ON WEIGHTED INEQUALITIES FOR CERTAIN FRACTIONAL INTEGRAL OPERATORS

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This paper considers the modified fractional integral operators involving the Gauss hypergeometric function and obtains weighted inequalities for these operators. Multidimensional fractional integral operators involving the H-function are also introduced.

1. Introduction and preliminaries

Tuan and Saigo [7] introduced the multidimensional modified fractional integrals of order α ($\mathrm{Re}(\alpha) > 0$) by

$$\begin{aligned} X^{\alpha}_{+;n} f(x) &= \frac{1}{\Gamma(\alpha+1)} \mathbf{D}^{\mathbf{n}} \int_{\mathbb{R}^{\mathbf{n}}_{+}} \left[\min\left\{\frac{x_1}{t_1},\ldots,\frac{x_n}{t_n}\right\} - 1\right]^{\alpha}_{+} f(t)dt, \\ X^{\alpha}_{-;n} f(x) &= \frac{(-1)^n}{\Gamma(\alpha+1)} \mathbf{D}^{\mathbf{n}} \int_{\mathbb{R}^{\mathbf{n}}_{+}} \left[1 - \max\left\{\frac{x_1}{t_1},\ldots,\frac{x_n}{t_n}\right\}\right]^{\alpha}_{+} f(t)dt, \end{aligned} \tag{1.1}$$

where $\mathbb{R}^{\mathbf{n}}_{+} = \{(t_1,\ldots,t_n) \mid t_i > 0\ (i = 1,\ldots,n)\}$, $\varphi_{+}(x)$ is a real-valued function defined in terms of the function $\varphi(x)$ by

$$\varphi_{+}(x) = \begin{cases} \varphi(x), & \varphi(x) > 0, \\ 0, & \varphi(x) \le 0, \end{cases} \tag{1.2}$$

and $\mathbf{D}^{\mathbf{n}}$ denotes the derivative operator $\partial^n/\partial x_1,\ldots,\partial x_n$.

The operators in (1.1) provide multidimensional generalizations to the well-known one-dimensional Riemann-Liouville and Weyl fractional integral operators defined in [5] (see also [1]). The paper [7] considers several formulas and interesting properties of (1.1). By invoking the Gauss hypergeometric function ${}_2F_1(\alpha,\beta;\gamma;x)$, the following generalizations of the multidimensional modified integral operators (1.1) of order α ($\mathrm{Re}(\alpha) > 0$)

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were studied in [6]:

$$S_{+;n}^{\alpha,\beta,\gamma}f(x)=\frac{1}{\Gamma(\alpha+1)}\mathbf{D}^{\mathbf{n}}\int_{\mathbb{R}_+^{\mathbf{n}}}\left[\min\left\{\frac{x_1}{t_1},\dots,\frac{x_n}{t_n}\right\}-1\right]_+^{\alpha} \cdot {}_2F_1\left(\alpha+\beta,\alpha+\eta;1+\alpha;1-\min\left\{\frac{x_1}{t_1},\dots,\frac{x_n}{t_n}\right\}\right)f(t)dt, \tag{1.3}$$

$$S_{-;n}^{\alpha,\beta,\gamma}f(x)=\frac{(-1)^n}{\Gamma(\alpha+1)}\mathbf{D}^{\mathbf{n}}\int_{\mathbb{R}_+^{\mathbf{n}}}\left[1-\max\left\{\frac{x_1}{t_1},\dots,\frac{x_n}{t_n}\right\}\right]_+^{\alpha} \cdot {}_2F_1\left(\alpha+\beta,-\eta;1+\alpha;1-\max\left\{\frac{x_1}{t_1},\dots,\frac{x_n}{t_n}\right\}\right)f(t)dt. \tag{1.4}$$

For $\beta=-\alpha$, the operators (1.3) and (1.4) reduce to the modified integral operators defined in (1.1), respectively. In [8], the integral operators $X_{+;n}^{\alpha}f(x)$ and $X_{-;n}^{\alpha}f(x)$ defined on the space $\mathcal{M}_\gamma(\mathbb{R}_+^{\mathbf{n}})$ are shown to satisfy some L_p-L_q weighted inequalities. The space $\mathcal{M}_\gamma(\mathbb{R}_+^{\mathbf{n}})$ represents the space of functions f which are defined on $\mathbb{R}_+^{\mathbf{n}}$, and are entire functions of exponential type (see [7]). The present paper is devoted to finding inequalities for the generalized multidimensional modified integral operators (1.3) and (1.4) by making use of the inequality stated in [8] (which was established with the aid of Pitt's inequality). Multidimensional operators have also been studied in [3, 4].

2. Inequalities for operators (1.3) and (1.4)

If $(\mathcal{K}f)(x)$ denotes the integral operator

$$(\mathcal{K}f)(x)=\int_{\mathbb{R}_+^{\mathbf{n}}}k(xy)f(y)dy, \tag{2.1}$$

then following [8], we have

$$\int_{\mathbb{R}_+^{\mathbf{n}}}k(xy)f(y)dy=\frac{1}{(2\pi i)^n}\int_{(1/2)}k^*(s)f^*(1-s)x^{-s}ds, \tag{2.2}$$

where the integral over (1/2) stands for the multiple integral

$$\int_{(1/2)}=\int_{1/2-i\infty}^{1/2+i\infty}\cdots\int_{1/2-i\infty}^{1/2+i\infty}, \tag{2.3}$$

and $k^*(s)$ and $f^*(1-s)$ are the Mellin transforms of the functions $k(y)$ and $f(y)$, respectively. It is proved in [8] that if

$$|k^*(s)|\leq C|s|^{-\alpha}\quad\left(s\in\left(\frac{1}{2}\right),\ \alpha\geq 0\right), \tag{2.4}$$

then there holds the inequality

$$\|\,|\log y-t|^{-b}y^{1/2-1/r}(\mathcal{K}f)(y)\|_{L_r(\mathbb{R}_+^{\mathbf{n}})}\leq C\|\,|\log y-t|^{d}y^{1/2-1/q}f(y)\|_{L_q(\mathbb{R}_+^{\mathbf{n}})}, \tag{2.5}$$

or equivalently,

$$\| |\log y - t|^{-b} y^{1/2-1/r} (\mathcal{K} f)(y) \|_{L_r(\mathbb{R}_+^{\mathbf{n}})} \le C \| |\log y - t|^{a-b+n(1/r-1/q)} y^{1/2-1/q} f(y) \|_{L_q(\mathbb{R}_+^{\mathbf{n}})}, \tag{2.6}$$

for all $t \in \mathbb{R}_+^{\mathbf{n}}$, provided that

$$\left[\begin{array}{c} \max\left\{\frac{2}{r}, \frac{\alpha}{n} + \frac{1}{r}\right\} - 1 \le \frac{b}{n} \le \min\left\{0, \frac{\alpha}{n} - \frac{1}{q}, \frac{\alpha}{n} + \frac{1}{q} - 1\right\} + \frac{1}{r}, \\ \max\left\{0, n\left(\frac{1}{r} + \frac{1}{q} - 1\right)\right\} \le \alpha \le n,\ 1 < q \le r < \infty. \end{array} \right]. \tag{2.7}$$

In the paper [6], it was established that if

$$\operatorname{Re}(\alpha) > 0, \qquad \operatorname{Re}(h_j) < \frac{1}{2} \quad (j = 1, \dots, n), \qquad \sum_{j=1}^{n} \operatorname{Re}(h_j) < \frac{n}{2} + \min\{\operatorname{Re}(\beta), \operatorname{Re}(\eta)\}, \tag{2.8}$$

then the operator $x^h S_{+;n}^{\alpha,\beta,\gamma} x^{-h} f(x)$ is a homeomorphism of the space $\mathcal{M}_{1/2}(\mathbb{R}_+^{\mathbf{n}})$ onto itself, and

$$x^h S_{+;n}^{\alpha,\beta,\gamma} x^{-h} f(x) = \frac{1}{(2\pi i)^n} \int_{(1/2)} \frac{\Gamma(\beta + n - \sum_{j=1}^n h_j - \sum_{j=1}^n s_j) \Gamma(\eta + n - \sum_{j=1}^n h_j - \sum_{j=1}^n s_j)}{\Gamma(n - \sum_{j=1}^n h_j - \sum_{j=1}^n s_j) \Gamma(\alpha + \beta + \eta + n - \sum_{j=1}^n h_j - \sum_{j=1}^n s_j)} f^*(s) x^{-s} ds. \tag{2.9}$$

We note that

$$\frac{\Gamma(\beta + n - \sum_{j=1}^n h_j - \sum_{j=1}^n s_j) \Gamma(\eta + n - \sum_{j=1}^n h_j - \sum_{j=1}^n s_j)}{\Gamma(n - \sum_{j=1}^n h_j - \sum_{j=1}^n s_j) \Gamma(\alpha + \beta + \eta + n - \sum_{j=1}^n h_j - \sum_{j=1}^n s_j)} = O(|s|^{-\alpha}), \tag{2.10}$$

so we can apply the inequality (2.6) to the multidimensional operator defined by (2.9), which leads to

$$\begin{aligned} &\| |\log y - t|^{-b} y^{1/2-1/r+h} S_{+;n}^{\alpha,\beta,\gamma} y^{-h} f(y) \|_{L_r(\mathbb{R}_+^{\mathbf{n}})} \\ &\quad \le C \| |\log y - t|^{a-b+n(1/r-1/q)} y^{1/2-1/q} f(y) \|_{L_q(\mathbb{R}_+^{\mathbf{n}})}, \end{aligned} \tag{2.11}$$

valid for all $t \in \mathbb{R}_+^{\mathbf{n}}$, provided that the constraints (2.7) and (2.8) are satisfied. On the other hand, (see [6]) if

$$\operatorname{Re}(\alpha) > 0, \qquad \operatorname{Re}(h_j) > \frac{1}{2} \quad (j = 1, \dots, n), \qquad \sum_{j=1}^{n} \operatorname{Re}(h_j) > \frac{n}{2} + \operatorname{Re}(\beta - \eta) - 1, \tag{2.12}$$

then the operator $x^h S^{\alpha,\beta,\gamma}_{-;n} x^{-h} f(x)$ is a homeomorphism of the space $\mathcal{M}_{1/2}(\mathbb{R}^{\mathbf{n}}_+)$ onto itself, and we obtain

$$\begin{aligned} x^h S^{\alpha,\beta,\gamma}_{-;n} x^{-h} f(x) &= \frac{1}{(2\pi i)^n} \int_{(1/2)} \frac{\Gamma(1-n+\sum_{j=1}^n h_j + \sum_{j=1}^n s_j)\Gamma(1-\beta+\eta-n+\sum_{j=1}^n h_j + \sum_{j=1}^n s_j)}{\Gamma(1-\beta-n+\sum_{j=1}^n h_j + \sum_{j=1}^n s_j)\Gamma(1+\alpha+\eta-n+\sum_{j=1}^n h_j + \sum_{j=1}^n s_j)} \\ &\quad \times f^*(s) x^{-s} ds. \end{aligned} \tag{2.13}$$

By noting the estimate that

$$\frac{\Gamma(1-n+\sum_{j=1}^n h_j + \sum_{j=1}^n s_j)\Gamma(1-\beta+\eta-n+\sum_{j=1}^n h_j + \sum_{j=1}^n s_j)}{\Gamma(1-\beta-n+\sum_{j=1}^n h_j + \sum_{j=1}^n s_j)\Gamma(1+\alpha+\eta-n+\sum_{j=1}^n h_j + \sum_{j=1}^n s_j)} = O(|s|^{-\alpha}), \tag{2.14}$$

we again apply the inequality (2.6) to the multidimensional operator defined by (2.13) to get

$$\begin{aligned} &\| |\log y - t|^{-b} y^{1/2-1/r+h} S^{\alpha,\beta,\gamma}_{-;n} y^{-h} f(y) \|_{L_r(\mathbb{R}^{\mathbf{n}}_+)} \\ &\quad \le C \| t |\log y - t|^{a-b+n(1/r-1/q)} y^{1/2-1/q} f(y) \|_{L_q(\mathbb{R}^{\mathbf{n}}_+)}, \end{aligned} \tag{2.15}$$

valid for all $t \in \mathbb{R}^{\mathbf{n}}_+$, provided that the constraints (2.7) and (2.12) are satisfied.

3. Classes of multidimensional operators

We introduce the following classes of multidimensional modified fractional integral operators involving the well-known H-function [2, Section 8.3] (see also [1, page 343]) defined by

$$\begin{aligned} \left(\mathbf{H}^{\mathbf{M,N}}_{\mathbf{P,Q},+;\mathbf{n}} \left| {}^{(\mathbf{a_P},\alpha_\mathbf{P})}_{(\mathbf{b_Q},\beta_\mathbf{Q})} \right. f\right)(x) &= \left(\mathbf{H}^{\mathbf{M,N}}_{\mathbf{P,Q},+;\mathbf{n}} \left| {}^{(\mathbf{a}_1,\alpha_1),\dots,(\mathbf{a_P},\alpha_\mathbf{P})}_{(\mathbf{b}_1,\beta_1),\dots,(\mathbf{b_Q},\beta_\mathbf{Q})} \right. \mathbf{f}\right)(x) \\ &= \mathbf{D}^{\mathbf{n}} \int_{\mathbb{R}^{\mathbf{n}}_+} H^{M,N}_{P,Q} \left[\min\left\{ \frac{x_1}{t_1}, \dots, \frac{x_n}{t_n} \right\} \left| {}^{(a_P,\alpha_P)}_{(b_Q,\beta_Q)} \right. \right] f(t) dt, \end{aligned} \tag{3.1}$$

$$\begin{aligned} \left(\mathbf{H}^{\mathbf{M,N}}_{\mathbf{P,Q},-;\mathbf{n}} \left| {}^{(\mathbf{a_P},\alpha_\mathbf{P})}_{(\mathbf{b_Q},\beta_\mathbf{Q})} \right. \mathbf{f}\right)(x) &= \left(\mathbf{H}^{\mathbf{M,N}}_{\mathbf{P,Q},-;\mathbf{n}} \left| {}^{(a_1,\alpha_1),\dots,(\mathbf{a_P},\alpha_\mathbf{P})}_{(\mathbf{b}_1,\beta_1),\dots,(\mathbf{b_Q},\beta_\mathbf{Q})} \right. \mathbf{f}\right)(x) \\ &= (-1)^n \mathbf{D}^{\mathbf{n}} \int_{\mathbb{R}^{\mathbf{n}}_+} H^{M,N}_{P,Q} \left[\max\left\{ \frac{x_1}{t_1}, \dots, \frac{x_n}{t_n} \right\} \left| {}^{(a_P,\alpha_P)}_{(b_Q,\beta_Q)} \right. \right] f(t) dt, \end{aligned} \tag{3.2}$$

where we assume that the parameters of the H-function involved in (3.1) and (3.2) satisfy the existence conditions as given in [2].

The special cases of the operators of interest in this paper are the operators which emerge from (3.1) and (3.2) in the case when $N = 0$, $P = M$, $Q = M$, and the parameters $\alpha_1 = \alpha_2 = \cdots = \alpha_m = 1$, and $\beta_1 = \beta_2 = \cdots = \beta_m = 1$. Thus, we have the following multidimensional fractional integral operators (defined in terms of Meijer's G-function) (see [6]):

$$\begin{aligned}\left(\mathbf{G}_{+;\mathbf{n}}^{(\mathbf{a_m});(\mathbf{b_m})}\mathbf{f}\right)(x) &= \left(\mathbf{G}_{+;\mathbf{n}}^{(\mathbf{a}_1,\dots,\mathbf{a_m});(\mathbf{b}_1,\dots,\mathbf{b_m})}\mathbf{f}\right)(x)\\ &= \mathbf{D}^{\mathbf{n}}\int_{\mathbb{R}_+^{\mathbf{n}}} G_{m,m}^{m,0}\left[\min\left\{\frac{x_1}{t_1},\dots,\frac{x_n}{t_n}\right\}\Big|\begin{matrix}(a_m)\\(b_m)\end{matrix}\right]f(t)dt,\\ \left(\mathbf{G}_{-;\mathbf{n}}^{(\mathbf{a_m});(\mathbf{b_m})}\mathbf{f}\right)(x) &= \left(\mathbf{G}_{+;\mathbf{n}}^{(\mathbf{a}_1,\dots,\mathbf{a_m});(\mathbf{b}_1,\dots,\mathbf{b_m})}\mathbf{f}\right)(x)\\ &= (-1)^n\mathbf{D}^{\mathbf{n}}\int_{\mathbb{R}_+^{\mathbf{n}}} G_{m,m}^{m,0}\left[\max\left\{\frac{x_1}{t_1},\dots,\frac{x_n}{t_n}\right\}\Big|\begin{matrix}(a_m)\\(b_m)\end{matrix}\right]f(t)dt.\end{aligned} \tag{3.3}$$

By setting the parameters

$$m=2,\qquad a_1=1-\beta,\qquad a_2=1-\eta,\qquad b_1=1-\alpha-\beta-\eta,\qquad b_2=0, \tag{3.4}$$

in (3.1), and

$$m=2,\qquad a_1=1-\beta,\qquad a_2=1+\alpha+\eta,\qquad b_1=1-\beta+\eta,\qquad b_2=0, \tag{3.5}$$

in (3.2), and noting the relation (see [1, equation (1.1.18), page 18])

$$\begin{aligned}G_{2,2}^{2,0}\left[\sigma\,\Big|\begin{matrix}a_1,a_2\\b_1,b_2\end{matrix}\right] &= \frac{\sigma^{b_2}(1-\sigma)^{a_1+a_2-b_1-b_2-1}}{\Gamma(a_1+a_2-b_1-b_2)}\\ &\quad\cdot {}_2F_1(a_2-b_1,a_1-b_1;a_1+a_2-b_1-b_2;1-\sigma)\quad(\sigma<1),\end{aligned} \tag{3.6}$$

we observe the following relationships:

$$\begin{aligned}\left(\mathbf{G}_{+;\mathbf{n}}^{(\mathbf{1}-\beta,\mathbf{1}-\eta);(\mathbf{1}-\alpha-\beta-\eta,\mathbf{0})}\mathbf{f}\right)(x) &= (-1)^{\alpha}S_{+;n}^{\alpha,\beta,\gamma}f(x),\\ \left(\mathbf{G}_{-;\mathbf{n}}^{(\mathbf{1}-\beta,\mathbf{1}+\alpha+\eta);(\mathbf{1}-\beta+\eta,\mathbf{0})}\mathbf{f}\right)(x) &= S_{-;n}^{\alpha,\beta,\gamma}f(x),\end{aligned} \tag{3.7}$$

in terms of the multidimensional modified fractional integral operators (1.3) and (1.4).

We state below two useful lemmas concerning the multidimensional Mellin transform of the functions $f(\max[x_1,\dots,x_n])$ and $f(\min[x_1,\dots,x_n])$ (see [3, 6]).

Lemma 3.1. *Let* $\operatorname{Re}(s_j)>0$ $(j=1,\dots,n)$ *and let* $\tau^{s\cdot 1-1}f(\tau)\in L_1(\mathbb{R}_+)$, *then*

$$\int_{\mathbb{R}_+^{\mathbf{n}}} x^{s-1}f(\max[x_1,\dots,x_n])dx = \frac{|s|}{s^1}f^*(|s|), \tag{3.8}$$

where s^1 *denotes the product* $s_1,\dots,s_n$, *and* $|s|=s_1+\cdots+s_n$.

Lemma 3.2. *Let* $\operatorname{Re}(s_j)<0$ $(j=1,\dots,n)$ *and let* $\tau^{s\cdot 1-1}f(\tau)\in L_1(\mathbb{R}_+)$, *then*

$$\int_{\mathbb{R}_+^{\mathbf{n}}} x^{s-1}f(\min[x_1,\dots,x_n])dx = (-1)^{n-1}\frac{|s|}{s^1}f^*(|s|). \tag{3.9}$$

Making use of (3.1), we have

$$\begin{aligned}\left(\mathbf{H}^{\mathbf{M},\mathbf{N}}_{\mathbf{P},\mathbf{Q},+;\mathbf{n}}\left|\begin{smallmatrix}(\mathbf{a_P},\alpha_\mathbf{P})\\(\mathbf{b_Q},\beta_\mathbf{Q})\end{smallmatrix}\right.\mathbf{x}^{-\mathbf{s}}\right)(x) &= \mathbf{D}^{\mathbf{n}}\int_{\mathbb{R}^{\mathbf{n}}_+} H^{M,N}_{P,Q}\left[\min\left\{\frac{x_1}{t_1},\dots,\frac{x_n}{t_n}\right\}\left|\begin{smallmatrix}(a_P,\alpha_P)\\(b_Q,\beta_Q)\end{smallmatrix}\right.\right]t^{-s}dt \\ &= \mathbf{D}^{\mathbf{n}}x^{1-s}\int_{\mathbb{R}^{\mathbf{n}}_+} t^{s-2}H^{M,N}_{P,Q}\left[\min\left\{t_1,\dots,t_n\right\}\left|\begin{smallmatrix}(a_P,\alpha_P)\\(b_Q,\beta_Q)\end{smallmatrix}\right.\right]dt.\end{aligned} \tag{3.10}$$

Applying now (3.9) of Lemma 3.2, and the following result giving the Mellin transform of the H-function [1, equation (E.20), page 348], namely,

$$\begin{aligned}\left\{H^{M,N}_{P,Q}\left[x\left|\begin{smallmatrix}(a_P,\alpha_P)\\(b_Q,\beta_Q)\end{smallmatrix}\right.\right]\right\}^*(s) &= \frac{\prod_{j=1}^{M}\Gamma(b_j+\beta_j s)\prod_{i=1}^{N}\Gamma(1-a_i-\alpha_i s)}{\prod_{i=N+1}^{P}\Gamma(a_i+\alpha_i s)\prod_{j=M+1}^{Q}\Gamma(1-b_j-\beta_j s)} \\ &\quad\times\left(-\min_{1\le j\le M}\left[\frac{\operatorname{Re}(b_j)}{\beta_j}\right]<\operatorname{Re}(s)<\min_{1\le i\le N}\left[\frac{1-\operatorname{Re}(a_i)}{\alpha_i}\right]\right),\end{aligned} \tag{3.11}$$

we obtain

$$\begin{aligned}&\left(\mathbf{H}^{\mathbf{M},\mathbf{N}}_{\mathbf{P},\mathbf{Q},+;\mathbf{n}}\left|\begin{smallmatrix}(\mathbf{a_P},\alpha_\mathbf{P})\\(\mathbf{b_Q},\beta_\mathbf{Q})\end{smallmatrix}\right.\mathbf{x}^{-\mathbf{s}}\right)(x) \\ &\quad= \frac{\Gamma(1+n-|s|)\prod_{j=1}^{M}\Gamma(b_j+\beta_j(|s|-n))\prod_{i=1}^{N}\Gamma(1-a_i-\alpha_i(|s|-n))}{\Gamma(n-|s|)\prod_{i=N+1}^{P}\Gamma(a_i+\alpha_i(|s|-n))\prod_{j=M+1}^{Q}\Gamma(1-b_j-\beta_j(|s|-n))}x^{-s}.\end{aligned} \tag{3.12}$$

Similarly, by using the multidimensional operator (3.2), we obtain

$$\begin{aligned}&\left(\mathbf{H}^{\mathbf{M},\mathbf{N}}_{\mathbf{P},\mathbf{Q},-;\mathbf{n}}\left|\begin{smallmatrix}(\mathbf{a_P},\alpha_\mathbf{P})\\(\mathbf{b_Q},\beta_\mathbf{Q})\end{smallmatrix}\right.\mathbf{x}^{-\mathbf{s}}\right)(x) \\ &\quad= \frac{(-1)^n\Gamma(1-n+|s|)\prod_{j=1}^{M}\Gamma(b_j+\beta_j(|s|-n))\prod_{i=1}^{N}\Gamma(1-a_i-\alpha_i(|s|-n))}{\Gamma(-n+|s|)\prod_{i=N+1}^{P}\Gamma(a_i+\alpha_i(|s|-n))\prod_{j=M+1}^{Q}\Gamma(1-b_j-\beta_j(|s|-n))}x^{-s}.\end{aligned} \tag{3.13}$$

The result (3.13) on specializing the parameters in accordance with (3.5) yields the formula [7, equation (3.6), page 148] involving the multidimensional modified integral operator (1.4). Similarly, we can deduce a result from (3.12) which involves the multidimensional modified integral operator (1.3).

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