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# From Fast to Slow Processes in the Evolution of Urban and Regional Settlement Structures*

WOLFGANG WEIDLICH

*Institute of Theoretical Physics, University of Stuttgart, Pfaffenwaldring 57/III, D-70550 Stuttgart, Germany*

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**Complex systems consist of many intertwined organizational levels starting from microstructures and ending with macrostructures. Their evolution takes place on different time scales: Micropatterns exhibit a fast dynamics whereas macropatterns develop slowly. Urban and regional science can make use of this fact by constructing a hierarchy of models on different spatio-temporal scales.**

**Based on this understanding two models are presented: One for the relatively fast urban evolution on the microscale and one for the relatively slow regional evolution on the macroscale.**

**The *micromodel* considers the urban structure as a system of sites on which different kinds of buildings (dwellings, schools, stores, service-stations, factories . . . ) can be erected. The step by step evolution of the city configuration is treated as a stochastic process guided by desirability considerations. The formalization of this concept leads to equations for the evolution of the urban city configuration. Numerical simulations illustrate this urban "microdynamics".**

**The *macromodel* treats the settlement formation in a region on a more global scale. The evolution of the density of economically active populations who produce and consume goods is considered. The driving force of density changes is the spatial difference of incomes motivating the individuals to migrate to locations of optimal income. This nonlinear process leads to the self-organization of spatially heterogeneous population distributions forming the settlements. Their micro-structure can thereupon be treated by the micromodel.**

*Keywords:* Urban dynamics, Settlement evolution, Sociodynamic modelling, Space–time windows of perception

## 0 INTRODUCTION

Human settlements belong to the most complex space–time structures in the world. In settlements there exist many different intertwined and interdependent organizational structures; the evolution of these structures takes place on different scales. Therefore the natural question arises whether this manifold of structures and processes can be ordered according to some principles, for

* This article is strongly based on a former publication of the author (*Discrete Dynamics in Nature and Society* **1**, 85–98). In the present article the different space–time windows of perception of the micromodel and macromodel are stressed.

instance the space–time scale or level on which they appear.

If this should prove true and if the relation between the levels could be formulated this would provide the justification for considering different windows of perception and for constructing different, nonetheless interrelated, models for each window.

In the following we shall see that indeed a separation of levels is indicated and that it is appropriate to comprehend and to connect the development of settlement structures on different scales with separate models. Only in a final stage the models can and should be fused into one integrated model.

## 1 THE SPACE–TIME WINDOWS OF PERCEPTION OF SETTLEMENT STRUCTURES

In synergetics there exists the fruitful "slaving principle" set up in high generality by Haken [1]. Verbally it can be formulated as follows: If in a system of nonlinear equations of motion for many variables these variables can be separated into slow ones and fast ones, a few of the slow variables (those with a trend to grow) are predestined to become "order parameters" dominating the dynamics of the whole system on the macroscale.

The reason for this remarkable system behavior is that the fast variables quickly adapt their values to the momentary state of the slow variables. Since they thereupon depend on the slow variables, the fast variables can be eliminated. As a consequence the slow variables *alone* obey a quasi-autonomous dynamics. Since all other variables depend, by adaptation, on the few slow variables which rise up to macroscopic size, the latter are denoted as order parameters and determine the macrodynamics of the system.

Let us now somewhat modify and generalize the slaving principle in view of its meaning for urban and regional structures and their dynamics.

In settlements one can easily identify fast and slow processes of change and evolution: The *fast processes* take place on the *local microlevel* of building sites where e.g. individual buildings are erected or teared down, and where the local traffic infrastructure of streets, subways, etc., is constructed. The *slow processes* take place on the *regional macrolevel*. They include the slow evolution of whole settlements like villages, towns and cities which can be considered as population agglomerations of different size, density and composition, furthermore the slow development of whole industries.

The relation between the fast development of local microstructures and the slow development of global regional macrostructures is rather simple and exhibits a strong similarity to the slaving principle.

On the one side the fast development of local microstructures is driven and guided by the quasi-constant regional macrostructure into which it is embedded. That means the global regional situation serves as the environment and the boundary condition under which each local urban microstructure evolves.

On the other hand, the (slowly developing) regional macrostructure is of course nothing but the global resultant of the many local structures of which an urban settlement is composed. However, similar to the longevity of the body of an animal, whose organs are regenerating on a shorter time scale than the lifetime of the whole body, the time of persistence of a regional macrostructure as a whole is much higher than the decay – and regeneration times of its local substructures.

Although this relation between urban microstructures and regional macrostructures is rather evident it has an important consequence for model builders: One can *separate* to some extent the microdynamic level from the macrodynamic level and make *separate adequate models for each space–time window of perception*. This means, in more detail:

In constructing a model for the *urban microevolution* it is allowed to consider some global regional parameters (e.g. referring to the global regional population or the global regional stage of industrialization) as *given environmental conditions* and to describe the fast local microdynamics as developing under these global conditions.

On the other hand, in constructing a model for the *regional macro-evolution* it is allowed to presume that a corresponding fast micro-evolution takes place which *adapts the local microstructures* to the respective slow variables of the global development.

In view of this possibility of a separate consideration of the micro- and macroperspective of settlement evolution we shall present in the next sections the design principles of a micromodel for the urban and of a macromodel for the regional evolution.

## 2 THE DESIGN PRINCIPLES OF A MICROMODEL OF URBAN EVOLUTION

In constructing a model for the urban evolution on the rather detailed level of individual building plots or sites we follow a general modelling strategy which has already proved its applicability in different sectors of sociodynamics [2]. The modelling scheme consists of the following steps:

1. A configuration space of variables characterizing the state of the urban system has to be set up.
2. A measure for the utility of each configuration under given environmental and populational conditions must be found.
3. Transition rates between neighboring configurations constitute the elements of the system dynamics. The "driving forces" behind these transitions are utility differences between the initial and the final configuration. Therefore the transition rates depend in an appropriate way on these utility differences.
4. Making use of the transition rates, evolution equations for the configurations can be derived on the stochastic and the quasi-deterministic level as well.
5. Selected scenario simulations demonstrate the evolution of characteristic urban structures.

*Step 1 The configuration space* The city landscape is considered to be tesselated into a square lattice of plots or sites $i(i_1, i_2)$, $j(j_1, j_2)$, where $(i_1, i_2)$, $(j_1, j_2)$ are integer lattice coordinates. One can introduce a distance between sites, for instance by the Manhattan metric

$$d(i,j) = |i_1 - j_1| + |i_2 - j_2|. \tag{2.1}$$

The sites can either be empty or filled with different kinds of buildings, e.g. $x_i$ lodgings, $y_i$ factories and perhaps other kinds of urban uses (service stations, store houses, parks, etc.). For simplicity we consider only lodgings and factories. The variables $x_i, y_i = 0, 1, 2, \ldots$ are integers denoting the number of (appropriately tailored) building units of the corresponding kind on site $i$. The city configuration

$$\{\mathbf{x}, \mathbf{y}\} = \{\ldots(x_i, y_i), \ldots(x_j, y_j), \ldots\} \tag{2.2}$$

characterizes the state of the city with respect to the kind, number and distribution of its buildings over the sites. It is the purpose of the model to give a formal mathematical description of the dynamics of the city configuration.

*Step 2 The utility of configurations* The utility of a given city configuration has now to be determined. The ansatz for $u(\mathbf{x}, \mathbf{y})$ comprises several terms designed to describe the main effects influencing this utility. The terms contain open coefficients to be calibrated according to the concrete case.

($\alpha$) *The local term* consists of the contributions of local utilities of erecting buildings on each site $j$:

$$u_L(\mathbf{x}, \mathbf{y}) = \sum_j u_j(x_j, y_j) \tag{2.3}$$

with

$$u_j(x_j, y_j) = p_j^{(x)} \ln(x_j + 1) + p_j^{(y)} \ln(y_j + 1) + p_j^{(z)} \ln(z_j). \tag{2.4}$$

The coefficients $p_j^{(k)} > 0$ are measures of the preferences to build on site $j$. The first two terms of (2.4) represent the increasing urban use of site $j$ with growing numbers $x_j$, $y_j$, a usefulness which however saturates if $x_j$, $y_j$ grow to high numbers. The third term describes the capacity constraint of site $j$. If $z_j$ is the empty disposable space on site $j$ and if one unit of lodging or factory needs one unit of the disposable space, respectively, the capacity $C_j$ of site $j$ is given by

$$C_j = x_j + y_j + z_j. \tag{2.5}$$

If the capacity of site $j$ tends to be exhausted for $z_j = C_j - x_j - y_j \to 0$ the third term of the utility $u_j(x_j, y_j)$ approaches $-\infty$. On the other hand, the first terms of $u_j(x_j, y_j)$ are zero for $x_j = 0$ and $y_j = 0$, respectively, and approach $-\infty$ for $x_j \to -1$ or $y_j \to -1$. As we shall see this has the consequence that states with negative values of $x_j$ or $y_j$ or with values for which $x_j + y_j \geq C_j$ can never be reached. That means, $x_j$ and $y_j$ are confined to values $x_j \geq 0$, $y_j \geq 0$ and $(x_j + y_j) < C_j$.

The value of the capacity $C_j$ on each site $j$ depends on how much this site is opened up for buildings. If the total urban population $n_c$ increases, more sites at the border of the city will be opened. In this way the size of the city area depends on its total population. We choose a Gaussian capacity distribution

$$C_j = C_0 \exp\left[-\frac{d^2(j, j_0)}{2\sigma^2(n_c)}\right] \tag{2.6}$$

where $d(j, j_0)$ is the Manhattan distance of site $j$ from the central site $j_0$ and $\sigma^2(n_c)$ is the population dependent variance: The factor $C_0$ has to be calibrated appropriately so that in the equilibrium state the population $n_c$ finds adequate total numbers $\sum_j x_j$ and $\sum_j y_j$ of lodgings and factories, respectively in the city.

($\beta$) *The interaction term* describes the – supportive or suppressive – utility influence between buildings on different sites $i$ and $j$. This term is assumed to have the form

$$u_I(\mathbf{x}, \mathbf{y}) = \sum_{i,j} a_{ij}^{xx} x_i x_j + \sum_{i,j} a_{ij}^{xy} x_i y_j + \sum_{i,j} a_{ij}^{yy} y_i y_j. \tag{2.7}$$

The signs of the coefficients decide about the interaction effect. If one choses for instance $a_{ii}^{xy} \ll 0$ this means that it is strongly disfavored and not considered useful to build lodgings *and* factories on the *same* site. If, on the other hand, $a_{ij}^{xy}$ is chosen as a *positive* parameter for $d(i, j) > d_0$ this means that lodgings on site $i$ lead to a high utility of factories on sites $j$ at a distance $d(i, j) \gtrsim d_0$, and vice versa. This is a plausible choice since workers living in the lodgings need working places in a not too distant neighborhood with $d(i, j) \gtrsim d_0$. On the other hand, the choice of positive coefficients $a_{ij}^{xx}$ and $a_{ij}^{yy}$ for $d(i, j) < d_0$ means that it is considered useful to have further lodgings in the near neighborhood ($d(i, j) < d_0$) of lodgings, and further factories in the near neighborhood of factories. In this manner the interaction term represents the dependance of the utility of a city configuration on the location of different kinds of buildings relative to each other.

The total utility of a city configuration $(\mathbf{x}, \mathbf{y})$ is now assumed to be the sum of the two terms (2.3) and (2.7):

$$u(\mathbf{x}, \mathbf{y}) - u_L(\mathbf{x}, \mathbf{y}) + u_I(\mathbf{x}, \mathbf{y}). \tag{2.8}$$

Here, the simplifying tacit assumption has been made in constructing (2.8), that *one objective* utility of a city configuration exists for all those citizens who make decisions about the development of the city.

*Step 3 The transition rates between configurations* The transition rates for a transition between the configuration $\mathbf{x}$, $\mathbf{y}$ and the neighboring configurations

$$\begin{aligned} \{\mathbf{x}^{j\pm}, \mathbf{y}\} &= \{\ldots, (x_j \pm 1; y_i), \ldots\}, \\ \{\mathbf{x}, \mathbf{y}^{j\pm}\} &= \{\ldots, (x_j; y_i \pm 1), \ldots\} \end{aligned} \tag{2.9}$$

must now be set up. Firstly they must be positive definite quantities. Secondly they should depend monotonously on the utility difference between the final and initial configuration, because these utility differences are the "driving forces" behind the activities effecting the transition.

The simplest and mathematically most appealing ansatz for the transition rates fulfilling these conditions is the following:

$$\left.\begin{aligned} \omega_{j\uparrow}^{(x)}(\mathbf{x},\mathbf{y}) &= \nu_{\uparrow}^{(x)}\cdot\exp\{\Delta_{j+}^{(x)}u(\mathbf{x},\mathbf{y})\}\\ \omega_{j\uparrow}^{(y)}(\mathbf{x},\mathbf{y}) &= \nu_{\uparrow}^{(y)}\cdot\exp\{\Delta_{j+}^{(y)}u(\mathbf{x},\mathbf{y})\} \end{aligned}\right] \quad \begin{array}{l}\text{(building up rates}\\ \text{for lodgings and}\\ \text{factories at site } j\text{),}\end{array} \tag{2.10}$$

$$\left.\begin{aligned} \omega_{j\downarrow}^{(x)}(\mathbf{x},\mathbf{y}) &= \nu_{\downarrow}^{(x)}\cdot\exp\{\Delta_{j-}^{(x)}u(\mathbf{x},\mathbf{y})\}\\ \omega_{j\downarrow}^{(y)}(\mathbf{x},\mathbf{y}) &= \nu_{\downarrow}^{(y)}\cdot\exp\{\Delta_{j-}^{(y)}u(\mathbf{x},\mathbf{y})\} \end{aligned}\right] \quad \begin{array}{l}\text{(tearing down rates}\\ \text{for lodgings and}\\ \text{factories at site } j\text{),}\end{array} \tag{2.11}$$

with

$$\begin{aligned} \Delta_{j\pm}^{(x)}u(\mathbf{x},\mathbf{y}) &= u(\mathbf{x}^{j\pm},\mathbf{y}) - u(\mathbf{x},\mathbf{y}),\\ \Delta_{j\pm}^{(y)}u(\mathbf{x},\mathbf{y}) &= u(\mathbf{x},\mathbf{y}^{j\pm}) - u(\mathbf{x},\mathbf{y}). \end{aligned} \tag{2.12}$$

Here we have taken into account that there will exist different global frequencies $\nu_{\uparrow}^{(x)}$, $\nu_{\uparrow}^{(y)}$ for building up processes and $\nu_{\downarrow}^{(x)}$, $\nu_{\downarrow}^{(y)}$ for tearing down processes.

*Step 4 Evolution equations for configurations* The transition rates which depend on utility differences between neighboring configurations are the starting point for setting up evolution equations for the configurations. Exactly speaking, the rates are probability transition rates per unit time. The exact equation corresponding to these quantitities is the master equation for the probability $P(\mathbf{x},\mathbf{y};t)$ to find the configuration $(\mathbf{x},\mathbf{y})$ at time $t$. It reads:

$$\begin{aligned} \frac{\mathrm{d}P(\mathbf{x},\mathbf{y};t)}{\mathrm{d}t} = &\sum_j \Big[\omega_{j\uparrow}^{(x)}(\mathbf{x}^{j-},\mathbf{y})P(\mathbf{x}^{j-},\mathbf{y};t) - \omega_{j\uparrow}^{(x)}(\mathbf{x},\mathbf{y})P(\mathbf{x},\mathbf{y};t)\Big]\\ &+ \sum_j \Big[\omega_{j\downarrow}^{(x)}(\mathbf{x}^{j+},\mathbf{y})P(\mathbf{x}^{j+},\mathbf{y};t) - \omega_{j\downarrow}^{(x)}(\mathbf{x},\mathbf{y})P(\mathbf{x},\mathbf{y};t)\Big]\\ &+ \sum_j \Big[\omega_{j\uparrow}^{(y)}(\mathbf{x},\mathbf{y}^{j-})P(\mathbf{x},\mathbf{y}^{j-};t) - \omega_{j\uparrow}^{(y)}(\mathbf{x},\mathbf{y})P(\mathbf{x},\mathbf{y};t)\Big]\\ &+ \sum_j \Big[\omega_{j\downarrow}^{(y)}(\mathbf{x},\mathbf{y}^{j+})P(\mathbf{x},\mathbf{y}^{j+};t) - \omega_{j\downarrow}^{(y)}(\mathbf{x},\mathbf{y})P(\mathbf{x},\mathbf{y};t)\Big]. \end{aligned} \tag{2.13}$$

From the master equation there can easily be derived exact equations of motion for the mean values $\bar{x}_j(t)$, $\bar{y}_j(t)$ of the components $x_j$, $y_j$ of the city configuration, which are defined by

$$\begin{aligned} \bar{x}_j(t) &= \sum_{\{\mathbf{x},\mathbf{y}\}} x_j P(\mathbf{x},\mathbf{y};t),\\ \bar{y}_j(t) &= \sum_{\{\mathbf{x},\mathbf{y}\}} y_j P(\mathbf{x},\mathbf{y};t). \end{aligned} \tag{2.14}$$

They read:

$$\begin{aligned} \frac{\mathrm{d}\bar{x}_j(t)}{\mathrm{d}t} &= \overline{\omega_{j\uparrow}^{(x)}(\mathbf{x},\mathbf{y})} - \overline{\omega_{j\downarrow}^{(x)}(\mathbf{x},\mathbf{y})},\\ \frac{\mathrm{d}\bar{y}_j(t)}{\mathrm{d}t} &= \overline{\omega_{j\uparrow}^{(y)}(\mathbf{x},\mathbf{y})} - \overline{\omega_{j\downarrow}^{(y)}(\mathbf{x},\mathbf{y})}, \end{aligned} \tag{2.15}$$

where the bars on the right hand side mean taking mean values with the probability distribution $P(\mathbf{x},\mathbf{y};t)$. The quasi-mean values $\tilde{x}_j(t), \tilde{y}_i(t)$ obey – in constrast to (2.15) – self-contained autonomous equations of motion which arise from (2.16) by substituting on the right hand side:

$$\overline{\omega(\mathbf{x},\mathbf{y})} \Rightarrow \omega(\tilde{\mathbf{x}}(t),\tilde{\mathbf{x}}(t)) \tag{2.16}$$

which leads to

$$\begin{aligned} \frac{\mathrm{d}\tilde{x}_j(t)}{\mathrm{d}t} &= \omega_{j\uparrow}^{(x)}(\tilde{\mathbf{x}}(t),\tilde{\mathbf{y}}(t)) - \omega_{j\downarrow}^{(x)}(\tilde{\mathbf{x}}(t),\tilde{\mathbf{y}}(t)),\\ \frac{\mathrm{d}\tilde{y}_j(t)}{\mathrm{d}t} &= \omega_{j\uparrow}^{(y)}(\tilde{\mathbf{x}}(t),\tilde{\mathbf{y}}(t)) - \omega_{j\downarrow}^{(y)}(\tilde{\mathbf{x}}(t),\tilde{\mathbf{y}}(t)). \end{aligned} \tag{2.17}$$

For unimodal probability distributions $P(\mathbf{x},\mathbf{y};t)$ the quasi-mean values approximate the true mean values; however, in the case of multimodal probability distributions the quasi-mean values no longer approximate the mean values; instead they approximate the true trajectories of the evolution of the city configuration.

## 3 SELECTED SIMULATIONS OF THE MICROMODEL OF URBAN EVOLUTION

In the following (Figs. 1 and 2) we exhibit selected results of simulations based on the micromodel of urban evolution discussed above.

## 4 THE DESIGN PRINCIPLE OF A MACROMODEL OF THE REGIONAL EVOLUTION OF SETTLEMENT STRUCTURES

The "space–time window of perception" of a macromodel is open towards the more coarse-grained spatial structures and the slow temporal evolutions. On the other hand, the fast processes on each local site are averaged out here, since we look at the slow evolution on the regional scale only. Therefore global variables are needed which represent the regional processes and structures.

The proposed macromodel is designed according to the following principles:

1. The economic and the population-dynamic migratory sector are integrated.
2. Populations are described by population densities distributed over the plane.
3. The populations produce goods; the local production costs including fixed costs and production costs, hence the local individual incomes depend on the population densities. The economy is assumed to be in equilibrium with the momentary population distribution.

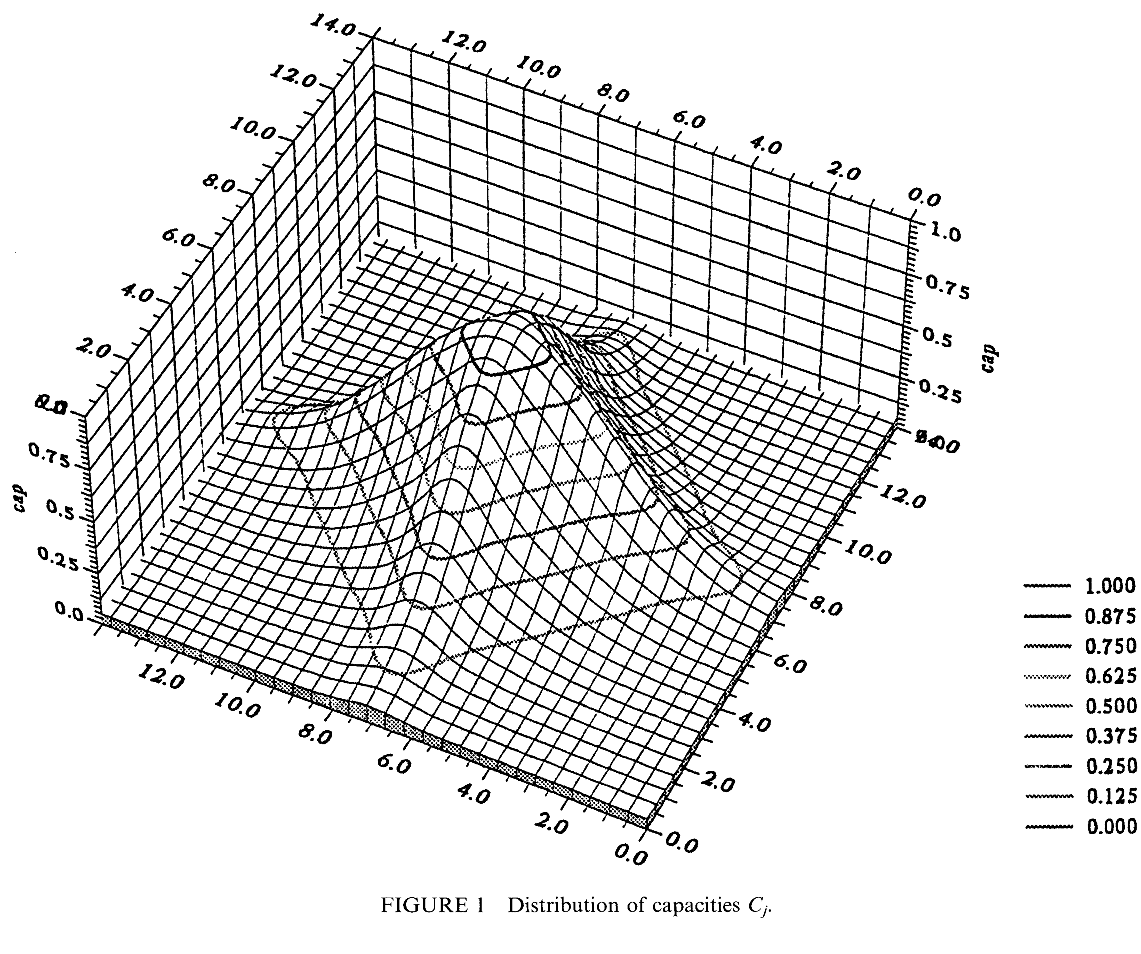


FIGURE 1 Distribution of capacities $C_j$.

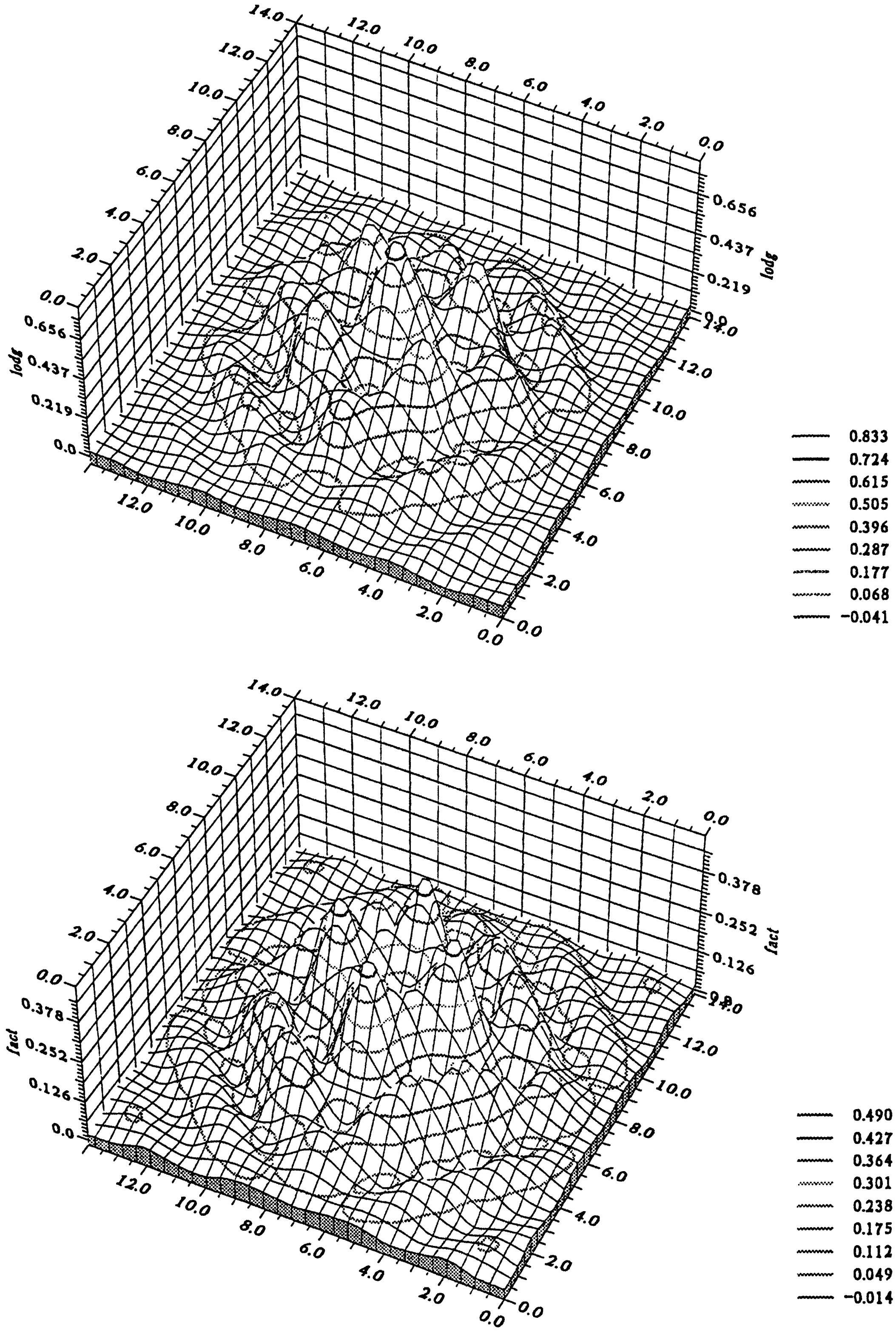


FIGURE 2 Distribution of (a) lodgings ($x_j$), (b) factories ($y_j$).

4. The members of the population migrate between different locations. Driving forces of this non-linear migration process are income differences between locations.
5. The migration leads to the formation of spatially heterogeneous population distributions; i.e. the settlements.

Let us now formulate these principles in mathematical form. We consider $A$ productive populations $\mathcal{P}_\alpha$, $\alpha = 1, 2, \ldots, A$, each producing for simplicity only one kind of commodity composed of units $C_\alpha$. Furthermore we assume two service populations, the landowners $\mathcal{P}_\lambda$ renting premises to the producers and the transporters $\mathcal{P}_\tau$ dispatching the goods of the producers.

Let $n_\alpha(\mathbf{x}, t)$ be the *density of population* $\mathcal{P}_\alpha$ at position $\mathbf{x}$ and time $t$ and $c_{pa}(\mathbf{x}, t)$ the production density of $\mathcal{P}_\alpha$, i.e. the number of units $C_\alpha$ produced per unit area and time. The *production density* is assumed to have the form

$$c_{pa}(\mathbf{x}, t) = \gamma_\alpha(\mathbf{x}, t) n_\alpha(\mathbf{x}, t) \tag{4.1}$$

with the *productivity factor*

$$\gamma_\alpha(\mathbf{x}, t) = \bar{\gamma}_\alpha \left[ \frac{n_\alpha(\mathbf{x}, t)}{\bar{n}_\alpha} \right]^{a_\alpha}. \tag{4.2}$$

The form (4.2) of $\gamma_\alpha$ expresses an "*economy of scale*" in the production of commodity $C_\alpha$. If the productivity exponent is $a_\alpha > 0$, the production density grows more than proportional to $n_\alpha(\mathbf{x}, t)$. This will be true for many industrial goods, whereas for agrarian goods the production density grows less than proportional to $n_\alpha(\mathbf{x}, t)$, which amounts to a productivity exponent $a_\alpha < 0$.

Further densities of economic quantities can now easily be introduced. If $\mathcal{P}_\alpha$ is the price of one unit $C_\alpha$, then the *gross income density* of population $\mathcal{P}_\alpha$ is given by

$$e_\alpha(\mathbf{x}, t) = P_\alpha c_{pa}(\mathbf{x}, t); \quad \alpha = 1, 2, \ldots, A. \tag{4.3}$$

The *net income density* of $\mathcal{P}_\alpha$

$$w_\alpha(\mathbf{x}, t) = e_\alpha(\mathbf{x}, t) - k_\alpha(\mathbf{x}, t) - t_\alpha(\mathbf{x}, t) \tag{4.4}$$

follows by *deducting the fixed costs density*

$$k_\alpha(\mathbf{x}, t) = \rho_\alpha P_\alpha \bar{\gamma}_\alpha n_\alpha(\mathbf{x}, t) \left[ \nu_{\alpha 0} + \nu_{\alpha 1} \left( \frac{n_\pi(\mathbf{x}, t)}{\bar{n}_\pi} \right) + \nu_{\alpha 2} \left( \frac{n_\pi(\mathbf{x}, t)}{\bar{n}_\pi} \right)^2 \right] \tag{4.5}$$

and the *transport density*

$$t_\alpha(\mathbf{x}, t) = \sigma_\alpha P_\alpha L^{-1} c_{pa}(\mathbf{x}, t) L^{-1} \bar{d}(\mathbf{x}, t) \tag{4.6}$$

from the *gross income density*.

In (4.5) and (4.6) a reasonable ansatz has been made for the dependance of the fixed costs and the transport costs on the partial and total population densities $n_\alpha(\mathbf{x}, t)$ and $n_\pi(\mathbf{x}, t)$, where

$$n_\pi(\mathbf{x}, t) = \sum_{\alpha=1}^{A} n_\alpha(\mathbf{x}, t). \tag{4.7}$$

The *fixed costs*, with $\rho_\alpha$ as fixed share coefficient, grow according to (4.5) over-proportionally with the total density $n_\pi(\mathbf{x}, t)$ of the local population, and the *transport costs*, with $\sigma_\alpha$ as transport share coefficient, are proportional to the mean transport distance $\bar{d}_\alpha(\mathbf{x}, t)$ of good $C_\alpha$ from the place of production $\mathbf{x}$.

The fixed costs (for renting premises) and transport costs (for dispatching goods) for the producing populations $\mathcal{P}_\alpha$ are simultaneously the *net incomes* $w_\lambda(\mathbf{x}, t)$ and $w_\tau(\mathbf{x}, t)$ for *the service populations* $\mathcal{P}_\lambda$ and $\mathcal{P}_\tau$, respectively:

$$w_\lambda(\mathbf{x}, t) = \sum_{\alpha=1}^{A} k_\alpha(\mathbf{x}, t) \tag{4.8}$$

and

$$w_\tau(\mathbf{x}, t) = \sum_{\alpha=1}^{A} t_\alpha(\mathbf{x}, t). \tag{4.9}$$

The relative prices $P_\alpha$ of the commodity units $C_\alpha$ can now be determined by taking into account that the goods are not only *produced* but also *consumed* by the same populations $\mathcal{P}_1, \ldots, \mathcal{P}_A, \mathcal{P}_\lambda, \mathcal{P}_\tau$ in the total area $\mathcal{A}$ under consideration: If $c_{c\alpha}(\mathbf{x}, t)$ is the *consumption density* of commodity $C_\alpha$, which corresponds to its production density, and which can be expressed by the local net incomes, then the equilibrium between production and consumption in the assumed closed economy of area $\mathcal{A}$ can be expressed by

$$\int_{\mathcal{A}} c_{p\alpha}(\mathbf{x}, t)\, d^2x = \int_{\mathcal{A}} c_{c\alpha}(\mathbf{x}, t)\, d^2x; \quad \alpha = 1, 2, \ldots, A. \tag{4.10}$$

From (4.10) there follow the relative prices (for details see [3,4]).

Finally, the *local net incomes per individual* are easily obtained:

$$\omega_\alpha(\mathbf{x}, t) = \frac{w_\alpha(\mathbf{x}, t)}{n_\alpha(\mathbf{x}, t)} \quad \text{with } \alpha = 1, \ldots, A; \lambda, \tau. \tag{4.11}$$

It is important to note that all economic quantities introduced so far, in particular the local individual net income (4.11), are expressed as functions of the population densities.

This means that the state of the simple economy described here (with production, income, and consumption densities, etc.) is well defined if the population densities are known. We now assume that this still holds if the $n_\alpha(\mathbf{x}, t)$ slowly evolve with time. This assumption implies that the adaptation of the economy to the momentary values of the $n_\alpha(\mathbf{x}, t)$ is *fast and flexible enough* to keep it always in *momentary equilibrium* with the population distribution.

We shall now see that a slow migration process governed by nonlinear migratory equations sets in if we let the motivations of the individuals to change their location depend on simple economic considerations. The result of the migration process is that a (perhaps initially existing) homogeneous density distribution of the populations becomes instable and that the different subpopulations $\mathcal{P}_\alpha$ segregate into agglomerations of different size and density. We conclude that already simple assumptions about economic and migratory structures lead to the self-organization of settlements.

The equations of motion for the population densities are obvious. They read

$$\frac{dn_\alpha(\mathbf{x}', t)}{dt} = \int_{\mathcal{A}} r_\alpha(\mathbf{x}', \mathbf{x}; t) n_\alpha(\mathbf{x}, t)\, d^2x - \int_{\mathcal{A}} r_\alpha(\mathbf{x}, \mathbf{x}'; t) n_\alpha(\mathbf{x}', t)\, d^2x \quad \text{for } \alpha = 1, 2, \ldots, A; \lambda, \tau, \tag{4.12}$$

where the decisive quantity is

$$d^2x' r_\alpha(\mathbf{x}', \mathbf{x}; t) = \textit{transition rate} \text{ of a member of } \mathcal{P}_\alpha \text{ from } \mathbf{x} \text{ to } \mathbf{x}' \text{ into the area element } d^2x'. \tag{4.13}$$

The form of this rate, which has been substantiated in [5], is

$$r_\alpha(\mathbf{x}', \mathbf{x}; t) = \mu_\alpha \exp[u_\alpha(\mathbf{x}', t) - u_\alpha(\mathbf{x}, t)] \tag{4.14}$$

where the "dynamic utility" $u_\alpha(\mathbf{x}, t)$, which is sometimes also denoted as motivation potential, is a measure of the attraction of location $\mathbf{x}'$ to a member of population $\mathcal{P}_\alpha$ at time $t$.

A plausible assumption in the frame of our simple model is that $u_\alpha(\mathbf{x}, t)$ is proportional to the local individual net income $\omega_\alpha(\mathbf{x}, t)$, i.e.

$$u_\alpha(\mathbf{x}, t) = \beta\omega_\alpha(\mathbf{x}, t) \tag{4.15}$$

where $\beta$ is a sensitivity factor calibrating the strength of migratory reactions to space-dependent income variations.

If (4.15) is inserted into (4.14) and (4.12), where $\omega_\alpha(\mathbf{x}, t)$ is to be expressed in terms of the population densities, Eq. (4.12) takes the form of a nonlinear integro-differential equation which can be solved numerically (see Section 5).

At the end of this short presentation of the macromodel we exhibit the interrelation of its construction elements in schematic form (Fig. 3).

A final remark should be made about the connection between the macromodel and the micromodel. As the simulations show (see Section 5) the macromodel demonstrates that global population agglomerations on a regional scale can arise by migration from a hinterland into an urban area taking shape due to certain economic production laws and migration decisions.

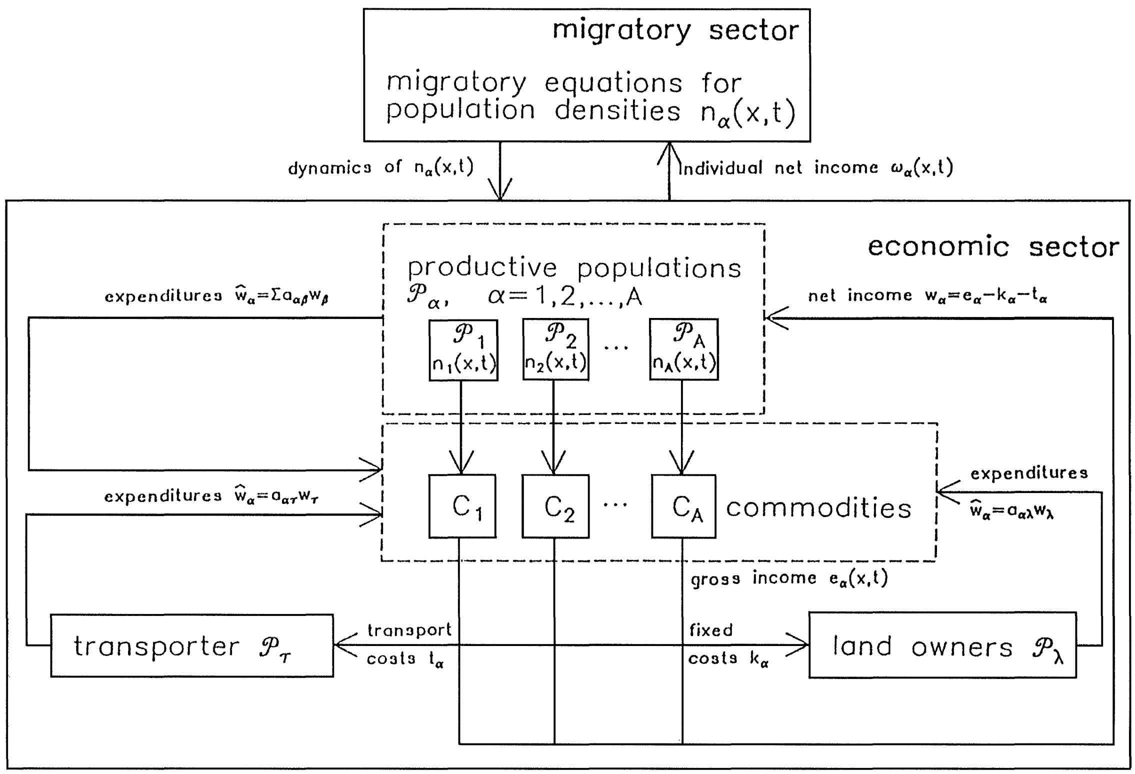


FIGURE 3 The interrelation of the elements of the macromodel for the formulation of settlement structures.

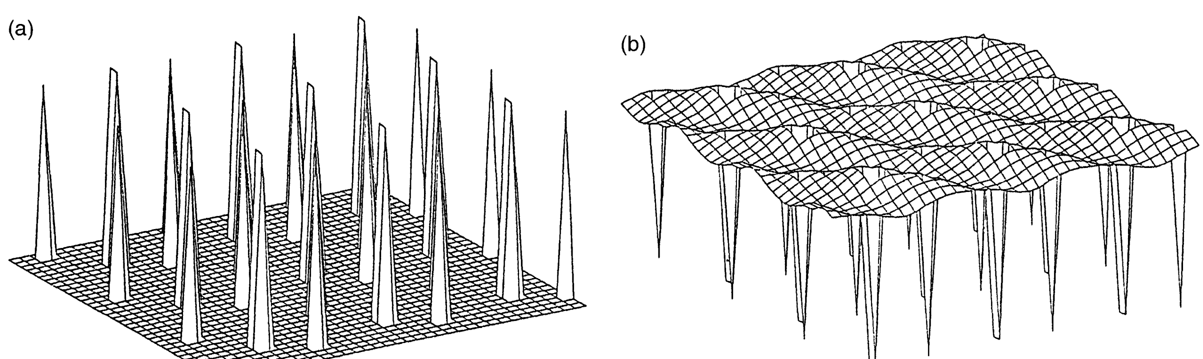


FIGURE 4 Parameters: inclusion of fixed costs ($\rho > 0$) and of transportation costs ($\sigma > 0$). (a) Stationary formation of more than one "town" per unit area in spatial neighborhood, settled by "craftsmen". (b) Stationary ring-shaped rural settlements of "peasants" around the "towns".

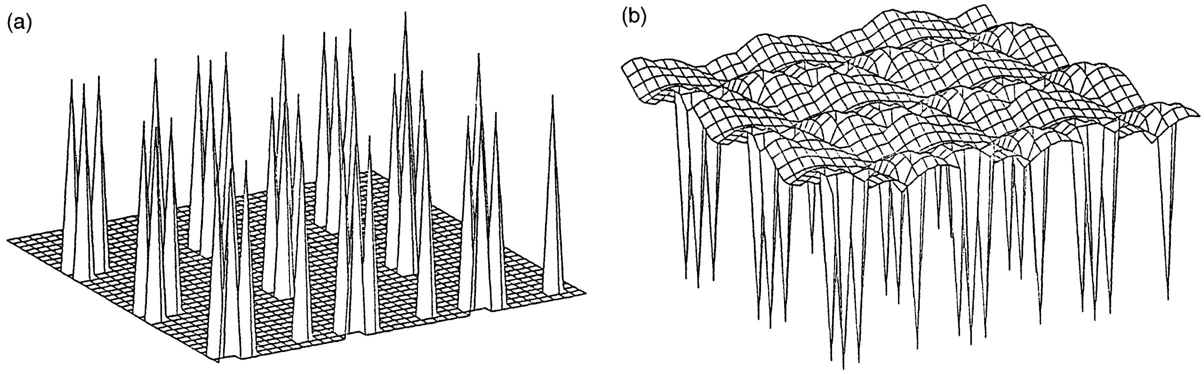


FIGURE 5 Parameters: inclusion of fixed costs ($\rho > 0$) and of transportation costs ($\sigma > 0$). (a) Stationary formation of differentiated "town-structures" settled by "craftsmen" in each unit area. (b) Differentiated ring-shaped rural settlements of "peasants" around the "town-structures".

Thereupon the micromodel considers the decision mechanisms how the – slowly varying – total urban population exerts a population pressure which in turn leads on the local level of sites to the organization of differentiated urban substructures.

## 5 SELECTED SIMULATIONS OF THE MACROMODEL

We present the result of numerical solutions of Eq. (4.12) in the case of only two productive populations which can be interpreted as "peasants" ($\alpha = \text{p}$) and "craftsmen" ($\alpha = \text{c}$). They are characterized by different productivity exponents (see Eq. (4.2)), namely

$$a_{\text{p}} = -0.1 < 0; \qquad a_{\text{c}} = 0.25 > 0. \tag{5.1}$$

Figures 4 and 5 depict nine equivalent unit areas (in which by construction the population distribution is periodically repeated). The densities in all figures are scaled to their maximum values (for details of the calibration of the model see [4]).

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