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Anomalous Resistance of the Fractal Current-carrying Corona of Z-pinch

A. KINGSEP

Russian Research Center "Kurchatov Institute", 123182 Moscow, Russia

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An effect of the random plasma inhomogeneity onto the scenario of ion-acoustic anomalous resistivity is considered. It is shown that such an inhomogeneity could be more efficient than nonlinear effects to determine the turbulent resistivity. In application to Z-pinches, some peculiarities of the skin-effect are considered, in particular, subsequent inhomogeneity of the current penetration into the conducting medium.

Keywords: Anomalous resistance, Ion-acoustic, Anomalous resistivity, Z-pinches

1 INTRODUCTION

In our recent paper [1] we have taught upon some features of the regime of anomalous resistivity in plasma opening switches. In particular, our statement was that in most pulsed plasmas ion-acoustic mechanism of anomalous resistivity seemed to be predominating one. Of course, some competing instabilities (low-hybrid, modified two-stream) can also join the game (we believe, the best candidate is the Bernstein mode). However, they can predominate in the resistive mechanism only under the following condition:

$$\omega_{\mathrm{Be}} \geq \omega_{\mathrm{pe}} \tag{1}$$

or, what is the same,

$$B^2 \geq 4\pi nmc^2. \tag{2}$$

The inequality (2) is not typical of the pulsed plasmas up-to-date (however, it Will be able to be more realistic at JUPITER's level of the basic parameters). The reason of importance of the condition (1) is the following. To magnetize the ion-acoustic waves, it is necessary to satisfy two conditions:

$$(a)\ \omega_{\mathrm{s}} \ll \omega_{\mathrm{Be}}; \quad (b)\ k_{\mathrm{s}}\rho_{\mathrm{Be}} \ll 1. \tag{3}$$

The first one is always true in Z-pinches while the second just for the typical unstable $k \sim r_{\mathrm{De}}^{-1}$ turns out to be equivalent to (1). Moreover, if it is violated, the Cherenkov (quasilinear) coupling becomes non-magnetized as well. So, if we operate with the range of the moderate magnetic field (opposite to (2)), we have to compare ion-acoustic quantum as competing to other possible momentum carriers. Then, in accordance with our paper

[1], we have very serious reason just for this choice.

The whole hierarchy of the anomalous resistivity includes, generally speaking, linear excitation of waves [2], quasilinear stage [3], nonlinear saturation [4] and anomalous transport [5]. (We have cited in this paragraph only first exact solutions.) However, on the level of very precise estimates it is not less known that such a hierarchy works as a whole only if the electric field is not so strong:

$$E \ll \frac{m}{M}(4\pi nT)^{1/2}, \tag{4}$$

where m/M is the mass ratio. If the inequality opposite to (4) is true, quasilinear effects are immaterial. This essentially nonlinear regime is just the case of the Kadomtsev's spectrum [6] and Sagdeev's formula for conductivity [7] we have used in [1]. There exists some especial reason of this preference in the case of Z-pinches. All the resistive plasma systems may be separated in two classes: (a) with a given electric field inside the gap and (b) with a net current completely determined by the outer circuit. The Z-pinches represent just the second case, hence, in such a system the current flow velocity cannot be restricted by the threshold of the instability. If this velocity exceeds essentially the threshold, the quasilinear (Cherenkov) effects do take place but cannot determine the effective collisional frequency.

At first sight, it means that only essentially nonlinear regime could be realized in the dynamics of anomalous resistivity of pulsed plasmas. However, it was noticed as early as in [4] that effect of the inhomogeneous plasma density $n(\mathbf{r})$ (or concentration of carriers in the general case) could effectively compete with the typical nonlinearity, i.e., nonlinear wave–particle scattering. The subsequent scenario in [4] was typical of the plasmas of mirror traps, that's why we have to "rebuild" it completely, in application to Z-pinches.

Let us emphasize: to "catch" proper instability in the unwieldy variety of instabilities and to construct the subsequent turbulent scenario, it would be rather useless to compare thresholds, growth rates, even levels of saturation. It looks much more reasonable, to use le Chatelier–Brown principle with respect to the momentum transfer which is always the main point of the problem of resistivity. In particular, it just provides the proper selection of the damping mechanism. In our case, by taking into account the plasma inhomogeneity, this damping may remain linear as well as the exitation of waves. The basic system of equations ("optical" or Hamiltonian) describing the trajectory of the ion-acoustic quantum in the phase space is the following one:

$$\frac{\partial \mathbf{r}}{\partial t} = \frac{\partial \omega}{\partial \mathbf{k}}, \quad \frac{\partial \mathbf{k}}{\partial t} = -\frac{\partial \omega}{\partial \mathbf{r}}. \tag{5}$$

Such a description is true if the outer conditions do not depend on time. The momentum transfer is conditioned by the second of Eq. (5) via the exchange of momentum between the quantum and the medium the predominating mass of which is being contained just in ions. As it will be shown hereafter, the random inhomogeneity of the plasma density can efficiently provide the momentum balance. Such an inhomogeneity may result as from the scenario of the plasma column production as from the nonlinear MHD evolution of Z-pinch (so-called regime of the "enhanced stability", see details in [8]).

Here and below, we will consider this random inhomogeneity as quasisteady from the "standpoint" of an ion-acoustic plasmon. That means, the redistribution of the density has to be slow compared to the ion-acoustic time scale:

$$\frac{v_A}{a} \ll \omega_{pi} \quad \text{where } v_A = \frac{B}{\sqrt{4\pi nMA}}, \ \ \omega_{pi} = \sqrt{\frac{4\pi ne^2}{MA}},$$

A is the mass number of the ion, M the proton mass, a the typical scale length of the spatial inhomogeneity. This inequality may be readily transformed into the following one:

$$a \gg \frac{B}{4\pi ne} \sim \frac{I}{4\pi nerc},$$

where r is the radius of Z-pinch. Let us take, as an example, $n \sim 10^{16}\,\mathrm{cm}^{-3}$, which corresponds to the rarefied corona of the fast Z-pinch. Then, let us estimate the condition of the quasistationarity for the SATURN parameters:

$$I_{\max} \sim 20\,\mathrm{MA} \;\Rightarrow\; r \sim 2\,\mathrm{cm} \;\Rightarrow\; a \gg 4 \cdot 10^{-2}\,\mathrm{cm}.$$

The same condition may be expressed in terms of the current flow velocity.

Let δ_{sk} be the space scale of the skin depth, then $I = 2\pi r \delta_{\mathrm{sk}} \cdot j$. Our inequality results in

$$a/\delta_{\mathrm{sk}} \gg j/nec,$$

as one easily can see, this restriction is not strong even if we take into account that j has to be much above the threshold of the ion-acoustic instability, $j \gg nec_{\mathrm{s}}$ where $c_{\mathrm{s}} = (ZT_{\mathrm{e}}/MA)^{1/2}$. It is useful to note that even when the temperature is of the order of several hundreds eV, the ratio $c_{\mathrm{s}}/c < 10^{-3}$.

To be sure in the efficiency of the collective phenomena, we have to keep Debye number $N_{\mathrm{D}} \gg 1 \Rightarrow T/(4\pi e^2 n^{1/3}) \gg 1$. Even if we take, e.g., $n \sim 10^{21}\,\mathrm{cm}^{-3}$, that means $T \gg 20\,\mathrm{eV}$.

2 ION-ACOUSTIC CURRENT FLOW INSTABILITY IN THE RANDOMLY INHOMOGENEOUS PLASMA

For simplicity, we will demonstrate all the basic effects on the 1-D model, i.e., $n = n(x)$, $\mathbf{j} \parallel \mathbf{E} \parallel \mathbf{O}_{\mathrm{X}}$. It has to be emphasized, however, that such a model is always more or less artificial since effect of anomalous resistivity is never one-dimensional. In pure 1-D plasma (e.g., presented by the PIC code) something like plateau on the particle distribution functions has to be formed cancelling the effect of resistivity (relative dynamics in a given electric field, see in [9,10]).

The dispersion law of the ion-acoustic waves is well known:

$$\omega_{\mathrm{s}}(k) = \frac{kc_{\mathrm{s}}}{\sqrt{1 + k^2 r_{\mathrm{De}}^2}} = \frac{\omega_{\mathrm{pi}}}{\sqrt{1 + (kr_{\mathrm{De}})^{-2}}}. \tag{6}$$

We will take into account only the spatial inhomogeneity of plasma density as the ion-acoustic waves are much less sensitive to the effect of inhomogeneous temperature. Equation (5) results in $\omega = \mathrm{const}$ which, together with (6), leads to the following result:

$$\begin{aligned} &k^{-2} + r_{\mathrm{De}}^2 = \mathrm{inv}, \quad r_{\mathrm{De}} \propto n^{-1}, \\ &\frac{\mathrm{d}n}{\mathrm{d}x} > 0 \;\Rightarrow\; k \to k_{\min}, \\ &\quad n \to 0 \;\Rightarrow\; k \to \infty. \end{aligned} \tag{7}$$

Thus, no reflection of an ion-acoustic plasmon may occur in the 1-D problem but it can disappear due to the ion Landau damping conditioned by the essential decrease of the density. It is interesting to note that within the frames of the self-consistent problem it disappears just in the opposite case of growing density (see below).

In the conventional case of multimode spectrum which can be presented in the form of the wave spectral density $W_{\mathbf{k}}$ or of the quasiparticle distribution $N_{\mathbf{k}} = W_{\mathbf{k}}/\omega_{\mathbf{k}}$, the system (5) is equivalent to the Liouville equation:

$$\frac{\partial N_{\mathbf{k}}}{\partial t} + \frac{\partial \omega}{\partial \mathbf{k}} \nabla N_{\mathbf{k}} - \frac{\partial N_{\mathbf{k}}}{\partial \mathbf{k}} \nabla \omega = 2\gamma_{\mathbf{k}} N_{\mathbf{k}}. \tag{8}$$

Equation (5) represents two characteristics of (8), the third one is the following:

$$N(\mathbf{k}, \mathbf{r}) = N(\mathbf{k}_0, \mathbf{r}_0) \cdot \exp\left[\int_{\mathbf{k}_0,\mathbf{r}_0}^{\mathbf{k},\mathbf{r}} \frac{2\gamma_{\mathbf{k}}}{|\partial\omega/\partial\mathbf{k}|}\,\mathrm{d}\mathbf{l}\right], \tag{9}$$

where $(\mathbf{k}, \mathbf{r})$ and $(\mathbf{k}_0, \mathbf{r}_0)$ are connected by the invariant (7) by given $n(\mathbf{r})$ dependence and the integration is being fulfilled along the plasmon trajectory given by the solution of the same system (5) or, in the 1-D case, by the invariant (7). In many cases, it turns out to be more useful to rewrite (9) in terms of ω, instead of $\mathbf{k}$, since ω is invariant.

The growth rate in (8) and (9) $\gamma = \gamma_{\mathrm{e}} + \gamma_{\mathrm{i}}$ includes both electron and ion increments of the current-carrying plasma, under the condition $\omega/k \gg v_{Te}$ they may be presented as follows:

$$\gamma_e \simeq \sqrt{\frac{\pi}{8}}(\mathbf{ku}-\omega)\left(\frac{\omega}{kv_{Te}}\right)^3\left(\frac{M}{m}\right),$$
$$\gamma_i \simeq -\pi^2\omega\cdot\left(\frac{\omega}{k}\right)^3 f_i\left(\frac{\omega}{k}\right), \tag{10}$$

where M/m is the mass ratio and $\mathbf{u}=\mathbf{j}/ne$ – current flow velocity. If we vary slightly this velocity (that just happens in a weakly inhomogeneous plasma), both increments have to vary as well. If one deals with the purely linear situation (in particular, Maxwellian distribution functions), the ion response is more essential since the $\gamma_e(\mathbf{k})$ dependence is linear while that of $f_i(\omega/k)$ is exponential one. In efficiency, resonant ions, due to their small number, are normally "overheated", as a result, the function $f_{ri}(\omega/k)$ is smooth enough. Thus, in the case of the well-developed current flow instability, it seems more reasonable to follow, first of all, $f_e(\mathbf{u}, \omega/k)$ and to compare $j/(n(x)e)$ with u_{cr}.

When they look for the threshold condition, the necessary equations that determine the point in the $\mathbf{k}$-space are the following:

$$\gamma(\mathbf{k},\mathbf{u})=0, \qquad \frac{\partial\gamma(\mathbf{k},\mathbf{u})}{\partial\mathbf{k}}=0, \tag{11}$$

which results in the threshold current flow velocity $\mathbf{u}_{cr}$. In principle, such a "threshold" regime is available in the weakly inhomogeneous plasma as well. In uniform plasmas balance of purely linear increments (10) hardly may provide the real steady state since the momentum acquired by the electrons from the electric field is being transformed to the small fraction of ions, i.e., resonant ions while the main fraction of ions being freely accelerated. That is why quasilinear regime [9] inevitably includes essential modification of both particle distributions and dispersion relations of the spectra. In the inhomogeneous plasmas another regime is possible.

Let us turn to the Fig. 1 where the plasma inhomogeneity is modeled, for simplicity, by the 1-D sinusoidal profile (which is quite immaterial for the subsequent conclusions). The current value I is determined by the outer circuit, hence, $|\mathbf{u}| \propto n^{-1}$. In Fig. 1, the dotted line corresponds to $|\mathbf{u}| \simeq u_{cr}$ determined by (11). More exactly, it has to follow from (9):

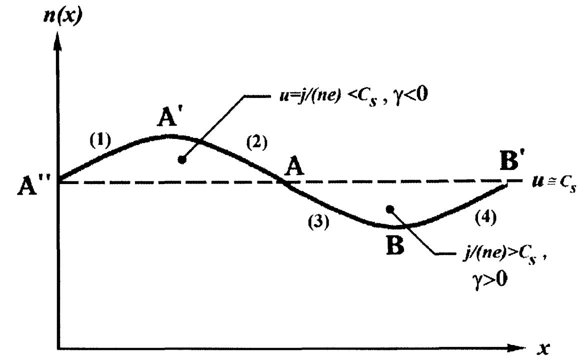

FIGURE 1

$$\int_{\mathbf{k}_0,\mathbf{r}_0}^{\mathbf{k},\mathbf{r}_1} \frac{2\gamma_{\mathbf{k}}}{|\partial\omega/\partial\mathbf{k}|}\,\mathrm{d}l = 0,$$

where (r_0, r_1) is just the period of the density perturbation. The exchange of momentum between the waves and medium (i.e., ions) follows from the second of Eq. (5) and also from Eqs. (6) and (7):

$$\nabla\omega \simeq \frac{1}{2}\omega_{pi}\frac{\nabla n}{n_0} = -\frac{\partial\mathbf{k}}{\partial t}.$$

At first sight, this exchange is impossible since each plasmon conserves its momentum after passing the whole period. Let us, however, compare the intervals $A''A'$ and $A'A$ in Fig. 1. In both $\gamma<0$, hence, according to (9), the number of quasiparticles in some point of $A''A'$ interval where they loss their momentum is less than in the point with the same density in $A'A$ where they acquire momentum. Now let us compare the intervals AB where plasmons acquire momentum and BB' where they loss it. The number of the latters is greater, according to (9), since in that region $\gamma>0$. Thus, in a whole, we can see that during the spectrum travelling along the inhomogeneous profile, it successfully transfers the momentum obtained from the electrons to the main body of the plasma ions. In accordance with the basic features of the resistive problem, it can provide the quasisteady state of the instability.

Unfortunately, this regime hardly could be typical of the systems with a given current as it hardly would correspond just to the dotted line in Fig. 1. Besides, this regime is too close to the threshold, and quasilinear effects can join the game cancelling the damping on the small angles $\angle(\mathbf{k},\mathbf{j}) \ll 1$ [4,11]. Another regime seems to be more realistic, which is presented in Fig. 2. We suppose that density fluctuations are great enough to provide the "trapping" of quanta, e.g., in some vicinity of the density wells. It cannot be real trapping since, as it has been noticed above, reflection of the ion-acoustic plasmons is impossible in the 1-D model. Their localization has to be conditioned by the efficient damping in the regions of higher density where $j/(ne) < u_{\rm cr}$. For convenience, let us introduce the quasiperiod of the inhomogeneity L which exceeds, in a general case, the typical space scale of the localization of the acoustic quanta:

$$J \equiv \int_L \frac{2\gamma_{\mathbf{k}}}{|\partial\omega/\partial\mathbf{k}|}\,\mathrm{d}l \leq 0.$$

An example of the localization of the plasma waves in the vicinity of the only well is shown in Fig. 3. The current flow velocity at the bottom $n(x)$ is supposed to exceed essentially $u_{\rm cr}$, the region of the instability (x_1, x_1') and the region of localization (x_1, x_2) are determined by the current value I supported by the outer circuit. The regions of the waves localization (see Fig. 2) turn out to be the regions of local heating and, what is more important, the regions of the enhanced field diffusion. If the net current I is so high that $J > 1$, the inhomogeneity cannot provide the stationary state of the instability and nonlinear effects join the game.

The spatial density of momentum varies in time according to Eq. (5):

$$\frac{\partial}{\partial t}\frac{\partial \mathbf{p}_{\rm e}}{\partial V} = -\frac{\partial}{\partial t}\frac{\partial \mathbf{p}_{\rm i}}{\partial V} = \sum_{\mathbf{k}} N_{\mathbf{k}} \frac{\partial \mathbf{k}}{\partial t} = \sum_{\omega} N_{\omega} \frac{\partial \mathbf{k}(\omega, \mathbf{r})}{\partial t} = -\sum_{\omega} N_{\omega} \nabla\omega. \tag{12}$$

On the other hand, balance of the electron momentum results in the following expression introducing the effective collisional frequency:

$$\int_{x_1}^{x_2} ne\mathbf{E}\,\mathrm{d}x = \mathbf{j}\frac{m}{e}\int_{x_1}^{x_2} \nu_{\rm eff}(x)\,\mathrm{d}x. \tag{13}$$

We have already replaced $\mathbf{k}$-representation by the ω-representation, let us, in addition, introduce the

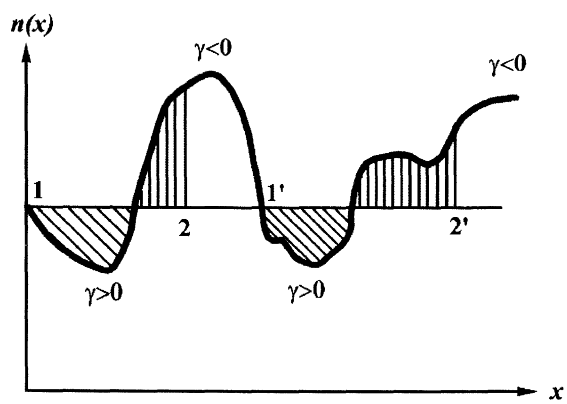

FIGURE 2

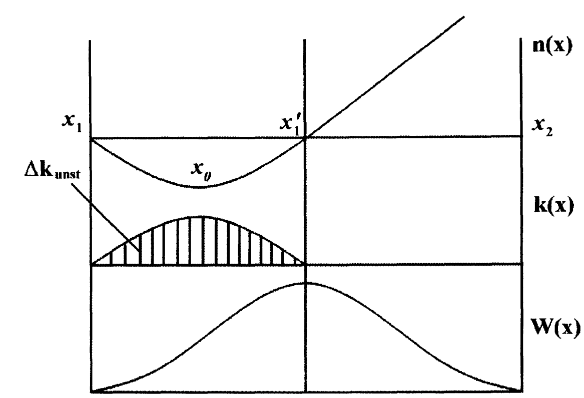

FIGURE 3

integral spatial density of the energy of noises, as follows:

$$\int N_\omega \, \mathrm{d}\omega \simeq W(x)/\omega$$

which allows to simplify essentially the estimate of ν_{eff}:

$$j\frac{m}{e}\int_{x_1}^{x_2} \nu_{\text{eff}}(x)\,\mathrm{d}x = \int_{x_1}^{x_2} \mathrm{d}x \int \mathrm{d}\omega N_\omega \frac{\partial \omega}{\partial x} \simeq \int_{x_1}^{x_2} \mathrm{d}x \frac{W(x)}{\omega}\frac{\partial \omega}{\partial x}. \quad (14)$$

In Fig. 3, the level of noises at the point x_1 has to be equal to that of thermal fluctuations,

$$W(x_1) \equiv W_0 \simeq nT/N_{\mathrm{D}},$$

where N_{D} is the Debye number (here taking into account the difference between T_{e} and T_{i} would be above the accuracy). At first sight, this value, $W(x)$, leads to more rough estimates than N_ω. In fact, as we will be able to see below, the spectrum would not be so broad since its typical frequency has to be close to the ion plasma frequency at the bottom, $\omega_{\mathrm{pi}}(x_0)$.

Including (9) and (10), one can readily obtain:

$$\begin{aligned} W(x) &= W_0 \exp\Gamma, \\ \Gamma &\equiv \sqrt{\frac{\pi}{8}}\frac{\omega}{v_{\mathrm{Te}}}\int_{x_1}^{x\le x_2}\left(\frac{ku}{\omega}-1\right)\mathrm{d}x \\ &\equiv \kappa\int_{x_1}^{x}\left(\frac{j}{e\omega}\frac{k(\xi)}{n(\xi)}-1\right)\mathrm{d}\xi, \\ \kappa &= \text{const} \sim \sqrt{\frac{m}{M}}r_D^{-1}(0), \quad \Gamma(x_1,x_2)=0, \end{aligned} \quad (15)$$

let $\Gamma_{\max} = \Gamma(x_1, x_2) \equiv \Gamma_0$.

In accordance with (6)and (7),

$$\frac{1}{\omega}\frac{\partial\omega}{\partial x} = \frac{1}{2}\frac{\omega^2}{\omega_{\mathrm{pi}}^2}\frac{1}{n}\frac{\partial n}{\partial x} \simeq \frac{n(x_0)}{2}\frac{1}{n^2}\frac{\partial n}{\partial x} \simeq \alpha\langle n\rangle\frac{\partial}{\partial x}\left(\frac{1}{n}\right), \quad \alpha \sim 1. \quad (16)$$

After that, we can modify (14):

$$j\frac{m}{e}\int_{x_1}^{x_2}\nu_{\text{eff}}(x)\,\mathrm{d}x = -\alpha\langle n\rangle\frac{\langle nT\rangle}{\langle N_{\mathrm{D}}\rangle}\int_{x_1}^{x_2}\frac{\partial}{\partial x}\left(\frac{1}{n}\right)\exp[\Gamma(x_1,x)]\,\mathrm{d}x. \quad (17)$$

Now we can define the effective collisional frequency throughout the region of instability ν_{B} (which has to determine the field diffusion) and also the effective collisional frequency averaged over the current-carrying layer ν_{R} (which has to determine the integral resistance) by the following relation:

$$\nu_{\mathrm{B}}(x_2-x_1) = \nu_{\mathrm{R}}\cdot L = -\beta\frac{\langle n\rangle^{5/2}e^4}{mj\sqrt{T}} \times \int_{x_1}^{x_2}\frac{\partial}{\partial x}\left(\frac{1}{n}\right)\exp[\Gamma(x_1,x)]\,\mathrm{d}x, \quad (18)$$

where L is the quasiperiod, $\beta > 1$. Let us take the RHS of (18) by parts, by taking into account that $\Gamma(x_1,x_1) = \Gamma(x_1,x_2) = 0$,

$$\int_{x_1}^{x_2}\frac{\partial}{\partial x}\left(\frac{1}{n}\right)\exp[\Gamma(x_1,x)]\,\mathrm{d}x = -\left(\frac{1}{n(x_1)}-\frac{1}{n(x_2)}+\kappa\int_{x_1}^{x_2}\frac{\mathrm{d}x}{n(x)}\left(\frac{j}{e\omega}\frac{k(x)}{n(x)}-1\right) \times \exp\left[\kappa\int_{x_1}^{x}\left(\frac{j}{e\omega}\frac{k(\xi)}{n(\xi)}-1\right)\mathrm{d}\xi\right]\right).$$

That allows to rewrite (18) in the following form:

$$\nu_{\mathrm{B}}(x_2-x_1) = \nu_{\mathrm{R}}\cdot L = \beta\frac{\langle n\rangle^{5/2}e^4}{mj\sqrt{T}}\left(\frac{1}{n(x_1)}-\frac{1}{n(x_2)}+\varepsilon\frac{\omega}{v_{\mathrm{Te}}} \times \int_{x_1}^{x_2}\frac{\mathrm{d}x}{n(x)}\left(\frac{j}{e\omega}\frac{k(x)}{n(x)}-1\right) \times \exp\left[\kappa\int_{x_1}^{x}\left(\frac{j}{e\omega}\frac{k(\xi)}{n(\xi)}-1\right)\mathrm{d}\xi\right]\right), \quad (19)$$

where $\varepsilon \leq 1$. Let us introduce the space scale of some partial well $a \leq L$, then close to the bottom

$$n = n(x_0)\left[1 + \frac{(x - x_0)^2}{a^2}\right].$$

After that, we will have to deal with rather few parameters in the subsequent estimates, i.e., $n_{\min}$ or $n(x_0)$, a and, may be, $L > a$. All the trajectories of quasiparticles have to obey the condition $k^{-2} + r_D^2 = \text{inv}$. To look for the selection rule to determine the kind of ion-acoustic plasmons just providing the resistance, let us turn to Fig. 4.

In accordance with all the suggestions declared above, our system has to be essentially above the threshold in the main part of the unstable interval $[x_1, x_1']$. That allows to neglect the quasilinear effects. In particular, that means, we cannot restrict ourselves by only ion plasma waves $\omega \to \omega_{\text{pi}}$ and have to deal with $j > nec_s$ within this region. Threshold points are determined by $j = n(x_1)ec_s = n(x_1')ec_s$.

The growth rate $\gamma \propto [(ku/\omega) - 1]$, hence, the more is ω (less ω/k), the more is γ. From this standpoint, just ion plasma waves seem to be most profitable. However, if we create a plasmon with $\omega \to \omega_{\text{pi}}(x)$ not at the bottom $x = x_0$ (double arrows (i, ii) in Fig. 4), it would keep $\omega = \text{inv}$ while travelling towards the bottom. Formally, at some point its frequency should exceed $\omega_{\text{pi}}(x)$. In efficiency, that means $\omega/k \to 0$ close to this point. As a result, strong ion Landau damping becomes sharply "switched" on and cuts off the trajectory. Let us consider another possibility, plasmon to be "born" after the bottom following the current flow (see double arrow (iii) in Fig. 4). It would be suppressed by the damping at some x_f where $\Gamma(x_i, x_f) = 0$, in such a case, $x_0 < x_i < x_f < x_2$. As the initial level of noises is close to that of thermal fluctuations while the maximal one $W(x_1')$ is not so far from nT, the maximal $\Gamma \sim \Gamma_0 \sim \Lambda = \ln N_D$ (e.g., ~ 10–20). In all the cases of reduced trajectories (i, ii, iii), their shortened length exponentially reduces the maximal level of noises:

$$\Gamma_{\text{i,ii,iii(max)}} < \Gamma_0 \quad \Rightarrow W_{\text{i,ii,iii(max)}} \ll W_{\max}.$$

As a result, we obtain quite obvious selection rule. The most profitable in respect of the momentum transfer is the ion-acoustic plasmons passing through the bottom but possessing as high frequency as possible, that means $\omega = \omega_{\text{pi}}(x_0)$. Such a trajectory coincides with the whole interval $[x_1, x_2]$ (solid arrow in Fig. 4).

After that, some useful relations can be added to go on our calculations:

$$k(x_1) \simeq \frac{\omega_{\text{pi}}(x_0)n_1 e}{j} \simeq \frac{\omega_{\text{pi}}(x_0)}{c_s} \simeq r_D^{-1}(x_0). \tag{20}$$

Note that $k(x_1)r_D(x_1) \sim (n_1 ec_s/j)(n_0/n_1)^{1/2} \ll 1$. In the case of quasiparabolic profile of the well, we can establish the relation between the "noisy" interval $[x_1, x_2]$ and the typical space scale a or L:

$$n_1 = \frac{j}{ec_s} = n_0\left(1 + \frac{(\Delta x)^2}{a^2}\right)$$
$$\Rightarrow (x_2 - x_1) \sim 2(x_1' - x_1) \sim 4\Delta x$$
$$\sim 4a\sqrt{\frac{j}{n_0 ec_s} - 1}.$$

At the level of accuracy we follow in our estimates, one can neglect unity in the RHS. Thus, we obtain the resulting estimate of the ratio of the length of the turbulent region which gives some input into

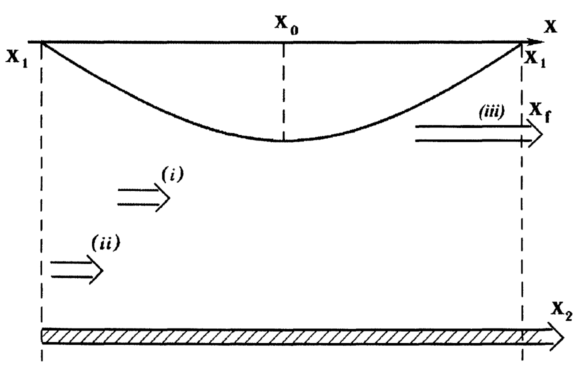

FIGURE 4

the resistance, to the typical space scale of the inhomogeneity:

$$\frac{(x_2 - x_1)}{a} \sim \sqrt{\frac{j}{n_{\min} e c_s}} \Rightarrow \nu_R \sim \nu_B \sqrt{\frac{j}{n_{\min} e c_s}}. \tag{21}$$

More exactly, this space scale can be determined by using the condition $\Gamma(x_1, x_2) = 0$, i.e.,

$$\int_{x_1}^{x_2} \left(\frac{j}{e\omega} \frac{k(\xi)}{n(\xi)} - 1 \right) \mathrm{d}\xi = 0$$
$$\Rightarrow \int_{x_1}^{x_2} \frac{j}{e\omega} \frac{k(\xi)}{n(\xi)} \mathrm{d}\xi = x_2 - x_1. \tag{22}$$

For convenience, let us denote for some particular well (see Fig. 4)

$$x_0 = 0, \;\; x_1 = -\Delta, \;\; x_1' = \Delta,$$
$$\frac{\Delta^2}{a^2} = \frac{j}{n_0 e c_s} - 1.$$

Invariant (7) may be rewritten in the form:

$$k^{-2} = r_D^2(0) \left[1 + \delta_* + \frac{n_0}{n(\xi)} \right],$$

where $\delta_* \ll 1$ is the cutoff factor to cancel the singularity in the integral (22). That allows to present the integrand of the Eq. (22) as follows:

$$\frac{j}{e\omega} \frac{k(\xi)}{n(\xi)} = \frac{n(\Delta)}{n} \frac{k}{k(\Delta)}$$
$$= \frac{a^2 + \Delta^2}{a^2 + \xi^2} \sqrt{\frac{1 + \delta_* - n_0/n(\Delta)}{1 + \delta_* - n_0/n(\xi)}}$$
$$= \sqrt{\frac{a^2 + \Delta^2}{a^2 + \xi^2}} \sqrt{\frac{\Delta^2 + a^2\delta}{\xi^2 + a^2\delta}},$$

where $\delta = \delta_*/(1 + \delta_*) \ll 1$. It is useful to introduce the dimensionless variables by taking a as the space scale:

$$(x) \equiv \frac{x_2 - x_1}{a}, \quad \Delta/a \to \Delta = \sqrt{\frac{j}{n_0 e c_s} - 1},$$

then the transcendental equation for (x) follows from (22):

$$(x) = \sqrt{1 + \Delta^2} \sqrt{\Delta^2 + \delta} \times \int_{-\Delta}^{(x)-\Delta} \frac{\mathrm{d}\xi}{\sqrt{(1 + \xi^2)(\xi^2 + \delta)}}. \tag{23}$$

Now we can rewrite the estimates (19) in the dimensionless form:

$$\begin{aligned}
\nu_R = (x)\nu_B = \beta \frac{\langle n \rangle^{5/2} e^4}{n_0 a m j \sqrt{T}} &\times \Bigg\{ \frac{(x)^2 - 2(x)\Delta}{(1 + \Delta^2)(1 + [(x) - \Delta]^2)} \\
&+ \varepsilon \sqrt{\frac{m}{M}} \frac{a}{r_D(0)} \int_{-\Delta}^{(x)-\Delta} \frac{\mathrm{d}x}{1 + x^2} \times \left[\sqrt{\frac{1 + \Delta^2}{1 + x^2}} \sqrt{\frac{\Delta^2 + \delta}{x^2 + \delta}} - 1 \right] \\
&\times \exp\left[\zeta \sqrt{\frac{m}{M}} \frac{a}{r_D(0)} \int_{-\Delta}^{x} \mathrm{d}\xi \times \left(\sqrt{\frac{1 + \Delta^2}{1 + \xi^2}} \sqrt{\frac{\Delta^2 + \delta}{\xi^2 + \delta}} - 1 \right) \right] \Bigg\},
\end{aligned}$$
$$\beta, \varepsilon, \zeta \sim 1. \tag{24}$$

In principle, Eqs. (23) and (24) solve the problem and such a result turns out to depend on the following parameters: j, a, n_0, $\langle n \rangle$, T.

Let us estimate the window of parameters providing the predominating role of the random inhomogeneity over the nonlinear effects. First, as it has been noticed above, roughly, $\Delta \sim (j/n_0 e c_s)^{1/2}$. Taking into account restrictions implied by the Buneman instability, we may believe $\Delta^2 \ll \sqrt{M/m}$. Then, our cutoff factor $\delta \sim k^{-2}(0) r_D^{-2}(0) \ll 1$, it cannot be, however, too small because of the ion Landau damping. Thus, we may put $\delta \sim T_i/T_e$.

In the nonlinear regime [6,7] with a given current the following estimate of the density of noises is true:

$$\left(\frac{W}{nT} \right)_{nl} \simeq \frac{j}{n e v_{Te}}.$$

In our case,

$$\left(\frac{W}{nT}\right)_{\max} \simeq \frac{\exp \Gamma_0}{N_D},$$

$$\Gamma_0 = \frac{a}{r_D}\sqrt{\frac{m}{M}} \int_{-\Delta}^{\Delta} \left(\sqrt{\frac{1+\Delta^2}{1+\xi^2}} \sqrt{\frac{\Delta^2+\delta}{\xi^2+\delta}} - 1 \right) d\xi.$$

Inhomogeneity turns out to be more efficient mechanism which determines the resistivity if

$$\left(\frac{W}{nT}\right)_{\max} \ll \left(\frac{W}{nT}\right)_{nl} \Rightarrow \Gamma_0 < \ln(\Delta^2 N_D),$$

$$\Delta^2 \sim \frac{j}{n_0 e c_s}, \quad N_D = \frac{1}{6\sqrt{\pi}} \frac{T_e^{3/2}}{n^{1/2} e^3}.$$

After all, the following inequality determines the applicability of our model:

$$\frac{a}{r_D}\sqrt{\frac{m}{M}}\, \Delta\sqrt{1+\Delta^2} \int_{-\Delta}^{\Delta} \frac{d\xi}{\sqrt{1+\xi^2}\sqrt{\xi^2+\delta}} < \ln(\Delta^2 n r_D^3). \qquad (25)$$

References

[1] A. Kingsep and A. Munier, *Journ. Plasma Physics*, **57**, 501 (1997).

[2] O. Penroze, *Phys. Fluids*, **3**, 258 (1960); J.B. Bernstein *et al.*, *ibid.*, **3**, 136 (1960).

[3] L.I. Rudakov and L.V. Korablev, *Sov. Phys. JETP*, **50**, 220 (1966).

[4] A.S. Kingsep, *ibid.*, **56**, 1309 (1969).

[5] R.C. Davidson and N.A. Krall, *Nuclear Fusion*, **17**, 1313 (1977).

[6] B.B. Kadomtsev, "*Reviews of Plasma Physics*", Consultants Bureau, NY, Vol. 4, 1968.

[7] R.Z. Sagdeev, *Proc. Simp. in Appl. Math.*, **18**, 281 (1967).

[8] P. Sheehey, J.E. Hammel, I.R. Lindemuth *et al.*, *Phys. Fluids B*, **4**(11), 3698 (1992); P. Sheehey and I.R. Lindemuth, *Phys. Plasmas*, **4**(1), 146 (1997).

[9] G.E. Vekshtein, D.D. Ryutov and R.Z. Sagdeev, *Sov. Phys. JETP*, **60**, 2142 (1971).

[10] A.S. Kingsep, *ibid.*, **63**, 498 (1972).

[11] A.S. Kingsep, *ibid.*, **58**, 1040 (1970).