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Research Article

Left $WMC2$ Rings

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We introduce in this paper the concept of left $WMC2$ rings and concern ourselves with rings containing an injective maximal left ideal. Some known results for left idempotent reflexive rings and left HI rings can be extended to left $WMC2$ rings. As applications, we are able to give some new characterizations of regular left self-injective rings with nonzero socle and extend some known results on strongly regular rings.

Throughout this paper, R denotes an associative ring with identity, and all modules are unitary. For any nonempty subset X of a ring R, $r(X) = r_R(X)$ and $l(X) = l_R(X)$ denote the set of right annihilators of X and the set of left annihilators of X, respectively. We use $J(R)$, $N_*(R)$, $N(R)$, $Z_l(R)$, $E(R)$, $Soc(_RR)$, and $Soc(R_R)$ for the Jacobson radical, the prime radical, the set of all nilpotent elements, the left singular ideal, the set of all idempotent elements, the left socle, and the right socle of R, respectively.

An element k of R is called left minimal if Rk is a minimal left ideal. An element e of R is called left minimal idempotent if $e^2 = e$ is left minimal. We use $M_l(R)$ and $ME_l(R)$ for the set of all left minimal elements and the set of all left minimal idempotent elements of R, respectively. Moreover, let $MP_l(R) = \{k \in M_l(R) \mid {}_RRk \text{ is projective}\}$.

A ring R is called left $MC2$ if every minimal left ideal which is isomorphic to a summand of $_RR$ is a summand. Left $MC2$ rings were initiated by Nicholson and Yousif in [1]. In [2–6], the authors discussed the properties of left $MC2$ rings. In [1], a ring R is called left mininjective if $rl(k) = kR$ for every $k \in M_l(R)$, and R is said to be left minsymmetric if $k \in M_l(R)$ always implies $k \in M_r(R)$. According to [1], left mininjective $\Rightarrow$ left minsymmetric $\Rightarrow$ left $MC2$, and no reversal holds.

A ring R is called left universally mininjective [1] if Rk is an idempotent left ideal of R for every $k \in M_l(R)$. The work in [2] uses the term left DS for the left universally mininjective. According to [1, Lemma 5.1], left DS rings are left mininjective.

A ring R is called left min-abel [3] if for each $e \in ME_l(R)$, e is left semicentral in R, and R is said to be strongly left min-abel [3, 7] if every element of $ME_l(R)$ is central in R.

A ring R is called left $WMC2$ if $gRe = 0$ implies $eRg = 0$ for $e \in ME_l(R)$ and $g \in E(R)$.

Let F be a field and $R = \{\left(\begin{smallmatrix} a & b \\ 0 & a \end{smallmatrix}\right) \mid a, b \in F\}$. Then $E(R) = \{\left(\begin{smallmatrix} 0 & 0 \\ 0 & 0 \end{smallmatrix}\right), \left(\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}\right)\}$ and $ME_l(R)$ is empty, so R is left $WMC2$. Now let $S = \left(\begin{smallmatrix} F & F \\ 0 & F \end{smallmatrix}\right)$. Then $ME_l(S) = \{\left(\begin{smallmatrix} 1 & u \\ 0 & 0 \end{smallmatrix}\right) \mid u \in F\ \}$ and $E(S) = \{\left(\begin{smallmatrix} 0 & 0 \\ 0 & 0 \end{smallmatrix}\right), \left(\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}\right), \left(\begin{smallmatrix} 1 & u \\ 0 & 0 \end{smallmatrix}\right), \left(\begin{smallmatrix} 0 & u \\ 0 & 1 \end{smallmatrix}\right) \mid u \in F\}$. Since $\left(\begin{smallmatrix} 0 & u \\ 0 & 1 \end{smallmatrix}\right) S \left(\begin{smallmatrix} 1 & 0 \\ 0 & 0 \end{smallmatrix}\right) = 0$ and $\left(\begin{smallmatrix} 1 & 0 \\ 0 & 0 \end{smallmatrix}\right) S \left(\begin{smallmatrix} 0 & u \\ 0 & 1 \end{smallmatrix}\right) \neq 0$, S is not left $WMC2$.

Let R be any ring and $S_1 = R[x]$ and $S_2 = R[[x]]$. Then $ME_l(S_1)$ and $ME_l(S_2)$ are all empties, so S_1 and S_2 are all left $WMC2$.

A ring R is called left idempotent reflexive [8] if $aRe = 0$ implies $eRa = 0$ for all $a \in R$ and $e \in E(R)$. Clearly, R is left idempotent reflexive if and only if for any $a \in N(R)$ and $e \in E(R)$, $aRe = 0$ implies $eRa = 0$ if and only if for any $a \in J(R)$ and $e \in E(R)$, $aRe = 0$ implies $eRa = 0$. Therefore, left idempotent reflexive rings are left $WMC2$.

In general, the existence of an injective maximal left ideal in a ring R can not guarantee the left self-injectivity of R. In [9], Osofsky proves that if R is a semiprime ring containing an injective maximal left ideal, then R is left self-injective. In [8], Kim and Baik prove that if R is left idempotent reflexive containing an injective maximal left ideal, then R is left self-injective. In [10], Wei and Li prove that if R is left $MC2$ containing an injective maximal left ideal, then R is left self-injective. Motivated by these results, in this paper, we show that if R is a left $WMC2$ ring containing an injective maximal left ideal, then R is left self-injective. As an application of this result, we show that a ring R is a semisimple Artinian ring if and only if R is a left WMC ring and left HI ring.

We start with the following theorem.

Theorem 1. *The following conditions are equivalent for a ring R:*

(1) *R is left $MC2$;*

(2) *for any $a \in R$ and $e \in ME_l(R)$, $eaRe = 0$ implies $ea = 0$;*

(3) *for any $e, g \in ME_l(R)$, $(g - e)Re = 0$ implies that $e = eg$;*

(4) *for any $k, l \in M_l(R)$, $kRl = 0$ implies $lRk = 0$.*

Proof. (1)⇒(2) Assume that $a \in R$ and $e \in ME_l(R)$ with $eaRe = 0$. If $ea \neq 0$, then $Rea \cong Re$. By (1), $Rea = Rg$ for some $g \in ME_l(R)$. Hence $Rg = RgRg = ReaRea = R(eaRe)a = 0$, which is a contradiction. Hence $ea = 0$.

(2)⇒(3) Let $e, g \in ME_l(R)$ such that $(g - e)Re = 0$. Then $e(g - e)Re = 0$. By (2), $e(g - e) = 0$. Hence $e = eg$.

(3)⇒(4) Assume that $k, l \in M_l(R)$ with $kRl = 0$. If $lRk \neq 0$, then $RlRk = Rk$. Hence $Rk = RlRk = (Rl)^2Rk$, which implies $Rl = Re$ for some $e \in ME_l(R)$. Since $ReRk = RlRk = Rk \neq 0$, there exists $b \in R$ such that $ebk \neq 0$. Let $g = e+ebk$. Then $g^2 = e+ebk+ebke+ebkebk = e+ebk = g \in ME_l(R)$ because $ebke \in RkRe = RkRl = 0$ and $g \neq 0$. Since $(g-e)Re = ebkRe = ebkRl = 0$, by (3), $e = eg$. Hence $g = eg = e$, which implies $ebk = 0$. It is a contradiction. Therefore $lRk = 0$.

(4)⇒(1) Let $a \in M_l(R)$ and $e \in ME_l(R)$ with $Ra \cong Re$. Then there exists $g \in ME_l(R)$ such that $a = ga$ and $l(a) = l(g)$. If $(Ra)^2 = 0$, then $RaR \subseteq l(a) = l(g)$, so $aRg = 0$, by (4), $gRa = 0$, which implies $a = ga = 0$. It is a contradiction. Hence $(Ra)^2 \neq 0$, so $Ra = Rh$ for some $h \in ME_l(R)$, which implies R is a left $MC2$ ring. □

Corollary 2. *Left $MC2$ rings are left $WMC2$.*

Proof. Let $e \in ME_l(R)$ and $g \in E(R)$ with $gRe = 0$. If $eRg \neq 0$, then $ebg \neq 0$ for some $b \in R$. Clearly, $ebg \in M_l(R)$ and $(ebg)Re = 0$. Since R is a left $MC2$ ring, by Theorem 1, $eR(ebg) = 0$, which implies $ebg = 0$, and this is a contradiction. Hence $eRg = 0$ and so R is a left $WMC2$ ring. □

We do not know whether the converse of Corollary 2 holds. However, we have the following characterization of left WMC2 rings.

Theorem 3. *Let R be a ring. Then the following conditions are equivalent:*

(1) *R is a left $WMC2$ ring;*

(2) *for any $e \in ME_l(R)$ and $g \in E(R)$, $eg \neq 0$ implies $gRe \neq 0$;*

(3) *for any $e \in ME_l(R)$, $l(Re) \cap E(R) \subseteq r(eR)$;*

(4) *for any $k \in MP_l(R)$ and $g \in E(R)$, $gRk = 0$ implies $kRg = 0$.*

Proof. (4)⇒(1)⇒(2) It is easy to show by the definition of left $WMC2$ ring.

(2)⇒(3) Let $g \in l(Re) \cap E(R)$. Then $gRe = 0$. We claim that $eRg = 0$. Otherwise, there exists $b \in R$ such that $ebg \neq 0$. Clearly, $h = ebg + g - eg \in E(R)$ and $eh = ebg \neq 0$. By (2), we have $hRe \neq 0$. But $hRe = 0$ because $gRe = 0$. This is a contradiction. Hence $eRg = 0$ and so $g \in r(eR)$. Therefore $l(Re) \cap E(R) \subseteq r(eR)$.

(3)⇒(4) Since $k \in MP_l(R)$, ${}_RRk$ is projective. It is easy to show that $k = ek$ and $l(k) = l(e)$ for some $e \in ME_l(R)$. Since $gRk = 0$, $gR \subseteq l(k)$. Therefore $gRe = 0$, which implies $g \in l(Re) \cap E(R)$. By (3), $eRg = 0$. Hence $kRg = ekRg \subseteq eRg = 0$. □

By Theorem 3, we have the following corollary.

Corollary 4. (1) *Let R be a left $WMC2$ ring. If $e \in E(R)$ satisfying $ReR = R$, then eRe is left $WMC2$.*

(2) *If R is a direct product of a family rings $\{R_i : i \in I\}$, then R is a left $WMC2$ ring if and only if every R_i is left $WMC2$.*

Theorem 5. (1) *If R is a subdirect product of a family left $WMC2$ rings $\{R_i : i \in I\}$, then R is a left $WMC2$ ring.*

(2) *If $R/Z_l(R)$ is a left $WMC2$ ring, so is R.*

Proof. (1) Let $R_i = R/A_i$, where A_i are ideals of R with $\bigcap_{i\in I} A_i = 0$. Let $e \in ME_l(R)$ and $g \in E(R)$ satisfying $gRe = 0$. For any $i \in I$, if $e \in A_i$, then $eRg \in A_i$; if $e \notin A_i$, then we can easily show that $e_i = e + A_i \in ME_l(R_i)$. Since R_i is a left $WMC2$ ring and $g_iR_ie_i = 0$, where $g_i = g + A_i$, $e_iR_ig_i = 0$. Hence $eRg \subseteq A_i$. In any case, we have $eRg \subseteq A_i$ for all $i \in I$. Therefore $eRg \subseteq \bigcap_{i\in I} A_i = 0$ and so $eRg = 0$. This shows that R is a left $WMC2$ ring.

(2) Let $e \in ME_l(R)$ and $g \in E(R)$ satisfying $eg \neq 0$. Clearly, in $\overline{R} = R/Z_l(R)$, $\overline{e} = e + Z_l(R) \in ME_l(\overline{R})$, $\overline{g} = g + Z_l(R) \in E(\overline{R})$. Since ${}_RReg \cong {}_RRe$, $eg \notin Z_l(R)$. Since $\overline{R}$ is a left $WMC2$ ring, by Theorem 3, $\overline{g}\overline{R}\overline{e} \neq 0$, which implies $gRe \neq 0$. Thus R is a left $WMC2$ ring by Theorem 3. □

Theorem 6. (1) *R is a strongly left min-abel ring if and only if R is a left min-abel left $WMC2$ ring.*

(2) *If $R/Z_l(R)$ is a strongly left min-abel ring, then so is R.*

Proof. (1) Theorem 1.8 in [3] shows that R is a strongly left min-abel ring if and only if R is a left min-abel left $MC2$ ring, so by Corollary 2, we obtain that strongly left min-abel ring is left min-abel left $WMC2$.

Conversely, let R be a left min-abel left $WMC2$ ring. Let $e \in ME_l(R)$ and $a \in R$ satisfying $eaRe = 0$. Set $g = 1 - e + ea$. Then, clearly, $g \in E(R)$ and $eg = ea$. Since R is a left min-abel ring, $(1-e)Re = (1-e)eRe = 0$, so $gRe = 0$. Since R is a left $WMC2$ ring, $eRg = 0$, which implies $ea = eg = 0$, by Theorem 1, R is a left $MC2$ ring. Hence R is a strongly left min-abel ring.

(2) It is an immediate corollary of (1), [3, Corollary 1.5(2)] and Theorem 5(2). □

A ring R is called left idempotent reflexive [8] if $aRe = 0$ implies $eRa = 0$ for all $a \in R$ and $e \in E(R)$. Clearly, left idempotent reflexive rings are left $WMC2$.

In general, the existence of an injective maximal left ideal in a ring R cannot guarantee the left self-injectivity of R. Proposition 5 in [8] proves that if R is a left idempotent reflexive ring containing an injective maximal left ideal, then R is a left self-injective ring. Theorem 4.1 in [10] proves that if R is a left MC2 ring containing an injective maximal left ideal, then R is a left self-injective ring. We can generalize the results as follows.

Theorem 7. *Let R be a left $WMC2$ ring. If R contains an injective maximal left ideal, then R is a left self-injective ring.*

Proof. Let M be an injective maximal left ideal of R. Then $R = M \oplus N$ for some minimal left ideal N of R. Hence we have $M = Re$ and $N = R(1-e)$ for some $e^2 = e \in R$. If $MN = 0$, then $eR(1-e) = 0$. Since R is left $WMC2$ and $1-e \in ME_l(R)$, $(1-e)Re = 0$. So e is central. Now let L be any proper essential left ideal of R and $f : L \to N$ any nonzero left R–homomorphism. Then $L/U \cong N$, where $U = \ker f$ is a maximal submodule of L. Now $L = U \oplus V$, where $V \cong N = R(1-e)$ is a minimal left ideal of R. Since e is central, $V = R(1-e)$. For any $z \in L$, let $z = x + y$, where $x \in U$, $y \in V$. Then $f(z) = f(x) + f(y) = f(y)$. Since $y = y(1-e) = (1-e)y$, $f(z) = f(y) = f(y(1-e)) = yf(1-e)$. Since $x(1-e) = (1-e)x \in V \cap U = 0$, $xf(1-e) = fx(1-e) = f(0) = 0$. Thus $f(z) = yf(1-e) = yf(1-e) + xf(1-e) = (y+x)f(1-e) = zf(1-e)$. Hence $_RN$ is injective. If $MN \neq 0$, by the proof of [11, Proposition 5], we have $_RN$ is injective. Hence $R = M \oplus N$ is left self-injective. □

A ring R is called strongly left DS [3] if $k^2 \neq 0$ for all $k \in M_l(R)$. Since strongly left $DS \Rightarrow$ left $DS \Rightarrow$ left mininjective $\Rightarrow$ left minsymmetric $\Rightarrow$ left $MC2 \Rightarrow$ left $WMC2$ and strongly left min-abel $\Rightarrow$ left $WMC2$, we have the following corollary.

Corollary 8. *Let R contain an injective maximal left ideal. If R satisfies one of the following conditions, then R is a left self-injective ring.*

(1) *R is a strongly left DS ring.*

(2) *R is a left DS ring.*

(3) *R is a left mininjective ring.*

(4) *R is a left minsymmetric ring.*

(5) *R is a strongly left min-abel ring.*

(6) *R is a left $MC2$ ring.*

It is well known that if R is a left self-injective ring, then $J(R) = Z_l(R)$. Therefore by [2, Theorem 5.1] and Corollary 8, we have the following corollary.

Corollary 9. *Let R contain an injective maximal left ideal. Then R is left self-injective if and only if $J(R) = Z_l(R)$.*

A ring R is called left nil-injective [5] if for any $a \in N(R)$, $rl(a) = aR$, and R is said to be left $NC2$ [5] if for any $a \in N(R)$, ${}_R Ra$ is projective implies that $Ra = Re$ for some $e \in E(R)$. By [5, Theorem 2.22], left nil-injective rings are left $NC2$ and left $NC2$ rings are left $MC2$. A ring R is right Kasch if every simple right R-module can be embedded in R_R, and R is said to be left $C2$ [12] if every left ideal that is isomorphic to a direct summand of ${}_R R$ is itself a direct summand. Clearly, left self-injective rings are left $C2$ [13] and left $C2$ rings are left $NC2$ and by [14, Lemma 1.15], right Kasch rings are left $C2$. Hence, we have the following corollary.

Corollary 10. (1) *Let R contain an injective maximal left ideal. Then the following conditions are equivalent:*

(a) *R is a left self-injective ring;*

(b) *R is a left nil-injective ring;*

(c) *R is a left $C2$ ring;*

(d) *R is a left $NC2$ ring.*

(2) *If R is a right Kasch ring containing an injective maximal left ideal, then R is a left self-injective ring.*

A ring R is called left min-AP-injective if for any $k \in M_l(R)$, $rl(k) = kR \oplus X_k$, where X_k is a right ideal of R. Clearly, left mininjective rings are left min AP-injective.

Lemma 11. (1) *If R is a left min -AP-injective ring, then R is left $WMC2$.*
(2) *If $Soc({}_R R) \subseteq Soc(R_R)$, then R is left $WMC2$.*

Proof. (1) Let $e \in ME_l(R)$ and $g \in E(R)$ satisfying $eg \neq 0$. Since R is a left min-AP-injective ring and $l(e) = l(eg)$, $eR = rl(e) = rl(eg) = egR \oplus X_{eg}$, where X_{eg} is a right ideal R. Set $e = egb + x$, $b \in R$ and $x \in X_{eg}$. Then $eg = e(eg) = egbeg + xeg$, so $xeg = eg - egbeg \in egR \cap X_{eg}$, which implies $xeg = 0$, so $eg = egbeg$. Let $h = egb$. Then $h \in ME_l(R)$ and $egR = hR$. Therefore $hR = hRhR = egRegR$ which implies $gRe \neq 0$. By Theorem 3, R is a left $WMC2$ ring.

(2) Assume that $e \in ME_l(R)$ and $a \in R$ satisfying $eaRe = 0$. If $ea \neq 0$, then $ea \in Soc({}_R R) \subseteq Soc(R_R)$. Thus there exists a minimal right ideal kR of R such that $kR \subseteq eaR$. Clearly, $l(k) = l(ea) = l(e)$ and $kRkR \subseteq eaReaR = 0$. Hence $RkR \subseteq l(k)$. Let I be a complement right ideal of RkR in R. Then $I \subseteq l(k)$ and $e \in Soc({}_R R) \subseteq Soc(R_R) \subseteq RkR \oplus I \subseteq l(k) = l(e)$, which is a contradiction. Hence $ea = 0$. By Theorem 1, R is a left $MC2$ ring, so R is left $WMC2$ by Corollary 2. □

Since left mininjective rings are left min-AP-injective and $Soc({}_R R) \subseteq Soc(R_R)$. Hence by Theorem 7, Corollary 8 and Lemma 11, we have the following theorem.

Theorem 12. *Let R contain an injective maximal left ideal. Then the following conditions are equivalent:*

(1) *R is left self-injective;*

(2) *R is left min-AP-injective;*

(3) *$Soc({}_R R) \subseteq Soc(R_R)$.*

A ring R is called

(1) strongly reflexive if $aRbRc = 0$ implies $aRcRb = 0$ for all $a, b, c \in R$;

(2) reflexive [8, 15] if $aRb = 0$ implies $bRa = 0$ for all $a, b \in R$;

(3) symmetric if $abc = 0$ implies $acb = 0$ for all $a, b, c \in R$;

(4) ZC [16] if $ab = 0$ implies $ba = 0$ for all $a, b \in R$;

(5) ZI [16] if $ab = 0$ implies $aRb = 0$ for all $a, b \in R$.

Evidently, we have the following proposition.

Proposition 13. (1) *The following conditions are equivalent for a ring R:*

(a) *R is semiprime;*

(b) *R is strongly reflexive and every proper essential right ideal of R contains no nonzero nilpotent ideal;*

(c) *R is reflexive and every proper essential right ideal of R contains no nonzero nilpotent ideal;*

(d) *R is strongly reflexive and $N_*(R) \cap Z_l(R) = 0$;*

(e) *R is reflexive and $N_*(R) \cap Z_l(R) = 0$.*

(2) *R is symmetric if and only if R is ZI and strongly reflexive.*
(3) *R is reversible if and only if R is ZI and reflexive.*
(4) *Strongly reflexive $\Rightarrow$ reflexive $\Rightarrow$ left idempotent reflexive.*

It is well known that if R is a left self-injective ring, then $Z_l(R) = J(R)$, so $R/Z_l(R)$ is semiprimitive. Hence $R/Z_l(R)$ is left $WMC2$ by Proposition 13. Thus, by Theorems 5 and 7, we have the following theorem.

Theorem 14. *Let R contain an injective maximal left ideal. Then*

(1) *R is a left self-injective ring if and only if $R/Z_l(R)$ is a left $WMC2$ ring.*

(2) *If R satisfies one of the following conditions, then R is a left self-injective:*

(a) *R is a semiprime ring;*
(b) *R is a strongly reflexive;*
(c) *R is reflexive;*
(d) *R is a left idempotent reflexive.*

Recall that a ring R is left pp if every principal left ideal of R is projective as left R-module. As an application of Theorem 7, we have the following result.

Theorem 15. *The following conditions are equivalent for a ring R:*

(1) *R is a von Neumann regular left self-injective ring with $Soc({}_RR) \neq 0$;*

(2) *R is a left $WMC2$ left pp ring containing an injective maximal left ideal;*

(3) *R is a left pp ring containing an injective maximal left ideal and $R/Z_l(R)$ is a left $WMC2$ ring.*

Proof. (1)$\Rightarrow$(3) is trivial.
(3)$\Rightarrow$(2) is a direct result of Theorem 5(2).

(2)⇒(1) By Theorem 7, R is left self-injective. Hence, by [13, Theorem 1.2], R is left $C2$, so R is von Neumann regular because R is left pp. In addition, we have $Soc({}_RR) \neq 0$ since there is an injective maximal left ideal. □

By [17], a ring R is said to be left HI if R is left hereditary containing an injective maximal left ideal. Osofsky [9] proves that a left self-injective left hereditary ring is semisimple Artinian. We can generalize the result as follows.

Corollary 16. *The following conditions are equivalent for a ring R:*

(1) *R is semisimple Artinian;*

(2) *R is left $WMC2$ left HI;*

(3) *R is left $NC2$ left HI;*

(4) *R is left min-AP-injective left HI;*

(5) *R is left idempotent reflexive left HI.*

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