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EQUILIBRIA OF GENERALIZED GAMES WITH *L*-MAJORIZED CORRESPONDENCES

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ABSTRACT. In this paper, we shall prove three equilibrium existence theorems for generalized games in Hausdorff topological vector spaces.

KEY WORDS AND PHRASES. Equilibrium, maximal element, generalized game, L-majorized correspondence, class L.
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1. INTRODUCTION. In 1976, Borglin and Keiding first introduced the notions of KF-correspondences and KF-majorized correspondences and generalized Lemma 4 of Fan [5] to KF-majorized correspondences. Recently, Yannelis and Prabhakar [10] introduced the notions of L-majorized correspondences which generalize KF-majorized correspondences and they obtained an existence theorem of an equilibrium for a compact abstract economy but not with L-majorized preference correspondences.

In this paper, we shall prove existence theorems of equilibria for compact abstract economies with L-majorized correspondences in Hausdorff topological vector space. These results generalize the corresponding results of Borglin-Keiding ([1], Corollaries 2 and 3) with KF-majorized preference correspondences.

2. PRELIMINARIES.

If A is a set, we shall denote by 2^A the family of all subsets of A. If A is a subset of a topological space X, we denote by $cl_X A$ the closure of A in X. If A is a subset of a vector space, we shall denote by coA the convex hull of A. Let E be a topological vector space and A, X be non-empty subsets of E. If $T: A \to 2^E$ and $S: A \to 2^X$ are correspondences, then $coT: A \to 2^E$ and

$clS: A \to 2^X$ are correspondences defined by $(coT)(x) = coT(x), (clS)(x) = cl_X S(x)$ for each $x \in A$, respectively.

Let X be a non-empty subset of a topological vector space. A correspondence $\phi: X \to 2^X$ is said to be of *class* L [10] if (i) for each $x \in X, x \notin co\phi(x)$, (ii) for each $y \in X, \phi^{-1}(y) = \{x \in X: y \in \phi(x)\}$ is open in X. Let $\phi: X \to 2^X$ be a given correspondence and $x \in X$; then a correspondence $\phi_x: X \to 2^X$ is said to be an *L-majorant of ϕ at x* [10] if ϕ_x is of class L and there exists an open neighborhood N_x of x in X such that for each $z \in N_x$, $\phi(z) \subset \phi_x(z)$. The correspondence ϕ is said to be *L-majorized* if for each $x \in X$ with $\phi(x) \neq \emptyset$ there exists an L-majorant of ϕ at x.

We remark here that the notions of a correspondence of class L and an L-majorized correspondence defined above by Yannelis-Prabhakar in [10] generalize the notions of a KF-correspondence and KF-majorized correspondence, respectively, introduced by Borglin-Keiding [1]. These notions have been further generalized in ([2],[9]).

Let I be any set of agents. A *generalized game* (or *an abstract economy)* $\Gamma = (X_i, A_i, B_i, P_i)_{i \in I}$ is defined as a family of ordered quadruples (X_i, A_i, B_i, P_i) where $A_i, B_i: \Pi_{j \in I} X_j \to 2^{X_i}$ are constraint correspondences and $P_i: \Pi_{j \in I} X_j \to 2^{X_i}$ is a preference correspondence. An *equilibrium* for Γ is a point $\hat{x} \in X = \Pi_{i \in I} X_i$ such that for each $i \in I$, $\hat{x}_i \in clB_i(\hat{x})$ and $A_i(\hat{x}) \cap P_i(\hat{x}) = \emptyset$. When $A_i = B_i$ for each $i \in I$, our definitions of an abstract economy and an equilibrium coincide with the standard definitions, e.g., in Borglin-Keiding ([1], p. 315) or in Yannelis-Prabhakar ([10], p. 242).

We shall need the following which is essentially Lemma 5.1 of Yannelis-Prabhakar [10]:

LEMMA 1. Let X be a topological space, Y be a vector space and $\phi: X \to 2^Y$ be a correspondence such that for each $y \in Y, \phi^{-1}(y)$ is open in X. Define $\psi: X \to 2^Y$ by $\psi(x) = co\ \phi(x)$ for each $x \in X$. Then for each $y \in Y, \psi^{-1}(y)$ is open in X.

The following maximal element existence result is Theorem 5.1 of Yannelis-Prabhakar [10]:

LEMMA 2. Let X be a non-empty compact convex subset of a Hausdorff topological vector space and $\phi: X \to 2^X$ be a correspondence of class L. Then there exists $\hat{x} \in X$ such that $\phi(\hat{x}) = \emptyset$.

3. EXISTENCE OF EQUILIBRIA FOR L-MAJORIZED PREFERENCE CORRESPONDENCES.

The following result is due to Yannelis-Prabhakar ([10], Corollary 5.1), which generalizes Lemma 2 to L-majorized correspondence; however they did not give a proof. For completeness, we shall give a proof.

THEOREM 1. Let X be a non-empty compact convex subset of a Hausdorff topological vector space and $\phi: X \to 2^X$ be an L-majorized correspondence. Then there exists a maximal element $\hat{x} \in X$, i.e., $\phi(\hat{x}) = \emptyset$.

PROOF. Suppose that for each $x \in X, \phi(x) \neq \emptyset$. Since ϕ is L-majorized for each $x \in X$, there exist a correspondence $\phi_x: X \to 2^X$ of class L and an open neighborhood N_x of x in X such that for each $z \in N_x, \phi(z) \subset \phi_x(z)$. The family $\{N_x: x \in X\}$ is an open covering of X, which by the compactness of X, contains a finite subcover $\{N_{x_i}: i \in I\}$, where I is a finite set. Let $\{G_{x_i}: i \in I\}$ be a closed refinement of $\{N_{x_i}: i \in I\}$. For each $i \in I$, define a correspondence $\phi_i: X \to 2^X$ by

$$\phi_i(z) = \begin{cases} \phi_{x_i}(z), & \text{if } z \in G_{x_i}, \\ X, & \text{if } x \notin G_{x_i}. \end{cases}$$

Let $\Phi: X \to 2^X$ be defined by

$$\Phi(z) = \bigcap_{i \in I} \phi_i(z) \quad \text{for each } z \in X.$$

Then for each $i \in I$ and each $y \in X$, we have

$$\begin{aligned} \phi_i^{-1}(y) &= \{z \in X : y \in \phi_i(z)\} \\ &= \{z \in G_{x_i} : y \in \phi_i(z)\} \cup \{z \in X \backslash G_{x_i} : y \in \phi_i(z)\} \\ &= \{z \in G_{x_i} : y \in \phi_i(z)\} \cup (X \backslash G_{x_i}) \\ &= (G_{x_i} \cap \phi_{x_i}^{-1}(y)) \cup (X \backslash G_{x_i}) \\ &= (X \backslash G_{x_i}) \cup \phi_{x_i}^{-1}(y) \end{aligned}$$

is open in X. Hence $\Phi^{-1}(y) = \bigcap_{i \in I} \phi_i^{-1}(y)$ is open in X for each $y \in X$. For each $z \in X$, there exists $i_0 \in I$ such that $z \in G_{x_{i_0}} \subset N_{x_{i_0}}$, so that $z \notin co\phi_{x_{i_0}}(z) = co\phi_{i_0}(z)$; thus $z \notin co\Phi(z)$. It follows that Φ is of class L. Therefore by Lemma 2, there exists $\widetilde{x} \in X$ such that $\Phi(\widetilde{x}) = \emptyset$. On the other hand, for each $z \in X$, if $z \in G_{x_i} \subset N_{x_i}$ for some $i \in I$ then $\phi(z) \subset \phi_{x_i}(z) = \phi_i(z)$ and if $z \notin G_{x_i}$ then $\phi_i(z) = X$ so that we have $\phi(z) \subset \bigcap_{i \in I} \phi_i(z) = \Phi(z)$ for each $z \in X$. Since $\Phi(\tilde{x}) = \phi$, we must have $\phi(\widetilde{x}) = \emptyset$ which contradicts the assumption that $\phi(x) \neq \emptyset$ for all $x \in X$. Hence there must exist $\widehat{x} \in X$ such that $\phi(\widehat{x}) = \emptyset$. This completes the proof.

The following simple example shows that Theorem 1 is suitable for an L-majorized correspondence, which is not of class L, to assure the existence of a maximal element.

EXAMPLE 1. Let $X = [0,1]$ and $\phi: X \rightarrow 2^X$ be defined by

$$\phi(x) = \begin{cases} \{y \in X : 0 \leq y \leq x^2\}, & \text{if } x \in (0,1), \\ \emptyset, & \text{if } x \in \{0,1\}. \end{cases}$$

Then ϕ is not of class L since $\phi^{-1}(y)$ is not open in X for any $y \in (0,1)$. For any $x \in (0,1)$, let $N_x = X$, an open neighborhood of x in X, and define $\phi_x: X \rightarrow 2^X$ by

$$\phi_x(z) = \begin{cases} \{y \in X : 0 \leq y \leq x\}, & \text{if } z \in (0,1), \\ \emptyset, & \text{if } z \in \{0,1\}. \end{cases}$$

Then it is easy to see that ϕ_x is an L-majorant of ϕ at x for each $x \in (0,1)$, and hence ϕ is an L-majorized correspondence. Therefore, by Theorem 1, there exists a maximal element.

As an application of Theorem 1, we shall prove the following existence theorem of equilibrium for an abstract economy with an L-majorized preference correspondence in a Hausdorff topological vector space.

THEOREM 2. Let X be a non-empty compact convex subset of a Hausdorff topological vector space (a choice set). Let $A, B: X \rightarrow 2^X$ be constraint correspondences and $P: X \rightarrow 2^X$ be a preference correspondence satisfying the following conditions:

(1) P is L-majorized,

(2) for each $x \in X, A(x)$ is non-empty and $co\ A(x) \subset B(x)$,

(3) for each $y \in X, A^{-1}(y)$ is open in X,

(4) the correspondence $clB: X \rightarrow 2^X$ is upper semicontinuous.

Then there exists an equilibrium $\widehat{x} \in X$, i.e.,

$$\widehat{x} \in cl_X B(\widehat{x}) \quad \text{and} \quad A(\widehat{x}) \cap P(\widehat{x}) = \emptyset.$$

PROOF. Let $F = \{x \in X : x \in cl_X B(x)\}$, then F is closed in X since clB is upper semicontinuous. Define $\psi: X \to 2^X$ by

$$\psi(x) = \begin{cases} co\ A(x) \cap P(x), & \text{if } x \in F, \\ co\ A(x), & \text{if } x \notin F. \end{cases}$$

Suppose $\psi(x) \neq \emptyset$ for all $x \in X$. Let $x \in X$ be arbitrarily given. If $x \notin F$, then $N_x = X \backslash F$ is an open neighborhood of x in X such that $z \notin coA(z)$ for all $z \in N_x$. Define $\psi_x: X \to 2^X$ by

$$\psi_x(z) = \begin{cases} \emptyset, & \text{if } x \in F, \\ co\ A(z), & \text{if } x \notin F. \end{cases}$$

Then $z \notin co\psi_x(z)$ for all $z \in X$ and, by (3) and Lemma 1, $\psi_x^{-1}(y) = (X \backslash F) \cap (coA)^{-1}(y)$ is open in X for each $y \in X$. It follows that ψ_x is of class L. Moreover, for each $z \in N_x, \psi(z) = coA(z) = \psi_x(z)$. Thus ψ_x is an L-majorant of ψ at x.

Now suppose that $x \in F$. Then $\psi(x) = coA(x) \cap P(x)$ so that $P(x) \neq \emptyset$; then by the assumption (1), there exist $\phi_x: X \to 2^X$ of class L and an open neighborhood N_x of x in X such that $P(z) \subset \phi_x(z)$ for all $z \in X$.

We now define $\psi_x: X \to 2^X$ by

$$\psi_x(z) = \begin{cases} co\ A(z) \cap \phi_x(z), & \text{if } x \in F, \\ co\ A(z), & \text{if } x \notin F. \end{cases}$$

Note that as $P(z) \subset \phi_x(z)$ for each $z \in N_x$, we have $\psi(z) \subset \psi_x(z)$ for each $z \in N_x$. Let $z \in X$; if $z \notin F$, by (2), we have $z \notin co\ A(z) = co\psi_x(z)$ and if $z \in F$, then $\psi_x(z) = co\ A(z) \cap \phi_x(z) \subset \phi_x(z)$ so that $z \notin co\ \psi_x(z)$ as $z \notin co\ \phi_x(z)$. Hence $z \notin co\ \psi_x(z)$ for all $z \in X$. Next, for each $y \in X$,

$$\begin{aligned} (\psi_x)^{-1}(y) &= \{z \in X: y \in \psi_x(z)\} \\ &= z \in F: y \in \psi_x(z)\} \cup \{z \in X \backslash F: y \in \psi_x(z)\} \\ &= \{z \in F: y \in [co\ A(z) \cap \phi_x(z)]\} \cup \{z \in X \backslash F: y \in co\ A(z)\} \\ &= [F \cap (co\ A)^{-1}(y) \cap \phi_x^{-1}(y)] \cup [(X \backslash F) \cap (co\ A)^{-1}(y)] \\ &= [\phi_x^{-1}(y) \cup (X \backslash F)] \cap (co\ A)^{-1}(y) \end{aligned}$$

is open in X by (3) and Lemma 1. Thus ψ_x is also an L-majorant of ψ at x. Therefore in both cases, ψ is L-majorized. By Theorem 1, there exists a point $\widetilde{x} \in X$ such that $\psi(\widetilde{x}) = \emptyset$, which is a contradiction.

Hence there must exist a point $\widehat{x} \in X$ such that $\psi(\widehat{x}) = \emptyset$. By (2), we must have $\widehat{x} \in cl_X B(\widehat{x})$ and $co\ A(\widehat{x}) \cap P(\widehat{x}) = \emptyset$ so that $A(\widehat{x}) \cap P(\widehat{x}) = \emptyset$. This completes the proof.

If A has an open graph in $X \times X$, then $A^{-1}(y)$ is open in X for each $y \in X$ (see Corollary 4.1 in [10]). Hence we can obtain Corollary 2 of Borglin-Keiding [1] as an easy consequence of Theorem 2:

COROLLARY 1. Let X be a non-empty compact convex subset of a Hausdorff topological vector space and let $P, A: X \to 2^X$ be two correspondences satisfying the following conditions:

(1) P is L-majorized,

(2) for each $x \in X$, $A(x)$ is a non-empty convex,

(3) the graph of A is open in $X \times X$,

(4) the correspondence $clA: X \to 2^X$ is upper semicontinuous.

Then there exists an equilibrium $\hat{x} \in X$, i.e.,

$$\hat{x} \in cl_X A(\hat{x}) \quad \text{and} \quad A(\hat{x}) \cap P(\hat{x}) = \emptyset.$$

By applying Theorem 2, we can obtain an equilibrium for the following 1-person game:

EXAMPLE 2. Let $X = [0,1]$ be a compact convex choice set, constraint correspondences $A, B: X \to 2^X$ and preference correspondence $P: X \to 2^X$ be defined by

$$A(x) = \begin{cases} \{1\}, & \text{if } x \in \{0,1\}, \\ (0,x) \cup \{1\}, & \text{if } x \in (0,1), \end{cases}$$

$$B(x) = \begin{cases} (0,1], & \text{if } x \in [0,1), \\ [0,1], & \text{if } x = 1, \end{cases}$$

$$P(x) = \begin{cases} \{y \in X : 0 \le y \le x^2\}, & \text{if } x \in (0,1), \\ \emptyset, & \text{if } x \in \{0,1\}. \end{cases}$$

Then P is L-majorized as in Example 1 and the whole assumptions of Theorem 2 are satisfied so that, by Theorem 2, there exists an equilibrium $1 \in X$ such that $1 \in clB(1)$ and $A(1) \cap P(1) = \emptyset$. As remarked before, equilibrium existence results for the correspondences of class L cannot be applicable in this setting.

Let I be a finite set of agents and X_i be a Hausdorff topological vector space. Let $X = \Pi_{i \in I} X_i$. For a given correspondence $A_i: X \to 2^{X_i}$, recall that a correspondence $A'_i: X \to 2^X$ is defined by $A'_i(x) = \{y \in X : y_i \in A_i(x)\} (= \pi_i^{-1}(A_i(x))$, where $\pi_i: X \to X_i$ is the i-th projection). Then it is easy to show that the following two conditions are equivalent:

(1) A'_i is a correspondence of class L;

(2) for each $x \in X$, $x_i \notin coA_i(x)$ and for each $y \in X_i$, $A_i^{-1}(y)$ is open in X.

Using the method in Borglin-Keiding [1], we shall now show that the case of n agents ($n > 1$) with preference correspondences of class L can be reduced to a 1-person game with L-majorized preference correspondence (i.e., Theorem 2).

THEOREM 3. Let $\Gamma = (X_i, A_i, B_i, P_i)_{i \in I}$ be a generalized game where I is a finite set such that for each $i \in I$,

(1) X_i is a non-empty compact convex subset of a Hausdorff topological vector space,

(2) for each $x \in X = \Pi_{i \in I} X_i$, $A_i(x)$ is non-empty and $coA_i(x) \subset B_i(x)$,

(3) for each $y \in X_i$, $A_i^{-1}(y)$ is open in X,

(4) the correspondence $clB_i: X \to 2^{X_i}$ is upper semicontinuous,

(5) the correspondence $P'_i: X \to 2^X$ is of class L (where $P'_i = \pi_i^{-1} \circ P_i$).

Then Γ has an equilibrium $\hat{x} \in X$, i.e. for each $i \in I$,

$$\hat{x}_i \in cl_X B_i(\hat{x}) \quad \text{and} \quad A_i(\hat{x}) \cap P_i(\hat{x}) = \emptyset.$$

PROOF. By (1), $X = \Pi_{\iota \in I} X_\iota$ is a non-empty compact convex subset of a Hausdorff topological vector space. Define the correspondences $A, B, P: X \to 2^X$ by

$$A(x) = \Pi_{\iota \in I} A_\iota(x),$$
$$B(x) = \Pi_{\iota \in I} B_\iota(x),$$

and

$$P(x) = \begin{cases} \cap_{\iota \in I(x)} P'_\iota(x) \cap A(x), & \text{if } I(x) \neq \emptyset, \\ \emptyset, & \text{if } I(x) = \emptyset, \end{cases}$$

where

$$I(x) = \{\iota \in I: P'_\iota(x) \cap A(x) \neq \emptyset\}.$$

By (2), for each $x \in X, A(x)$ is non-empty and $coA(x) \subset B(x)$. By (3), for each $y \in X, A^{-1}(y) = \cap_{\iota \in I} A_\iota^{-1}(y_\iota)$ is open in X. Moreover, since for each $x \in X, cl_X B(x) = cl_X[\Pi_{\iota \in I} B_\iota(x)] = \Pi_{\iota \in I} cl_X B_\iota(x)$, e.g., see ([3], p. 99), it follows from (4) and Lemma 3 of Fan ([4], p. 124) that $clB: X \to 2^X$ is also upper semicontinuous.

Now let $x \in X$ and suppose that $P(x) \neq \emptyset$. It follows that $I(x) \neq \emptyset$. We shall first show that there exists an open neighborhood N_x of x in X such that $I(x) \subset I(z)$ (and hence also $I(z) \neq \emptyset$) for all $z \in N_x$. Indeed, let $i \in I(x)$; as $P'_\iota(x) \cap A(x) \neq \emptyset$, take any $y \in P'_\iota(x) \cap A(x)$, then $x \in (P'_\iota)^{-1}(y) \cap A^{-1}(y)$. Let $N_\iota = (P'_\iota)^{-1}(y) \cap A^{-1}(y)$, then N_i is an open neighborhood of x in X since P'_ι is of class L and $A^{-1}(y)$ is open. Let $N_x = \cap_{\iota \in I(x)} N_\iota$, then N_x is an open neighborhood of x in X. If $z \in N_x$, then for each $i \in I(x), z \in N_\iota = (P'_\iota)^{-1}(y) \cap A^{-1}(y)$ so that $y \in P'_\iota(z) \cap A(z)$ and hence $P'_\iota(z) \cap A(z) \neq \emptyset$; that is $i \in I(z)$. This shows that $I(x) \subset I(z)$ for all $z \in N_x$. Next fix $i_0 \in I(x)$. Then for any $z \in N_x$, we have

$$\begin{aligned} P(z) &= \cap_{\iota \in I(z)} P'_\iota(z) \cap A(z) \\ &\subset \cap_{\iota \in I(x)} P'_\iota(z) \cap A(z) \qquad \text{(since } I(x) \subset I(z)) \\ &\subset P'_{\iota_0}(z) \cap A(z). \end{aligned}$$

Now we define a correspondence $P_x: X \to 2^X$ by

$$P_x(z) = P'_{\iota_0}(z) \cap A(z) \text{ for each } z \in X.$$

Then for any $z \in N_x$ we have $P(z) \subset P_x(z)$ and P_x is of class L. Therefore P_x is an L-majorant of P at x. This shows that P is L-majorized. Hence all the hypotheses of Theorem 2 are satisfied so that there exists $\hat{x} \in X$ such that $\hat{x} \in cl_X B(\hat{x})$ and $A(\hat{x}) \cap P(\hat{x}) = \emptyset$. It follows that $\hat{x}_\iota \in cl_X B_\iota(\hat{x})$ for each $i \in I$. We shall now show that $I(\hat{x}) = \emptyset$. Suppose $I(\hat{x}) \neq \emptyset$. Note that $P(\hat{x}) = (\Pi_{\iota \in I} M_\iota) \cap A(\hat{x})$, where

$$M_\iota = \begin{cases} X_\iota, & \text{if } i \notin I(x), \\ P_i(\hat{x}), & \text{if } i \in I(x). \end{cases}$$

Thus $A(\hat{x}) \cap P(\hat{x}) = \emptyset$ implies $A_\iota(\hat{x}) \cap P_\iota(\hat{x}) = \emptyset$ for at least one $i \in I(\hat{x})$ so that $A(\hat{x}) \cap P'_i(\hat{x}) = \emptyset$ for at least one $i \in I(\hat{x})$ which contradicts the definition of $I(\hat{x})$. Therefore we must have $I(\hat{x}) = \emptyset$, i.e., $A(\hat{x}) \cap P'_\iota(\hat{x}) = \emptyset$ for all $i \in I$, and hence $A_\iota(\hat{x}) \cap P_\iota(\hat{x}) = \emptyset$ for each $i \in I$. This completes the proof.

REMARK. Theorem 3 is closely related to Theorem 6.1 of Yannelis-Prabhakar [10]. In fact, in Theorem 3, X_i need not be a metrizable subset of a locally convex space; but in Theorem 6.1 in [10], the set of agents I need not be finite.

The following result is a special case of Lemma 1 in [2]:

LEMMA 3. Let X be a non-empty convex subset of a topological vector space and $P: X \to 2^X$ be L-majorized. If every open subset of X containing the set $\{x \in X: P(x) \neq \emptyset\}$ is paracompact, then there exists a correspondence $\phi: X \to 2^X$ of class L such that $P(x) \subset \phi(x)$ for all $x \in X$.

We shall now generalize Theorem 3 to the case $P'_i: X \to 2^X$ is L-majorized as follows:

THEOREM 4. Let $\Gamma = (X_i, A_i, B_i, P_i)_{i \in I}$ be a generalized game where I is a finite set such that for each $i \in I$.

(1) X_i is a non-empty compact convex subset of a Hausdorff topological vector space such that every open subset of $X = \Pi_{i \in I} X_i$ containing the set $\{x \in X: P'_i(x) \neq \emptyset\}$ is paracompact,

(2) for each $x \in X, A_i(x)$ is non-empty and $coA_i(x) \subset B_i(x)$,

(3) for each $y \in X_i, A_i^{-1}(y)$ is open in X,

(4) the correspondence $clB_i: X \to 2^{X_i}$ is upper semicontinuous,

(5) the correspondence $P'_i: X \to 2^X$ is L-majorized (where $P'_i = \pi_i^{-1} \circ P_i$).

Then Γ has an equilibrium $\widehat{x} \in X$, i.e., for each $i \in I$,

$$\widehat{x}_i \in cl_X B_i(\widehat{x}) \quad \text{and} \quad A_i(\widehat{x}) \cap P_i(\widehat{x}) = \emptyset.$$

PROOF. By the assumptions (1) and (5), the whole hypotheses of Lemma 1 in [2] are satisfied, so that for each $i \in I$, there exists a correspondence $Q'_i: X \to 2^X$ of class L (where $Q'_i = \pi_i^{-1} \circ Q_i$ for some $Q_i: X \to 2^{X_i}$) such that $P'_i(x) \subset Q'_i(x)$ for each $x \in X$. Therefore the conclusion follows from Theorem 3.

Theorem 4 is a generalization of Corollary 3 of Borglin-Keiding [1] to infinite dimensional spaces as well as to L-majorized preference correspondences.

Finally we remark that the condition "every open subset of X containing the set $\{x \in X: P'_i(x) \neq \emptyset\}$ is paracompact" in Theorem 4 is satisfied if X is perfectly normal (i.e., every open subset of X is an F_σ-set).

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