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FUZZY Θ-CLOSURE OPERATOR ON FUZZY TOPOLOGICAL SPACES

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ABSTRACT. The paper contains a study of fuzzy Θ-closure operator, Θ-closures of fuzzy sets in a fuzzy topological space are characterized and some of their properties along with their relation with fuzzy δ-closures are investigated. As applications of these concepts, certain functions as well as some spaces satisfying certain fuzzy separation axioms are characterized in terms of fuzzy Θ-closures and δ-closures.

KEY WORDS AND PHRASES. Fuzzy Θ-cluster point, fuzzy Θ-closure, fuzzy δ-closure, q-coincidence, q-neighbourhood.
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1. INTRODUCTION.

It is well-known that the concepts of Θ-closure and δ-closure are useful tools in standard topology in the study of H-closed spaces, Katetov's and H-closed extensions, generalizations of Stone-Weierstrass' theorem etc. For basic results and some applications of Θ-closure and δ-closure operators we refer to Veličko [1], Dickman and Porter [2], Espelie and Joseph [3] and Sivaraj [4]. Due to varied applicabilities of these operators in formulating various important set-topological concepts, it is natural to try for their extensions to fuzzy topological spaces. With this motivation in mind the concept of Θ-closure operator in a fuzzy topological space (due to Chang [5]) was introduced by us in [6] in the light of the notions of quasi-coincidence and q-neighbourhoods of Pu and Liu [7,8]. In the present paper our aim is to continue the same study which ultimately shows that different fuzzy topological concepts can effectively be characterized in terms of fuzzy Θ-closure and δ-closure operators.

In Section 2 of this paper we develop the concept of fuzzy Θ-closure operators and characterize fuzzy Θ-closures of fuzzy sets in a fuzzy topological space in different ways. In literature there can be found several definitions of T_2-spaces in fuzzy setting. We take the definition of fuzzy T_2-space as given by Ganguly and Saha [9] and become able to successfully characterize it in our context. Fuzzy regularity has been introduced by many workers from different view points, including one by us in [6]. Since our fuzzy regularity along with the fuzzy T_1-axiom (of [9]) does not yield the above fuzzy T_2-axiom, we propose to call it "strong T_2" in fuzzy setting. Fuzzy semiregularity and almost regularity were also defined in [6]. We characterize fuzzy regularity and these weaker forms of fuzzy regularity in terms of fuzzy Θ-closure and δ-closure. All these characterizations are incorporated in Section 3 of the paper. Fuzzy weakly continuous functions were first introduced by Azad [10] and were further investigated in [11], whereas the concept of fuzzy Θ-continuous functions was initiated in [6]. Section 3 also includes the characterizations of these functions with the help of the notion of fuzzy Θ-closures.

We now recall some definitions and results of a fuzzy topological space (henceforth fts, for short) (X,T) to be used in this paper excepting very standard ones for which we refer to Zadeh [12], Chang [5] and Pu and Liu [7,8]. The interior and closure of a fuzzy set A in an fts (X,T) will be denoted by Int A and Cl A respectively. A fuzzy point [7] with a singleton support x (say) and value $\alpha (0 < \alpha \leq 1)$ at x is denoted by x_α. For a fuzzy set A, the support and complement of A are denoted by A_o and A' (or $1 - A$) respectively. For a fuzzy point x_α and a fuzzy set A, we write $x_\alpha \in A$ iff $\alpha \leq A(x)$, and x_α is said to be quasi-coincident (q-coincident, for short) with A, denoted by $x_\alpha q A$, iff $\alpha > A'(x)$. A is said to be a q-neighbourhood (q-nbd, for short) of x_α iff there exists a fuzzy open set B such that $x_\alpha q B \leq A$. For two fuzzy sets A and B, $A \leq B$ iff $A \not{q} B'$, and a fuzzy point $x_\alpha \in$ ClA iff each q-nbd of x_α is q-coincident with A [7]. For the definitions of fuzzy regularly open, regularly closed, semi-open and semi-closed sets we refer to Azad [10]. Simply by X and Y we shall mean the fuzzy topological spaces (X,T) and (Y,T_1) respectively. The constant fuzzy sets 0_X and 1_X are defined by $0_X(y) = 0$ and $1_X(y) = 1$, for each $y \in X$.

2. FUZZY Θ-CLOSURE AND ITS PROPERTIES.

DEFINITION 2.1. A fuzzy point x_α is said to be a fuzzy Θ-cluster point (δ-cluster point [13]) of a fuzzy set A iff closures of every open q-nbd (resp. iff every regularly open q-nbd) of x_α is q-coincident with A.

The union of all fuzzy Θ-cluster (δ-cluster) points of A is called the fuzzy Θ-closure of A and is denoted by $[A]_\Theta$ (resp. $[A]_\delta$). A fuzzy set A will be called fuzzy Θ-closed (δ-closed) iff $A = [A]_\Theta$ (resp. $A = [A]_\delta$). It is known [6] that for any fuzzy set A in an fts X, Cl $A \leq [A]_\delta \leq [A]_\Theta$, but the reverse implications are false (see [6] and [13]). However, it is true (see [6]) that for a fuzzy open set A in an fts X, Cl $A = [A]_\delta = [A]_\Theta$.

THEOREM 2.2. In an fts (X,T), the following hold:

(a) Finite union and arbitrary intersection of Θ-closed sets in X is fuzzy Θ-closed.

(b) For two fuzzy sets A and B in X, if $A \leq B$ then $[A]_\Theta \leq [B]_\Theta$.

(c) The fuzzy sets 0_X and 1_X are fuzzy Θ-closed.

PROOF. The straightforward proofs are omitted.

REMARK 2.3. The complements of fuzzy Θ-closed sets in an fts (X,T) induce a fuzzy topology T_Θ (say) which is coarser than the fuzzy topology T of the space. Again, for a fuzzy set A in X, $[A]_\Theta$ is evidently fuzzy closed but not necessarily fuzzy Θ-closed as is seen from the next example. Thus, fuzzy Θ-closure operator is not a Kuratowski closure operator. However, it will be shown in the next section that for any fuzzy set A in an fts X, $[A]_\Theta$ is fuzzy Θ-closed if the space X is fuzzy regular (see Corollary 3.6), or iff the space X is fuzzy almost regular (see Theorem 3.10).

EXAMPLE 2.4. Let $X = \{a,b,c\}$ and $T = \{0_X, 1_X, A, B\}$, where

$$A(a) = 0.5, A(b) = 0.6, A(c) = 0.2$$

and

$$B(a) = 0.4, B(b) = 0.5, B(c) = 0.1.$$

Let U be any fuzzy set given by, $U(a) = U(b) = 0.3$ and $U(c) = 0$. Then, $a_{.6} \in [U]_\Theta, a_{.8} \notin [U]_\Theta$, but $a_{.8} \in [a_{.6}]_\Theta \leq [[U]]_\Theta]_\Theta$. Thus, $[U]_\Theta \neq [[U]_\Theta]_\Theta$. Hence, $[U]_\Theta$ is not fuzzy Θ-closed.

In the following example, we observe a deviation from the corresponding established result [3] in general topology that $x \in [y]_\Theta$ iff $y \in [x]_\Theta$, if x,y are two points in a topological space.

EXAMPLE 2.5. Let X be an ordinary set with at least two distinct points a, b. Consider the fuzzy topology $T = \{0_X, 1_X A\}$, where $A(a) = \frac{1}{2}$, $A(b) = \frac{2}{5}$ and $A(x) = 0$, for $x \neq a, b (x \in X)$. Let us consider the fuzzy points $a_{\frac{1}{12}}$ and $b_{\frac{4}{5}}$. It can be checked that $a_{\frac{1}{12}} \in [b_{\frac{4}{5}}]_\Theta$, but $b_{\frac{4}{5}} \notin [a_{\frac{1}{12}}]_\Theta$,

THEOREM 2.6. For any fuzzy set A in an fts (X,T), $[A]_\Theta = \cap\{[U]_\Theta : U \in T \text{ and } A \leq U\}$.

PROOF. Obviously, L.H.S. $\leq$ R.H.S. Now, if possible let $x_\alpha \in$ R.H.S. but $x_\alpha \notin [A]_\Theta$. Then there exists an open q-nbd V of x_α such that Cl $\not q A$ and hence $A \leq 1 - C1V$. Then $x_\alpha \in [1 - C1V]_\Theta$ and consequently, Cl $Vq(1 - C1V)$ which is impossible.

According to Pu and Liu [7] a function $S: D \to J$ is called a fuzzy net in X, where $(D, \geq)$ is a directed set and J denote the collection of all fuzzy points in X. It is denoted by $\{S_n, n \in D\}$ or simply by (S, D). We now set the following:

DEFINITION 2.7. Let $\{S_n, n \in D\}$ be a fuzzy net and x_α a fuzzy point in X.

(a) x_α is called a Θ-cluster point of the fuzzy net iff for every open q-nbd W of x_α and for any $n \in D$, there exists $m \geq n$ $(m \in D)$ such that $S_m q ClW$.

(b) The fuzzy net is said to be Θ-converge to x_α if for any open q-nbd U of x_α, exists $m \in D$ such that $S_n q C1U$, for all $n \geq m (n \in D)$. This is denoted by $S \xrightarrow{\Theta} x_\alpha$.

THEOREM 2.8. A fuzzy point x_α is a Θ-cluster point of a fuzzy net $\{s_n, n \in D\}$ in X iff there is a subnet of $\{S_n, n \in D\}$, which Θ-converges to x_α.

PROOF. Let x_α be a Θ-cluster point of the given fuzzy net. Let Qx_α denote the set of closures of all open q-nbds of x_α. Now for any member A of Qx_α, there exists an element S_n of the net such that $S_n q A$. Let E denote the set of all ordered pairs (n, A) with the above property, i.e., $n \in D, A \in Qx_\alpha$ and $S_n q A$. Then $(E, \gg)$ is a directed set, where $(m, A) \gg (n, B)$, $((m, A), (n, B) \in E)$ iff $m \geq n$ in D and $A \leq B$. Then $T: (E, \gg) \to (X, T)$ given by $T(m, A) = S_m$ can be checked by a subnet of $\{S_n, n \in D\}$. To show that $T \xrightarrow{\Theta} x_\alpha$, let V be any open q-nbd of x_α. Then there exists $n \in D$ such that $(n: C1V) \in E$ and then $S_n q ClV$. Now, for any $(m, A) \gg (n, C1V), T(m, A) = S_m q A \leq C1V$. Hence, $T \xrightarrow{\Theta} x_\alpha$. Converse is clear.

THEOREM 2.9. Let A be a fuzzy set in X. A fuzzy point $x_\alpha \in [A]_\Theta$ iff there exists a fuzzy net in A, Θ-converging to x_α.

PROOF. Let $x_\alpha \in [A]_\Theta$. For each open q-nbd U of x_α, $C1UqA$. That is , there exist $y^U \in A_o$ and real number β_U with $0 < \beta_U \leq A(y^U)$ such that $y^U_{\beta_U} \in A$ and $y^U_{\beta_U} qC1U$. We choose and fix one such $y^U_{\beta_U}$ for each U. Let D denote the set of all open q-nbds of x_α. Then $(D, \geq)$ is directed under inclusion relation, i.e., for $B, C \in D, B \geq C$ iff $B \leq C$. Then $\{y^U_{\beta_U} \in A : y^U_{\beta_U} qC1U$ and $U \in D\}$ is a fuzzy net in A such that it Θ-converges to x_α. Converse is straightforward even if x_α is a Θ-cluster point of the fuzzy net in A.

3. CHARACTERIZATIONS OF CERTAIN SEPARATION AXIOMS AND FUNCTIONS IN TERMS OF FUZZY Θ-CLOSURE AND δ-CLOSURE.

DEFINITION 3.1. [9] An fts X is called fuzzy strongly T_2 iff for any two distinct fuzzy points x_α and y_β in X : whenever $x \neq y, x_\alpha$ and y_β have fuzzy open nbds U and V respectively such that $U \not q V$; and when $x = y, \alpha < \beta$ (say), there exist fuzzy open sets U and V such that $x_\alpha \in U, y_\beta q V$ and $U \not q V$.

LEMMA 3.2. For any two fuzzy open sets A and B in an fts (X,T), $A \not q B \Rightarrow C1A \not q B$ and $A \not q C1B$.

THEOREM 3.3. An fts (X,T) is fuzzy strongly T_2 iff every fuzzy point of X is fuzzy Θ-closed, and for $x, y \in X$ with $x \neq (C1U)_o$.

PROOF. Let X be fuzzy strongly T_2, and let x_α be a fuzzy point in X. In order to show that $[x_\alpha]_\Theta = x_\alpha$, it suffices to establish that for any fuzzy point y_β, $y_\beta \notin [x_\alpha]_\Theta$ when either $x \neq y$, or $x = y$ and $\beta > \alpha$. In the first case, there exist fuzzy open nbds U and V of y_1 and x_α respectively such that $U \not q V$ and then $C1U \not q V$ (by Lemma 3.2). Then U is an open q-nbd of y_β with $C1U \not q x_\alpha$ so that $Y_\beta \notin [x_\alpha]_\Theta$. In the second case, there exist a fuzzy open nbd U of x_α and an open q-nbd V of y_β such that $U \not q V$. Then $C1V \not q U$ so that $C1V \not q x_\alpha$ and hence $y_\beta \notin [x_\alpha]_\Theta$. Finally, for two distinct points x, y of X, there exist fuzzy open nbds U of x_1 and V of y_1 such that $U \not q V$ and hence $C1U \not q V$, i.e.,

$y_1 \in V \leq 1 - C1U$. Then $(1-C1U)(y) = 1 \Rightarrow (C1U)(y) = 0 \Rightarrow y \notin (C1U)_o$. Conversely, let x_α and y_β be two distinct fuzzy points in X.

CASE I. Let $x \neq y$. First suppose that at least one of α and β is less than 1, say $\alpha < 1$. Then there exists $\lambda > 0$ such that $\alpha + \lambda < 1$. Now $x_\lambda \notin [y_\beta]_\Theta$ and hence there exists a fuzzy open nbd U of y_β such that $x_\lambda \notin [U]_\Theta$ (by Theorem 2.6). Then $U \not q C1V$, for an open q-nbd V of x_λ. Since $V(x) > 1 - \lambda > \alpha$, V and U are fuzzy open nbds of x_α and y_β respectively such that $U \not q V$.

Next, suppose $\alpha - \beta - 1$. By hypothesis, there exists a fuzzy open nbd U of x_1 such that $(C1U)(y) = 0$. Then $(1 - C1U)$ is a fuzzy open nbd of $y_1$such that $U \not q (1 - C1U)$.

CASE II. Let $x = y$. Suppose $\alpha < \beta$. Then $y_\beta \notin [s_\alpha]_\Theta$ and so $y_\beta \notin [U]_\Theta$, for some fuzzy open nbd U of x_α. Then for an open q-nbd V of y_β, $C1V \not q U$ and hence $V \not q U$.

DEFINITION 3.4. [6] An fts X is said to be:

(a) fuzzy regular (semi-regular) iff for each fuzzy point x_α in X and each open q-nbd U of x_α, there exists an open q-nbd V of x_α such that $C1V \leq U$ (resp. Int $C1V \leq U$);

(B) fuzzy almost regular iff for each fuzzy point x_α in X and each regularly open q-nbd U of x_α, there exists a regularly open q-nbd V of x_α such that $C1V \leq U$.

THEOREM 3.5. An fts X is:

(a) fuzzy regular iff for any fuzzy set A in X, $C1A = [A]_\Theta$;

(b) fuzzy semi-regular iff $[A]_\delta = C1A$, for any fuzzy set A in X.

PROOF. Let X be fuzzy regular. For any fuzzy set A in X it is always true that $C1A \leq [A]_\Theta$. Now, let x_α be a fuzzy point in X such that $x_\alpha \in [A]_\Theta$ and let U be any open q-nbd of x_α. Then by fuzzy regularity of X, there exists an open q-nbd V of x_α such that $C1V \leq U$. Now, $x_\alpha \in [A]_\Theta \Rightarrow C1VqA \Rightarrow UqA \Rightarrow x_\alpha \in C1A$. Thus $[A]_\Theta = C1A$.

Conversely, let x_α be a fuzzy point in X and U an open q-nbd of x_α. Then $x_\alpha \notin (1-U) = C1(1-U) = [1-U]_\Theta$. Thus there exists an open q-nbd V of x_α such that $C1V \not q (1-U)$ and then $C1V \leq U$. Hence X is fuzzy regular. (b) Similar to (a) and is omitted.

COROLLARY 3.6. In a fuzzy regular space (X,T), a fuzzy closed set is fuzzy Θ-closed, and hence for any fuzzy set A in X, $[A]_\Theta$ is fuzzy Θ-closed.

LEMMA 3.7. For any fuzzy semi-open set A in X, $[A]_\delta = C1A$.

PROOF. It suffices to show that $[A]_\delta \leq C1A$. Let $x_\alpha \notin C1A$. Then there exists an open q-nbd V of x_α such that $V \not q A$. Then IntC1 $V \leq$ Int $C1(1-A) = 1 - C1$ IntA $\leq 1 - A$ (since A is fuzzy semi-open). Thus Int C1 $V \not q A$ and consequently, $x_\alpha \notin [A]_\delta$.

THEOREM 3.8. An fts X is fuzzy almost regular iff $[A]_\Theta = C1A$, for every fuzzy semi-open set A in X.

PROOF. Let X be fuzzy almost regular and A any fuzzy semi-open set in X. It is enough to show that $[A]_\Theta \leq C1A$. Suppose $x_\alpha \notin C1A$. By Lemma 3.7, there exists an open q-nbd V of x_α such that Int $C1V \not q A$. Since X is fuzzy almost regular, there is a fuzzy regularly open set U such that $x_\alpha U \leq C1U \leq$ Int $C1V \leq 1 - A$. Then $C1U \not q A$ and hence $x_\alpha \notin [A]_\Theta$. Conversely, let U be any fuzzy regularly open q-nbd of a fuzzy point x_α. Then $x_\alpha \notin 1 - U = C1(1-U) = [1-U]_\Theta$, since a fuzzy regularly closed set is fuzzy semi-open. Hence, there is an open q-nbd V of x_α such that $C1V \not q (1-U)$. Since $V \leq$ Int $C1V$, Int $C1V$ is a regularly open q-nbd of x_α such that Cl Int $C1V = C1V \leq U$ and X is fuzzy almost regular.

THEOREM 3.9. In an fts X, the following statements are equivalent:

(a) For any fuzzy open set A in X, $[[A]_\Theta]_\Theta = [A]_\Theta$.

(b) For any fuzzy set A in X, $[[A]_\Theta]_\Theta = [A]_\Theta$.

(c) For any fuzzy set A in X, $[A]_\Theta = [A]_\delta$.

(d) X is fuzzy almost regular.

PROOF. (a) $\Rightarrow$ (d): We first show that for any fuzzy regularly closed set F in X, $F = [F]_\Theta$. In fact, F being fuzzy regularly closed, $F = C1U$, for some fuzzy open set U. Now, $[F]_\Theta = [C1U]_\Theta][[U]_\Theta]_\Theta$ (since U is fuzzy open) $= [U]_\Theta = C1U = F$. Next, let x_α be a fuzzy point in X and A any fuzzy regularly open set in X with $x_\alpha\, qA$. Then $x_\alpha \notin (1-A) = [1-A]_\Theta$, since $(1-A)$ is fuzzy regularly closed. Hence, there exist a fuzzy open set V such that $x_\alpha qV$, but $ClV \not q (1-A)$. Let $W =$ Int C 1 V . Then $x_\alpha qW$, and $ClW = ClV \not q (1-A)$. Thus, W is a regularly open q-nbd of x_α such that $ClW \leq A$. Hence, X is fuzzy almost regular.

(d) $\Rightarrow$ (c): For any fuzzy set A, it is clear that $[A]_\delta \leq [A]_\Theta$. Now, let $x_\alpha \in [A]_\Theta$ and U an open q-nbd of x_α. Then $x_\alpha q$ Int ClU. By (d), there exists a regularly open q-nbd V of x_α such that $ClV \leq$ Int ClU. Now, $x_\alpha \in [A]_\Theta \Rightarrow ClVqA \Rightarrow$ Int $ClUqA \Rightarrow x_\alpha \in [A]_\delta$.

(c) $\Rightarrow$ (b): $[[A]_\Theta]_\Theta = [[A]_\delta]_\Theta = [[A]_\delta]_\delta = [A]_\delta = [A]_\Theta$.

(b) $\Rightarrow$ (a): Obvious.

DEFINITION 3.10. A function $f: X \to Y$ from an fts (X,T) to another fts (Y,T_1) is called

(i) fuzzy weakly continuous [10] iff for each fuzzy open set A in $Y, f^{-1}(A) \leq$ Int $f^{-1}(ClA)$.

(ii) fuzzy Θ-continuous [6] iff for each fuzzy point x_α in X and each open q-nbd V of $x_\alpha, f(ClU) \leq ClV$, for some open q-nbd U of x_α.

LEMMA 3.11. Let $f: X \to Y$ be a function. Then for a fuzzy set B in Y, $f(1 - f^{-1}(B)) \leq 1 - B$, where equality holds if f is onto.

PROOF. Let $y \in Y$. If $f^{-1}(y) = \emptyset$, then $[f(1-f^{-1}(B))](y) = 0 \leq (1-B)(y)$. If $f^{-1}(y) \neq \emptyset$, then

$$[f(1-f^{-1}(B))](y) = \operatorname*{Sup}_{x \in f^{-1}(y)} [1-f^{-1}(B)](x) = \operatorname*{Sup}_{x \in f^{-1}(y)} \{1 - B(f(x))\}$$

$$= \operatorname*{Sup}_{x \in f^{-1}(y)} \{1 - B(y)\} = 1 - B(y) = (1-B)(y).$$

If f is onto , then for each $y \in Y, f^{-1}(y) \neq \emptyset$, and hence we have $f(1-f^{-1}(B)) = 1 - B$.

THEOREM 3.12. A function $f: X \to Y$ is:

(a) fuzzy weakly continuous iff $f(ClU) \leq [f(U)]_\Theta$, for each fuzzy set U in X.

(b) fuzzy Θ-continuous iff $f([A]_\Theta) \leq [f(A)]_\Theta$, for any fuzzy set A in X.

PROOF. (a) Let f be a fuzzy weakly continuous and U any fuzzy set in X. Suppose $x_\alpha \in ClU$. It is enough to show that $f(x_\alpha) \in [f(U)]_\Theta$. Let A be any open q-nbd of $f(x_\alpha)$. Then $f^{-1}(A)qx_\alpha$. By fuzzy weak continuity of $f, f^{-1}(A) \leq$ Int $f^{-1}(ClA)$ and hence Int $f^{-1}(ClA)$ is an open q-nbd of x_α. Since $x_\alpha \in ClU$, we have Int $f^{-1}(ClA)qU$. Then $f^{-1}(ClA)qU$ and hence $ClAqf(U)$. Thus $f(x_\alpha) \in [f(U)]_\Theta$.

Conversely, for any fuzzy open set U in Y, $f(1-$ Int $f^{-1}(ClU))$

$$= f(Cl(1-f^{-1}(ClU))) \leq [f(1-f^{-1}(ClU)]_\Theta \leq [1-ClU]_\Theta$$

(by Lemma 3.11)

$= Cl(1-ClU) = 1-$ Int $ClU \leq 1 - U \Rightarrow f(1-$ Int $f^{-1}(ClU))\not q U \Rightarrow 1-$ Int $f^{-1}(ClU) \not q f^{-1}(U)$

$\Rightarrow f^{-1}(U) \leq$ Int $f^{-1}(ClU)$

Hence f is fuzzy weakly continuous.

(b) Let the condition hold. For any fuzzy point x_α in X and any open q-nbd A of $f(x_\alpha)$ in Y, we have by Lemma 3.11, $f(1-f^{-1}(ClA)) \leq 1 - ClA$. Thus, $ClA \not q f(1-f^{-1}(ClA))$ so that $f(x_\alpha) \notin [f(1-f^{-1}(ClA))]_\Theta$. By hypothesis, $f(x_\alpha) \notin f([1-f^{-1}(ClA)]_\Theta)$ and hence $x_\alpha \notin [1-f^{-1}(ClA)]_\Theta$. Then there is an open q-nbd V of x_α such that $ClV \not q (1-f^{-1}(ClA))$ and hence $f(ClV) \leq ff^{-1}(ClA) \leq ClA$. Thus f is fuzzy Θ-continuous.

The converse part was proved in [6].

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