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THE EQUIVALENCE OF MANN ITERATION AND ISHIKAWA ITERATION FOR ψ-UNIFORMLY PSEUDOCONTRACTIVE OR ψ-UNIFORMLY ACCRETIVE MAPS

B. E. RHOADES and ŞTEFAN M. ŞOLTUZ

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We show that the Ishikawa iteration and the corresponding Mann iteration are equivalent when applied to ψ-uniformly pseudocontractive or ψ-uniformly accretive maps.

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1. Introduction. Let X be a real Banach space, B a nonempty, convex subset of X, and T a self-map of B, and let $x_0 = u_0 \in B$. The Mann iteration (see [2]) is defined by

$$u_{n+1} = (1-\alpha_n)u_n + \alpha_n T u_n, \quad n = 0,1,2,\ldots. \tag{1.1}$$

The Ishikawa iteration is defined (see [1]) by

$$\begin{aligned} x_{n+1} &= (1-\alpha_n)x_n + \alpha_n T y_n, \\ y_n &= (1-\beta_n)x_n + \beta_n T x_n, \quad n = 0,1,2,\ldots. \end{aligned} \tag{1.2}$$

The sequences $\{\alpha_n\} \subset (0,1)$, $\{\beta_n\} \subset [0,1)$ satisfy

$$\lim_{n\to\infty} \alpha_n = \lim_{n\to\infty} \beta_n = 0, \quad \sum_{n=1}^{\infty} \alpha_n = +\infty. \tag{1.3}$$

The map $J : X \to 2^{X^*}$ given by

$$Jx := \{f \in X^* : \langle x, f\rangle = \|x\|, \ \|f\| = \|x\|\}, \quad \forall x \in X, \tag{1.4}$$

is called *the normalized duality mapping*.

Remark 1.1. The above J satisfies

$$\langle x, j(y)\rangle \le \|x\|\|y\|, \quad \forall x \in X, \ \forall j(y) \in J(y). \tag{1.5}$$

Proof. Denote $j(y)$ by f. Since $f \in X^*$, we have

$$\langle x, f\rangle \leq \|f\|\|x\|. \tag{1.6}$$

From (1.4), we know that $\|f\| = \|y\|$. Hence (1.5) holds. □

Let

$$\Psi := \{\psi \mid \psi : [0,+\infty) \longrightarrow [0,+\infty) \text{ is a nondecreasing map such that } \psi(0) = 0\}. \tag{1.7}$$

The following definition is from [3].

Definition 1.2. Let X be a real Banach space. Let B be a nonempty subset of X. A map $T : B \to B$ is called ψ-uniformly pseudocontractive if there exist the map $\psi \in \Psi$ and $j(x-y) \in J(x-y)$ such that

$$\langle Tx - Ty, j(x-y)\rangle \leq \|x-y\|^2 - \psi(\|x-y\|), \quad \forall x, y \in B. \tag{1.8}$$

The map $S : X \to X$ is called ψ-uniformly accretive if there exist the map $\psi \in \Psi$ and $j(x-y) \in J(x-y)$ such that

$$\langle Sx - Sy, j(x-y)\rangle \geq \psi(\|x-y\|), \quad \forall x, y \in X. \tag{1.9}$$

Taking $\psi(a) := \psi(a) \cdot a$, for all $a \in [0,+\infty)$, $\psi \in \Psi$, we get the usual definitions of ψ-strongly pseudocontractivity and ψ-strongly accretivity. Taking $\psi(a) := \gamma \cdot a^2$, $\gamma \in (0,1)$, for all $a \in [0,+\infty)$, $\psi \in \Psi$, we get the usual definitions of strong pseudocontractivity and strong accretivity.

Denote by I the identity map.

Remark 1.3. T is ψ-uniformly pseudocontractive if and only if $S = (I-T)$ is ψ-uniformly accretive.

Let $F(T)$ denote the fixed point set with respect to B for the map T.

In [9], the following conjecture was given: "if the Mann iteration converges, then so does the Ishikawa iteration." In a series of papers [5, 6, 7, 8, 9, 10], the authors have given a positive answer to this conjecture, showing the equivalence between Mann and Ishikawa iterations for several classes of maps. In this paper, we show that the convergence of Mann iteration is equivalent to the convergence of Ishikawa iteration, for the most general class of ψ-uniformly pseudocontractive and ψ-uniformly accretive maps.

Lemma 1.4 [4]. *Let X be a real Banach space and let $J : X \to 2^{X^*}$ be the duality mapping. Then the following relation is true:*

$$\|x+y\|^2 \leq \|x\|^2 + 2\langle y, j(x+y)\rangle, \quad \forall x, y \in X,\ \forall j(x+y) \in J(x+y). \tag{1.10}$$

Lemma 1.5 [3]. *Let $\{\theta_n\}$ be a sequence of nonnegative real numbers, let $\{\lambda_n\}$ be a real sequence satisfying*

$$0 \le \lambda_n \le 1, \quad \sum_{n=0}^{\infty} \lambda_n = +\infty, \tag{1.11}$$

and let $\psi \in \Psi$. If there exists a positive integer n_0 such that

$$\theta_{n+1}^2 \le \theta_n^2 - \lambda_n \psi(\theta_{n+1}) + \sigma_n, \tag{1.12}$$

for all $n \ge n_0$, with $\sigma_n \ge 0$, for all $n \in \mathbb{N}$, and $\sigma_n = o(\lambda_n)$, then $\lim_{n\to\infty} \theta_n = 0$.

2. Main result.

We are now able to prove the following result.

Theorem 2.1. *Let X be a real Banach space, let B be a nonempty, convex subset of X, and let $T : B \to B$ be a uniformly continuous and ψ-uniformly pseudocontractive map with $T(B)$ bounded. If $\{\alpha_n\}$, $\{\beta_n\}$ satisfy (1.3), and $u_0 = x_0 \in B$, then the following are equivalent:*

(i) *the Mann iteration (1.1) converges (to $x^* \in F(T)$),*

(ii) *the Ishikawa iteration (1.2) converges (to the same $x^* \in F(T)$).*

Proof. The implication (ii)⇒(i) is obvious by setting, in (1.2), $\beta_n = 0$, for all $n \in \mathbb{N}$. We will prove the implication (i)⇒(ii). Let x^* be the fixed point of T. Suppose that $\lim_{n\to\infty} u_n = x^*$. Using

$$\lim_{n\to\infty} ||x_n - u_n|| = 0, \tag{2.1}$$

$$0 \le ||x^* - x_n|| \le ||u_n - x^*|| + ||x_n - u_n||, \tag{2.2}$$

we get

$$\lim_{n\to\infty} x_n = x^*. \tag{2.3}$$

The proof is complete if we prove the relation (2.1).

Set

$$M := \{||x_0 - u_0|| + \sup\{\|Tx - Ty\|,\ x, y \in B\}\} \ge 0. \tag{2.4}$$

The condition that $T(B)$ is bounded leads to

$$0 \le M < +\infty. \tag{2.5}$$

It is clear that $\|x_0 - u_0\| \le M$. Supposing that $\|x_n - u_n\| \le M$, we will prove that $\|x_{n+1} - u_{n+1}\| \le M$. Indeed, from (1.1) and (1.2), we have

$$\begin{aligned} ||x_{n+1} - u_{n+1}|| &\le (1-\alpha_n)||x_n - u_n|| + \alpha_n ||Ty_n - Tu_n|| \\ &\le (1-\alpha_n)M + \alpha_n M = M. \end{aligned} \tag{2.6}$$

That is,

$$\|x_n - u_n\| \le M, \quad \forall n \in \mathbb{N}. \tag{2.7}$$

The real function $f : [0,+\infty) \to [0,+\infty)$, $f(t) = t^2$, is increasing and convex. For all $\lambda \in [0,1]$ and $t_1, t_2 > 0$, we have

$$\left((1-\lambda)t_1 + \lambda t_2\right)^2 \le (1-\lambda)t_1^2 + \lambda t_2^2. \tag{2.8}$$

Set $t_1 := \|x_n - u_n\|$, $t_2 := Ty_n - Tu_n$, $\lambda := \alpha_n$ in (2.8), to obtain

$$\begin{aligned}\|x_{n+1} - u_{n+1}\|^2 &= \|(1-\alpha_n)(x_n - u_n) + \alpha_n(Ty_n - Tu_n)\|^2 \\ &\le \left((1-\alpha_n)\|x_n - u_n\| + \alpha_n\|Ty_n - Tu_n\|\right)^2 \\ &\le (1-\alpha_n)\|x_n - u_n\|^2 + \alpha_n M^2 \\ &\le \|x_n - u_n\|^2 + \alpha_n M^2.\end{aligned} \tag{2.9}$$

From (1.1), (1.2), (1.5), and (1.10), with

$$\begin{aligned} x &:= (1-\alpha_n)(x_n - u_n), \\ y &:= \alpha_n(Ty_n - Tu_n), \\ x + y &= x_{n+1} - u_{n+1}, \end{aligned} \tag{2.10}$$

we get

$$\begin{aligned}
&\|x_{n+1} - u_{n+1}\|^2 \\
&\quad = \|(1-\alpha_n)(x_n - u_n) + \alpha_n(Ty_n - Tu_n)\|^2 \\
&\quad \le (1-\alpha_n)^2\|x_n - u_n\|^2 + 2\alpha_n\langle Ty_n - Tu_n, j(x_{n+1} - u_{n+1})\rangle \\
&\quad = (1-\alpha_n)^2\|x_n - u_n\|^2 + 2\alpha_n\langle Tx_{n+1} - Tu_{n+1}, j(x_{n+1} - u_{n+1})\rangle \\
&\qquad + 2\alpha_n\langle Ty_n - Tx_{n+1}, j(x_{n+1} - u_{n+1})\rangle \\
&\qquad + 2\alpha_n\langle Tu_{n+1} - Tu_n, j(x_{n+1} - u_{n+1})\rangle \\
&\quad \le (1-\alpha_n)^2\|x_n - u_n\|^2 + 2\alpha_n\|x_{n+1} - u_{n+1}\|^2 - 2\alpha_n\psi(\|x_{n+1} - u_{n+1}\|) \\
&\qquad + 2\alpha_n\langle Ty_n - Tx_{n+1}, j(x_{n+1} - u_{n+1})\rangle \\
&\qquad + 2\alpha_n\langle Tu_{n+1} - Tu_n, j(x_{n+1} - u_{n+1})\rangle \\
&\quad \le (1-\alpha_n)^2\|x_n - u_n\|^2 + 2\alpha_n\|x_{n+1} - u_{n+1}\|^2 - 2\alpha_n\psi(\|x_{n+1} - u_{n+1}\|) \\
&\qquad + 2\alpha_n\|Ty_n - Tx_{n+1}\|\|x_{n+1} - u_{n+1}\| \\
&\qquad + 2\alpha_n\|Tu_{n+1} - Tu_n\|\|x_{n+1} - u_{n+1}\| \\
&\quad \le (1-\alpha_n)^2\|x_n - u_n\|^2 + 2\alpha_n\|x_{n+1} - u_{n+1}\|^2 - 2\alpha_n\psi(\|x_{n+1} - u_{n+1}\|) \\
&\qquad + 2\alpha_n\|Ty_n - Tx_{n+1}\|M + 2\alpha_n\|Tu_{n+1} - Tu_n\|M \\
&\quad = (1-\alpha_n)^2\|x_n - u_n\|^2 + 2\alpha_n\|x_{n+1} - u_{n+1}\|^2 - 2\alpha_n\psi(\|x_{n+1} - u_{n+1}\|) \\
&\qquad + 2\alpha_n(b_n + c_n),
\end{aligned} \tag{2.11}$$

where

$$b_n := ||Ty_n - Tx_{n+1}||M,$$
$$c_n := ||Tu_{n+1} - Tu_n||M. \tag{2.12}$$

From (1.2), we have

$$\begin{aligned} ||x_{n+1} - y_n|| &= ||(\beta_n - \alpha_n)x_n + \alpha_n Ty_n - \beta_n Tx_n|| \\ &\le (\beta_n - \alpha_n)||x_n|| + \alpha_n||Ty_n|| + \beta_n||Tx_n||. \end{aligned} \tag{2.13}$$

Analogously as for (2.6), we obtain the boundedness of $\{x_n\}$. Conditions (2.13) and (1.3) lead to

$$\lim_{n\to\infty} ||x_{n+1} - y_n|| = 0; \tag{2.14}$$

the uniform continuity of T leads to

$$\lim_{n\to\infty} ||Ty_n - Tx_{n+1}|| = 0; \tag{2.15}$$

thus, we have

$$\lim_{n\to\infty} b_n = 0. \tag{2.16}$$

The convergence of the Mann iteration $\{u_n\}$ implies $\lim_{n\to\infty} \|u_{n+1} - u_n\| = 0$. The uniform continuity of T implies $\lim_{n\to\infty} \|Tu_{n+1} - Tu_n\| = 0$, that is,

$$\lim_{n\to\infty} c_n = 0. \tag{2.17}$$

Substituting (2.9) in (2.11) and using (2.7), we get

$$\begin{aligned} &(1-\alpha_n)^2||x_n - u_n||^2 + 2\alpha_n||x_{n+1} - u_{n+1}||^2 \\ &\quad \le (1-\alpha_n)^2||x_n - u_n||^2 + 2\alpha_n\left(||x_n - u_n||^2 + \alpha_n M^2\right) \\ &\quad = \left[(1-\alpha_n)^2 + 2\alpha_n\right]||x_n - u_n||^2 + 2\alpha_n^2 M^2 \\ &\quad = (1+\alpha_n^2)||x_n - u_n||^2 + 2\alpha_n^2 M^2 \\ &\quad = ||x_n - u_n||^2 + \alpha_n^2||x_n - u_n||^2 + 2\alpha_n^2 M^2 \\ &\quad \le ||x_n - u_n||^2 + 3\alpha_n^2 M^2. \end{aligned} \tag{2.18}$$

Substituting (2.18) into (2.11), we obtain

$$\begin{aligned} &||x_{n+1} - u_{n+1}||^2 \\ &\quad \le (1-\alpha_n)^2||x_n - u_n||^2 + 2\alpha_n||x_{n+1} - u_{n+1}||^2 \\ &\qquad - 2\alpha_n\psi(||x_{n+1} - u_{n+1}||) + 2\alpha_n(b_n + c_n) \\ &\quad \le ||x_n - u_n||^2 + 3\alpha_n^2 M^2 - 2\alpha_n\psi(||x_{n+1} - u_{n+1}||) + 2\alpha_n(b_n + c_n) \\ &\quad = ||x_n - u_n||^2 - 2\alpha_n\psi(||x_{n+1} - u_{n+1}||) + \alpha_n(3\alpha_n M^2 + 2b_n + 2c_n). \end{aligned} \tag{2.19}$$

Denote

$$\begin{aligned}\theta_n &:= \|x_n - u_n\|^2,\\ \lambda_n &:= 2\alpha_n,\\ \sigma_n &:= \alpha_n(3\alpha_n M^2 + 2b_n + 2c_n).\end{aligned} \tag{2.20}$$

Condition (1.3) assures the existence of a positive integer n_0 such that $\lambda_n = 2\alpha_n \leq 1$, for all $n \geq n_0$. Relations (1.3), (2.16), (2.17), (2.19), (2.20), and Lemma 1.5 lead to $\lim_{n\to\infty}\theta_n = 0$; hence $\lim_{n\to\infty}\|x_n - u_n\| = 0$. □

The above result does not completely generalize the main result, stated below, from [8], because the map T in this result is not uniformly continuous.

Theorem 2.2 [8]. *Let X be a real Banach space with a uniformly convex dual and B a nonempty, closed, convex, bounded subset of X. Let $T : B \to B$ be a continuous and strongly pseudocontractive operator. Then for $u_1 = x_1 \in B$, the following assertions are equivalent:*

(i) *the Mann iteration (1.1) converges to the fixed point of T;*
(ii) *the Ishikawa iteration (1.2) converges to the fixed point of T.*

Remark 2.3 [8]. (i) If T has a fixed point, then Theorem 2.2 holds without the continuity of T.

(ii) If B is not bounded, then Theorem 2.2 holds if $\{x_n\}$ is bounded.

3. The Lipschitzian case.

The following result can be found in [6].

Corollary 3.1 [6]. *Let X be a real Banach space, B a nonempty, convex subset of X, and $T : B \to B$ a Lipschitzian and ψ-uniformly pseudocontractive map with $T(B)$ bounded. If $\{\alpha_n\}$, $\{\beta_n\}$ satisfy (1.3), then the following are equivalent:*

(i) *the Mann iteration (1.1) converges (to $x^* \in F(T)$),*
(ii) *the Ishikawa iteration (1.2) converges (to the same $x^* \in F(T)$).*

Proof. If the Lipschitzian constant $L \in (0,1)$, then the conclusion holds on basis of [9, Theorem 3]. If $L \geq 1$, then all the assumptions in Theorem 2.1 are satisfied because a Lipschitzian map is uniformly continuous. □

Corollary 3.1 does not completely generalize the main result, stated below, from [9], because neither boundedness of B nor that of $T(B)$ is required.

Theorem 3.2 [9]. *Let B be a closed, convex subset of an arbitrary Banach space X and let T be a Lipschitzian strongly pseudocontractive self-map of B. Consider the Mann iteration and the Ishikawa iteration with the same initial point and $\{\alpha_n\}$, $\{\beta_n\}$ satisfying (1.3). Then the following conditions are equivalent:*

(i) *the Mann iteration (1.1) converges to $x^* \in F(T)$,*
(ii) *the Ishikawa iteration (1.2) converges to $x^* \in F(T)$.*

4. Application. Let S be a ψ-uniformly accretive map. Suppose the equation $Sx = f$ has a solution for a given $f \in X$. Remark 1.3 assures that

$$Tx = x + f - Sx, \quad \forall x \in X, \tag{4.1}$$

is a ψ-uniformly pseudocontractive map. A fixed point for T is a solution of $Sx = f$, and conversely. For the same $\{\alpha_n\} \subset (0,1)$, $\{\beta_n\} \subset [0,1)$ as in (1.3), the iterations (1.2) and (1.1) become

$$\begin{aligned} x_{n+1} &= (1-\alpha_n)x_n + \alpha_n\big(f + (I-S)y_n\big), \\ y_n &= (1-\beta_n)x_n + \beta_n\big(f + (I-S)x_n\big), \quad n = 0,1,2,\dots, \end{aligned} \tag{4.2}$$

$$u_{n+1} = (1-\alpha_n)u_n + \alpha_n\big(f + (I-S)u_n\big), \quad n = 0,1,2,\dots. \tag{4.3}$$

We are now able to give the following result.

Corollary 4.1. *Let X be a real Banach space and $S : X \to X$ a uniformly continuous and ψ-uniformly accretive map with $(I-S)(X)$ bounded. If $\{\alpha_n\}$, $\{\beta_n\}$ satisfy (1.3) and $u_0 = x_0 \in B$, then the following are equivalent:*
(i) *the Mann iteration (4.3) converges to a solution of $Sx = f$,*
(ii) *the Ishikawa iteration (4.2) converges to a solution of $Sx = f$.*

Proof. Set $Tx := f + (I-S)x$. If S is uniformly continuous, then T is also uniformly continuous. The boundedness of $(I-S)(X)$ assures the boundedness of $\{\|y_n + f - Sy_n\|\}$ and $\{\|x_n + f - Sx_n\|\}$. Hence Theorem 2.1 gives our conclusion. □

From Corollary 3.1, we obtain, (see [6]) the following result.

Corollary 4.2 [6]. *Let X be a real Banach space and $S : X \to X$ a Lipschitzian and ψ-uniformly accretive map with $(I-S)(X)$ bounded. If $\{\alpha_n\}$, $\{\beta_n\}$ satisfy (1.3), then the following are equivalent:*
(i) *the Mann iteration (4.3) converges to a solution of $Sx = f$,*
(ii) *the Ishikawa iteration (4.2) converges to a solution of $Sx = f$.*

Proof. Set, in Corollary 3.1, $Tx := (I-S)x$ and use Remark 1.3. □

5. The equivalence between T-stabilities of Mann and Ishikawa iterations. All the arguments for the equivalence between T-stabilities of Mann and Ishikawa iterations are similar to those from [5]. The following nonnegative sequences are well defined for all $n \in \mathbb{N}$:

$$\varepsilon_n := \|x_{n+1} - (1-\alpha_n)x_n - \alpha_n T y_n\|, \tag{5.1}$$

$$\delta_n := \|u_{n+1} - (1-\alpha_n)u_n - \alpha_n T u_n\|. \tag{5.2}$$

Definition 5.1. If $\lim_{n\to\infty} \varepsilon_n = 0$ (resp., $\lim_{n\to\infty} \delta_n = 0$) implies that $\lim_{n\to\infty} x_n = x^*$ (resp., $\lim_{n\to\infty} u_n = x^*$), then (1.2) (resp., (1.1)) is said to be T-stable.

Remark 5.2 [5]. Let X be a normed space, B a nonempty, convex, closed subset of X, and $T : B \to B$ a continuous map. If the Mann (resp., Ishikawa) iteration converges, then $\lim_{n\to\infty} \delta_n = 0$ (resp., $\lim_{n\to\infty} \varepsilon_n = 0$).

Theorem 5.3. *Let X be a real Banach space, B a nonempty, convex subset of X, and $T : B \to B$ a uniformly continuous and ψ-uniformly pseudocontractive map with $T(B)$ bounded. If $\{\alpha_n\}$, $\{\beta_n\}$ satisfy (1.3) and $u_0 = x_0 \in B$, then the following are equivalent:*
(i) *the Mann iteration (1.1) is T-stable,*
(ii) *the Ishikawa iteration (1.2) is T-stable.*

Proof. The equivalence (i)⇔(ii) means that $\lim_{n\to\infty} \varepsilon_n = 0 \Leftrightarrow \lim_{n\to\infty} \delta_n = 0$. The implication $\lim_{n\to\infty} \varepsilon_n = 0 \Rightarrow \lim_{n\to\infty} \delta_n = 0$ is obvious by setting $\beta_n = 0$, for all $n \in \mathbb{N}$, in (1.2) and using (5.2). Conversely, we suppose that (1.1) is T-stable. Using Definition 5.1, we get

$$\lim_{n\to\infty} \delta_n = 0 \Longrightarrow \lim_{n\to\infty} u_n = x^*. \tag{5.3}$$

Theorem 2.1 assures that $\lim_{n\to\infty} u_n = x^*$ leads us to $\lim_{n\to\infty} x_n = x^*$. Using Remark 5.2, we have $\lim_{n\to\infty} \varepsilon_n = 0$. Thus, we get $\lim_{n\to\infty} \delta_n = 0 \Rightarrow \lim_{n\to\infty} \varepsilon_n = 0$. □

Set $Tx = f + (I - S)x$ in Theorem 5.3. Corollary 3.1 leads to the following result.

Corollary 5.4. *Let X be a real Banach space and $S : X \to X$ a uniformly continuous and ψ-uniformly accretive map with $(I - S)(X)$ bounded. If $\{\alpha_n\}$, $\{\beta_n\}$ satisfy (1.3) and $u_0 = x_0 \in B$, then the following are equivalent:*
(i) *the Mann iteration (4.3) is T-stable,*
(ii) *the Ishikawa iteration (4.2) is T-stable.*

Analogously, we obtain the following corollary.

Corollary 5.5 [5]. *Let X be a real Banach space and $S : X \to X$ a Lipschitzian and ψ-uniformly accretive map with $(I - S)(X)$ bounded. If $\{\alpha_n\}$, $\{\beta_n\}$ satisfy (1.3), then the following are equivalent:*
(i) *the Mann iteration (4.3) is T-stable,*
(ii) *the Ishikawa iteration (4.2) is T-stable.*

If the map T is multivalued, then the definition of a ψ-uniformly pseudocontractive map has the following form.

Definition 5.6. Let X be a real Banach space. Let B be a nonempty subset. A map $T : B \to 2^B$ is called ψ-uniformly pseudocontractive if there exist $\psi \in \Psi$ and $j(x - y) \in J(x - y)$ such that

$$\langle \xi - \theta, j(x - y)\rangle \le \|x - y\|^2 - \psi(\|x - y\|), \tag{5.4}$$

for all $x, y \in B$, $\xi \in Tx$, $\theta \in Ty$.

Let $S : X \to 2^X$. The map S is called ψ-uniformly accretive if there exist $\psi \in \Psi$ and $j(x - y) \in J(x - y)$ such that

$$\langle \xi - \theta, j(x - y)\rangle \ge \psi(\|x - y\|), \tag{5.5}$$

for all $x, y \in X$, $\xi \in Sx$, $\theta \in Sy$.

We remark that all the results from this paper hold in the multivalued case, provided that these multivalued maps admit an appropriate selection.

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B. E. Rhoades: Department of Mathematics, Indiana University, Bloomington, IN 47405-7106, USA

E-mail address: rhoades@indiana.edu

Ştefan M. Şoltuz: "Tiberiu Popoviciu" Institute of Numerical Analysis, P.O. Box 68-1, 400110 Cluj-Napoca, Romania

E-mail address: soltuzul@yahoo.com