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Research Article

Compact Weighted Composition Operators and Fixed Points in Convex Domains

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Let D be a bounded, convex domain in $\mathbb{C}^n$, and suppose that $\phi : D \to D$ is holomorphic. Assume that $\psi : D \to \mathbb{C}$ is analytic, bounded away from zero toward the boundary of D, and not identically zero on the fixed point set of D. Suppose also that the weighted composition operator $W_{\psi,\phi}$ given by $W_{\psi,\phi}(f) = \psi(f \circ \phi)$ is compact on a holomorphic, functional Hilbert space (containing the polynomial functions densely) on D with reproducing kernel K satisfying $K(z,z) \to \infty$ as $z \to \partial D$. We extend the results of J. Caughran/H. Schwartz for unweighted composition operators on the Hardy space of the unit disk and B. MacCluer on the ball by showing that ϕ has a unique fixed point in D. We apply this result by making a reasonable conjecture about the spectrum of $W_{\psi,\phi}$ based on previous results.

1. Introduction

Let ϕ be a holomorphic self-map of a bounded domain D in $\mathbb{C}^n$, and suppose that ψ is a holomorphic function on D. We define the linear operator $W_{\psi,\phi}$ on the linear space of complex-valued, holomorphic functions $\mathcal{H}(D)$ by

$$W_{\psi,\phi}(f) = \psi(f \circ \phi). \tag{1.1}$$

$W_{\psi,\phi}$ is called the *weighted composition operator* induced by the *weight symbol* ψ and *composition symbol* ϕ. Note that $W_{\psi,\phi}$ is the (unweighted) composition operator C_ϕ given by $C_\phi(f) = f \circ \phi$, when $\psi = 1$.

It is natural to consider the dynamics of the sequence of iterates of a composition symbol of a weighted composition operator and the spectra of such operators. The following classical result of [1] began this line of investigation for compact, unweighted composition operators in the one-variable case. The reader is referred to [2, Chapter 2] for basic facts about composition operators and the definition of the Hardy space of the unit disk.

Theorem 1.1. *Let $\phi : \Delta \to \Delta$ be an analytic self-map of the unit disk Δ in $\mathbb{C}$. If C_ϕ is compact or power compact on the Hardy space $H^2(\Delta)$, then the following statements hold.*

(a) *ϕ has a unique fixed point in Δ (this point turns out to be the so-called Denjoy-Wolff point a of ϕ in Δ; see [2, Chapter 2]).*

(b) *The spectrum of C_ϕ is the set consisting of 0, 1, and all powers of $\phi'(a)$.*

The analogue of this result for Hardy spaces of the unit ball $\mathbb{B}_n$ in $\mathbb{C}^n$ was obtained by MacCluer in [3].

Theorem 1.2. *Let $\phi : \mathbb{B}_n \to \mathbb{B}_n$ be a holomorphic self-map of $\mathbb{B}_n$ and suppose that $p \geq 1$. If C_ϕ is compact or power compact on the Hardy space $H^p(\mathbb{B}_n)$, then*

(a) *ϕ must have a unique fixed point in $\mathbb{B}_n$ (again, this point is the so-called Denjoy-Wolff point a of ϕ in $\mathbb{B}_n$; see [2, Chapter 2]);*

(b) *the spectrum of C_ϕ is the set consisting of 0, 1, and all products of eigenvalues of $\phi'(a)$.*

This result also holds for weighted Bergman spaces of $\mathbb{B}_n$ [2]. The proofs of parts (a) of Theorems 1.1 and 1.2 appeal to the Denjoy-Wolff theorems in Δ and $\mathbb{B}_n$. Therefore, it is natural to consider whether Theorem 1.1 holds when $\mathbb{B}_n$ is replaced by more general bounded symmetric domains or even the polydisk Δ^n. It has been shown that the Denjoy-Wolff theorem fails in Δ^n for $n > 1$; nevertheless, it is shown in [4] that MacCluer's results can be generalized from $\mathbb{B}_n$ to arbitrary bounded symmetric domains that are either reducible or irreducible.

Recently, in [5] (additionally, see [6–8] for related results), Theorem 1.1 has been extended to weighted composition operators on a certain class of weighted Hardy spaces of Δ, when ψ is bounded away from 0 toward the unit circle in $\mathbb{C}$.

Theorem 1.3. *Let $(b_j)_{j\in\mathbb{N}}$ be a sequence of positive numbers such that $\liminf_{j\to\infty} b_j^{1/j} \geq 1$, and let $H_b^2(\Delta)$ be the weighted Hardy space of analytic functions $f : \Delta \to \mathbb{C}$ whose MacClaurin series $f(z) = \sum_{j=0}^{\infty} a_j z^j$ satisfy $\sum_{j=0}^{\infty} |a_j|^2 b_j^2 < \infty$. Suppose that $\phi : \Delta \to \Delta$ is analytic, and let $\psi : \Delta \to \mathbb{C}$ be an analytic map that is bounded away from zero toward the unit circle. Assume that $W_{\psi,\phi}$ is compact on $H_b^2(\Delta)$. Then the following statements hold:*

(a) *ϕ has a unique fixed point $a \in \Delta$;*

(b) *the spectrum of $W_{\psi,\phi}$ is the set*

$$\left\{0, \psi(a)\right\} \cup \left\{\psi(a)[\phi'(a)]^j : j \in \mathbb{N}\right\}. \tag{1.2}$$

In Section 2, we will introduce some basic notation. The main objective of this paper is to obtain a version of part (a) of Theorem 1.3 that applies to a large class of functional Hilbert spaces on convex domains in one or more variables. This result will be stated and proved in Section 3. In Section 4, we apply our main result to Hardy and weighted Bergman spaces of bounded symmetric domains and make a natural conjecture about the spectrum of $W_{\psi,\phi}$ when it is compact in the general setting of our main result.

2. Notation and definitions

As in [2, page 2], a Hilbert space $\mathcal{Y}$ is called a *functional Hilbert space* on a given set X if the following conditions hold.

(1) Its underlying vector space consists of complex-valued functions on X, with vector addition given by pointwise addition of functions, and scalar multiplication given by $(\alpha f)(x) = \alpha f(x)$ for $\alpha \in \mathbb{C}$, $f \in \mathcal{Y}$, and $x \in X$.

(2) Whenever $f, g \in \mathcal{Y}$ and $f(x) = g(x)$ for all $x \in X$, we have that $f = g$.

(3) Whenever $f, g \in \mathcal{Y}, x, y \in X$, and $f(x) = f(y)$ for all $f \in \mathcal{Y}$, we have that $x = y$.

(4) For each $x \in X$, the *point evaluation* functional P_x on $\mathcal{Y}$, given by $P_x(f) = f(x)$ for all $f \in \mathcal{Y}$, is bounded.

Fix $n \in \mathbb{N}$. We denote the usual Euclidean distance from $z \in \mathbb{C}^n$ to $A \subset \mathbb{C}^n$ by $d(z,A)$, and we say that $z \to A$ if and only if $d(z,A) \to 0$.

Let D be a bounded domain in $\mathbb{C}^n$, and suppose that $\psi : D \to \mathbb{C}$. We say that ψ *is bounded away from zero toward the boundary of* D if and only if

$$\liminf_{z \to v} |\psi(z)| > 0 \quad \text{for each } v \in \partial D. \tag{2.1}$$

If $\mathcal{Y}$ is a functional Hilbert space of holomorphic functions defined on a domain $D \subset \mathbb{C}^n$, then for each $z \in D$, there is a unique $K_z \in \mathcal{Y}$ such that

$$f(z) = \langle f, K_z \rangle \quad \forall f \in \mathcal{Y}. \tag{2.2}$$

This uniqueness allows one to define the *reproducing kernel* $K : D \times D \to \mathbb{C}$ for $\mathcal{Y}$ by $K(z,w) = K_z(w)$.

3. The main result

The following result continues ideas in [1] and the fixed point portion of [4, Theorem 4.2]. In preparation for the proof that follows, we refer the reader to [4] for the definition of compact divergence.

Theorem 3.1. *Let $D \subset \mathbb{C}^n$ be a bounded, convex domain, and suppose that $\mathcal{Y}$ is a functional Hilbert space of holomorphic functions on D with reproducing kernel $K : D \times D \to \mathbb{C}$. Assume that $K(z,z) \to \infty$ as $z \to \partial D$, and assume that the polynomial functions on D are dense in $\mathcal{Y}$. Suppose that $\psi : D \to \mathbb{C}$ is holomorphic and bounded away from zero toward the boundary of D, and let $\phi : D \to D$ be holomorphic, with ψ not identically zero on the fixed point set of ϕ. Assume that $W_{\psi,\phi}$ is compact on $\mathcal{Y}$. Then ϕ has a unique fixed point in D.*

Proof. Let $k_z = K_z / \|K_z\|\mathcal{Y}$. Since $K(z,z) \to \infty$ as $z \to \partial D$ and the polynomials functions on D are dense in $\mathcal{Y}$, one can show, using an argument identical to that of the proof of [4, Lemma 3.1], that $k_z \to 0$ weakly as $z \to \partial D$. From the linearity of $W_{\psi,\phi}$ and the identity

$$W^*_{\psi,\phi} K_z = \overline{\psi(z)} K_{\phi(z)}, \tag{3.1}$$

it immediately follows that

$$\|W_{\psi,\phi}^* k_z\|_{\mathcal{Y}}^2 = |\psi(z)|^2 K(z,z)^{-1} K[\phi(z),\phi(z)]. \tag{3.2}$$

Since $k_z \to 0$ weakly as $z \to \partial D$, we then have that

$$\lim_{z\to\partial D} |\psi(z)|^2 K(z,z)^{-1} K[\phi(z),\phi(z)] = 0. \tag{3.3}$$

First, suppose that ϕ has no fixed point in D. We will obtain a contradiction. Let $z \in D$. Since D is convex, the sequence of iterates $\phi^{(j)}$ of ϕ is compactly divergent [9, page 274]. Thus, for every compact $K \subset D$, there is an $N \in \mathbb{N}$ such that $\phi^{(j)}(z) \in D\backslash K$ for all $j \geq N$.

Since for any $\varepsilon > 0$, the set K_ε of all $w \in D$ such that $d(w,\partial D) \geq \varepsilon$ is compact, it follows from the statement above that for all $\varepsilon > 0$, there is an $N \in \mathbb{N}$ such that for all $j \geq N$, $\phi^{(j)}(z) \notin K_\varepsilon$; alternatively, $d(\phi^{(j)}(z),\partial D) < \varepsilon$ for $j \geq N$. Hence we have that $\phi^{(j)}(z) \to \partial D$ as $j \to \infty$ for all $z \in D$. This sequence has a subsequence, which we relabel again without loss of generality as $\phi^{(j)}(z)$, such that $\phi^{(j)}(z) \to \nu$ for some $\nu \in \partial D$. Since $K(z,z) \to \infty$ as $z \to \partial D$ by assumption, it must be the case that

$$\lim_{j\to\infty} K[\phi^{(j)}(z),\phi^{(j)}(z)] = \infty. \tag{3.4}$$

Consequently, for any $z \in D$, and for infinitely many values of j, we have that

$$K\{\phi[\phi^{(j)}(z)],\phi[\phi^{(j)}(z)]\} > K[\phi^{(j)}(z),\phi^{(j)}(z)] > 0. \tag{3.5}$$

This statement and the assumption that ψ is bounded away from 0 toward the boundary of D together imply that there must be $\mu > 0$ and $\delta > 0$ such that whenever $w \in D$ and $d(w,\nu) < \delta$, we have that $|\psi(w)| > \mu$. In addition, for sufficiently large j, we have that $d(\phi^{(j)}(z),\nu) < \delta$, so that for these values of j, $|\psi[\phi^{(j)}(z)]| > \mu$. Therefore, for any $z \in D$, there is an $N \in \mathbb{N}$ such that the following inequality holds for infinitely many $j \geq N$:

$$|\psi(\phi^{(j)}(z))|^2 K\{\phi[\phi^{(j)}(z)],\phi[\phi^{(j)}(z)]\} > \mu^2 K[\phi^{(j)}(z),\phi^{(j)}(z)] > 0. \tag{3.6}$$

In particular, for any $z \in D$, there are infinitely many values of j such that

$$|\psi[\phi^{(j)}(z)]|^2 K[\phi^{(j)}(z),\phi^{(j)}(z)]^{-1} K\{\phi[\phi^{(j)}(z)],\phi[\phi^{(j)}(z)]\} > \mu^2. \tag{3.7}$$

Denote this sequence of values of j by $(j_k)_{k\in\mathbb{N}}$. Then, we have that $\phi^{(j_k)}(z) \to \nu$ as $k \to \infty$. This fact, in combination with the fact that the above inequality holds for the subsequence $(j_k)_{k\in\mathbb{N}}$ of $\mathbb{N}$ for our arbitrary choice of $z \in D$, leads to a contradiction of (1.1). Hence the assumption that ϕ has no fixed points is false.

To show that ϕ has only one fixed point, assume to the contrary that ϕ has more than one fixed point. By a result of Vigué, the fixed point set of a holomorphic self-map of a bounded, convex domain in $\mathbb{C}^n$ is a connected, analytic submanifold of that domain (see [4, Theorem 4.1] or [10]). Since the fixed point set of ϕ is not a singleton by assumption, we must have in particular that the fixed point set of ϕ is uncountable. Denote this set of fixed points by $\mathcal{F}$. We then have that

$$W^*_{\psi,\phi}(K_a) = \overline{\psi(a)}K_{\phi(a)} = \overline{\psi(a)}K_a \quad \forall a \in \mathcal{F}. \tag{3.8}$$

Therefore, for all $a \in \mathcal{F}$, we have that $\overline{\psi(a)}$ is an eigenvalue of the compact operator $W^*_{\psi,\phi}$. Since ψ is continuous and $\mathcal{F}$ is a connected, analytic manifold in $\mathbb{C}^n$, $\psi(\mathcal{F})$ must be either a singleton or uncountable.

First, assume that $\psi(\mathcal{F})$ is a singleton $\{\lambda\}$, so that Condition (3.8) becomes

$$W^*_{\psi,\phi}(K_a) = \bar{\lambda}K_{\phi(a)} = \bar{\lambda}K_a \quad \forall a \in \mathcal{F}. \tag{3.9}$$

By the assumption that ψ is not identically zero on $\mathcal{F}$, we have that $\lambda \neq 0$. Since $\{K_a : a \in D\}$ is a linearly independent set, it follows that the $\bar{\lambda}$-eigenspace of $W^*_{\psi,\phi}$ has infinite dimension. However, by [11, Proposition 4.13], this infiniteness contradicts the compactness of $W^*_{\psi,\phi}$ on $\mathcal{Y}^*$.

Next, assume that $\psi(\mathcal{F})$ is uncountable. Then, by Condition (3.8), $W^*_{\psi,\phi}$ has uncountably many eigenvalues $\overline{\psi(a)}$ with $a \in \mathcal{F}$. Now, since $\mathcal{Y}$ contains the polynomials and is, therefore, infinite-dimensional, $\mathcal{Y}^*$ is also infinite-dimensional. Therefore, the compact operator $W^*_{\psi,\phi}$ has countably many eigenvalues [11, Theorem. 7.1, page 214], and we have again obtained a contradiction.

Hence our assumption that ϕ has more than one fixed point is false. □

4. Remarks

Based on the results to date, it is obviously natural to consider whether or not the following conjecture holds.

Conjecture 1. Suppose that $D \subset \mathbb{C}^n$ is a bounded, convex domain such that a given functional Hilbert space of holomorphic functions $\mathcal{Y}$ in which the polynomials are contained densely has reproducing kernel K satisfying $K(z,z) \to \infty$ as $z \to \partial D$. Let $\psi : D \to \mathbb{C}$ be holomorphic and suppose that ψ is bounded away from 0 toward ∂D. Assume that $\phi : D \to D$ is a holomorphic map and that $W_{\psi,\phi}$ is compact on $\mathcal{Y}$. Then, the spectrum of $W_{\psi,\phi}$ is the set $\{\psi(a)\sigma : \sigma \in E\}$, where E is the set consisting of 0, 1, and all possible products of eigenvalues of $\phi'(a)$.

The resolution of whether this conjecture holds is open even for classical function spaces in the multivariable case. It would also be of interest to determine whether or not one can remove the assumption in Theorem 3.1 that ψ does not vanish on the fixed point set of ϕ. Notice, for example, that this assumption is not needed in Theorem 3.1.

B. MacCluer has pointed out to the author that by using [2, Exercise 5.1.1], it can be shown that under the hypotheses of Theorem 3.1 in the case of the Bergman space $A^2(D)$, $W_{\psi,\phi}$ cannot be compact unless C_ϕ is compact. It is, therefore, natural to consider whether or not this statement holds for other functional Hilbert spaces on Δ or other domains, under the hypotheses of Theorem 3.1.

Note that if $D = \Delta$ or $\mathbb{B}_n$, the fixed point of ϕ in Theorem 3.1 is precisely the so-called Denjoy-Wolff point of ϕ, to which the iterates of ϕ converge uniformly on compacta. One can consider the question of whether or not this uniform convergence holds in the general setting of Theorem 3.1. However, as is stated in [4], an interesting aspect of the above result is that in the case when $D = \Delta^n$, the Denjoy-Wolff theorem fails, and there is no unique "Denjoy-Wolff point". Nevertheless, Theorem 3.1 holds even for reducible convex domains such as Δ^n.

The convexity of D in the proof of Theorem 3.1 is used in two places: (a) to establish that if ϕ has no interior fixed points, the iterates of ϕ diverge compactly, and (b) to establish the assertion that when D is convex, the fixed point set of ϕ is a *connected*, analytic submanifold of D. It is, therefore, of interest to determine to what extent the hypothesis of convexity can be weakened in such a way that tasks (a) and (b) can still be simultaneously completed.

Let G be a simply connected region that is properly contained in $\mathbb{C}$, and suppose that $\tau : \Delta \to \mathbb{C}$ is the Riemann mapping for G. Let $H^2(G)$ be the Hardy space of functions $f : G \to \mathbb{C}$ that are analytic and satisfy

$$\sup_{0<r<1} \int_{\tau(\{z\in\Delta:|z|=r\})} |f(z)|^2 |dz| < \infty. \tag{4.1}$$

In [7], it is shown that if C_ϕ is compact on $H^2(G)$ for some analytic $\phi : \Delta \to \Delta$, then ϕ must have a unique fixed point in G. Of course, such a domain G can have boundary portions that are concave though all domains in $\mathbb{C}$ are trivially pseudoconvex [12]. On the other hand, as is well known, the Riemann mapping theorem does not extend to several complex variables, and the proof in [7] does seem to rely on the Denjoy-Wolff theory that is inherent from the convexity of Δ.

Note that in the proof of Theorem 3.1, all that was needed from Vigué's theorem is the assertion that if the fixed point set of a holomorphic self-map of a convex domain is nonempty, then, it either contains one point or uncountably many points. Vigué, in [13], has shown that the fixed point set of a holomorphic self-map of any bounded domain D (note that "convex" is omitted!) in $\mathbb{C}^n$ is also an analytic submanifold of D, but it is an interesting and open question as to whether or not the fixed point set in this case is necessarily connected for general bounded domains besides the convex ones.

M. Abate has conjectured that the answer is affirmative for a topologically contractible, strictly pseudoconvex domain. A resolution of this conjecture, together with a compact divergence result appearing in [14], would imply that Theorem 3.1 extends to these domains.

For the weighted Hardy spaces $H_b^2(\Delta)$ of the unit disk in $\Delta \in \mathbb{C}$, the Hardy spaces $H^2(D)$ and weighted Bergman spaces $A_\alpha^2(D)$, where D is either $\mathbb{B}_n$, Δ^n, or more generally, any bounded symmetric domain in its Harish-Chandra realization (see [4]), the reproducing kernel K satisfies $K(z,z) \to \infty$ as $z \to \Delta$ (resp., $z \to D$), so the following fact, which extends the fixed point results in [1, 5], is an immediate consequence of Theorem 3.1.

Corollary 4.1. *Suppose that $\mathcal{Y}$ is either the Hardy space $H^2(D)$ or the weighted Bergman space $A_\alpha^2(D)$ of a bounded symmetric domain D with $\alpha < \alpha_D$, where α_D is a certain critical value that depends on D (cf. [4]), and assume that $\psi : D \to \mathbb{C}$ is analytic, bounded away* *from zero, and not identically zero on the fixed point set of* ϕ. *Suppose that* $\phi : D \to D$ *is holomorphic, and let* $W_{\psi,\phi}$ *be compact on* $\mathcal{Y}$. *Then,* ϕ *has a unique fixed point in D. This result also holds when* $D = \Delta$ *and* $\mathcal{Y} = H_b^2(\Delta)$.

Proof. The assertions about $H^2(D)$ and $A_\alpha^2(D)$ immediately follow from Theorem 3.1 and the fact that their reproducing kernels approach infinity along $\{(z,z) : z \in D\}$ as $z \to D$ (see [4]). The assertion about $\mathcal{Y} = H_b^2(\Delta)$ also immediately follows from Theorem 3.1 and the fact that the assumed condition on the sequence $(b_j)_{j\in\mathbb{N}}$ implies that the reproducing kernel K for $H_b^2(\Delta)$ satisfies the same singularity property toward the boundary along the diagonal (cf. [4]). □

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