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Letter to the Editor

Cusp Forms in $S_4(\Gamma_0(47))$ and the Number of Representations of Positive Integers by Some Direct Sum of Binary Quadratic Forms with Discriminant -47

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A basis of $S_4(\Gamma_0(47))$ is given and the formulas for the number of representations of positive integers by some direct sum of the quadratic forms $x_1^2+x_1x_2+12x_2^2$, $2x_1^2\pm x_1x_2+6x_2^2$, $3x_1^2\pm x_1x_2+4x_2^2$ are determined.

1. Introduction

This paper is the correction of the paper [1].

(1) It is stated that $\dim S_4(\Gamma_0(47),1) = 5$ at page 643 in [1]. But this dimension is 11 as stated at page 299 in [2]. Therefore, the coefficients of power series in (2.2), (2.3), (2.5), (2.6), (2.7), (2.8), (2.10), (2.12), (2.14), (2.15), (2.20), and (2.22) have to be calculated up to z^{11}, and Theorem 2.4 and consequently Theorem 2.7 are false since 5 vectors cannot be a basis of 11-dimensional vector space.

(2) The notations $G_k(\Gamma,\chi)$ and (k,Γ,χ) are both used as if they are different like at line 11 at page 638.

(3) The definitions of

$$\Gamma_1(N),\ \Gamma(N) \text{ and } \wp(\tau;Q(X),P_v(X),h) = \sum_{n_i\equiv h_i \pmod{N}} P_v(n_1,n_2,\dots,n_k) z_N^{(1/N)Q(n_1,n_2,\dots,n_k)} \tag{1.1}$$

have never been used in the paper.

(4) The class number of $\mathbb{Q}(\sqrt{-23})$ is 3; therefore, only F_1, Φ_1, and their combinations have been examined and a basis of $S_4(\Gamma_0(23),1)$ could be obtained. The authors in [1] also examined only two quadratic forms

$$F_1 = x_1^2 + x_1x_2 + 12x_2^2, \qquad G_1 = 2x_1^2 + x_1x_2 + 6x_2^2 \tag{1.2}$$

and their combinations. But, by simple calculations, it is possible to see that these quadratic forms are not enough to get a basis of $S_4(\Gamma_0(47),1)$. The class number of $\mathbb{Q}(\sqrt{-47})$ is 5; therefore,

$$F_1 = x_1^2 + x_1x_2 + 12x_2^2, \qquad G_1 = 2x_1^2 + x_1x_2 + 6x_2^2, \qquad H_1 = 2x_1^2 + x_1x_2 + 6x_2^2 \tag{1.3}$$

and their combinations have to be examined. Only in that case, it is possible to obtain a basis of $S_4(\Gamma_0(47),1)$ as we have done in the following.

2. Determination of a Basis of $S_4(\Gamma_0(47))$

We can calculate all reduced forms of a positive definite quadratic form

$$Q = ax^2 + bxy + cy^2, \quad a > 0, \tag{2.1}$$

with discriminant $\Delta = -47$ as follows:

$$\begin{gathered} F_1 = x_1^2 + x_1x_2 + 12x_2^2, \qquad H_1 = 2x_1^2 + x_1x_2 + 6x_2^2, \qquad G_1 = 3x_1^2 + x_1x_2 + 4x_2^2, \\ G_1' = 3x_1^2 - x_1x_2 + 4x_2^2, \qquad H_1' = 2x_1^2 - x_1x_2 + 6x_2^2. \end{gathered} \tag{2.2}$$

Here G_1' is the inverse of G_1, and they represent the same integers. Similarly, H_1' is the inverse of H_1 and they represent the same integers. Therefore, the theta series of H_1 and H_1' are the same with the theta series of G_1 and G_1', respectively. F_1 is the identity element. It can be seen easily that, the group of these quadratic forms is a group of order 5 and can be described as

$$H_1^2 = G_1', \qquad H_1^3 = G_1, \qquad H_1^4 = H_1', \qquad H_1^5 = F_1. \tag{2.3}$$

We can easily see that for the quadratic forms

$$F_1, G_1, H_1, \tag{2.4}$$

the determinant, the discriminant, and the character are

$$D = 47, \qquad \Delta = (-1)^{2/2}47 = -47, \qquad \chi(d) = \left(\frac{-47}{d}\right). \tag{2.5}$$

Consequently, their theta series are in

$$M_1\left(\Gamma_0(47), \left(\frac{-47}{d}\right)\right). \tag{2.6}$$

Hence by Theorem 2.1 in [3], F_2, H_2, G_2, $F_1 \oplus H_1$, $F_1 \oplus G_1$, and $H_1 \oplus G_1$ are quadratic forms whose theta series are in

$$M_2(\Gamma_0(47)). \tag{2.7}$$

We immediately obtain the following Corollary by Theorem 2.2 in [3].

Corollary 2.1. *Let Q be a positive definite form of 8 variables whose theta series Θ_Q is in*

$$M_4(\Gamma_0(47)). \tag{2.8}$$

Then the Eisenstein part of Θ_Q is

$$E(q:Q) = 1 + \sum_{n=1}^{\infty}\left(\alpha\sigma_3(n)q^n + \beta\sigma_3(n)q^{47n}\right), \tag{2.9}$$

where

$$\rho_4 = \frac{3!}{(2\pi)^4}\zeta(4) = \frac{1}{240}, \qquad \alpha = 240\frac{47^2-1}{47^4-1} = \frac{24}{221}, \qquad \beta = 240\frac{47^4-47^2}{47^4-1} = 47^2\frac{24}{221},$$

$$\begin{aligned} E(q:F_4) &= E(q:F_3\oplus H_1) = E(q:F_2\oplus H_2) = E(q:F_1\oplus H_3) = E(q:H_4) \\ &= E(q:F_3\oplus G_1) = E(q:F_2\oplus G_2) = E(q:F_1\oplus G_3) = E(q:G_4) = E(q:H_3\oplus G_1) \\ &= E(q:H_2\oplus G_2) = E(q:H_1\oplus G_3) = 1 + \frac{24}{221}\sum_{n=1}^{\infty}\left(q^n + 47^2q^{47n}\right)\sigma_3(n) \\ &= 1 + \frac{24}{221}\sum_{n=1}^{\infty}\sigma_3^*(n)q^n \\ &= 1 + \frac{24}{221}q + \frac{24\cdot 9}{221}q^2 + \frac{24\cdot 28}{221}q^3 + \frac{24\cdot 73}{221}q^4 + \frac{24\cdot 126}{221}q^5 + \frac{24\cdot 252}{221}q^6 \\ &\quad + \frac{24\cdot 344}{221}q^7 + \frac{24\cdot 585}{221}q^8 + \frac{24\cdot 757}{221}q^9 + \frac{24\cdot 1134}{221}q^{10} + \frac{24\cdot 1332}{221}q^{11} + \cdots. \end{aligned} \tag{2.10}$$

Here

$$\sigma_3^*(n) = \begin{cases} \sigma_3(n) & \text{if } n \geq 1 \text{ and } 47 \nmid n, \\ \sigma_3(n) + 47^2\sigma_3\left(\dfrac{n}{47}\right) & \text{if } 47 \mid n. \end{cases} \tag{2.11}$$

Now we will determine the sum of quadratic forms F_1, H_1, and G_1 and select 11 spherical functions such that the corresponding cusp forms are linearly independent.

(1) For quadratic form

$$2F_2 = 2x_1^2 + 2x_1x_2 + 24x_2^2 + 2x_3^2 + 2x_3x_4 + 24x_4^2 = (x_1, x_2, x_3, x_4)\begin{pmatrix} 2 & 1 & & \\ 1 & 24 & & \\ & & 2 & 1 \\ & & 1 & 24 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}, \tag{2.12}$$

the determinant and a cofactor are

$$D = 47^2, \qquad A_{11} = 24 \cdot 47. \tag{2.13}$$

By putting $2k = 4$, $Q = F_2$, and appropriate i, j in Theorem 2.1 in [3], we get

$$\varphi_{11} = x_1^2 - \frac{1}{4}\frac{24 \cdot 47}{47^2}2F_2 = x_1^2 - \frac{12}{47}F_2, \tag{2.14}$$

which will be spherical functions of second order with respect to F_2.

(2) Similarly, for

$$2H_2 = 4x_1^2 + 2x_1x_2 + 12x_2^2 + 4x_3^2 + 2x_3x_4 + 12x_4^2, \tag{2.15}$$

the determinant and some cofactors are

$$D = 47^2, \qquad A_{11} = 12 \cdot 47, \qquad A_{12} = -47, \qquad A_{13} = 12 \cdot 12. \tag{2.16}$$

By putting $2k = 4$, $Q = H_2$, and appropriate i, j in Theorem 2.1 in [3], we get

$$\varphi_{11} = x_1^2 - \frac{1}{4}\frac{12 \cdot 47}{47^2}2H_2 = x_1^2 - \frac{6}{47}H_2, \qquad \varphi_{12} = x_1x_2 + \frac{1}{4}\frac{47}{47^2}2H_2 = x_1x_2 + \frac{1}{2 \cdot 47}H_2,$$
$$\varphi_{13} = x_1x_3 - \frac{1}{4}\frac{12 \cdot 12}{47^2}2H_2 = x_1x_3 - \frac{72}{47^2}H_2, \tag{2.17}$$

which will be spherical functions of second order with respect to H_2.

(3) Similarly, for quadratic form

$$2G_2 = 6x_1^2 + 2x_1x_2 + 8x_2^2 + 6x_3^2 + 2x_3x_4 + 8x_4^2, \tag{2.18}$$

the determinant and some cofactors are

$$D = 47^2, \qquad A_{11} = 8 \cdot 47, \qquad A_{22} = 6 \cdot 47, \qquad A_{33} = 8 \cdot 47, \qquad A_{34} = -47. \tag{2.19}$$

By putting $2k = 4$, $Q = G_2$, and appropriate i, j in Theorem 2.1 in [3], we get

$$\begin{aligned} \varphi_{11} &= x_1^2 - \frac{1}{4}\frac{8 \cdot 47}{47^2} 2G_2 = x_1^2 - \frac{4}{47} G_2, \qquad \varphi_{22} = x_2^2 - \frac{1}{4}\frac{6 \cdot 47}{47^2} 2G_2 = x_2^2 - \frac{3}{47} G_2, \\ \varphi_{33} &= x_3^2 - \frac{1}{4}\frac{8 \cdot 47}{47^2} 2G_2 = x_3^2 - \frac{4}{47} G_2, \qquad \varphi_{34} = x_3x_4 + \frac{1}{4}\frac{47}{47^2} 2G_2 = x_3x_4 + \frac{1}{2 \cdot 47} G_2, \end{aligned} \tag{2.20}$$

which will be spherical functions of second order with respect to G_2.

(4) Similarly, for quadratic form

$$2(H_1 \oplus G_1) = 4x_1^2 + 2x_1x_2 + 12x_2^2 + 6x_3^2 + 2x_3x_4 + 8x_4^2, \tag{2.21}$$

the determinant and some cofactors are

$$D = 47^2, \qquad A_{11} = 12 \cdot 47, \qquad A_{22} = 4 \cdot 47, \qquad A_{33} = 8 \cdot 47. \tag{2.22}$$

By putting $2k = 4$, $Q = H_1 \oplus G_1$, and appropriate i, j in Theorem 2.1 in [3], we get

$$\begin{aligned} \varphi_{11} &= x_1^2 - \frac{1}{4}\frac{12 \cdot 47}{47^2} 2(H_1 \oplus G_1) = x_1^2 - \frac{6}{47}(H_1 \oplus G_1), \\ \varphi_{22} &= x_2^2 - \frac{1}{4}\frac{4 \cdot 47}{47^2} 2(H_1 \oplus G_1) = x_2^2 - \frac{2}{47}(H_1 \oplus G_1), \\ \varphi_{33} &= x_2^2 - \frac{1}{4}\frac{8 \cdot 47}{47^2} 2(H_1 \oplus G_1) = x_2^2 - \frac{4}{47}(H_1 \oplus G_1), \end{aligned} \tag{2.23}$$

which will be spherical functions of second order with respect to $H_1 \oplus G_1$.

Now we can determine a basis of $S_4(\Gamma_0(47))$ whose dimension is 11, see [2].

Theorem 2.2. *The following generalized 11 theta series:*

$$\begin{aligned}
\Theta_{F_2,\varphi_{11}}(q) &= \frac{1}{47}\sum_{n=1}^{\infty}\sum_{F_2=n}\left(47x_1^2 - 12F_2\right)q^n,\\
\Theta_{H_2,\varphi_{11}}(q) &= \frac{1}{47}\sum_{n=1}^{\infty}\sum_{H_2=n}\left(47x_1^2 - 6H_2\right)q^n,\\
\Theta_{H_2,\varphi_{12}}(q) &= \frac{1}{2\cdot 47}\sum_{n=1}^{\infty}\sum_{H_2=n}(2\cdot 47x_1x_2 + H_2)q^n,\\
\Theta_{H_2,\varphi_{13}}(q) &= \frac{1}{47^2}\sum_{n=1}^{\infty}\sum_{H_2=n}\left(47^2x_1x_3 - 72H_2\right)q^n,\\
\Theta_{G_2,\varphi_{11}}(q) &= \frac{1}{47}\sum_{n=1}^{\infty}\sum_{G_2=n}\left(47x_1^2 - 4G_2\right)q^n,\\
\Theta_{G_2,\varphi_{22}}(q) &= \frac{1}{47}\sum_{n=1}^{\infty}\sum_{G_2=n}\left(47x_2^2 - 3G_2\right)q^n,\\
\Theta_{G_2,\varphi_{33}}(q) &= \frac{1}{47}\sum_{n=1}^{\infty}\sum_{G_2=n}\left(47x_3^2 - 4G_2\right)q^n,\\
\Theta_{G_2,\varphi_{34}}(q) &= \frac{1}{2\cdot 47}\sum_{n=1}^{\infty}\sum_{G_2=n}(2\cdot 47x_3x_4 + G_2)q^n,\\
\Theta_{H_1\oplus G_1,\varphi_{11}}(q) &= \frac{1}{47}\sum_{n=1}^{\infty}\sum_{H_1\oplus G_1=n}\left(47x_1^2 - 6(H_1\oplus G_1)\right)q^n,\\
\Theta_{H_1\oplus G_1,\varphi_{22}}(q) &= \frac{1}{47}\sum_{n=1}^{\infty}\sum_{H_1\oplus G_1=n}\left(47x_2^2 - 2(H_1\oplus G_1)\right),\\
\Theta_{H_1\oplus G_1,\varphi_{33}}(q) &= \frac{1}{47}\sum_{n=1}^{\infty}\sum_{H_1\oplus G_1=n}\left(47x_3^2 - 4(H_1\oplus G_1)\right)
\end{aligned} \tag{2.24}$$

are a basis of $S_4(\Gamma_0(47))$.

Proof. The series are cusp forms because of Theorem 2.1 in [3]. Moreover, by simple calculations, we have

$$\begin{aligned}
\Theta_{F_2,\varphi_{11}}(q) &= \frac{1}{79}\left(46q + 192q^2 + 184q^4 + 460q^5 + 368q^8 + 414q^9 + 920q^{10} + 0q^{11} + \cdots\right),\\
\Theta_{H_2,\varphi_{11}}(q) &= \frac{1}{47}\left(46q^2 + 92q^4 - 144q^6 - 74q^7 - 12q^8 - 178q^9 + 460q^{10} - 152q^{11} + \cdots\right),\\
\Theta_{H_2,\varphi_{12}}(q) &= \frac{1}{2\cdot 47}\left(8q^2 + 16q^4 + 24q^6 - 160q^7 + 96q^8 - 80q^9 + 80q^{10} + 464q^{11} + \cdots\right),
\end{aligned}$$

$$\begin{aligned}
\Theta_{H_2,\varphi_{13}}(q) &= \frac{1}{47^2}\left(8q^2+16q^4+24q^6+28q^7+96q^8+108q^9+80q^{10}+88q^{11}+\cdots\right),\\
\Theta_{G_2,\varphi_{11}}(q) &= \frac{1}{47}\left(46q^3-64q^4-4q^6-36q^7-162q^8+88q^9-132q^{10}+24q^{11}+\cdots\right),\\
\Theta_{G_2,\varphi_{22}}(q) &= \frac{1}{47}\left(-48q^3+30q^4-98q^6-36q^7+26q^8-100q^9+56q^{10}-164q^{11}+\cdots\right),\\
\Theta_{G_2,\varphi_{33}}(q) &= \frac{1}{47}\left(46q^3-64q^4+90q^6-36q^7-162q^8+840q^9-132q^{10}+24q^{11}+\cdots\right),\\
\Theta_{G_2,\varphi_{34}}(q) &= \frac{1}{47}\left(-48q^3-64q^4-286q^6-36q^7-162q^8-476q^9-508q^{10}-164q^{11}+\cdots\right),\\
\Theta_{H_1\oplus G_1,\varphi_{11}}(q) &= \frac{1}{47}\left(70q^2-36q^3-48q^4+68q^5-100q^6+10q^7+180q^8-230q^9-344q^{10}\right.\\
&\qquad \left.-152q^{11}+\cdots\right),\\
\Theta_{H_1\oplus G_1,\varphi_{22}}(q) &= \frac{1}{47}\left(-24q^2-36q^3-48q^4-120q^5-194q^6+10q^7-384q^8-42q^9-344q^{10}\right.\\
&\qquad \left.-340q^{11}+\cdots\right),\\
\Theta_{H_1\oplus G_1,\varphi_{33}}(q) &= \frac{1}{47}\left(-16q^2+70q^3-32q^4+108q^5-98q^6-56q^7+26q^8-28q^9-104q^{10}\right.\\
&\qquad \left.-164q^{11}+\cdots\right).
\end{aligned} \tag{2.25}$$

The determinant of the coefficients matrix is 5321 028 802 318 956 232 704/47^{11}. So, the set of theta series in the Theorem is a basis of $S_4(\Gamma_0(47))$. □

3. Representation Numbers of n

Proposition 3.1. *The theta series of the quadratic forms are*

$$\begin{aligned}
\Theta_{F_4}(q) &= \Theta_{F_2}(q)\Theta_{F_2}(q) = 1+8q+24q^2+32q^3+24q^4+48q^5+96q^6+64q^7\\
&\qquad +24q^8+104q^9+144q^{10}+96q^{11}+\cdots,\\
\Theta_{H_4}(q) &= \Theta_{H_2}(q)\Theta_{H_2}(q) = 1+8q^2+24q^4+40q^6+8q^7+72q^8+56q^9+144q^{10}\\
&\qquad +144q^{11}+\cdots,\\
\Theta_{G_4}(q) &= \Theta_{G_2}(q)\Theta_{G_2}(q) = 1+8q^3+8q^4+32q^6+48q^7+32q^8+80q^9+144q^{10}\\
&\qquad +144q^{11}+\cdots,\\
\Theta_{F_3\oplus H_1}(q) &= \Theta_{F_3}(q)\Theta_{H_1}(q) = 1+6q+14q^2+20q^3+30q^4+40q^5+38q^6+62q^7\\
&\qquad +98q^8+84q^9+112q^{10}+184q^{11}+\cdots,
\end{aligned}$$

$$\begin{aligned}
\Theta_{F_2\oplus H_2}(q) &= \Theta_{F_2}(q)\Theta_{H_2}(q) = 1+4q+8q^2+16q^3+24q^4+24q^5+36q^6+52q^7\\
&\quad +64q^8+112q^9+144q^{10}+152q^{11}+\cdots,\\
\Theta_{F_1\oplus H_3}(q) &= \Theta_{F_1}(q)\Theta_{H_3}(q) = 1+2q+6q^2+12q^3+14q^4+24q^5+26q^6+34q^7\\
&\quad +66q^8+92q^9+136q^{10}+168q^{11}+\cdots,\\
\Theta_{F_3\oplus G_1}(q) &= \Theta_{F_3}(q)\Theta_{G_1}(q) = 1+6q+12q^2+10q^3+20q^4+60q^5+66q^6+40q^7\\
&\quad +98q^8+154q^9+108q^{10}+112q^{11}+\cdots,\\
\Theta_{F_2\oplus G_2}(q) &= \Theta_{F_2}(q)\Theta_{G_2}(q) = 1+4q+4q^2+4q^3+24q^4+40q^5+24q^6+56q^7\\
&\quad +124q^8+108q^9+112q^{10}+184q^{11}+\cdots,\\
\Theta_{F_1\oplus G_3}(q) &= \Theta_{F_1}(q)\Theta_{G_3}(q) = 1+2q+6q^3+20q^4+12q^5+18q^6+72q^7+78q^8\\
&\quad +70q^9+148q^{10}+192q^{11}+\cdots,\\
\Theta_{H_3\oplus G_1}(q) &= \Theta_{H_3}(q)\Theta_{G_1}(q) = 1+6q^2+2q^3+14q^4+12q^5+28q^6+30q^7+68q^8\\
&\quad +58q^9+124q^{10}+120q^{11}+\cdots,\\
\Theta_{H_2\oplus G_2}(q) &= \Theta_{H_2}(q)\Theta_{G_2}(q) = 1+4q^2+4q^3+8q^4+16q^5+28q^6+28q^7+68q^8\\
&\quad +68q^9+112q^{10}+144q^{11}+\cdots,\\
\Theta_{H_1\oplus G_3}(q) &= \Theta_{H_1}(q)\Theta_{G_3}(q) = 1+2q^2+6q^3+6q^4+12q^5+32q^6+26q^7+56q^8+94q^9\\
&\quad +108q^{10}+136q^{11}+\cdots,\\
\Theta_{F_2\oplus H_1\oplus G_1}(q) &= \Theta_{F_2}(q)\cdot\Theta_{H_1\oplus G_1}(q) = 1+4q+6q^2+10q^3+22q^4+28q^5+40q^6+74q^7\\
&\quad +76q^8+82q^9+148q^{10}+168q^{11}+\cdots,\\
\Theta_{F_1\oplus H_2\oplus G_1}(q) &= \Theta_{H_2}(q)\cdot\Theta_{F_1\oplus G_1}(q) = 1+2q+4q^2+10q^3+12q^4+20q^5+38q^6+44q^7\\
&\quad +66q^8+98q^9+108q^{10}+152q^{11}+\cdots,\\
\Theta_{F_1\oplus H_1\oplus G_2}(q) &= \Theta_{G_2}(q)\cdot\Theta_{F_1\oplus H_1}(q) = 1+2q+2q^2+8q^3+14q^4+16q^5+38q^6+54q^7\\
&\quad +54q^8+104q^9+144q^{10}+144q^{11}+\cdots,
\end{aligned} \tag{3.1}$$

and the substraction of the any one of these theta series by the Eisenstein series

$$\begin{aligned}
E(q:F_4) &= \cdots = E(q:F_1\oplus H_1\oplus G_2) = 1+\frac{24}{221}\sum_{n=1}^{\infty}\sigma_3^*(n)q^n\\
&= 1+\frac{24}{221}q+\frac{24\cdot 9}{221}q^2+\frac{24\cdot 28}{221}q^3+\frac{24\cdot 73}{221}q^4+\frac{24\cdot 126}{221}q^5+\frac{24\cdot 252}{221}q^6\\
&\quad +\frac{24\cdot 344}{221}q^7+\frac{24\cdot 585}{221}q^8+\frac{24\cdot 757}{221}q^9+\frac{24\cdot 1134}{221}q^{10}+\frac{24\cdot 1332}{221}q^{11}+\cdots
\end{aligned} \tag{3.2}$$

is a linear combinations of the theta series in the preceding theorem. The coefficients are given in table [4].

Proof. By determination of solutions of

$$F_1 = n, \quad H_1 = n, \quad G_1 = n \quad \text{for } n = 1, 2, \ldots, 11, \tag{3.3}$$

we easily calculate the theta series

$$\begin{gathered}\Theta_{F_1}, \Theta_{H_1}, \Theta_{G_1}, \Theta_{F_2}, \Theta_{F_3}, \Theta_{F_4}, \Theta_{H_2}, \Theta_{H_3}, \Theta_{H_4}, \Theta_{G_2}, \Theta_{G_3}, \Theta_{G_4}, \Theta_{F_1 \oplus H_1}, \Theta_{F_3 \oplus \mathrm{H}_1}, \Theta_{F_2 \oplus H_2}, \\ \Theta_{F_1 \oplus H_3}, \Theta_{F_1 \oplus G_1}, \Theta_{F_3 \oplus G_1}, \Theta_{F_4}, \Theta_{F_2 \oplus G_2}, \Theta_{F_1 \oplus G_3}, \Theta_{H_1 \oplus G_1}, \Theta_{H_3 \oplus G_1}, \Theta_{H_2 \oplus G_2}, \Theta_{H_1 \oplus G_3}, \\ \Theta_{F_2 \oplus H_1 \oplus G_1}, \Theta_{F_1 \oplus H_2 \oplus G_1}, \Theta_{F_1 \oplus H_1 \oplus G_2}.\end{gathered} \tag{3.4}$$

For the second part, now let us look at the case:

$$\begin{aligned}\Theta_{F_4}(q) - E(q : F) &= \frac{1744}{221} q + \frac{5088}{221} q^2 + \frac{6400}{221} q^3 + \frac{3552}{221} q^4 + \frac{7584}{221} q^5 + \frac{15168}{221} q^6 \\ &\quad + \frac{5888}{221} q^7 - \frac{672}{221} q^8 + \frac{4816}{221} q^9 + \frac{4608}{221} q^{10} - \frac{10752}{221} q^{11} + \cdots \\ &= c_1 \Theta_{F_2, \varphi_{11}}(q) + c_2 \Theta_{H_2, \varphi_{11}}(q) + c_3 \Theta_{H_2, \varphi_{12}}(q) + c_4 \Theta_{H_2, \varphi_{13}}(q) \\ &\quad + c_5 \Theta_{G_2, \varphi_{11}}(q) + c_6 \Theta_{G_2, \varphi_{22}}(q) + c_7 \Theta_{G_2, \varphi_{33}}(q) + c_8 \Theta_{G_2, \varphi_{34}}(q) \\ &\quad + c_9 \Theta_{H_1 \oplus G_1, \varphi_{11}}(q) + c_{10} \Theta_{H_1 \oplus G_1, \varphi_{22}}(q) + c_{11} \Theta_{H_1 \oplus G_1, \varphi_{33}}(q).\end{aligned} \tag{3.5}$$

By equating the coefficients of q^n in both sides for $n = 1, 2, 3, \ldots, 11$, we get an equation in coefficients

$$\begin{gathered}(c_1 \quad c_2 \quad c_3 \quad c_4 \quad c_5 \quad c_6), \\ (c_7 \quad c_8 \quad c_9 \quad c_{10} \quad c_{11}).\end{gathered} \tag{3.6}$$

We repeat the same procedure for the other cases. At the end, by solving 11 linear equations in 11 variables, we get the coefficients in table [4]. □

Corollary 3.2. *The representation numbers for the quadratic forms*

$$\begin{aligned}\Omega = {} & F_4, H_4, H_4', G_4, G_4', F_3 \oplus H_1, F_3 \oplus H_1', F_2 \oplus H_2, F_2 \oplus H_2', F_1 \oplus H_3, F_1 \oplus H_3', \\ & F_3 \oplus G_1, F_3 \oplus G_1', F_2 \oplus G_2, F_2 \oplus G_2', F_1 \oplus G_3, F_1 \oplus G_3', \ H_3 \oplus G_1, \ H_3' \oplus G_1, \ H_3 \oplus G_1', H_3' \oplus G_1', \\ & H_2 \oplus G_2, \ H_2' \oplus G_2, H_2 \oplus G_2', H_2' \oplus G_2', H_1 \oplus G_3, H_1' \oplus G_3, H_1 \oplus G_3', H_1' \oplus G_3', \\ & F_2 \oplus H_1 \oplus G_1, F_1 \oplus H_2 \oplus G_1, F_1 \oplus H_1 \oplus G_2\end{aligned} \tag{3.7}$$

are

$$
\begin{aligned}
r(n;H) = {} & \frac{24}{221}\sigma_3^*(n) + \frac{c_1}{47}\sum_{F_2=n}\left(47x_1^2 - 12n\right) + \frac{c_2}{47}\sum_{H_2=n}\left(47x_1^2 - 6n\right) \\
& + \frac{c_3}{2\cdot 47}\sum_{H_2=n}(2\cdot 47x_1x_2 + n) + \frac{c_4}{47^2}\sum_{H_2=n}\left(47^2x_1x_3 - 72n\right) + \frac{c_5}{47}\sum_{G_2=n}\left(47x_1^2 - 4n\right) \\
& + \frac{c_6}{47}\sum_{G_2=n}\left(47x_2^2 - 3n\right) + \frac{c_7}{47}\sum_{G_2=n}\left(47x_3^2 - 4n\right) + \frac{c_8}{2\cdot 47}\sum_{G_2=n}(2\cdot 47x_3x_4 + n) \\
& + \frac{c_9}{47}\sum_{H_1\oplus G_1=n}\left(47x_1^2 - 6n\right) + \frac{c_{10}}{47}\sum_{H_1\oplus G_1=n}\left(47x_2^2 - 2n\right) + \frac{c_{11}}{47}\sum_{H_1\oplus G_1=n}\left(47x_3^2 - 4n\right).
\end{aligned}
\tag{3.8}
$$

The coefficients

$$
\begin{gathered}
\left(c_1 \;\; c_2 \;\; c_3 \;\; c_4 \;\; c_5 \;\; c_6\right), \\
\left(c_7 \;\; c_8 \;\; c_9 \;\; c_{10} \;\; c_{11}\right)
\end{gathered}
\tag{3.9}
$$

corresponding to the quadratic form Ω are given in [4].

Proof. It follows from the preceding theorem. □

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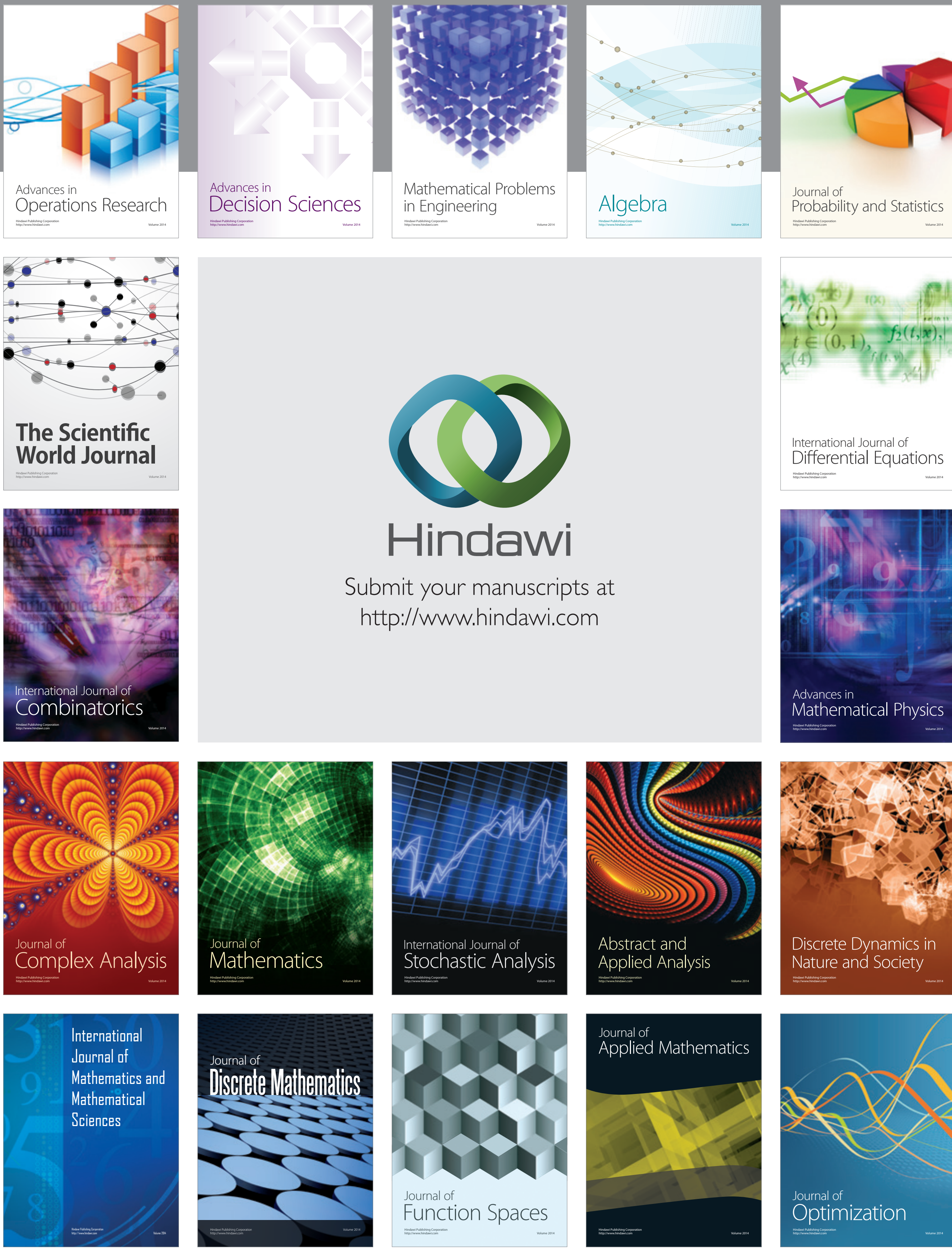
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