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A NOTE ON SOME APPLICATIONS OF α-OPEN SETS

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The object of this note is to introduce and study topological properties of α-derived, α-border, α-frontier, and α-exterior of a set using the concept of α-open sets. Moreover, we study some further properties of the well-known notions of α-closure and α-interior. We also obtain a new decomposition of α-continuous functions.

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1. Introduction. The notion of α-open set (originally called α-sets) in topological spaces was introduced by Njåstad [2] in 1965. Since then, it has been widely investigated in the literature. For these sets, we introduce the notions of α-derived, α-border, α-frontier, and α-exterior of a set and show that some of their properties are analogous to those for open sets. Also, we give some additional properties of α-closure and α-interior of a set due to Njåstad [2].

Throughout this paper, (X,τ) (simply X) always mean topological spaces. A subset A of (X,τ) is called α-open [2] if $A \subset \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(A)))$. The complement of an α-open set is called α-closed. The intersection of all α-closed sets containing A is called the α-closure of A, denoted by $\mathrm{Cl}_\alpha(A)$. A subset A is also α-closed if and only if $A = \mathrm{Cl}_\alpha(A)$. We denote the family of α-open sets of (X,τ) by τ^α. It is shown in [2] (see also [4]) that each of $\tau \subset \tau^\alpha$ and τ^α is a topology on X.

2. Applications of α-open sets

Definition 2.1. Let A be a subset of a space X. A point $x \in A$ is said to be α-limit point of A if for each α-open set U containing x, $U \cap (A\backslash\{x\}) \neq \emptyset$. The set of all α-limit points of A is called an α-derived set of A and is denoted by $D_\alpha(A)$.

Theorem 2.2. *For subsets A,B of a space X, the following statements hold:*
(1) $D_\alpha(A) \subset D(A)$, *where $D(A)$ is the derived set of A;*
(2) *if $A \subset B$, then $D_\alpha(A) \subset D_\alpha(B)$;*
(3) $D_\alpha(A) \cup D_\alpha(B) \subset D_\alpha(A \cup B)$ *and* $D_\alpha(A \cap B) \subset D_\alpha(A) \cap D_\alpha(B)$;
(4) $D_\alpha(D_\alpha(A))\backslash A \subset D_\alpha(A)$;
(5) $D_\alpha(A \cup D_\alpha(A)) \subset A \cup D_\alpha(A)$.

Proof. (1) It suffices to observe that every open set is α-open.

(3) Follows by (2).

(4) If $x \in D_\alpha(D_\alpha(A))\backslash A$ and U is an α-open set containing x, then $U \cap (D_\alpha(A)\backslash\{x\}) \neq \emptyset$. Let $y \in U \cap (D_\alpha(A)\backslash\{x\})$. Then, since $y \in D_\alpha(A)$ and $y \in U$, $U \cap (A\backslash\{y\}) \neq \emptyset$. Let $z \in U \cap (A\backslash\{y\})$. Then, $z \neq x$ for $z \in A$ and $x \notin A$. Hence, $U \cap (A\backslash\{x\}) \neq \emptyset$. Therefore, $x \in D_\alpha(A)$.

(5) Let $x \in D_\alpha(A \cup D_\alpha(A))$. If $x \in A$, the result is obvious. So, let $x \in D_\alpha(A \cup D_\alpha(A))\backslash A$, then, for α-open set U containing x, $U \cap (A \cup D_\alpha(A)\backslash\{x\}) \neq \emptyset$. Thus, $U \cap (A\backslash\{x\}) \neq \emptyset$ or $U \cap (D_\alpha(A)\backslash\{x\}) \neq \emptyset$. Now, it follows similarly from (4) that $U \cap (A\backslash\{x\}) \neq \emptyset$. Hence, $x \in D_\alpha(A)$. Therefore, in any case, $D_\alpha(A \cup D_\alpha(A)) \subset A \cup D_\alpha(A)$. □

In general, the converse of (1) may not be true and the equality does not hold in (3) of Theorem 2.2.

Example 2.3. Let $X = \{a,b,c\}$ with topology $\tau = \{\emptyset,\{a\},X\}$. Thus, $\tau^\alpha = \{\emptyset,\{a\},\{a,b\},\{a,c\},X\}$. Take the following:

(i) $A = \{c\}$. Then, $D(A) = \{b\}$ and $D_\alpha(A) = \emptyset$. Hence, $D(A) \not\subset D_\alpha(A)$;

(ii) $C = \{a\}$ and $E = \{b,c\}$. Then, $D_\alpha(C) = \{b,c\}$ and $D_\alpha(E) = \emptyset$. Hence, $D_\alpha(C \cup E) \neq D_\alpha(C) \cup D_\alpha(E)$.

Theorem 2.4. *For any subset A of a space X, $\mathrm{Cl}_\alpha(A) = A \cup D_\alpha(A)$.*

Proof. Since $D_\alpha(A) \subset \mathrm{Cl}_\alpha(A)$, $A \cup D_\alpha(A) \subset \mathrm{Cl}_\alpha(A)$. On the other hand, let $x \in \mathrm{Cl}_\alpha(A)$. If $x \in A$, then the proof is complete. If $x \notin A$, each α-open set U containing x intersects A at a point distinct from x; so $x \in D_\alpha(A)$. Thus, $\mathrm{Cl}_\alpha(A) \subset A \cup D_\alpha(A)$, which completes the proof. □

Corollary 2.5. *A subset A is α-closed if and only if it contains the set of its α-limit points.*

Definition 2.6. A point $x \in X$ is said to be an α-interior point of A if there exists an α-open set U containing x such that $U \subset A$. The set of all α-interior points of A is said to be α-interior of A [1] and denoted by $\mathrm{Int}_\alpha(A)$.

Theorem 2.7. *For subsets A,B of a space X, the following statements are true:*

(1) $\mathrm{Int}_\alpha(A)$ *is the largest α-open set contained in A;*

(2) *A is α-open if and only if* $A = \mathrm{Int}_\alpha(A)$;

(3) $\mathrm{Int}_\alpha(\mathrm{Int}_\alpha(A)) = \mathrm{Int}_\alpha(A)$;

(4) $\mathrm{Int}_\alpha(A) = A\backslash D_\alpha(X\backslash A)$;

(5) $X\backslash \mathrm{Int}_\alpha(A) = \mathrm{Cl}_\alpha(X\backslash A)$;

(6) $X\backslash \mathrm{Cl}_\alpha(A) = \mathrm{Int}_\alpha(X\backslash A)$;

(7) $A \subset B$, *then* $\mathrm{Int}_\alpha(A) \subset \mathrm{Int}_\alpha(B)$;

(8) $\mathrm{Int}_\alpha(A) \cup \mathrm{Int}_\alpha(B) \subset \mathrm{Int}_\alpha(A \cup B)$;

(9) $\mathrm{Int}_\alpha(A) \cap \mathrm{Int}_\alpha(B) \supset \mathrm{Int}_\alpha(A \cap B)$.

Proof. (4) If $x \in A\backslash D_\alpha(X\backslash A)$, then $x \notin D_\alpha(X\backslash A)$ and so there exists an α-open set U containing x such that $U \cap (X\backslash A) = \emptyset$. Then, $x \in U \subset A$ and hence $x \in \text{Int}_\alpha(A)$, that is, $A\backslash D_\alpha(X\backslash A) \subset \text{Int}_\alpha(A)$. On the other hand, if $x \in \text{Int}_\alpha(A)$, then $x \notin D_\alpha(X\backslash A)$ since $\text{Int}_\alpha(A)$ is α-open and $\text{Int}_\alpha(A) \cap (X\backslash A) = \emptyset$. Hence, $\text{Int}_\alpha(A) = A\backslash D_\alpha(X\backslash A)$.

(5) $X\backslash \text{Int}_\alpha(A) = X\backslash(A\backslash D_\alpha(X\backslash A)) = (X\backslash A) \cup D_\alpha(X\backslash A) = \text{Cl}_\alpha(X\backslash A)$. □

Definition 2.8. $b_\alpha(A) = A\backslash \text{Int}_\alpha(A)$ is said to be the α-border of A.

Theorem 2.9. *For a subset A of a space X, the following statements hold:*

(1) $b_\alpha(A) \subset b(A)$ *where $b(A)$ denotes the border of A;*
(2) $A = \text{Int}_\alpha(A) \cup b_\alpha(A)$;
(3) $\text{Int}_\alpha(A) \cap b_\alpha(A) = \emptyset$;
(4) *A is an α-open set if and only if $b_\alpha(A) = \emptyset$;*
(5) $b_\alpha(\text{Int}_\alpha(A)) = \emptyset$;
(6) $\text{Int}_\alpha(b_\alpha(A)) = \emptyset$;
(7) $b_\alpha(b_\alpha(A)) = b_\alpha(A)$;
(8) $b_\alpha(A) = A \cap \text{Cl}_\alpha(X\backslash A)$;
(9) $b_\alpha(A) = D_\alpha(X\backslash A)$.

Proof. (6) If $x \in \text{Int}_\alpha(b_\alpha(A))$, then $x \in b_\alpha(A)$. On the other hand, since $b_\alpha(A) \subset A$, $x \in \text{Int}_\alpha(b_\alpha(A)) \subset \text{Int}_\alpha(A)$. Hence, $x \in \text{Int}_\alpha(A) \cap b_\alpha(A)$, which contradicts (3). Thus, $\text{Int}_\alpha(b_\alpha(A)) = \emptyset$.

(8) $b_\alpha(A) = A\backslash \text{Int}_\alpha(A) = A\backslash(X\backslash \text{Cl}_\alpha(X\backslash A)) = A \cap \text{Cl}_\alpha(X\backslash A)$.

(9) $b_\alpha(A) = A\backslash \text{Int}_\alpha(A) = A\backslash(A\backslash D_\alpha(X\backslash A)) = D_\alpha(X\backslash A)$. □

Example 2.10. Consider the topological space (X,τ) given in Example 2.3. If $A = \{a,b\}$, then $b_\alpha(A) = \emptyset$ and $b(A) = \{b\}$. Hence, $b(A) \not\subset b_\alpha(A)$, that is, in general, the converse Theorem 2.9(1) may not be true.

Definition 2.11. $\text{Fr}_\alpha(A) = \text{Cl}_\alpha(A)\backslash \text{Int}_\alpha(A)$ is said to be the α-frontier of A.

Theorem 2.12. *For a subset A of a space X, the following statements hold:*

(1) $\text{Fr}_\alpha(A) \subset \text{Fr}(A)$ *where $\text{Fr}(A)$ denotes the frontier of A;*
(2) $\text{Cl}_\alpha(A) = \text{Int}_\alpha(A) \cup \text{Fr}_\alpha(A)$;
(3) $\text{Int}_\alpha(A) \cap \text{Fr}_\alpha(A) = \emptyset$;
(4) $b_\alpha(A) \subset \text{Fr}_\alpha(A)$;
(5) $\text{Fr}_\alpha(A) = b_\alpha(A) \cup D_\alpha(A)$;
(6) *A is an α-open set if and only if $\text{Fr}_\alpha(A) = D_\alpha(A)$;*
(7) $\text{Fr}_\alpha(A) = \text{Cl}_\alpha(A) \cap \text{Cl}_\alpha(X\backslash A)$;
(8) $\text{Fr}_\alpha(A) = \text{Fr}_\alpha(X\backslash A)$;
(9) $\text{Fr}_\alpha(A)$ *is α-closed;*
(10) $\text{Fr}_\alpha(\text{Fr}_\alpha(A)) \subset \text{Fr}_\alpha(A)$;
(11) $\text{Fr}_\alpha(\text{Int}_\alpha(A)) \subset \text{Fr}_\alpha(A)$;
(12) $\text{Fr}_\alpha(\text{Cl}_\alpha(A)) \subset \text{Fr}_\alpha(A)$;
(13) $\text{Int}_\alpha(A) = A\backslash \text{Fr}_\alpha(A)$.

Proof. (2) $\mathrm{Int}_\alpha(A)\cup\mathrm{Fr}_\alpha(A)=\mathrm{Int}_\alpha(A)\cup(\mathrm{Cl}_\alpha(A)\setminus\mathrm{Int}_\alpha(A))=\mathrm{Cl}_\alpha(A)$.

(3) $\mathrm{Int}_\alpha(A)\cap\mathrm{Fr}_\alpha(A)=\mathrm{Int}_\alpha(A)\cap(\mathrm{Cl}_\alpha(A)\setminus\mathrm{Int}_\alpha(A))=\varnothing$.

(5) Since $\mathrm{Int}_\alpha(A)\cup\mathrm{Fr}_\alpha(A)=\mathrm{Int}_\alpha(A)\cup b_\alpha(A)\cup D_\alpha(A)$, $\mathrm{Fr}_\alpha(A)=b_\alpha(A)\cup D_\alpha(A)$.

(7) $\mathrm{Fr}_\alpha(A)=\mathrm{Cl}_\alpha(A)\setminus\mathrm{Int}_\alpha(A)=\mathrm{Cl}_\alpha(A)\cap\mathrm{Cl}_\alpha(X\setminus A)$.

(9) $\mathrm{Cl}_\alpha(\mathrm{Fr}_\alpha(A))=\mathrm{Cl}_\alpha(\mathrm{Cl}_\alpha(A)\cap\mathrm{Cl}_\alpha(X\setminus A))\subset\mathrm{Cl}_\alpha(\mathrm{Cl}_\alpha(A))\cap\mathrm{Cl}_\alpha(\mathrm{Cl}_\alpha(X\setminus A))=\mathrm{Fr}_\alpha(A)$.

Hence, $\mathrm{Fr}_\alpha(A)$ is α-closed.

(10) $\mathrm{Fr}_\alpha(\mathrm{Fr}_\alpha(A))=\mathrm{Cl}_\alpha(\mathrm{Fr}_\alpha(A))\cap\mathrm{Cl}_\alpha(X\setminus\mathrm{Fr}_\alpha(A))\subset\mathrm{Cl}_\alpha(\mathrm{Fr}_\alpha(A))=\mathrm{Fr}_\alpha(A)$.

(12) $\mathrm{Fr}_\alpha(\mathrm{Cl}_\alpha(A))=\mathrm{Cl}_\alpha(\mathrm{Cl}_\alpha(A))\setminus\mathrm{Int}_\alpha(\mathrm{Cl}_\alpha(A))=\mathrm{Cl}_\alpha((A))\setminus\mathrm{Int}_\alpha(\mathrm{Cl}_\alpha(A))=\mathrm{Cl}_\alpha(A)\setminus\mathrm{Int}_\alpha(A)=\mathrm{Fr}_\alpha(A)$.

(13) $A\setminus\mathrm{Fr}_\alpha(A)=A\setminus(\mathrm{Cl}_\alpha(A)\setminus\mathrm{Int}_\alpha(A))=\mathrm{Int}_\alpha(A)$. □

The converses of (1) and (4) of Theorem 2.12 are not true in general, as shown by Example 2.13.

Example 2.13. Consider the topological space (X,τ) given in Example 2.3. If $A=\{c\}$, then $\mathrm{Fr}(A)=\{b,c\}\not\subseteq\{c\}=\mathrm{Fr}_\alpha(A)$, and if $B=\{a,b\}$, then $\mathrm{Fr}_\alpha(B)=\{c\}\not\subseteq b_\alpha(B)$.

Definition 2.14. A function $f:(X,\tau)\to(Y,\sigma)$ is said to be α-continuous [1] if $f^{-1}(V)\in\tau^\alpha$ for every $V\in\sigma$ and, equivalently, if for each $x\in X$ and each open set V of Y containing $f(x)$, there exists $U\in\tau^\alpha$ with $x\in U$ such that $f(U)\subset V$.

In the following theorem, $\sharp\alpha$-c. denotes the set of points x of X for which a function $f:(X,\tau)\to(Y,\sigma)$ is not α-continuous.

Theorem 2.15. *$\sharp\alpha$-c. is identical with the union of the α-frontiers of the inverse images of α-open sets containing $f(x)$.*

Proof. Suppose that f is not α-continuous at a point x of X. Then, there exists an open set $V\subset Y$ containing $f(x)$ such that $f(U)$ is not a subset of V for every $U\in\tau^\alpha$ containing x. Hence, we have $U\cap(X\setminus f^{-1}(V))\neq\varnothing$ for every $U\in\tau^\alpha$ containing x. It follows that $x\in\mathrm{Cl}_\alpha(X\setminus f^{-1}(V))$. We also have $x\in f^{-1}(V)\subset\mathrm{Cl}_\alpha(f^{-1}(V))$. This means that $x\in\mathrm{Fr}_\alpha(f^{-1}(V))$.

Now, let f be α-continuous at $x\in X$ and $V\subset Y$ any open set containing $f(x)$. Then, $x\in f^{-1}(V)$ is an α-open set of X. Thus, $x\in\mathrm{Int}_\alpha(f^{-1}(V))$ and therefore $x\notin\mathrm{Fr}_\alpha(f^{-1}(V))$ for every open set V containing $f(x)$. □

Definition 2.16. $\mathrm{Ext}_\alpha(A)=\mathrm{Int}_\alpha(X\setminus A)$ is said to be an α-exterior of A.

Theorem 2.17. *For a subset A of a space X, the following statements hold:*

(1) $\mathrm{Ext}(A)\subset\mathrm{Ext}_\alpha(A)$ *where* $\mathrm{Ext}(A)$ *denotes the exterior of* A;

(2) $\mathrm{Ext}_\alpha(A)$ *is α-open;*

(3) $\mathrm{Ext}_\alpha(A)=\mathrm{Int}_\alpha(X\setminus A)=X\setminus\mathrm{Cl}_\alpha(A)$;

(4) $\mathrm{Ext}_\alpha(\mathrm{Ext}_\alpha(A))=\mathrm{Int}_\alpha(\mathrm{Cl}_\alpha(A))$;

(5) *If* $A \subset B$, *then* $\mathrm{Ext}_\alpha(A) \supset \mathrm{Ext}_\alpha(B)$;
(6) $\mathrm{Ext}_\alpha(A \cup B) \subset \mathrm{Ext}_\alpha(A) \cup \mathrm{Ext}_\alpha(B)$;
(7) $\mathrm{Ext}_\alpha(A \cap B) \supset \mathrm{Ext}_\alpha(A) \cap \mathrm{Ext}_\alpha(B)$;
(8) $\mathrm{Ext}_\alpha(X) = \emptyset$;
(9) $\mathrm{Ext}_\alpha(\emptyset) = X$;
(10) $\mathrm{Ext}_\alpha(A) = \mathrm{Ext}_\alpha(X \setminus \mathrm{Ext}_\alpha(A))$;
(11) $\mathrm{Int}_\alpha(A) \subset \mathrm{Ext}_\alpha(\mathrm{Ext}_\alpha(A))$;
(12) $X = \mathrm{Int}_\alpha(A) \cup \mathrm{Ext}_\alpha(A) \cup \mathrm{Fr}_\alpha(A)$.

Proof. (4) $\mathrm{Ext}_\alpha(\mathrm{Ext}_\alpha(A)) = \mathrm{Ext}_\alpha(X \setminus \mathrm{Cl}_\alpha(A)) = \mathrm{Int}_\alpha(X \setminus (X \setminus \mathrm{Cl}_\alpha(A))) = \mathrm{Int}_\alpha(\mathrm{Cl}_\alpha(A))$.

(10) $\mathrm{Ext}_\alpha(X \setminus \mathrm{Ext}_\alpha(A)) = \mathrm{Ext}_\alpha(X \setminus \mathrm{Int}_\alpha(X \setminus A)) = \mathrm{Int}_\alpha(X \setminus (X \setminus \mathrm{Int}_\alpha(X \setminus A))) = \mathrm{Int}_\alpha(\mathrm{Int}_\alpha(X \setminus A)) = \mathrm{Int}_\alpha(X \setminus A) = \mathrm{Ext}_\alpha(A)$.

(11) $\mathrm{Int}_\alpha(A) \subset \mathrm{Int}_\alpha(\mathrm{Cl}_\alpha(A)) = \mathrm{Int}_\alpha(X \setminus \mathrm{Int}_\alpha(X \setminus A)) = \mathrm{Int}_\alpha(X \setminus \mathrm{Ext}_\alpha(A)) = \mathrm{Ext}_\alpha(\mathrm{Ext}_\alpha(A))$. □

Example 2.18. Let $X = \{a, b, c, d\}$ with topology $\tau = \{\emptyset, \{c, d\}, X\}$. Hence, $\tau^\alpha = \{\emptyset, \{c, d\}, \{b, c, d\}, \{a, c, d\}, X\}$ If $A = \{a\}$ and $B = \{b\}$. Then, $\mathrm{Ext}_\alpha(A) \nsubseteq \mathrm{Ext}(A)$, $\mathrm{Ext}_\alpha(A \cap B) \neq \mathrm{Ext}_\alpha(A) \cap \mathrm{Ext}_\alpha(B)$, and $\mathrm{Ext}_\alpha(A \cup B) \neq \mathrm{Ext}_\alpha(A) \cup \mathrm{Ext}_\alpha(B)$.

3. A new decomposition of α-continuity

Definition 3.1. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be weakly α-continuous [3] if, for each $x \in X$ and each open set V of Y containing $f(x)$, there exists $U \in \tau^\alpha$ containing x such that $f(U) \subset \mathrm{Cl}(V)$.

Theorem 3.2 (Noiri [3]). *A function* $f : (X, \tau) \to (Y, \sigma)$ *is weakly α-continuous if and only if, for every open set* V *of* Y, $f^{-1}(V) \subset \mathrm{Int}_\alpha(f^{-1}(\mathrm{Cl}(V)))$.

The following notion is motivated by the above theorem.

Definition 3.3. A function $f : (X, \tau) \to (Y, \sigma)$ is relatively weakly α-continuous, if for each $x \in X$ and each open set V in Y containing $f(x)$, the set $f^{-1}(V)$ is α-open in the subspace $f^{-1}(\mathrm{Cl}(V))$.

Theorem 3.4. *An α-continuous function is relatively weakly α-continuous.*

Proof. Straightforward. □

The following example shows that the converse of Theorem 3.4 is not true.

Example 3.5. Let X be the set of all real numbers, τ the indiscrete topology for X, and σ the discrete topology for X. Let $f : (X, \tau) \to (X, \sigma)$ be the identity function. Then, f is relatively weakly α-continuous but it is not weakly α-continuous (hence it is not α-continuous) because $\mathrm{Int}_\alpha(f^{-1}(\mathrm{Cl}(V))) = \emptyset$ for any subset V of (X, σ).

Example 3.6. Let $X = \{a,b,c\}$, $\tau = \{\emptyset, X, \{c\}\}$, and $\sigma = \{\emptyset, X, \{a\}, \{b\}, \{a,b\}\}$. Let $f : (X,\tau) \to (X,\sigma)$ be the identity function. Then, f is weakly α-continuous but not relatively weakly α-continuous.

Examples 3.5 and 3.6 show that weakly α-continuous and relatively weakly α-continuous are independent.

The significance of relatively weakly α-continuous is that it yields a decomposition of α-continuous with weakly α-continuous as the other factor.

Theorem 3.7. *A function $f : (X,\tau) \to (Y,\sigma)$ is α-continuous if and only if it is weakly α-continuous and relatively weakly α-continuous.*

Proof. The necessity is given by Theorem 3.4 and by the fact that every α-continuous function is weakly α-continuous.

Sufficiency. Let V be an open set in Y. Since f is relatively weakly α-continuous, we have $f^{-1}(V) = f^{-1}(\mathrm{Cl}(V)) \cap W$, where W is an α-open set of X. Suppose that $x \in f^{-1}(V)$. This means that $f(x) \in V$ and also $x \in W$. By the fact that f is weakly α-continuous, there exists $U \in \tau^{\alpha}$ containing x such that $f(U) \subset \mathrm{Cl}(V)$. Therefore, $U \subset f^{-1}(\mathrm{Cl}(V))$. We can take U to be a subset of W. It follows that $x \in U \subset f^{-1}(\mathrm{Cl}(V)) \cap W = f^{-1}(V)$ and thus the claim follows. □

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