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Research Article

Unsteady Stagnation-Point Flow of a Viscoelastic Fluid in the Presence of a Magnetic Field

F. Labropulu

Luther College - Mathematics, University of Regina, Regina, SK, Canada S4S 0AZ

Correspondence should be addressed to F. Labropulu, fotini.labropulu@uregina.ca

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The unsteady two-dimensional stagnation point flow of the Walters B′ fluid impinging on an infinite plate in the presence of a transverse magnetic field is examined and solutions are obtained. It is assumed that the infinite plate at $y = 0$ is making harmonic oscillations in its own plane. A finite difference technique is employed and solutions for small and large frequencies of the oscillations are obtained for various values of the Hartmann's number and the Weissenberg number.

1. Introduction

During the past few decades, non-Newtonian viscoelastic fluids have become more and more important industrially. Among these fluids are the fluids of diffferential type such as the Walters B′ fluid [1]. Behaviour of viscoelastic fluids cannot be accurately described by the Newtonian fluid model. The equations of motion of non-Newtonian fluids are highly non-linear and one order higher than the Navier-Stokes equations. For this reason, boundary conditions in addition to the non-slip condition are required to have a well-posed problem. Only in some special cases where the higher order nonlinear terms in these equations can be neglected thereby reducing their order, are the "no-slip" condition sufficient to yield unique solutions. In general, Rajagopal [2], Rajagopal and Gupta [3], Rajagopal [4] and Rajagopal and Kaloni [5] have shown that the absence of this additional boundary condition leads to non-unique solutions for problems involving the flow of fluids of differential type such as Walters' B′ fluid in a bounded domain.

The flow of an incompressible viscous fluid over a moving plate has its importance in many industrial applications. The extrusion of plastic sheets, fabrication of adhesive tapes and application of coating layers onto rigid substrates are some of the examples. If a magnetic field is present, viscous flows due to a moving plate in an electro-magnetic field,

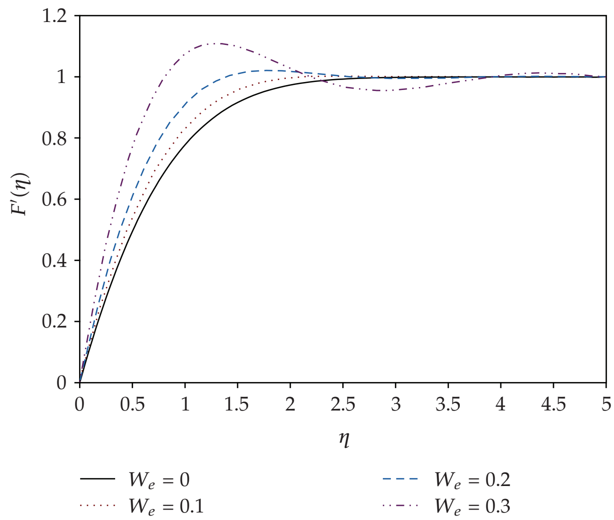

Figure 1: Variation of $F'(\eta)$ for various W_e and $M = 0$.

ie magnetohydrodymanic (MHD) flows, are relevant to many practical applications in the metallurgy industry, such as the cooling of continuous strips and filaments drawn through a quiescent fluid and the purification of molten metals from non-metallic inclusions.

In the history of fluid dynamics, considerable attention has been given to the study of 2-D stagnation point flow. Hiemenz [6] derived an exact solution of the steady flow of a Newtonian fluid impinging orthogonally on an infinite flat plate. Stuart [7], Tamada [8] and Dorrepaal [9] independently investigated the solutions of a stagnation point flow when the fluid impinges obliquely on the plate. Beard and Walters [10] used boundary-layer equations to study two-dimensional flow near a stagnation point of a viscoelastic fluid. Dorrepaal et al. [11] investigated the behaviour of a viscoelastic fluid impinging on a flat rigid wall at an arbitrary angle of incidence. Labropulu et al. [12] studied the oblique flow of a viscoelastic fluid impinging on a porous wall with suction or blowing. The Hiemenz flow of a Newtonian fluid in the presence of a magnetic field was first considered by Na [13] and later by Ariel [14]. The flow of the non-Newtonian Walters' B' fluid in the presence of a transverse magnetic field was studied by Ariel [15]. Attia [16] investigated the steady flow of a non-Newtonian second grade fluid at a stagnation point with heat transfer in an an external uniform magnetic field.

Furthermore, the unsteady or time dependent viscous flow near a stagnation-point has also been widely investigated. Glauert [17] and Rott [18] studied the stagnation-point flow of a Newtonian fluid when the plate performs harmonic oscillations in its own plane. Srivastava [19] investigated the same problem for a non-Newtonian second grade fluid using the Karman-Pohlhausen method. Labropulu et al. [20] used series methods to solve the unsteady stagnation point flow of a viscoelastic fluid impinging on an oscillating flat plate.

In this work, the two-dimensional unsteady stagnation point flow of the Walters B' fluid impinging on an infinite plate in the presence of a magnetic field is examined and solutions are obtained. It is assumed that the infinite plate at $y = 0$ is making harmonic oscillations in its own plane and the magnetic field is transverse or perpendicular everywhere in the flow field. Solutions for small and large frequencies of the oscillations are obtained using a finite difference technique.

2. Flow equations and boundary conditions

We consider the two-dimensional flow of an incompressible non-Newtonian Walters' B′ fluid against an infinite plate normal to the flow in the presence of a magnetic field. The x-axis is along the plate and the y-axis is normal to the plate. We assume that the plate makes harmonic oscillations on its own plane with velocity in the x-direction equal to $ae^{i\omega t}$ where a and ω are constants. The unsteady two-dimensional flow of a viscous incompressible non-Newtonian Walters B′ fluid in the presence of a magnetic field is governed by (see Beard and Walters [10] and Na [13])

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{2.1}$$

$$\begin{aligned}\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+\frac{1}{\rho}\frac{\partial p}{\partial x}&=\nu\nabla^2 u-\frac{\alpha_1}{\rho}\left\{\frac{\partial}{\partial t}(\nabla^2 u)+\left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)\nabla^2 u-\frac{\partial u}{\partial x}\nabla^2 u-\frac{\partial u}{\partial y}\nabla^2 v\right.\\&\quad\left.-2\left[\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x^2}+\frac{\partial v}{\partial y}\frac{\partial^2 u}{\partial y^2}+\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\frac{\partial^2 u}{\partial x\partial y}\right]\right\}-\frac{\sigma B_0}{\rho}u,\end{aligned} \tag{2.2}$$

$$\begin{aligned}\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}++\frac{1}{\rho}\frac{\partial p}{\partial y}&=\nu\nabla^2 v-\frac{\alpha_1}{\rho}\left\{\frac{\partial}{\partial t}(\nabla^2 v)+\left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)\nabla^2 v-\frac{\partial v}{\partial x}\nabla^2 u-\frac{\partial v}{\partial y}\nabla^2 v\right.\\&\quad\left.-2\left[\frac{\partial u}{\partial x}\frac{\partial^2 v}{\partial x^2}+\frac{\partial v}{\partial y}\frac{\partial^2 v}{\partial y^2}+\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\frac{\partial^2 v}{\partial x\partial y}\right]\right\},\end{aligned} \tag{2.3}$$

where $u=u(x,y,t)$, $v=v(x,y,t)$ are the velocity components, $p=p(x,y,t)$ is the pressure, $\nu=\mu/\rho$ is the kinematic viscosity, ρ is the fluid density, σ is the electrical conductivity, B_0 is the magnetic field and α_1 is a measure of the viscoelasticity of the fluid. It is assumed that $\sigma B_0 \ll 1$, so that it is possible to neglect the effect of the induced magnetic field.

For this problem, the boundary conditions are given by

$$\begin{aligned}u&=ae^{i\omega t},\quad v=0\quad\text{at } y=0,\\u&=cx,\quad v=-cy\quad\text{as } y\longrightarrow\infty,\end{aligned} \tag{2.4}$$

where $c>0$ is a constant which has the units of inverse time. The quantity $U=cx$ is the velocity of the fluid outside the boundary layer.

Continuity (2.1) implies the existence of a streamfunction $\psi(x,y,t)$ such that

$$u=\frac{\partial\psi}{\partial y},\qquad v=-\frac{\partial\psi}{\partial x}. \tag{2.5}$$

Substitution of (2.5) in (2.2) and (2.3) and elimination of pressure from the resulting equations using $p_{xy} = p_{yx}$ yields

$$\frac{\partial}{\partial t}(\nabla^2\psi) + \frac{\alpha_1}{\rho}\frac{\partial}{\partial t}(\nabla^4\psi) - \frac{\partial(\psi,\nabla^2\psi)}{\partial(x,y)} - \frac{\alpha_1}{\rho}\frac{\partial(\psi,\nabla^4\psi)}{\partial(x,y)} - \nu\nabla^4\psi + \frac{\sigma B_0}{\rho}\frac{\partial^2\psi}{\partial y^2} = 0. \tag{2.6}$$

Having obtained a solution of (2.6), the velocity components are given by (2.5).

The boundary conditions in terms of the streamfunction $\psi(x,y,t)$ take the form

$$\begin{aligned} &\frac{\partial\psi}{\partial y} = ae^{i\omega t}, \quad \frac{\partial\psi}{\partial x} = 0 \quad \text{at } y = 0, \\ &\frac{\partial\psi}{\partial y} = cx, \quad \frac{\partial\psi}{\partial x} = cy \quad \text{as } y \longrightarrow \infty. \end{aligned} \tag{2.7}$$

The shear stress component τ_{12} is given by

$$\begin{aligned} \tau_{12} = \mu&\left\{\frac{\partial^2\psi}{\partial y^2} - \frac{\partial^2\psi}{\partial x^2}\right\} \\ &- \alpha_1\left[\frac{\partial\psi}{\partial y}\left(\frac{\partial^3\psi}{\partial x\partial y^2} - \frac{\partial^3\psi}{\partial x^3}\right) - \frac{\partial\psi}{\partial x}\left(\frac{\partial^3\psi}{\partial y^3} - \frac{\partial^3\psi}{\partial x^2\partial y}\right) + 2\frac{\partial^2\psi}{\partial x\partial y}\frac{\partial^2\psi}{\partial y^2} + 2\frac{\partial^2\psi}{\partial x^2}\frac{\partial^2\psi}{\partial x\partial y}\right]. \end{aligned} \tag{2.8}$$

3. Solutions

Following Glauert [17], we seek a solution in the form

$$\psi = cxf(y) + ae^{i\omega t}g(y) \tag{3.1}$$

The boundary conditions take the form

$$\begin{aligned} f(0) = f'(0) = 0, &\qquad g'(0) = 1, \\ f'(\infty) = 1, \qquad f(\infty) = y, &\qquad g'(\infty) = 0. \end{aligned} \tag{3.2}$$

Using (3.1) in (2.6), we obtain

$$\begin{aligned} &\nu f^{(iv)} + c\left(ff''' - f'f''\right) + \frac{\alpha_1 c}{\rho}\left(ff^{(v)} - f'f^{(iv)}\right) - \frac{\sigma B_0}{\rho}f'' = 0, \\ &\nu g^{(iv)} - i\omega g'' + \frac{\alpha_1}{\rho}i\omega g^{(iv)} + c\left(fg''' - f''g'\right) + \frac{\alpha c}{\rho}\left(fg^{(v)} - f^{(iv)}g'\right) - \frac{\sigma B_0}{\rho}g'' = 0. \end{aligned} \tag{3.3}$$

Non-dimensionalizing using

$$\eta = \sqrt{\frac{c}{\nu}}y, \qquad f(y) = \sqrt{\frac{\nu}{c}}F(\eta), \qquad g(y) = \sqrt{\frac{\nu}{c}}G(\eta) \tag{3.4}$$

we get

$$F^{(iv)} + FF''' - F'F'' + W_e\left(FF^{(v)} - F'F^{(iv)}\right) - MF'' = 0,$$
$$G^{(iv)} + FG''' - F''G' + W_e\left(FG^{(v)} - F^{(iv)}G'\right) - \frac{i\omega}{c}G'' - \frac{i\omega W_e}{c}G^{(iv)} - MG'' = 0, \quad (3.5)$$

where $W_e = a_1 c/\mu$, the Weissenberg number, is the ratio of the elastic effects over the viscous effects and $M = \sigma B_0/\rho c$, is the Hartmann's number.

The asymptotic behaviour of $F(y)$ far away from the plate (outside the boundary layer) is given by

$$F(y) \sim y - A \quad \text{as } y \longrightarrow \infty, \quad (3.6)$$

where A is a constant that accounts for the boundary layer displacement.

Using this asymptotic behaviour, we obtain

$$F'(\infty) = 1, \qquad F''(\infty) = F'''(\infty) = F^{(iv)}(\infty) = 0 \quad (3.7)$$

Integrating (3.5) once with respect to η and using the conditions at infinity, we have (see [10])

$$F''' + FF'' - F'^2 + W_e\left(FF^{(iv)} - 2F'F''' + F''^2\right) - MF' = -1 - M,$$
$$F(0) = 0, \qquad F'(0) = 0, \qquad F'(\infty) = 1, \quad (3.8)$$

$$G''' + FG'' - F'G' + W_e\left(FG^{(iv)} - F'G''' + F''G'' - F'''G'\right) - \frac{i\omega}{c}\left(G' + W_e G'''\right) - MG' = 0,$$
$$G'(0) = 1, \qquad G'(\infty) = 0. \quad (3.9)$$

System (3.8) with $M = 0$ has been solved numerically by many authors (Beard and Walter [10], Ariel [21]). Using the shooting method with the finite difference technique described by Ariel [21], we find that $F''(0) = 1.23259$ when $W_e = 0$ and $M = 0$ which is in good agreement with the values obtained by Hiemenz [6] and Glauert [17]. Numerical values of $F''(0)$ for different values of W_e and M are shown in Table 1. These values are in good agreement with the values obtained by Ariel [21] when $M = 0$, Ariel [15] for all M and Labropulu et al. [20] when $M = 0$. These values are also in good agreement with the values obtained by Attia [16] when $K = W_e = 0.0$. Figure 1 shows the profiles of F' for various W_e when $M = 0$. Figure 2 shows the profiles of F' for various M when $W_e = 0$. Figure 3 depicts the profiles of F' for various M when $W_e = 0.2$. We observed that as the elasticity of the fluid and the Hartman's number increase, the velocity near the wall increases.

Letting $\phi(\eta) = G'(\eta)$, then system (3.9) becomes

$$\phi'' + F\phi' - F'\phi + W_e\left(F\phi''' - F'\phi'' + F''\phi' - F'''\phi\right) - \frac{i\omega}{c}\left(\phi + W_e\phi''\right) - M\phi = 0,$$
$$\phi(0) = 1, \qquad \phi(\infty) = 0. \quad (3.10)$$

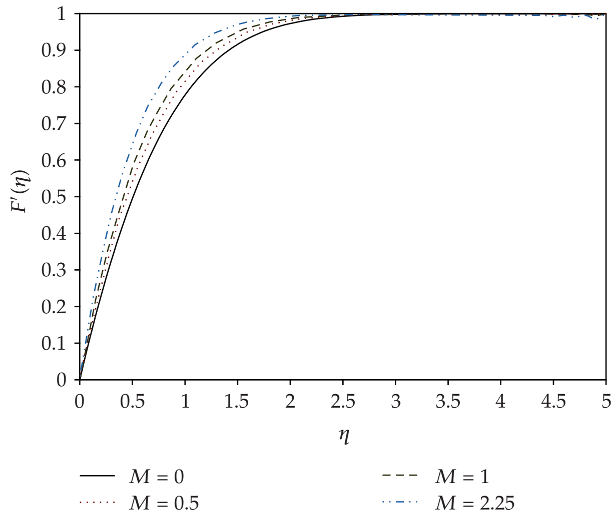

Figure 2: Variation of $F'(\eta)$ for various M and $W_e = 0$.

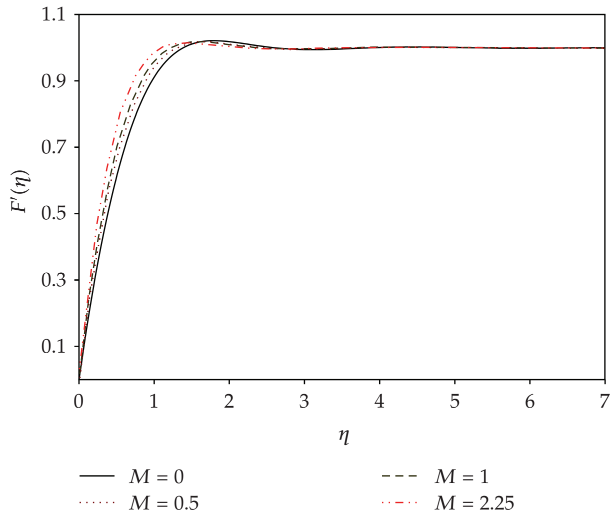

Figure 3: Variation of $F'(\eta)$ for various M and $W_e = 0.2$.

Table 1: Numerical values of $F''(0)$, $\phi_0'(0)$, $\phi_1'(0)$ and $\phi_2'(0)$ for different values of M and $W_e = 0$.

M	$F''(0)$	$\phi_0'(0)$	$\phi_1'(0)$	$\phi_2'(0)$
0.0	1.23259	−0.811318	−0.49307	0.0945276
0.5	1.41976	−1.05648	−0.40999	0.0945488
1.0	1.58533	−1.2615	−0.357149	0.0582149
2.0	1.87353	−1.60113	−0.29228	0.0400542
2.25	1.93895	−1.67603	−0.280805	0.020476

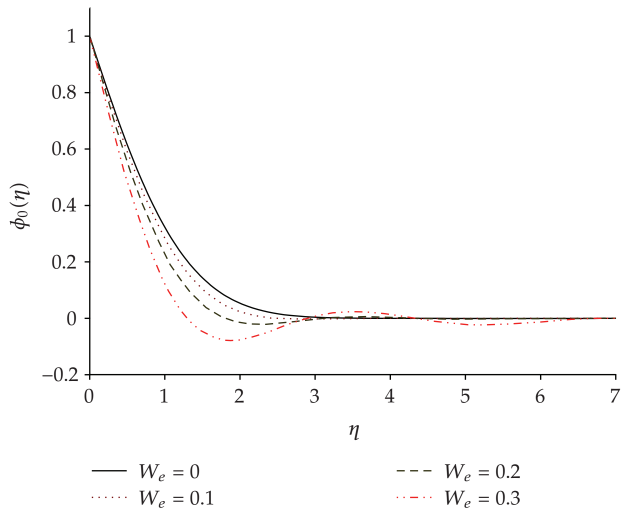

Figure 4: Variation of $\phi_0(\eta)$ for various W_e and $M = 0$.

The only parameter in system (3.10) is the frequency ratio ω/c. Series solutions will be developed, valid for small and large values of ω/c, respectively.

3.1. Small values of ω/c

Consider the case where $\omega = 0$, which implies that the plate velocity has the constant value a. Letting $\phi = \phi_0$, then system (3.10) gives

$$\phi_0'' + F\phi_0' - F'\phi_0 + W_e\left(F\phi_0''' - F'\phi_0'' + F''\phi_0' - F'''\phi_0\right) - M\phi_0 = 0,$$
$$\phi_0(0) = 1, \qquad \phi_0(\infty) = 0. \tag{3.11}$$

This system is solved numerically using a shooting method and it is found that for $W_e = 0$ and $M = 0$, $\phi_0'(0) = -0.811318$ which is in good agreement with the value obtained by Glauert [17]. Numerical values of $\phi_0'(0)$ for different values of W_e and M are shown in Table 1. These values are in good agreement with the values obtained by Labropulu et al. [20] for $M = 0$. Figure 4 depicts the profiles of ϕ_0 for various values of W_e and $M = 0$. Figure 5 shows the profiles of ϕ_0 for various M when $W_e = 0.1$.

For small but non-zero values of ω/c, we let

$$\phi(\eta) = \sum_{n=0}^{\infty}\left(\frac{i\omega}{c}\right)^n \phi_n(\eta) = \phi_0(\eta) + \frac{i\omega}{c}\phi_1(\eta) + \left(\frac{i\omega}{c}\right)^2\phi_2(\eta) + \cdots . \tag{3.12}$$

Substituting (3.12) into (3.10), we get for $n \geq 1$

$$\phi_n'' + F\phi_n' - F'\phi_n + W_e\left(F\phi_n''' - F'\phi_n'' + F''\phi_n' - F'''\phi_n\right) - M\phi_n = \phi_{n-1} + W_e\phi_{n-1}'',$$
$$\phi_n(0) = 0, \qquad \phi_n(\infty) = 0. \tag{3.13}$$

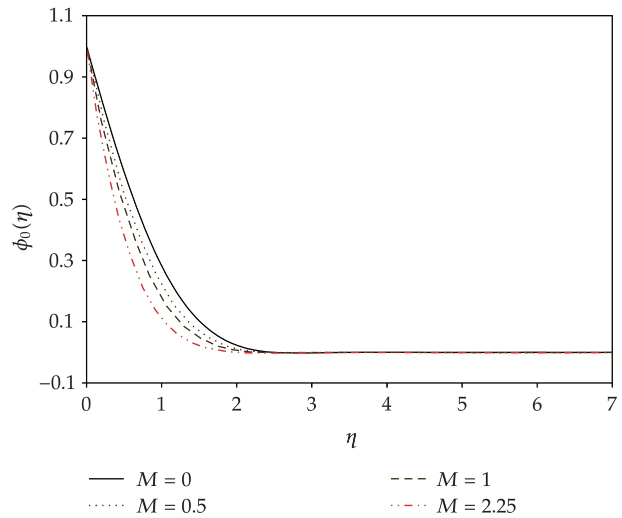

Figure 5: Variation of $\phi_0(\eta)$ for various M and $W_e = 0.1$.

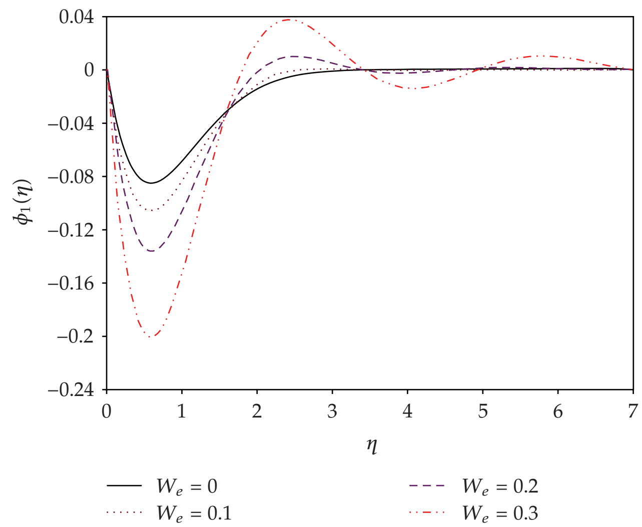

Figure 6: Variation of $\phi_1(\eta)$ for various W_e and $M = 1$.

This system can be solved numerically either by using the perturbation technique or by a finite difference scheme. Numerical integration of system (3.13) for $n = 1$ using a finite difference technique gives for $W_e = 0$ and $M = 0$, $\phi_1'(0) = -0.49307$ which is in good agreement with Glauert's value [17]. Numerical values of $\phi_1'(0)$ for different values of W_e and M are shown in Table 1. These values are in good agreement with Labropulu et al. [20] for $M = 0$. Figure 6 shows the profiles of ϕ_1 for various values of W_e and $M = 1$ and Figure 7 depicts the profiles of ϕ_1 for various values of M and $W_e = 0.2$.

Numerical integration of system (3.13) for $n = 2$ using a finite difference technique gives for $W_e = 0$ and $M = 0$, $\phi_2'(0) = 0.0945488$ which is in good agreement with Glauert's

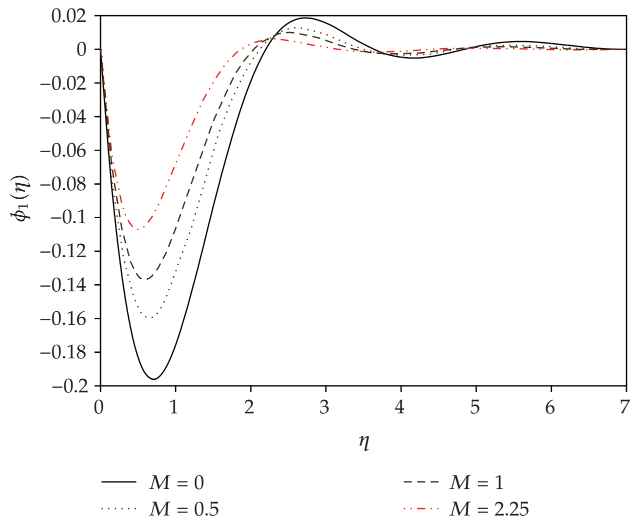

Figure 7: Variation of $\phi_1(\eta)$ for various M and $W_e = 0.2$.

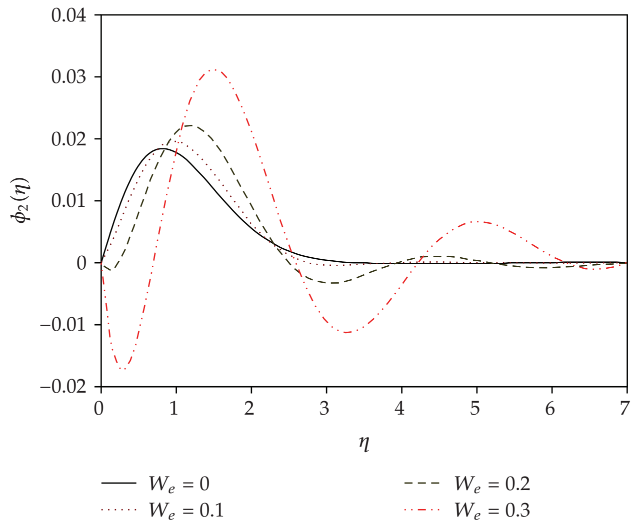

Figure 8: Variation of $\phi_2(\eta)$ for various W_e and $M = 1$.

Table 2: Numerical values of $F''(0)$, $\phi_0'(0)$, $\phi_1'(0)$, and $\phi_2'(0)$ for different values of M and $We = 0.1$.

M	$F''(0)$	$\phi_0'(0)$	$\phi_1'(0)$	$\phi_2'(0)$
0.0	1.36946	−0.86708	−0.547301	0.065851
0.5	1.56814	−1.11269	−0.47879	0.0334295
1.0	1.74431	−1.31954	−0.43719	0.0170392
2.0	2.05118	−1.66405	−0.390321	0.0014389
2.25	2.12082	−1.74024	−0.38277	−0.0008256

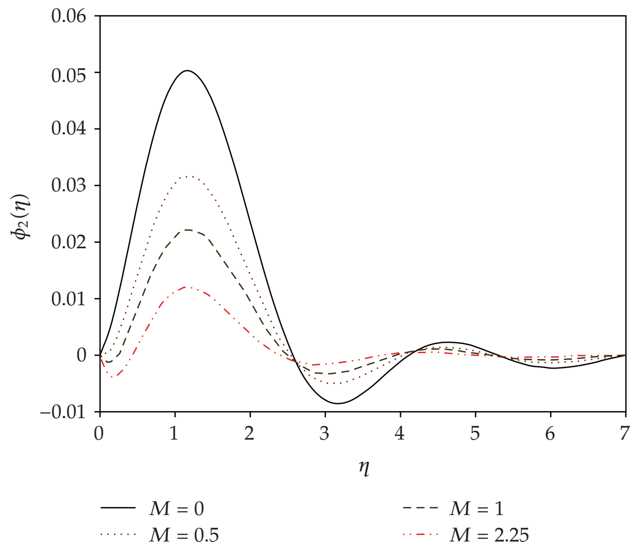

Figure 9: Variation of $\phi_2(\eta)$ for various M and $W_e = 0.2$.

Table 3: Numerical values of $F''(0)$, $\phi_0'(0)$, $\phi_1'(0)$, and $\phi_2'(0)$ for different values of M and $W_e = 0.2$.

M	$F''(0)$	$\phi_0'(0)$	$\phi_1'(0)$	$\phi_2'(0)$
0.0	1.58676	−0.947459	−0.633892	0.0221997
0.5	1.80411	−1.19217	−0.582324	−0.0092379
1.0	1.99811	−1.40046	−0.553854	−0.0265088
2.0	2.33916	−1.75019	−0.528391	−0.044982
2.25	2.41702	−1.82787	−0.525596	−0.0480166

value [17]. Numerical values of $\phi_2'(0)$ for different values of W_e and M are shown in Table 1. These values are in good agreement with Labropulu et al. [20] when $M = 0$. Figure 8 depicts the profiles of ϕ_2 for various values of W_e and $M = 1$ and Figure 9 shows the profiles of ϕ_2 for various values of M and $W_e = 0.2$.

The oscillating component of the shear stress on the wall is given by

$$\frac{\tau_{12}}{\rho a^2} = \sqrt{\frac{c\nu}{a^2}} e^{i\omega t} \left[\phi_0'(0) + \frac{i\omega}{c} \phi_1'(0) - W_e F''(0) \right], \tag{3.14}$$

where $F''(0), \phi_0'(0), \phi_1'(0)$ and $\phi_2'(0)$ are given in Tables 1, 2, 3, and 4 for different values of W_e and M. When $W_e = 0$ and $M = 0$, the value of the shear stress on the wall is in good agreement with the value obtained by Glauert [17].

Table 4: Numerical values of $F''(0)$, $\phi_0'(0)$, $\phi_1'(0)$, and $\phi_2'(0)$ for different values of M and $W_e = 0.3$.

M	$F''(0)$	$\phi_0'(0)$	$\phi_1'(0)$	$\phi_2'(0)$
0.0	2.11092	−1.10873	−0.842848	−0.076115
0.5	2.34531	−1.34347	−0.806961	−0.11492
1.0	2.56139	−1.5492	−0.791711	−0.13911
2.0	2.95028	−1.90173	−0.79062	−0.170032
2.25	3.0402	−1.98085	−0.79368	−0.175938

3.2. Large values of ω/c

When ω/c is large, we let

$$Y = \sqrt{\frac{i\omega}{c}}\eta = \sqrt{\frac{i\omega}{\nu}}y. \tag{3.15}$$

Letting $\sqrt{c/i\omega} = \alpha$, then $d/d\eta = d/\alpha dY$ and (3.10) takes the form

$$\begin{gathered}\frac{1}{\alpha^2}\frac{d^2\phi}{dY^2} + \frac{1}{\alpha}\left[F\frac{d\phi}{dY} - \frac{dF}{dY}\phi\right] + \frac{1}{\alpha^3}W_e\left[F\frac{d^3\phi}{dY^3} - \frac{dF}{dY}\frac{d^2\phi}{dY^2} + \frac{d^2F}{dY^2}\frac{d\phi}{dY} - \frac{d^3F}{dY^3}\phi\right] \\ -\frac{1}{\alpha^2}\phi - \frac{W_e}{\alpha^2}\frac{d^2\phi}{dY^2} - M\phi = 0.\end{gathered} \tag{3.16}$$

Since W_e is small for most fluids which behave as second order fluids (see Markovitz and Coleman [22]), we follow Srivastava [19] and take W_e to be of the order of α^2. Thus, $W_e = m\alpha^2$ and (3.16) becomes

$$(1-m)\frac{d^2\phi}{dY^2} + \alpha\left[F\frac{d\phi}{dY} - \frac{dF}{dY}\phi\right] + m\alpha\left[F\frac{d^3\phi}{dY^3} - \frac{dF}{dY}\frac{d^2\phi}{dY^2} + \frac{d^2F}{dY^2}\frac{d\phi}{dY} - \frac{d^3F}{dY^3}\phi\right] - \phi - M\alpha^2\phi = 0. \tag{3.17}$$

The expansion for $F(Y)$ near the wall $Y = 0$ is

$$\begin{gathered}F(Y) = \frac{1}{2}A\alpha^2Y^2 - \frac{1}{6}(1+M)\alpha^3Y^3 + \frac{1}{24}MA\alpha^4Y^4 - \frac{1}{6}mA^2\alpha^5Y^3 \\ + \frac{1}{120}(A^2 - M^2 - M)\alpha^5Y^5 + \cdots,\end{gathered} \tag{3.18}$$

where $A = F''(0)$.

Since for large values of ω/c the parameter α is small, we let

$$\phi = \sum_{n=0}^{\infty}\alpha^n\phi_n(Y) = \phi_0(Y) + \alpha\phi_1(Y) + \alpha^2\phi_2(Y) + \cdots. \tag{3.19}$$

The boundary conditions are

$$\phi_0(0) = 1, \quad \phi_n(0) = 0 \quad \text{if } n \geq 1, \qquad \phi_n(\infty) = 0 \quad \forall n. \tag{3.20}$$

Substituting (3.20) in (3.17) and equating the coefficients of different powers of α to zero, we find that the boundary-value problem for $\phi_0(Y)$ is

$$(1-m)\frac{d^2\phi_0}{dY^2} - \phi_0 = 0, \qquad \phi_0(0) = 1, \qquad \phi_0(\infty) = 0 \tag{3.21}$$

with solution $\phi_0(Y) = \exp[-(Y/\sqrt{1-m})]$ provided $0 \leq m < 1$.

The second equation gives that ϕ_1 is zero. The next four equations for $\phi_2(Y)$, $\phi_3(Y)$, $\phi_4(Y)$ and $\phi_5(Y)$ are

$$\begin{aligned}
&(1-m)\frac{d^2\phi_2}{dY^2} - \phi_2 = M\phi_0,\\
&(1-m)\frac{d^2\phi_3}{dY^2} - \phi_3 = M\phi_1 - \frac{1}{2}mAY^2\frac{d^3\phi_0}{dY^3} + mAY\frac{d^2\phi_0}{dY^2} + \left(-\frac{1}{2}AY^2 - mA\right)\frac{d\phi_0}{dY} + AY\phi_0,\\
&(1-m)\frac{d^2\phi_4}{dY^2} - \phi_4 = M\phi_2 + \frac{1}{6}m(1+M)Y^3\frac{d^3\phi_0}{dY^3} - \frac{1}{2}m(1+M)Y^2\frac{d^2\phi_0}{dY^2}\\
&\qquad + \left[\frac{1}{6}(1+M)Y^3 + m(1+M)Y\right]\frac{d\phi_0}{dY} - \left[\frac{1}{2}(1+M)Y^2 + m(1+M)\right]\phi_0\\
&\qquad - \frac{1}{2}mAY^2\frac{d^3\phi_1}{dY^3} + mAY\frac{d^2\phi_1}{dY^2} - \left(\frac{1}{2}AY^2 - Am\right)\frac{d\phi_1}{dY} + AY\phi_1,\\
&(1-m)\frac{d^2\phi_5}{dY^2} - \phi_5 = M\phi_3 - \frac{1}{2}mAY^2\frac{d^3\phi_3}{dY^3} = mA\frac{d^2\phi_2}{dY^2} - \left(\frac{1}{2}AY^2 + mA\right)\frac{d\phi_2}{dY}\\
&\qquad + AY\phi_2 + \frac{1}{6}m(1+M)Y^3\frac{d^3\phi_1}{dY^3} - \frac{1}{2}m(1+M)Y^2\frac{d^2\phi_1}{dY^2}\\
&\qquad + \left[\frac{1}{6}(1+M)Y^3 + m(1+M)Y\right]\frac{d\phi_1}{dY} - \left[\frac{1}{2}(1+M)Y^2 + m(1+M)\right]\phi_1\\
&\qquad - \frac{1}{24}mAMY^4\frac{d^3\phi_0}{dY^3} + \frac{1}{6}mAMY^3\frac{d^2\phi_0}{dY^2}\\
&\qquad - \left(\frac{1}{24}MAY^4 + \frac{1}{2}mAMY^2\right)\frac{d\phi_0}{dY} + \left(\frac{1}{6}MAY^3 + mAMY\right)\phi_0.
\end{aligned} \tag{3.22}$$

Solving these equations and using the boundary conditions, we obtain

$$\phi_2(Y) = -\frac{M}{2\sqrt{1-m}} Y e^{-(1/\sqrt{1-m})Y},$$

$$\phi_3(Y) = -\frac{A}{1-m} e^{-(Y/\sqrt{1-m})} \left[\frac{3+4m}{8} Y + \frac{3}{8\sqrt{1-m}} Y^2 + \frac{1}{12(1-m)} Y^3\right],$$

$$\phi_4(Y) = \frac{1}{8\sqrt{1-m}} e^{-(Y/\sqrt{1-m})} \left\{ \left[\frac{1}{2}(3+4m)(1+M) + M^2\right] Y + \left[\frac{1}{2}(3+4m)(1+M) + M^2\right] Y^2 + \frac{1+M}{\sqrt{1-m}} Y^3 + \frac{1}{6(1-m)^{3/2}} Y^4 \right\},$$

$$\phi_5(Y) = \frac{AM}{8\sqrt{1-m}} e^{-(Y/\sqrt{1-m})} \left\{ \frac{-12m^3 + 25m^2 - 30m + 9}{4\sqrt{1-m}} Y + \frac{12m^2 - 13m + 9}{8} Y^2 + \frac{4m^2 - 5m + 6}{6(1-m)^{3/2}} Y^3 + \frac{3m+1}{12(1-m)^2} Y^4 - \frac{1}{30(1-m)^{3/2}} Y^5 \right\} \tag{3.23}$$

provided $0 \leq m < 1$. If $m = 0$ and $M = 0$, we recover the solutions for the Newtonian fluid obtained by Glauert [13] and if $M = 0$, we recover the solutions obtained by Labropulu et al. [16].

The oscillating component of the shear stress on the wall is given by

$$\frac{\tau_{12}}{\rho a^2} = -\sqrt{\frac{c\nu}{a^2}} e^{i\omega t} \left[\frac{1}{\alpha\sqrt{1-m}} + \frac{M}{2\sqrt{1-m}}\alpha + \frac{(3+12m-8m^2)A}{8(1-m)}\alpha^2 - \frac{(3+4m)(1+M)) + 2M^2}{16\sqrt{1-m}}\alpha^3 + \cdots\right]. \tag{3.24}$$

If $m = 0$ and $M = 0$, the shear stress is in good agreement with the result obtained by Glauert [13].

4. Conclusions

The unsteady stagnation-point flow of a viscoelastic Walters' B' fluid in the presence of a magnetic field is examined. Results for this flow are obtained for various values of the Hartmann's number M and the Weissenberg number W_e. At the higher frequencies, the perturbation is a shear layer, exactly as on a plate oscillating in a fluid at rest. Figure 1 shows the variation of $F'(\eta)$ for various Weissenberg numbers when the Hartmann's number $M = 0$. The effect of the Weissenberg number is to increase the velocity $F'(\eta)$ near the wall as it increases. Figures 2 and 3 show the variation of $F'(\eta)$ for various Hartmann's number M when $W_e = 0$ and $W_e = 0.2$. The effect of the Hartmann's number is to increase the velocity $F'(\eta)$ near the wall as it increases. Figures 4 and 5 show that $\phi_0(\eta)$ decreases near the wall as M and W_e are increasing. Also, from Tables 1 to 4, $F''(0)$ increases with Hartmann's number M and Weissenberg number W_e. The reason for this behaviour is that the magnetic field B_0 induces a force along the surface which supports the motion. As a result, the velocity along the surface increases everywhere and hence the shear stress on the wall increases with increasing Hartmann's number and Weissenberg number.

These conclusions are presented for the Walters' B′ fluid. Future works will examine the unsteady stagnation-point flow in the presence of a magnetic field of other viscoelastic fluids.

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