Hindawi Publishing Corporation
International Journal of Mathematics and Mathematical Sciences
Volume 2012, Article ID 670410, 12 pages
doi:10.1155/2012/670410

Research Article

Coupled Fixed Point Results in Complete Partial Metric Spaces

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Received 23 March 2012; Revised 18 September 2012; Accepted 25 September 2012

Academic Editor: A. Zayed

We establish some coupled fixed point theorems for a mapping satisfying some contraction conditions in complete partial metric spaces. Our consequences extend the results of H. Aydi (2011).

1. Introduction and Mathematical Preliminaries

The notion of a partial metric space (PMS) was introduced in 1992 by Matthews [1, 2]. Matthews proved a fixed point theorem on this spaces, analogous to the Banach's fixed point theorem. Recently, many authors have focused on partial metric spaces and their topological properties (see e.g. [3–9]).

The definition of a partial metric space is given by Matthews (see [1, 2]) as follows:

Definition 1.1. Let X be a nonempty set and let $p : X \times X \to R^+$ satisfies

(P1) $x = y \Leftrightarrow p(x,x) = p(y,y) = p(x,y)$, for all $x, y \in X$,

(P2) $p(x,x) \le p(x,y)$, for all $x, y \in X$,

(P3) $p(x,y) = p(y,x)$, for all $x, y \in X$,

(P4) $p(x,y) \le p(x,z) + p(z,y) - p(z,z)$, for all $x, y, z \in X$.

Then the pair (X, p) is called a partial metric space and p is called a partial metric on X.

The function $d_p : X \times X \to R^+$ defined by

$$d_p(x,y) = 2p(x,y) - p(x,x) - p(y,y) \tag{1.1}$$

satisfies the conditions of a metric on X; therefore it is a (usual) metric on X.

Remark 1.2. if $x = y$, $p(x,y)$ may not be 0.

(1) A famous example of partial metric spaces is the pair (R^+, p), where $p(x,y) = \max\{x,y\}$ for all $x, y \in R^+$. In this case, d_p is the Euclidian metric $d_p(x,y) = |x-y|$.

(2) Each partial metric p on X generates a T_0 topology τ_p on X which has a base of open p-balls $B_p(x,\varepsilon)$, where $x \in X$ and $\varepsilon > 0$ ($B_p(x,\varepsilon) = \{y \in X : p(x,y) < p(x,x) + \varepsilon\}$).

The following concepts has been defined as follows on a partial metric space.

Definition 1.3 (see e.g., [1, 2]). (i) A sequence $\{x_n\}$ in a PMS (X,p) converges to $x \in X$ if and only if $p(x,x) = \lim_{n\to\infty} p(x,x_n)$.

(ii) A sequence $\{x_n\}$ in a PMS (X,p) is called Cauchy if and only if $\lim_{n,m\to\infty} p(x_n,x_m)$ exists (and is finite).

(iii) A PMS $\{X,p\}$ is said to be complete if every Cauchy sequence $\{x_n\}$ in X converges, with respect to τ_p, to a point $x \in X$ such that $p(x,x) = \lim_{n,m\to\infty} p(x_n,x_m)$.

The concept of coupled fixed point have been introduced in [10] by Bhaskar and Lakshmikantham as follows.

Definition 1.4 (see [10]). An element $(x,y) \in X \times X$ is called a coupled fixed point of mapping $F : X \times X \to X$ if $x = F(x,y)$ and $y = F(y,x)$.

Aydi in [11] has obtained some coupled fixed point results for mappings satisfying different contractive conditions on complete partial metric spaces. Some of these results are the following cases.

Theorem 1.5 (see [11, Theorem 2.1]). *Let (X,p) be a complete partial metric space. Suppose that the mapping $F : X \times X \to X$ satisfies the following contractive condition:*

$$p(F(x,y), F(u,v)) \le kp(x,u) + lp(y,v), \tag{1.2}$$

for all $x,y,u,v \in X$, where k,l are nonnegative constants with $k + l < 1$. Then, F has a unique coupled fixed point.

Theorem 1.6 (see [11, Theorem 2.4]). *Let (X, p) be a complete partial metric space. Suppose that the mapping $F : X \times X \to X$ satisfies the following contractive condition:*

$$p(F(x,y), F(u,v)) \le kp(F(x,y),x) + lp(F(u,v),u), \tag{1.3}$$

for all $x,y,u,v \in X$, where k,l are nonnegative constants with $k + l < 1$. Then, F has a unique coupled fixed point.

Theorem 1.7 (see [11, Theorem 2.5]). *Let (X, p) be a complete partial metric space. Suppose that the mapping $F : X \times X \to X$ satisfies the following contractive condition:*

$$p(F(x,y),F(u,v)) \le kp(F(x,y),u) + lp(F(u,v),x), \tag{1.4}$$

for all $x,y,u,v \in X$, where k,l are nonnegative constants with $k + l < 1$. Then, F has a unique coupled fixed point.

For a survey of fixed point theory, its applications, and related results in partial metric spaces we refer the reader to [4, 5, 12–20] and the references mentioned therein. Also, many researchers have obtained coupled fixed point results for mappings under various contractive conditions in the framework of partial metric spaces (see, e.g., [21, 22]).

In this paper we establish some coupled fixed point results of contractive mappings in the framework of complete partial metric spaces. Our results extend and generalize the results of Aydi [11].

2. Main Results

We recall three easy lemmas which have an essential role in the proof of the main result. These results can be derived easily (see, e.g., [1, 2, 6]).

Lemma 2.1. (1) *A sequence $\{x_n\}$ is a Cauchy sequence in the PMS (X,p) if and only if it is a Cauchy sequence in the metric space (X,d_p).*

(2) *A PMS (X,p) is complete if and only if the metric space (X,d_p) is complete. Moreover,*

$$\lim_{n\longrightarrow\infty} d_p(x,x_n) = 0 \Longleftrightarrow p(x,x) = \lim_{n\longrightarrow\infty} p(x,x_n) = \lim_{n,m\longrightarrow\infty} p(x_n,x_m). \tag{2.1}$$

Lemma 2.2 (see [3]). *Assume that $x_n \to z$ as $n \to \infty$ in a PMS (X,p) such that $p(z,z) = 0$. Then, $\lim_{n\to\infty} p(x_n,y) = p(z,y)$, for every $y \in X$.*

Lemma 2.3 (see, e.g., [3, 4]). *Let (X, p) be a complete PMS. Then,*

(a) *if $p(x,y) = 0$ then, $x = y$,*

(b) *if $x \ne y$, then $p(x,y) > 0$.*

Throughout this paper, we assume that all of the constants are nonnegative. Our main result is the following. The method of the proof can be found in [11].

Theorem 2.4. *Let (X,p) be a complete partial metric space and $F : X \times X \to X$ be a mapping such that*

$$\begin{aligned} p(F(x,y),F(u,v)) \le{}& \alpha_1 p(x,u) + \alpha_2 p(y,v) \\ &+ \alpha_3 p(F(x,y),x) + \alpha_4 p(F(y,x),y) \\ &+ \alpha_5 p(F(x,y),u) + \alpha_6 p(F(y,x),v) + \alpha_7 p(F(u,v),x) \\ &+ \alpha_8 p(F(v,u),y) + \alpha_9 p(F(u,v),u) + \alpha_{10} p(F(v,u),v), \end{aligned} \tag{2.2}$$

for every pairs $(x,\ y), (u,\ v) \in X \times X$, *where* $\sum_{i=1}^{10} \alpha_i < 1$. *Then,* F *has a unique coupled fixed point in* X.

Proof. Let $x_0, y_0 \in X$ be arbitrary. Define $x_1, y_1 \in X$ such that $x_1 = F(x_0, y_0)$ and $y_1 = F(y_0, x_0)$ and in this way, we construct the sequences $\{x_n\}$ and $\{y_n\}$ as $x_n = F(x_{n-1}, y_{n-1})$ and $y_n = F(y_{n-1}, x_{n-1})$, for all $n \geq 0$.

We will complete the proof in three steps.

Step I. Let $\delta_n = p(x_{n-1}, x_n) + p(y_{n-1}, y_n)$. We will show that $\lim_{n\to\infty} \delta_n = 0$.

Using (2.2) we obtain that

$$
\begin{aligned}
p(x_n, x_{n+1}) &= p(F(x_{n-1}, y_{n-1}), F(x_n, y_n)) \\
&\leq \alpha_1 p(x_{n-1}, x_n) + \alpha_2 p(y_{n-1}, y_n) + \alpha_3 p(F(x_{n-1}, y_{n-1}), x_{n-1}) \\
&\quad + \alpha_4 p(F(y_{n-1}, x_{n-1}), y_{n-1}) \\
&\quad + \alpha_5 p(F(x_{n-1}, y_{n-1}), x_n) + \alpha_6 p(F(y_{n-1}, x_{n-1}), y_n) + \alpha_7 p(F(x_n, y_n), x_{n-1}) \\
&\quad + \alpha_8 p(F(y_n, x_n), y_{n-1}) + \alpha_9 p(F(x_n, y_n), x_n) + \alpha_{10} p(F(y_n, x_n), y_n) \\
&= \alpha_1 p(x_{n-1}, x_n) + \alpha_2 p(y_n, y_{n-1}) + \alpha_3 p(x_n, x_{n-1}) + \alpha_4 p(y_n, y_{n-1}) + \alpha_5 p(x_n, x_n) \\
&\quad + \alpha_6 p(y_n, y_n) + \alpha_7 p(x_{n+1}, x_{n-1}) + \alpha_8 p(y_{n+1}, y_{n-1}) + \alpha_9 p(x_{n+1}, x_n) \\
&\quad + \alpha_{10} p(y_{n+1}, y_n) \\
&\leq \alpha_1 p(x_{n-1}, x_n) + \alpha_2 p(y_{n-1}, y_n) + \alpha_3 p(x_n, x_{n-1}) + \alpha_4 p(y_n, y_{n-1}) \\
&\quad + (\alpha_5 - \alpha_7) p(x_n, x_n) + (\alpha_6 - \alpha_8) p(y_n, y_n) \\
&\quad + \alpha_7 \left[p(x_{n+1}, x_n) + p(x_n, x_{n-1})\right] + \alpha_8 \left[p(y_{n+1}, y_n) + p(y_n, y_{n-1})\right] \\
&\quad + \alpha_9 p(x_{n+1}, x_n) + \alpha_{10} p(y_{n+1}, y_n).
\end{aligned}
\tag{2.3}
$$

Analogously, starting from $p(x_{n+1}, x_n) = p(F(x_n, y_n), F(x_{n-1}, y_{n-1}))$, we have

$$
\begin{aligned}
p(x_{n+1}, x_n) &= p(F(x_n, y_n), F(x_{n-1}, y_{n-1})) \\
&\leq \alpha_1 p(x_n, x_{n-1}) + \alpha_2 p(y_n, y_{n-1}) + \alpha_3 p(F(x_n, y_n), x_n) + \alpha_4 p(F(y_n, x_n), y_n) \\
&\quad + \alpha_5 p(F(x_n, y_n), x_{n-1}) + \alpha_6 p(F(y_n, x_n), y_{n-1}) + \alpha_7 p(F(x_{n-1}, y_{n-1}), x_n) \\
&\quad + \alpha_8 p(F(y_{n-1}, x_{n-1}), y_n) + \alpha_9 p(F(x_{n-1}, y_{n-1}), x_{n-1}) + \alpha_{10} p(F(y_{n-1}, x_{n-1}), y_{n-1}) \\
&= \alpha_1 p(x_{n-1}, x_n) + \alpha_2 p(y_n, y_{n-1}) + \alpha_3 p(x_{n+1}, x_n) + \alpha_4 p(y_{n+1}, y_n) + \alpha_5 p(x_{n+1}, x_{n-1}) \\
&\quad + \alpha_6 p(y_{n+1}, y_{n-1}) + \alpha_7 p(x_n, x_n) + \alpha_8 p(y_n, y_n) + \alpha_9 p(x_n, x_{n-1}) + \alpha_{10} p(y_n, y_{n-1}) \\
&\leq \alpha_1 p(x_{n-1}, x_n) + \alpha_2 p(y_{n-1}, y_n) + \alpha_3 p(x_{n+1}, x_n) + \alpha_4 p(y_{n+1}, y_n) \\
&\quad + \alpha_5 \left[p(x_{n+1}, x_n) + p(x_n, x_{n-1})\right] + \alpha_6 \left[p(y_{n+1}, y_n) + p(y_n, y_{n-1})\right]
\end{aligned}
$$

$$+ (\alpha_7 - \alpha_5)p(x_n, x_n) + (\alpha_8 - \alpha_6)p(y_n, y_n) + \alpha_9 p(x_n, x_{n-1}) + \alpha_{10} p(y_n, y_{n-1}). \tag{2.4}$$

In a similar way, we have

$$\begin{aligned} p(y_n, y_{n+1}) &= p(F(y_{n-1}, x_{n-1}), F(y_n, x_n)) \\ &\leq \alpha_1 p(y_{n-1}, y_n) + \alpha_2 p(x_{n-1}, x_n) + \alpha_3 p(y_n, y_{n-1}) + \alpha_4 p(x_n, x_{n-1}) \\ &\quad + (\alpha_5 - \alpha_7)p(y_n, y_n) + (\alpha_6 - \alpha_8)p(x_n, x_n) \\ &\quad + \alpha_7 [p(y_{n+1}, y_n) + p(y_n, y_{n-1})] + \alpha_8 [p(x_{n+1}, x_n) + p(x_n, x_{n-1})] \\ &\quad + \alpha_9 p(y_{n+1}, y_n) + \alpha_{10} p(x_{n+1}, x_n). \end{aligned} \tag{2.5}$$

Analogously, starting from $p(y_{n+1}, y_n) = p(F(y_n, x_n), F(y_{n-1}, x_{n-1}))$, we have

$$\begin{aligned} p(y_{n+1}, y_n) &= p(F(y_n, x_n), F(y_{n-1}, x_{n-1})) \\ &\leq \alpha_1 p(y_{n-1}, y_n) + \alpha_2 p(x_{n-1}, x_n) + \alpha_3 p(y_{n+1}, y_n) + \alpha_4 p(x_{n+1}, x_n) \\ &\quad + \alpha_5 [p(y_{n+1}, y_n) + p(y_n, y_{n-1})] + \alpha_6 [p(x_{n+1}, x_n) + p(x_n, x_{n-1})] \\ &\quad + (\alpha_7 - \alpha_5)p(y_n, y_n) + (\alpha_8 - \alpha_6)p(x_n, x_n) \\ &\quad + \alpha_9 p(y_n, y_{n-1}) + \alpha_{10} p(x_n, x_{n-1}). \end{aligned} \tag{2.6}$$

Adding (2.3), (2.4), (2.5), and (2.6) we obtain that

$$\begin{aligned} 2\delta_{n+1} &\leq [2\alpha_1 + 2\alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 + \alpha_7 + \alpha_8 + \alpha_9 + \alpha_{10}]\delta_n \\ &\quad + [\alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 + \alpha_7 + \alpha_8 + \alpha_9 + \alpha_{10}]\delta_{n+1}, \end{aligned} \tag{2.7}$$

or, equivalently,

$$\delta_{n+1} \leq \lambda \delta_n, \tag{2.8}$$

where, $\lambda = [2\alpha_1+2\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8+\alpha_9+\alpha_{10}]/(2-[\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8+\alpha_9+\alpha_{10}])$.

Repeating the above mentioned process, we have

$$\delta_{n+1} \leq \lambda \delta_n \leq \lambda^2 \delta_{n-1} \leq \cdots \leq \lambda^{n+1} \delta_0, \tag{2.9}$$

where, from our assumption about coefficients α_i, $\lambda \in [0, 1)$; hence,

$$\lim_{n \longrightarrow \infty} \delta_n = 0. \tag{2.10}$$

Step II. $\{x_n\}$ and $\{y_n\}$ are Cauchy.

If $\delta_0 = 0$ then, $p(x_0, x_1) + p(y_0, y_1) = 0$. Hence, we get $x_0 = x_1 = F(x_0, y_0)$ and $y_0 = y_1 = F(y_0, x_0)$; that is, (x_0, y_0) is a coupled fixed point of F. Now, let $\delta_0 > 0$. For each $m \geq n$, we have

$$\begin{aligned} p(x_m, x_n) + p(y_m, y_n) &\leq p(x_m, x_{m-1}) + p(y_m, y_{m-1}) \\ &\quad + p(x_{m-1}, x_{m-2}) + p(y_{m-1}, y_{m-2}) \\ &\quad + \cdots \\ &\quad + p(x_{n+1}, x_n) + p(y_{n+1}, y_n) \\ &= \delta_m + \delta_{m-1} + \cdots + \delta_{n+1} \\ &\leq \left[\lambda^m + \lambda^{m-1} + \cdots + \lambda^{n+1}\right]\delta_0 \leq \frac{\lambda^{n+1}}{1-\lambda}\delta_0. \end{aligned} \tag{2.11}$$

So, we have $\lim_{n,m\to\infty} p(x_n, x_m) + p(y_n, y_m) = 0$. This proves that $\{x_n\}$ and $\{y_n\}$ are Cauchy sequences in (X, p) and hence $\{x_n\}$ and $\{y_n\}$ are Cauchy sequences in the metric space (X, d_p). From Lemma 2.1, (X, d_p) is complete, so $\{x_n\}$ and $\{y_n\}$ converge to some $x, y \in X$, respectively; that is, $\lim_{n\to\infty} d_p(x_n, x) = 0$ and $\lim_{n\to\infty} d_p(y_n, y) = 0$. Therefore, from Lemma 2.1 and (2.10), we have

$$p(x, x) = \lim_{n\longrightarrow\infty} p(x_n, x) = \lim_{n,m\longrightarrow\infty} p(x_n, x_m) = 0, \tag{2.12}$$

$$p(y, y) = \lim_{n\longrightarrow\infty} p(y_n, y) = \lim_{n,m\longrightarrow\infty} p(y_n, y_m) = 0. \tag{2.13}$$

Step III. We will show that F has a unique coupled fixed point.

From the above step,

$$\lim_{n\longrightarrow\infty} p(F(x_n, y_n), x) = \lim_{n\longrightarrow\infty} p(F(y_n, x_n), y) = 0. \tag{2.14}$$

Next, we will prove that $x = F(x, y)$ and $y = F(y, x)$.

We have

$$p(x, F(x, y)) \leq p(x, x_{n+1}) + p(x_{n+1}, F(x, y)) - p(x_{n+1}, x_{n+1}). \tag{2.15}$$

Taking the limit as $n \to \infty$ in the above inequality, as $x_{n+1} = F(x_n, y_n)$ and using triangle inequality and (2.12), we have

$$\begin{aligned} p(x, F(x, y)) &\leq \lim_{n\longrightarrow\infty} p(x, x_{n+1}) + \lim_{n\longrightarrow\infty} p(F(x_n, y_n), F(x, y)) \\ &= \lim_{n\longrightarrow\infty} p(F(x_n, y_n), F(x, y)). \end{aligned} \tag{2.16}$$

But, for all $n \geq 0$, from (2.2),

$$\begin{aligned} p(F(x_n, y_n), F(x, y)) &\leq \alpha_1 p(x_n, x) + \alpha_2 p(y_n, y) \\ &\quad + \alpha_3 p(F(x_n, y_n), x_n) + \alpha_4 p(F(y_n, x_n), y_n) \\ &\quad + \alpha_5 p(F(x_n, y_n), x) + \alpha_6 p(F(y_n, x_n), y) + \alpha_7 p(F(x, y), x_n) \\ &\quad + \alpha_8 p(F(y, x), y_n) + \alpha_9 p(F(x, y), x) + \alpha_{10} p(F(y, x), y) \\ &= \alpha_1 p(x_n, x) + \alpha_2 p(y_n, y) + \alpha_3 p(x_{n+1}, x) \\ &\quad + \alpha_4 p(y_{n+1}, y) + \alpha_5 p(x_{n+1}, x) + \alpha_6 p(y_{n+1}, y) + \alpha_7 p(F(x, y), x_n) \\ &\quad + \alpha_8 p(F(y, x), y_n) + \alpha_9 p(F(x, y), x) + \alpha_{10} p(F(y, x), y). \end{aligned} \tag{2.17}$$

In the above inequality, if $n \to \infty$, using (2.12) and Lemma 2.2 we have

$$\lim_{n \to \infty} p(F(x_n, y_n), F(x, y)) \leq (\alpha_7 + \alpha_9) p(F(x, y), x) + (\alpha_8 + \alpha_{10}) p(F(y, x), y). \tag{2.18}$$

Analogously,

$$p(y, F(y, x)) \leq p(y, y_{n+1}) + p(y_{n+1}, F(y, x)) - p(y_{n+1}, y_{n+1}). \tag{2.19}$$

Taking the limit as $n \to \infty$ in the above inequality, since $y_{n+1} = F(y_n, x_n)$ and using triangle inequality and (2.13), we have

$$\begin{aligned} p(y, F(y, x)) &\leq \lim_{n \to \infty} p(y, y_{n+1}) + \lim_{n \to \infty} p(F(y_n, x_n), F(y, x)) \\ &= \lim_{n \to \infty} p(F(y_n, x_n), F(y, x)). \end{aligned} \tag{2.20}$$

Similar to (2.17), we have

$$\lim_{n \to \infty} p(F(y_n, x_n), F(y, x)) \leq (\alpha_7 + \alpha_9) p(F(y, x), y) + (\alpha_8 + \alpha_{10}) p(F(x, y), x). \tag{2.21}$$

Adding (2.18) and (2.21) and using (2.15) and (2.19), we obtain that

$$\begin{aligned} &p(x, F(x, y)) + p(y, F(y, x)) \\ &\quad \leq (\alpha_7 + \alpha_8 + \alpha_9 + \alpha_{10}) [p(x, F(x, y)) + p(y, F(y, x))]. \end{aligned} \tag{2.22}$$

Therefore, $p(x, F(x, y)) + p(y, F(y, x)) = 0$; that is, $F(x, y) = x$ and $F(y, x) = y$. □

Remark 2.5. (1) If in the above theorem, we assume that $\alpha_i = 0$, for all $3 \leq i \leq 10$, then we obtain the result of Aydi in [11] which is noted here in Theorem 1.5.

(2) If in the above theorem, $\alpha_i = 0$, for all $1 \leq i \leq 10$, unless $i = 3, 9$, then we obtain the result of Aydi in [11] which is mentioned here in Theorem 1.6.

(3) If in the above theorem, we assume that $\alpha_i = 0$, for all $3 \leq i \leq 10$, except that $i \neq 5, 7$, then we obtain the result of Aydi in [11] (Theorem 1.7).

Many results can be deduced from the above theorem as follows.

Corollary 2.6. *Let* (X, p) *be a complete partial metric space and* $F : X \times X \to X$ *be a mapping such that*

$$p(F(x,y), F(u,v)) \leq \alpha_1 p(F(x,y), x) + \alpha_2 p(F(y,x), y) + \alpha_3 p(F(u,v), u) + \alpha_4 p(F(v,u), v), \tag{2.23}$$

for every pairs $(x,y), (u,v) \in X \times X$, *where* $\sum_{i=1}^{4} \alpha_i < 1$. *Then,* F *has a unique coupled fixed point in* X.

Corollary 2.7. *Let* (X, p) *be a complete partial metric space and* $F : X \times X \to X$ *be a mapping such that*

$$p(F(x,y), F(u,v)) \leq \alpha_1 p(F(x,y), u) + \alpha_2 p(F(y,x), v) + \alpha_3 p(F(u,v), x) + \alpha_4 p(F(v,u), y), \tag{2.24}$$

for every pairs $(x,y), (u,v) \in X \times X$, *where* $\sum_{i=1}^{4} \alpha_i < 1$. *Then,* F *has a unique coupled fixed point in* X.

Corollary 2.8. *Let* (X, p) *be a complete partial metric space and* $F : X \times X \to X$ *be a mapping such that*

$$p(F(x,y), F(u,v)) \leq \alpha_1 p(F(x,y), x) + \alpha_2 p(F(y,x), y) + \alpha_3 p(F(u,v), x) + \alpha_4 p(F(v,u), y), \tag{2.25}$$

for every pairs $(x,y), (u,v) \in X \times X$, *where* $\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 < 1$. *Then,* F *has a unique coupled fixed point in* X.

Corollary 2.9. *Let* (X, p) *be a complete partial metric space and* $F : X \times X \to X$ *be a mapping such that*

$$p(F(x,y), F(u,v)) \leq \alpha_1 p(F(x,y), u) + \alpha_2 p(F(y,x), v) + \alpha_3 p(F(u,v), u) + \alpha_4 p(F(v,u), v), \tag{2.26}$$

for every pairs $(x,y), (u,v) \in X \times X$, *where* $\sum_{i=1}^{4} \alpha_i < 1$. *Then,* F *has a unique coupled fixed point in* X.

Also, we have the following results, when the constants in the above corollaries are equal.

Corollary 2.10. *Let* (X, p) *be a complete partial metric space and* $F : X \times X \to X$ *be a mapping such that*

$$\begin{aligned} p(F(x,y), F(u,v)) \le{}& \frac{k}{2}[p(x,u) + p(y,v)] \\ &+ \frac{l}{2}[p(F(x,y),x) + p(F(y,x),y)] \\ &+ \frac{r}{2}[p(F(x,y),u) + p(F(y,x),v)] + \frac{s}{2}[p(F(u,v),x) + p(F(v,u),y)] \\ &+ \frac{t}{2}[p(F(u,v),u) + p(F(v,u),v)], \end{aligned} \tag{2.27}$$

for every pairs $(x, y), (u, v) \in X \times X$*, where* $k + l + r + s + t < 1$*. Then,* F *has a unique coupled fixed point in* X*.*

Corollary 2.11. *Let* (X, p) *be a complete partial metric space and* $F : X \times X \to X$ *be a mapping such that*

$$\begin{aligned} p(F(x,y), F(u,v)) \le{}& \frac{k}{2}[p(F(x,y),x) + p(F(y,x),y)] \\ &+ \frac{l}{2}[p(F(u,v),u) + p(F(v,u),v)], \end{aligned} \tag{2.28}$$

for every pairs $(x, y), (u, v) \in X \times X$*, where* $k + l < 1$*. Then,* F *has a unique coupled fixed point in* X*.*

Corollary 2.12. *Let* (X, p) *be a complete partial metric space and* $F : X \times X \to X$ *be a mapping such that*

$$\begin{aligned} p(F(x,y), F(u,v)) \le{}& \frac{k}{2}[p(F(x,y),u) + p(F(y,x),v)] \\ &+ \frac{l}{2}[p(F(u,v),x) + p(F(v,u),y)], \end{aligned} \tag{2.29}$$

for every pairs $(x, y), (u, v) \in X \times X$*, where* $k + l < 1$*. Then,* F *has a unique coupled fixed point in* X*.*

Corollary 2.13. *Let* (X, p) *be a complete partial metric space and* $F : X \times X \to X$ *be a mapping such that*

$$\begin{aligned} p(F(x,y), F(u,v)) \le{}& \frac{k}{2}[p(F(x,y),x) + p(F(y,x),y)] \\ &+ \frac{l}{2}[p(F(u,v),x) + p(F(v,u),y)], \end{aligned} \tag{2.30}$$

for every pairs $(x, y), (u, v) \in X \times X$*, where* $k + l < 1$*. Then,* F *has a unique coupled fixed point in* X*.*

Corollary 2.14. *Let (X, p) be a complete partial metric space and $F : X \times X \to X$ be a mapping such that*

$$\begin{aligned} p(F(x,y),F(u,v)) \le{}& \frac{k}{2}[p(x,u)+p(y,v)] \\ &+\frac{l}{4}[p(F(x,y),x)+p(F(y,x),y)+p(F(u,v),u)+p(F(v,u),v)] \\ &+\frac{r}{4}[p(F(x,y),u)+p(F(y,x),v)+p(F(u,v),x)+p(F(v,u),y)], \end{aligned} \tag{2.31}$$

for every pairs $(x, y), (u, v) \in X \times X$, where $k + l + r < 1$. Then, F has a unique coupled fixed point in X.

Corollary 2.15. *Let (X, p) be a complete partial metric space and $F : X \times X \to X$ be a mapping such that*

$$\begin{aligned} p(F(x,y),F(u,v)) \le{}& \frac{k}{2}[p(F(x,y),u)+p(F(y,x),v)] \\ &+\frac{l}{2}[p(F(u,v),u)+p(F(v,u),v)], \end{aligned} \tag{2.32}$$

for every pairs $(x, y), (u, v) \in X \times X$, where $k + l < 1$. Then F has a unique coupled fixed point in X.

Example 2.16. Let $X = [0, \infty)$ and p on X be given as $p(a, b) = \max\{a, b\}$. Obviously, the partial metric space (X, p) is complete (see, e.g., Example 2.3 of [11]).

Define $F : X \times X \to X$ as $F(x, y) = (x + y)/30$, for all $x, y \in X$.

Now, we have

$$\begin{aligned} p(F(x,y),F(u,v)) &= \frac{1}{30}\max\{x+y,u+v\} \\ &\le \frac{1}{27}[\max\{x,u\}+\max\{y,v\}] \\ &\le \frac{1}{27}[\max\{x,u\}+\max\{y,v\}] \\ &\quad+\frac{1}{27}\left[\max\left\{\frac{x+y}{30},x\right\}+\max\left\{\frac{y+x}{30},y\right\}\right] \\ &\quad+\frac{1}{27}\left[\max\left\{\frac{x+y}{30},u\right\}+\max\left\{\frac{y+x}{30},v\right\}\right] \\ &\quad+\frac{1}{27}\left[\max\left\{\frac{u+v}{30},x\right\}+\max\left\{\frac{v+u}{30},y\right\}\right] \end{aligned}$$

$$\begin{aligned}
&+\frac{1}{27}\left[\max\left\{\frac{u+v}{30},u\right\}+\max\left\{\frac{v+u}{30},v\right\}\right]\\
&=\alpha_1 p(x,u)+\alpha_2 p(y,v)+\alpha_3 p(F(x,y),x)+\alpha_4 p(F(y,x),y)\\
&\quad+\alpha_5 p(F(x,y),u)+\alpha_6 p(F(y,x),v)+\alpha_7 p(F(u,v),x)\\
&\quad+\alpha_8 p(F(v,u),y)+\alpha_9 p(F(u,v),u)+\alpha_{10} p(F(v,u),v).
\end{aligned} \tag{2.33}$$

Thus, (2.2) is satisfied with $\alpha_i = 1/27$. Obviously, all the conditions of Theorem 2.4 are satisfied. Moreover, $(0,0)$ is the unique coupled fixed point of F.

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