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Research Article

Identities on the Bernoulli and Genocchi Numbers and Polynomials

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We give some interesting identities on the Bernoulli numbers and polynomials, on the Genocchi numbers and polynomials by using symmetric properties of the Bernoulli and Genocchi polynomials.

1. Introduction

Let p be a fixed odd prime number. Throughout this paper $\mathbb{Z}_p, \mathbb{Q}_p$, and $\mathbb{C}_p$ will denote the ring of p-adic rational integers, the field of p-adic rational numbers, and the completion of the algebraic closure of $\mathbb{Q}_p$. Let $\mathbb{N}$ be the set of natural numbers and $\mathbb{Z}_+ = \mathbb{N} \cup \{0\}$. The p-adic norm on C_p is normalized so that $|p|_p = p^{-1}$. Let $C(\mathbb{Z}_p)$ be the space of continuous functions on $\mathbb{Z}_p$. For $f \in C(\mathbb{Z}_p)$, the fermionic p-adic integral on $\mathbb{Z}_p$ is defined by Kim as follows:

$$I_{-1}(f) = \int_{\mathbb{Z}_p} f(x)d\mu_{-1}(x) = \lim_{N\to\infty} \sum_{x=0}^{p^N-1} f(x)(-1)^x \tag{1.1}$$

(see [1–16]). From (1.1), we have

$$I_{-1}(f_1) = -I_{-1}(f) + 2f(0) \tag{1.2}$$

(see [1–16]), where $f_1(x) = f(x+1)$.

Let us take $f(x) = e^{xt}$. Then, by (1.2), we get

$$t\int_{\mathbb{Z}_p} e^{xt} d\mu_{-1}(x) = \frac{2t}{e^t+1} = \sum_{n=0}^{\infty} G_n \frac{t^n}{n!}, \tag{1.3}$$

where G_n are the nth ordinary Genocchi numbers (see [8, 15]).

From the same method of (1.3), we can also derive the following equation:

$$t\int_{\mathbb{Z}_p} e^{(x+y)t} d\mu_{-1}(y) = \frac{2t}{e^t+1} e^{xt} = \sum_{n=0}^{\infty} G_n(x) \frac{t^n}{n!}, \tag{1.4}$$

where $G_n(x)$ are called the nth Genocchi polynomials (see [14, 15]).

By (1.3), we easily see that

$$G_n(x) = \sum_{l=0}^{n} \binom{n}{l} G_l x^{n-l} \tag{1.5}$$

(see [15]). By (1.3) and (1.4), we get Witt's formula for the nth Genocchi numbers and polynomials as follows:

$$\int_{\mathbb{Z}_p} x^n d\mu_{-1}(x) = \frac{G_{n+1}}{n+1}, \quad \int_{\mathbb{Z}_p} (x+y)^n d\mu_{-1}(y) = \frac{G_{n+1}(x)}{n+1}, \quad \text{for } n \in \mathbb{Z}_+. \tag{1.6}$$

From (1.2), we have

$$\int_{\mathbb{Z}_p} (x+1)^n d\mu_{-1}(x) + \int_{\mathbb{Z}_p} x^n d\mu_{-1}(x) = 2\delta_{0,n}, \tag{1.7}$$

where the symbol $\delta_{0,n}$ is the Kronecker symbol (see [4, 5]).

Thus, by (1.5) and (1.7), we get

$$(G+1)^n + G_n = 2\delta_{1,n} \tag{1.8}$$

(see [15]). From (1.4), we can derive the following equation:

$$\int_{\mathbb{Z}_p} (1-x+y)^n d\mu_{-1}(y) = (-1)^n \int_{\mathbb{Z}_p} (x+y)^n d\mu_{-1}(y). \tag{1.9}$$

By (1.6) and (1.9), we see that

$$\frac{G_{n+1}(1-x)}{n+1} = (-1)^n \frac{G_{n+1}(x)}{n+1}. \tag{1.10}$$

Thus, by (1.10), we get $G_{n+1}(2)/(n+1) = (-1)^n (G_{n+1}(-1)/(n+1))$.

From (1.5) and (1.8), we have

$$\frac{G_{n+1}(2)}{n+1} = 2 - \frac{G_{n+1}(1)}{n+1} = 2 + \frac{G_{n+1}}{n+1} - 2\delta_{1,n+1}. \tag{1.11}$$

The Bernoulli polynomials $B_n(x)$ are defined by

$$\frac{t}{e^t - 1}e^{xt} = e^{B(x)t} = \sum_{n=0}^{\infty} B_n(x)\frac{t^n}{n!} \tag{1.12}$$

(see [6, 9, 12]) with the usual convention about replacing $B^n(x)$ by $B_n(x)$.

In the special case, $x = 0$, $B_n(0) = B_n$ is called the n-th Bernoulli number. By (1.12), we easily see that

$$B_n(x) = \sum_{l=0}^{n}\binom{n}{l}x^{n-l}B_l = (B + x)^n \tag{1.13}$$

(see [6]). Thus, by (1.12) and (1.13), we get reflection symmetric formula for the Bernoulli polynomials as follows:

$$B_n(1 - x) = (-1)^n B_n(x), \tag{1.14}$$

$$B_0 = 1, \quad (B + 1)^n - B_n = \delta_{1,n} \tag{1.15}$$

(see [6, 9, 12]). From (1.14) and (1.15), we can also derive the following identity:

$$(-1)^n B_n(-1) = B_n(2) = n + B_n(1) = n + B_n + \delta_{1,n}. \tag{1.16}$$

In this paper, we investigate some properties of the fermionic p-adic integrals on $\mathbb{Z}_p$. By using these properties, we give some new identities on the Bernoulli and the Euler numbers which are useful in studying combinatorics.

2. Identities on the Bernoulli and Genocchi Numbers and Polynomials

Let us consider the following fermionic p-adic integral on $\mathbb{Z}_p$ as follows:

$$\begin{aligned} I_1 &= \int_{\mathbb{Z}_p} B_n(x)d\mu_{-1}(x) = \sum_{l=0}^{n}\binom{n}{l}B_{n-l}\int_{\mathbb{Z}_p} x^l d\mu_{-1}(x) \\ &= \sum_{l=0}^{n}\binom{n}{l}B_{n-l}\frac{G_{l+1}}{l+1}, \quad \text{for } n \in \mathbb{Z}_+ = \mathbb{N} \cup \{0\}. \end{aligned} \tag{2.1}$$

On the other hand, by (1.14) and (1.15), we get

$$\begin{aligned}
I_1 &= (-1)^n \int_{\mathbb{Z}_p} B_n(1-x)\,d\mu_{-1}(x) \\
&= (-1)^n \sum_{l=0}^{n} \binom{n}{l} B_{n-l} \int_{\mathbb{Z}_p} (1-x)^l d\mu_{-1}(x) \\
&= (-1)^n \sum_{l=0}^{n} \binom{n}{l} B_{n-l} (-1)^l \frac{G_{l+1}(-1)}{l+1} \\
&= (-1)^n \sum_{l=0}^{n} \binom{n}{l} B_{n-l} \left(2 + \frac{G_{l+1}}{l+1} - 2\delta_{1,l+1} \right) \\
&= 2(-1)^n (B_n + \delta_{1,n}) + (-1)^n \sum_{l=0}^{n} \binom{n}{l} B_{n-l} \frac{G_{l+1}}{l+1} + 2(-1)^{n+1} B_n.
\end{aligned} \tag{2.2}$$

Equating (2.1) and (2.2), we obtain the following theorem.

Theorem 2.1. *For $n \in \mathbb{Z}_+$, one has*

$$\left(1 + (-1)^{n+1}\right) \sum_{l=0}^{n} \binom{n}{l} B_{n-l} \frac{G_{l+1}}{l+1} = 2(-1)^n \delta_{1,n}. \tag{2.3}$$

By using the reflection symmetric property for the Euler polynomials, we can also obtain some interesting identities on the Euler numbers.

Now, we consider the fermionic p-adic integral on $\mathbb{Z}_p$ for the polynomials as follows:

$$\begin{aligned}
I_2 &= \int_{\mathbb{Z}_p} G_n(x)\,d\mu_{-1}(x) \\
&= \sum_{l=0}^{n} \binom{n}{l} G_{n-l} \int_{\mathbb{Z}_p} x^l d\mu_{-1}(x) \\
&= \sum_{l=0}^{n} \binom{n}{l} G_{n-l} \frac{G_{l+1}}{l+1}, \quad \text{for } n \in \mathbb{Z}_+.
\end{aligned} \tag{2.4}$$

On the other hand, by (1.8), (1.10), and (1.11), we get

$$\begin{aligned}
I_2 &= (-1)^{n-1} \int_{\mathbb{Z}_p} G_n(1-x)\,d\mu_{-1}(x) \\
&= (-1)^{n-1} \sum_{l=0}^{n} \binom{n}{l} G_{n-l} \int_{\mathbb{Z}_p} (1-x)^l d\mu_{-1}(x) \\
&= (-1)^{n-1} \sum_{l=0}^{n} \binom{n}{l} G_{n-l} (-1)^l \frac{G_{l+1}(-1)}{l+1}
\end{aligned}$$

$$
\begin{aligned}
&= (-1)^{n-1}\sum_{l=0}^{n}\binom{n}{l}G_{n-l}\left(2+\frac{G_{l+1}}{l+1}-2\delta_{1,l+1}\right)\\
&= 2(-1)^{n-1}(2\delta_{1,n}-G_n)+2(-1)^{n}G_n\\
&\quad + (-1)^{n-1}\sum_{l=0}^{n}\binom{n}{l}G_{n-l}\frac{G_{l+1}}{l+1}.
\end{aligned}
\tag{2.5}
$$

Equating (2.4) and (2.5), we obtain the following theorem.

Theorem 2.2. *For $n \in \mathbb{Z}_+$, one has*

$$
(1+(-1)^n)\sum_{l=0}^{n}\binom{n}{l}G_{n-l}\frac{G_{l+1}}{l+1} = 4(-1)^n G_n + 4(-1)^{n+1}\delta_{1,n}. \tag{2.6}
$$

Let us consider the fermionic p-adic integral on $\mathbb{Z}_p$ for the product of $B_n(x)$ and $G_n(x)$ as follows:

$$
\begin{aligned}
I_3 &= \int_{\mathbb{Z}_p} B_m(x)G_n(x)\,d\mu_{-1}(x)\\
&= \sum_{k=0}^{m}\sum_{l=0}^{n}\binom{m}{k}\binom{n}{l}B_{m-k}G_{n-l}\int_{\mathbb{Z}_p} x^{k+l}\,d\mu_{-1}(x)\\
&= \sum_{k=0}^{m}\sum_{l=0}^{n}\binom{m}{k}\binom{n}{l}B_{m-k}G_{n-l}\frac{G_{k+l+1}}{k+l+1}.
\end{aligned}
\tag{2.7}
$$

On the other hand, by (1.10) and (1.14), we get

$$
\begin{aligned}
I_3 &= \int_{\mathbb{Z}_p} B_m(x)G_n(x)\,d\mu_{-1}(x)\\
&= (-1)^{n+m-1}\int_{\mathbb{Z}_p} B_m(1-x)G_n(1-x)\,d\mu_{-1}(x)\\
&= (-1)^{n+m-1}\sum_{k=0}^{m}\sum_{l=0}^{n}\binom{m}{k}\binom{n}{l}B_{m-k}G_{n-l}\int_{\mathbb{Z}_p} (1-x)^{k+l}\,d\mu_{-1}(x)\\
&= 2(-1)^{n+m-1}B_m(1)G_n(1) + 2(-1)^{m+n}B_mG_n\\
&\quad + (-1)^{n+m-1}\sum_{k=0}^{m}\sum_{l=0}^{n}\binom{m}{k}\binom{n}{l}B_{m-k}G_{n-l}\frac{G_{k+l+1}}{k+l+1}.
\end{aligned}
\tag{2.8}
$$

By (2.7) and (2.8), we easily see that

$$\begin{aligned}
&\left(1+(-1)^{n+m+1}\right)\sum_{k=0}^{m}\sum_{l=0}^{n}\binom{m}{k}\binom{n}{l}B_{m-k}G_{n-l}\frac{G_{k+l+1}}{k+l+1}\\
&\quad=2(-1)^{m+n-1}(\delta_{1,m}+B_m)(2\delta_{1,n}-G_n)+2(-1)^{m+n}B_mG_n\\
&\quad=4(-1)^{m+n-1}B_m\delta_{1,n}+2(-1)^{m+n}B_mG_n+4(-1)^{m+n-1}\delta_{1,m}\delta_{1,n}\\
&\qquad+2(-1)^{m+n}\delta_{1,m}G_n+2(-1)^{m+n}B_mG_n.
\end{aligned}\tag{2.9}$$

Therefore, by (2.9), we obtain the following theorem.

Theorem 2.3. *For $n,m\in\mathbb{Z}_+$, one has*

$$\begin{aligned}
&\left(1+(-1)^{n+m+1}\right)\sum_{k=0}^{m}\sum_{l=0}^{n}\binom{m}{k}\binom{n}{l}B_{m-k}\frac{G_{n-l+1}}{n-l+1}\frac{G_{k+l+1}}{k+l+1}\\
&\quad=4(-1)^{m+n}B_mG_n+4(-1)^{m+n-1}B_m\delta_{1,n}+4(-1)^{m+n-1}\delta_{1,m}\delta_{1,n}\\
&\qquad+2(-1)^{m+n}\delta_{1,m}G_n.
\end{aligned}\tag{2.10}$$

Corollary 2.4. *For $n,m\in\mathbb{N}$, one has*

$$\sum_{k=0}^{2m}\sum_{l=0}^{2n}\binom{2m}{k}\binom{2n}{l}B_{2m-k}G_{2n-l}\frac{G_{k+l+1}}{k+l+1}=2B_{2m}G_{2n}.\tag{2.11}$$

Let us consider the fermionic p-adic integral on $\mathbb{Z}_p$ for the product of the Bernoulli polynomials and the Bernstein polynomials. For $n,k\in\mathbb{Z}_+$, with $0\le k\le n$, $B_{k,n}(x)=\binom{n}{k}x^k(1-x)^{n-k}$ are called the Bernstein polynomials of degree n, see [11]. It is easy to show that $B_{k,n}(x)=B_{n-k,n}(1-x)$,

$$\begin{aligned}
I_4&=\int_{\mathbb{Z}_p}B_m(x)B_{k,n}(x)\,d\mu_{-1}(x)\\
&=\binom{n}{k}\sum_{l=0}^{m}\binom{m}{l}B_{m-l}\int_{\mathbb{Z}_p}x^{k+l}(1-x)^{n-k}\,d\mu_{-1}(x)\\
&=\binom{n}{k}\sum_{l=0}^{m}\sum_{j=0}^{n-k}\binom{m}{l}\binom{n-k}{j}(-1)^jB_{m-l}\int_{\mathbb{Z}_p}x^{k+l+j}\,d\mu_{-1}(x)\\
&=\binom{n}{k}\sum_{l=0}^{m}\sum_{j=0}^{n-k}\binom{m}{l}\binom{n-k}{j}(-1)^jB_{m-l}\frac{G_{k+l+j+1}}{k+l+j+1}.
\end{aligned}\tag{2.12}$$

On the other hand, by (1.14) and (2.12), we get

$$
\begin{aligned}
I_4 &= (-1)^m \int_{\mathbb{Z}_p} B_m(1-x) B_{n-k,n}(1-x)\, d\mu_{-1}(x) \\
&= (-1)^m \binom{n}{k} \sum_{l=0}^{m} \sum_{j=0}^{k} \binom{m}{l} \binom{k}{j} (-1)^j B_{m-l} \int_{\mathbb{Z}_p} (1-x)^{n-k+l+j}\, d\mu_{-1}(x) \\
&= (-1)^m \binom{n}{k} \sum_{l=0}^{m} \sum_{j=0}^{k} \binom{m}{l} \binom{k}{j} (-1)^j B_{m-l} \\
&\quad \times \left(2 - 2\delta_{1,n-k+l+j+1} + \frac{G_{n-k+l+j+1}}{n-k+l+j+1} \right) \\
&= 2(-1)^m \binom{n}{k} B_m(1) \delta_{0,k} + 2(-1)^{m+1} \binom{n}{k} B_m \delta_{k,n} \\
&\quad + (-1)^m \binom{n}{k} \sum_{l=0}^{m} \sum_{j=0}^{k} \binom{m}{l} \binom{k}{j} (-1)^j B_{m-l} \frac{G_{n-k+l+j+1}}{n-k+l+j+1}.
\end{aligned}
\tag{2.13}
$$

Equating (2.12) and (2.13), we see that

$$
\begin{aligned}
&\sum_{l=0}^{m} \sum_{j=0}^{n-k} \binom{m}{l} \binom{n-k}{j} (-1)^j B_{m-l} \frac{G_{k+l+j+1}}{k+l+j+1} \\
&\quad = 2(-1)^m B_m(1) \delta_{0,k} + 2(-1)^{m+1} B_m \delta_{k,n} \\
&\qquad + (-1)^m \sum_{l=0}^{m} \sum_{j=0}^{k} \binom{m}{l} \binom{k}{j} (-1)^j B_{m-l} \frac{G_{n-k+l+j+1}}{n-k+l+j+1}.
\end{aligned}
\tag{2.14}
$$

Thus, from (2.14), we obtain the following theorem.

Theorem 2.5. *For $n, m \in \mathbb{N}$, one has*

$$
\sum_{l=0}^{2m} \sum_{j=0}^{n} \binom{2m}{l} \binom{n}{j} (-1)^j B_{2m-l} \frac{G_{l+j+1}}{l+j+1} = 2B_{2m}(1) + \sum_{l=0}^{2m} \binom{2m}{l} B_{2m-l} \frac{G_{n+l+1}}{n+l+1}. \tag{2.15}
$$

Finally, we consider the fermionic p-adic integral on $\mathbb{Z}_p$ for the product of the Euler polynomials and the Bernstein polynomials as follows:

$$
\begin{aligned}
I_5 &= \int_{\mathbb{Z}_p} G_m(x) B_{k,n}(x)\, d\mu_{-1}(x) \\
&= \binom{n}{k} \sum_{l=0}^{m} \binom{m}{l} G_{m-l} \int_{\mathbb{Z}_p} x^{k+l} (1-x)^{n-k}\, d\mu_{-1}(x)
\end{aligned}
$$

$$= \binom{n}{k}\sum_{l=0}^{m}\sum_{j=0}^{n-k}\binom{m}{l}\binom{n-k}{j}(-1)^j G_{m-l}\int_{\mathbb{Z}_p} x^{k+l+j}\,d\mu_{-1}(x)$$
$$= \binom{n}{k}\sum_{l=0}^{m}\sum_{j=0}^{n-k}\binom{m}{l}\binom{n-k}{j}(-1)^j G_{m-l}\frac{G_{k+l+j+1}}{k+l+j+1}. \tag{2.16}$$

On the other hand, by (1.10) and (2.12), we get

$$\begin{aligned}
I_5 &= (-1)^{m-1}\int_{\mathbb{Z}_p} G_m(1-x)B_{n-k,n}(1-x)\,d\mu_{-1}(x)\\
&= (-1)^{m-1}\binom{n}{k}\sum_{l=0}^{m}\binom{m}{l}G_{m-l}\sum_{j=0}^{k}\binom{k}{j}(-1)^j\int_{\mathbb{Z}_p}(1-x)^{n-k+l+j}\,d\mu_{-1}(x)\\
&= (-1)^{m-1}\binom{n}{k}\sum_{l=0}^{m}\sum_{j=0}^{k}\binom{m}{l}\binom{k}{j}(-1)^j G_{m-l}\\
&\quad\times\left(2+\frac{G_{n-k+l+j+1}}{n-k+l+j+1}-2\delta_{1,n-k+l+j+1}\right)\\
&= 2(-1)^{m-1}\binom{n}{k}G_m(1)\delta_{0,k}+2(-1)^m\binom{n}{k}G_m\delta_{k,n}\\
&\quad+(-1)^{m-1}\binom{n}{k}\sum_{l=0}^{m}\sum_{j=0}^{k}\binom{m}{l}\binom{k}{j}(-1)^j G_{m-l}\frac{G_{n-k+l+j+1}}{n-k+l+j+1}.
\end{aligned} \tag{2.17}$$

Equating (2.16) and (2.17), we obtain

$$\begin{aligned}
&\sum_{l=0}^{m}\sum_{j=0}^{n-k}\binom{m}{l}\binom{n-k}{j}(-1)^j G_{m-l}\frac{G_{k+l+j+1}}{k+l+j+1}\\
&\quad= 2(-1)^{m-1}G_m(1)\delta_{0,k}+2(-1)^m G_m\delta_{k,n}\\
&\qquad+(-1)^{m-1}\sum_{l=0}^{m}\sum_{j=0}^{k}\binom{m}{l}\binom{k}{j}(-1)^j G_{m-l}\frac{G_{n-k+l+j+1}}{n-k+l+j+1}.
\end{aligned} \tag{2.18}$$

Therefore, by (2.18), we obtain the following theorem.

Theorem 2.6. *For $n, m \in \mathbb{N}$, one has*

$$\sum_{l=0}^{2m}\sum_{j=0}^{n}\binom{2m}{l}\binom{n}{j}(-1)^j G_{2m-l}\frac{G_{l+j+1}}{l+j+1} = -2G_{2m}(1)-\sum_{l=0}^{2m}\binom{2m}{l}G_{2m-l}\frac{G_{n+l+1}}{n+l+1}. \tag{2.19}$$

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References

[1] T. Kim, "q-Volkenborn integration," *Russian Journal of Mathematical Physics*, vol. 9, no. 3, pp. 288–299, 2002.
[2] L. Carlitz, "Note on the integral of the product of several Bernoulli polynomials," *Journal of the London Mathematical Society*, vol. 34, pp. 361–363, 1959.
[3] T. Kim, "A note on q-Volkenborn integration," *Proceedings of the Jangjeon Mathematical Society*, vol. 8, no. 1, pp. 13–17, 2005.
[4] T. Kim, "On the multiple q-Genocchi and Euler numbers," *Russian Journal of Mathematical Physics*, vol. 15, no. 4, pp. 481–486, 2008.
[5] T. Kim, "On the q-extension of Euler and Genocchi numbers," *Journal of Mathematical Analysis and Applications*, vol. 326, no. 2, pp. 1458–1465, 2007.
[6] T. Kim, "Symmetry p-adic invariant integral on $\mathbb{Z}_p$ for Bernoulli and Euler polynomials," *Journal of Difference Equations and Applications*, vol. 14, no. 12, pp. 1267–1277, 2008.
[7] T. Kim, "Symmetry of power sum polynomials and multivariate fermionic p-adic invariant integral on $\mathbb{Z}_p$," *Russian Journal of Mathematical Physics*, vol. 16, no. 1, pp. 93–96, 2009.
[8] A. Bayad and T. Kim, "Identities for the Bernoulli, the Euler and the Genocchi numbers and polynomials," *Advanced Studies in Contemporary Mathematics*, vol. 20, no. 2, pp. 247–253, 2010.
[9] H. M. Srivastava, T. Kim, and Y. Simsek, "q-Bernoulli numbers and polynomials associated with multiple q-zeta functions and basic L-series," *Russian Journal of Mathematical Physics*, vol. 12, no. 2, pp. 241–268, 2005.
[10] T. Kim, "Barnes-type multiple q-zeta functions and q-Euler polynomials," *Journal of Physics A*, vol. 43, no. 25, Article ID 255201, 11 pages, 2010.
[11] T. Kim, "A note on q-Bernstein polynomials," *Russian Journal of Mathematical Physics*, vol. 18, no. 1, pp. 73–82, 2011.
[12] L. C. Jang, W. J. Kim, and Y. Simsek, "A study on the p-adic integral representation on $\mathbb{Z}_p$ associated with Bernstein and Bernoulli polynomials," *Advances in Difference Equations*, vol. 2010, Article ID 163217, 6 pages, 2010.
[13] Y. Simsek, "Special functions related to Dedekind-type DC-sums and their applications," *Russian Journal of Mathematical Physics*, vol. 17, no. 4, pp. 495–508, 2010.
[14] L.-C. Jang, T. Kim, D.-H. Lee, and D.-W. Park, "An application of polylogarithms in the analogs of Genocchi numbers," *Notes on Number Theory and Discrete Mathematics*, vol. 7, no. 3, pp. 65–70, 2001.
[15] S.-H. Rim, K.-H. Park, and E.-J. Moon, "On Genocchi numbers and polynomials," *Abstract and Applied Analysis*, vol. 2008, Article ID 898471, 7 pages, 2008.
[16] T. Kim, J. Choi, Y. H. Kim, and C.-S. Ryoo, "On the fermionic p-adic integral representation of Bernstein polynomials associated with Euler numbers and polynomials," *Journal of Inequalities and Applications*, vol. 2010, Article ID 864247, 12 pages, 2010.