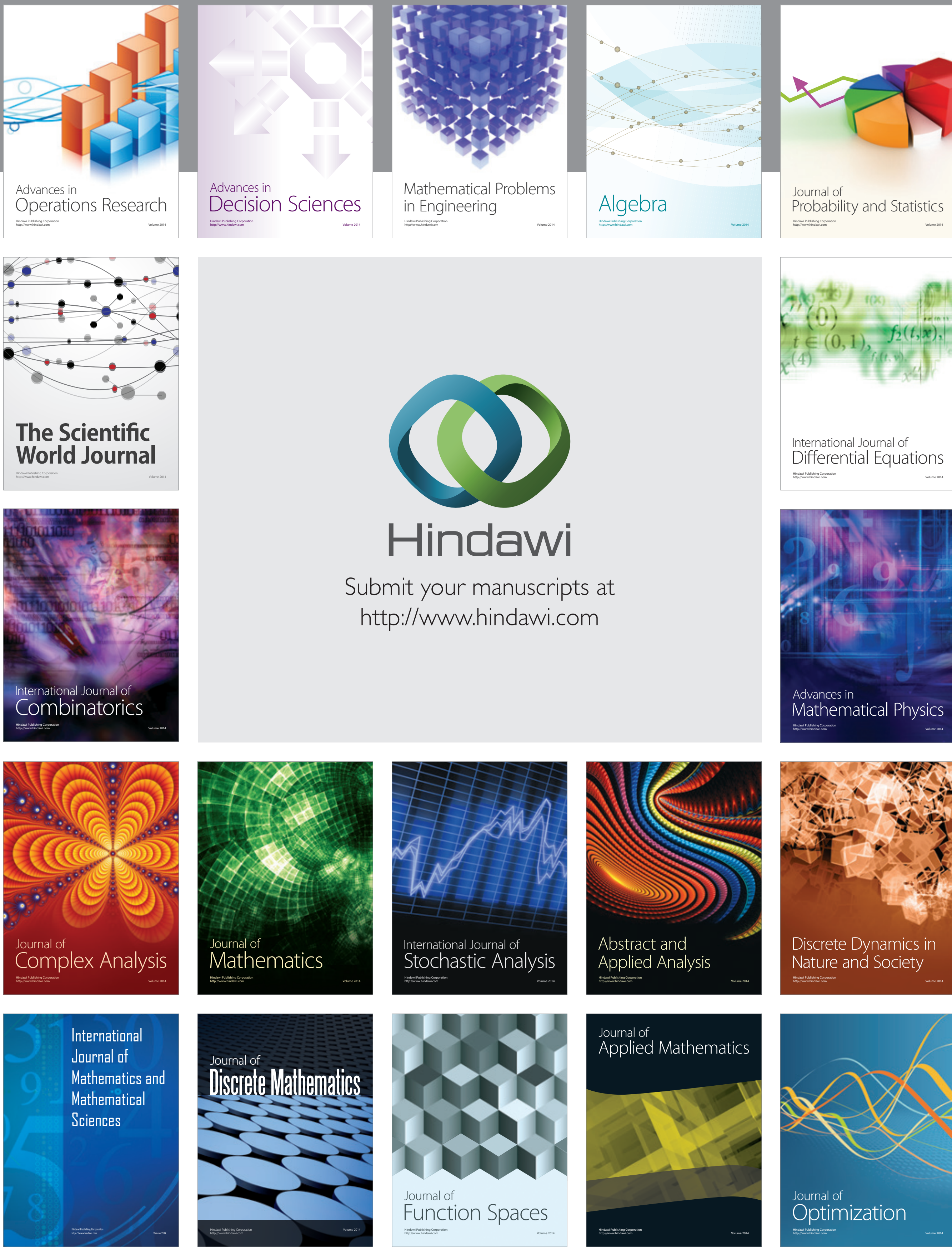
[31] T. Z. Xu, J. M. Rassias, and W. X. Xu, "A fixed point approach to the stability of a general mixed AQCQ-functional equation in non-Archimedean normed spaces," *Discrete Dynamics in Nature and Society*, vol. 2010, Article ID 812545, 24 pages, 2010.

[32] T. Z. Xu, "Stability of multi-Jensen mappings in non-Archimedean normed spaces," *Journal of Mathematical Physics*, vol. 53, Article ID 10.1063/1.3684746, 9 pages, 2012.

[33] P. Găvruta and L. Găvruta, "A new method for the generalized Hyer-Ulam-Rassias stability," *International Journal of Nonlinear Analysis and Applications*, vol. 1, no. 2, pp. 11–18, 2010.

[34] L. M. Arriola and W. A. Beyer, "Stability of the Cauchy functional equation over p-adic fields," *Real Analysis Exchange*, vol. 31, no. 1, pp. 125–132, 2005.

[35] K. Hensel, "Uber eine neue Begrundung der Theorie der algebraischen Zahlen," *Jahresbericht der Deutschen Mathematiker-Vereinigung*, vol. 6, pp. 83–88, 1897.


Advances in Operations Research
Advances in Decision Sciences
Mathematical Problems in Engineering
Algebra
Journal of Probability and Statistics
The Scientific World Journal
International Journal of Differential Equations
Hindawi
Submit your manuscripts at
http://www.hindawi.com
International Journal of Combinatorics
Advances in Mathematical Physics
Journal of Complex Analysis
Journal of Mathematics
International Journal of Stochastic Analysis
Abstract and Applied Analysis
Discrete Dynamics in Nature and Society
International Journal of Mathematics and Mathematical Sciences
Journal of Discrete Mathematics
Journal of Function Spaces
Journal of Applied Mathematics
Journal of Optimization