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*Research Article*

# Combined Algebraic Properties of IP* and Central* Sets Near 0

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It is known that for an IP* set $A$ in $\mathbb{N}$ and a sequence $\langle x_n \rangle_{n=1}^{\infty}$ there exists a sum subsystem $\langle y_n \rangle_{n=1}^{\infty}$ of $\langle x_n \rangle_{n=1}^{\infty}$ such that $FS(\langle y_n \rangle_{n=1}^{\infty}) \cup FP(\langle y_n \rangle_{n=1}^{\infty}) \subseteq A$. Similar types of results also have been proved for central* sets. In this present work we will extend the results for dense subsemigroups of $((0,\infty),+)$.

## 1. Introduction

One of the famous Ramsey theoretic results is Hindman's Theorem.

**Theorem 1.1.** *Given a finite coloring* $\mathbb{N} = \bigcup_{i=1}^{r} A_i$ *there exists a sequence* $\langle x_n \rangle_{n=1}^{\infty}$ *in* $\mathbb{N}$ *and* $i \in \{1,2,\ldots,r\}$ *such that*

$$FS(\langle x_n \rangle_{n=1}^{\infty}) = \left\{ \sum_{n\in F} x_n : F \in \mathcal{P}_f(\mathbb{N}) \right\} \subseteq A_i, \tag{1.1}$$

*where for any set* $X$, $\mathcal{P}_f(X)$ *is the set of finite nonempty subsets of* $X$.

The original proof of this theorem was combinatorial in nature. But later using algebraic structure of $\beta\mathbb{N}$ a very elegant proof of this theorem was established in [1, Corollary 5.10]. First we give a brief description of algebraic structure of $\beta S_d$ for a discrete semigroup $(S,\cdot)$.

We take the points of $\beta S_d$ to be the ultrafilters on $S$, identifying the principal ultrafilters with the points of $S$ and thus pretending that $S \subseteq \beta S_d$. Given $A \subseteq S$,

$$c\ell A = \overline{A} = \{p \in \beta S_d : A \in p\} \tag{1.2}$$

is a basis for the closed sets of $\beta S_d$. The operation $\cdot$ on $S$ can be extended to the Stone-Čech compactification $\beta S_d$ of $S$ so that $(\beta S_d, \cdot)$ is a compact right topological semigroup (meaning that for any $p \in \beta S_d$, the function $\rho_p : \beta S_d \to \beta S_d$ defined by $\rho_p(q) = q \cdot p$ is continuous) with $S$ contained in its topological center (meaning that for any $x \in S$, the function $\lambda_x : \beta S_d \to \beta S_d$ defined by $\lambda_x(q) = x \cdot q$ is continuous). A nonempty subset $I$ of a semigroup $T$ is called a *left ideal of* $S$ if $TI \subset I$, a *right ideal* if $IT \subset I$, and a *two-sided ideal* (or simply an *ideal*) if it is both a left and right ideal. A *minimal left ideal* is the left ideal that does not contain any proper left ideal. Similarly, we can define *minimal right ideal* and *smallest ideal*.

Any compact Hausdorff right topological semigroup $T$ has a smallest two-sided ideal:

$$\begin{aligned} K(T) &= \bigcup\{L : L \text{ is a minimal left ideal of } T\} \\ &= \bigcup\{R : R \text{ is a minimal right ideal of } T\}. \end{aligned} \tag{1.3}$$

Given a minimal left ideal $L$ and a minimal right ideal $R$, $L \cap R$ is a group, and in particular contains an idempotent. An idempotent in $K(T)$ is a *minimal* idempotent. If $p$ and $q$ are idempotents in $T$ we write $p \leq q$ if and only if $pq = qp = p$. An idempotent is minimal with respect to this relation if and only if it is a member of the smallest ideal.

Given $p, q \in \beta S$, and $A \subseteq S$, $A \in p \cdot q$ if and only if $\{x \in S : x^{-1}A \in q\} \in p$, where $x^{-1}A = \{y \in S : x \cdot y \in A\}$. See [1] for an elementary introduction to the algebra of $\beta S$ and for any unfamiliar details.

$A \subseteq \mathbb{N}$ is called an IP* set if it belongs to every idempotent in $\beta\mathbb{N}$. Given a sequence $\langle x_n \rangle_{n=1}^{\infty}$ in $\mathbb{N}$, we let $FP(\langle x_n \rangle_{n=1}^{\infty})$ be the product analogue of Finite Sum. Given a sequence $\langle x_n \rangle_{n=1}^{\infty}$ in $\mathbb{N}$, we say that $\langle y_n \rangle_{n=1}^{\infty}$ is a *sum subsystem* of $\langle x_n \rangle_{n=1}^{\infty}$ provided there is a sequence $\langle H_n \rangle_{n=1}^{\infty}$ of nonempty finite subsets of $\mathbb{N}$ such that $\max H_n < \min H_{n+1}$ and $y_n = \sum_{t \in H_n} x_t$ for each $n \in \mathbb{N}$.

**Theorem 1.2.** *Let $\langle x_n \rangle_{n=1}^{\infty}$ be a sequence in $\mathbb{N}$ and let $A$ be an $IP^*$ set in $(\mathbb{N}, +)$. Then there exists a sum subsystem $\langle y_n \rangle_{n=1}^{\infty}$ of $\langle x_n \rangle_{n=1}^{\infty}$ such that*

$$FS(\langle y_n \rangle_{n=1}^{\infty}) \cup FP(\langle y_n \rangle_{n=1}^{\infty}) \subseteq A. \tag{1.4}$$

*Proof.* See [2, Theorem 2.6] or see [1, Corollary 16.21]. □

*Definition 1.3.* A subset $C \subseteq S$ is called central if and only if there is an idempotent $p \in K(\beta S)$ such that $C \in p$.

The algebraic structure of the smallest ideal of $\beta S$ has played a significant role in Ramsey Theory. It is known that any central subset of $(\mathbb{N}, +)$ is guaranteed to have substantial additive structure. But Theorem 16.27 of [1] shows that central sets in $(\mathbb{N}, +)$ need not have any multiplicative structure at all. On the other hand, in [2] we see that sets which belong to every minimal idempotent of N, called central* sets, must have significant multiplicative structure. In fact central* sets in any semigroup $(S,\cdot)$ are defined to be those sets which meet every central set.

**Theorem 1.4.** *If $A$ is a central* set in $(\mathbb{N},+)$ then it is central in $(\mathbb{N},\cdot)$.*

*Proof.* See [2, Theorem 2.4]. □

In case of central* sets a similar result has been proved in [3] for a restricted class of sequences called minimal sequences, where a sequence $\langle x_n\rangle_{n=1}^{\infty}$ in $\mathbb{N}$ is said to be a minimal sequence if

$$\bigcap_{m=1}^{\infty}\overline{FS(\langle x_n\rangle_{n=m}^{\infty})}\cap K(\beta\mathbb{N})\neq\emptyset. \tag{1.5}$$

**Theorem 1.5.** *Let $\langle y_n\rangle_{n=1}^{\infty}$ be a minimal sequence and let $A$ be a central* set in $(\mathbb{N},+)$. Then there exists a sum subsystem $\langle x_n\rangle_{n=1}^{\infty}$ of $\langle y_n\rangle_{n=1}^{\infty}$ such that*

$$FS(\langle x_n\rangle_{n=1}^{\infty})\cup FP(\langle x_n\rangle_{n=1}^{\infty})\subseteq A. \tag{1.6}$$

*Proof.* See [3, Theorem 2.4]. □

A strongly negative answer to the partition analogue of Hindman's theorem was presented in [4]. Given a sequence $\langle x_n\rangle_{n=1}^{\infty}$ in $\mathbb{N}$, let us denote $PS(\langle x_n\rangle_{n=1}^{\infty})=\{x_m+x_n : m,n\in\mathbb{N}$ and $m\neq n\}$ and $PP(\langle x_n\rangle_{n=1}^{\infty})=\{x_m\cdot x_n : m,n\in\mathbb{N}$ and $m\neq n\}$.

**Theorem 1.6.** *There exists a finite partition $\mathcal{R}$ of $\mathbb{N}$ with no one-to-one sequence $\langle x_n\rangle_{n=1}^{\infty}$ in $\mathbb{N}$ such that $PS(\langle x_n\rangle_{n=1}^{\infty})\cup PP(\langle x_n\rangle_{n=1}^{\infty})$ is contained in one cell of the partition $\mathcal{R}$.*

*Proof.* See [4, Theorem 2.11]. □

A similar result in this direction in the case of dyadic rational numbers has been proved by V. Bergelson et al..

**Theorem 1.7.** *There exists a finite partition $\mathbb{D}\setminus\{0\}=\bigcup_{i=1}^{r}A_i$ such that there do not exist $i\in\{1,2,\ldots,r\}$ and a sequence $\langle x_n\rangle_{n=1}^{\infty}$ with*

$$FS(\langle x_n\rangle_{n=1}^{\infty})\cup PP(\langle x_n\rangle_{n=1}^{\infty})\subseteq A_i. \tag{1.7}$$

*Proof.* See [5, Theorem 5.9]. □

In [5], the authors also presented the following conjecture and question.

**Conjecture 1.8.** *There exists a finite partition $\mathbb{Q}\setminus\{0\}=\bigcup_{i=1}^{r}A_i$ such that there do not exists $i\in\{1,2,\ldots,r\}$ and a sequence $\langle x_n\rangle_{n=1}^{\infty}$ with*

$$FS(\langle x_n\rangle_{n=1}^{\infty})\cup FP(\langle x_n\rangle_{n=1}^{\infty})\subseteq A_i. \tag{1.8}$$

*Question 1.* Does there exist a finite partition $\mathbb{R} \setminus \{0\} = \bigcup_{i=1}^{r} A_i$ such that there do not exist $i \in \{1, 2, \ldots, r\}$ and a sequence $\langle x_n \rangle_{n=1}^{\infty}$ with

$$FS(\langle x_n \rangle_{n=1}^{\infty}) \cup FP(\langle x_n \rangle_{n=1}^{\infty}) \subseteq A_i? \tag{1.9}$$

In the present paper our aim is to extend Theorems 1.2 and 1.5 for dense subsemigroups of $((0, \infty), +)$.

*Definition 1.9.* If $S$ is a dense subsemigroup of $((0, \infty), +)$ one defines $0^+(S) = \{p \in \beta S_d :$ (for all $\epsilon > 0)((0, \epsilon) \in p)\}$.

It is proved in [6], that $0^+(S)$ is a compact right topological subsemigroup of $(\beta S_d, +)$ which is disjoint from $K(\beta S_d)$ and hence gives some new information which are not available from $K(\beta S_d)$. Being compact right topological semigroup $0^+(S)$ contains minimal idempotents of $0^+(S)$. A subset $A$ of $S$ is said to be IP*-set near 0 if it belongs to every idempotent of $0^+(S)$ and a subset $C$ of $S$ is said to be central* set near 0 if it belongs to every minimal idempotent of $0^+(S)$. In [7] the authors applied the algebraic structure of $0^+(S)$ on their investigation of image partition regularity near 0 of finite and infinite matrices. Article [8] used algebraic structure of $0^+(\mathbb{R})$ to investigate image partition regularity of matrices with real entries from $\mathbb{R}$.

## 2. IP* and Central* Set Near 0

In the following discussion, we will extend Theorem 1.2 for a dense subsemigroup of $((0, \infty), +)$ in the appropriate context.

*Definition 2.1.* Let $S$ be a dense subsemigroup of $((0, \infty), +)$. A subset $A$ of $S$ is said to be an IP *set near* 0 if there exists a sequence $\langle x_n \rangle_{n=1}^{\infty}$ such that $\sum_{n=1}^{\infty} x_n$ converges and such that $FS(\langle x_n \rangle_{n=1}^{\infty}) \subseteq A$. One calls a subset $D$ of $S$ an IP**set near* 0 if for every subset $C$ of $S$ which is IP *set near* 0, $C \cap D$ is IP *set near* 0.

From [6, Theorem 3.1] it follows that for a dense subsemigroup $S$ of $((0, \infty), +)$ a subset $A$ of $S$ is an IP set near 0 if and only if there exists some idempotent $p \in 0^+(S)$ with $A \in p$. Further it can be easily observed that a subset $D$ of $S$ is an IP* set near 0 if and only if it belongs to every idempotent of $0^+(S)$.

Given $c \in \mathbb{R} \setminus \{0\}$ and $p \in \beta\mathbb{R}_d \setminus \{0\}$, the product $c \cdot p$ is defined in $(\beta\mathbb{R}_d, \cdot)$. One has $A \subseteq \mathbb{R}$ is a member of $c \cdot p$ if and only if $c^{-1}A = \{x \in \mathbb{R} : c \cdot x \in A\}$ is a member of $p$.

**Lemma 2.2.** *Let $S$ be a dense subsemigroup of $((0, \infty), +)$ such that $S \cap (0, 1)$ is a subsemigroup of $((0, 1), \cdot)$. If $A$ is an IP set near 0 in $S$ then $sA$ is also an IP set near 0 for every $s \in S \cap (0, 1)$. Further if $A$ is a an IP* set near 0 in $(S, +)$ then $s^{-1}A$ is also an IP* set near 0 for every $s \in S \cap (0, 1)$.*

*Proof.* Since $A$ is an IP set near 0 then by [6, Theorem 3.1] there exists a sequence $\langle x_n \rangle_{n=1}^{\infty}$ in $S$ with the property that $\sum_{n=1}^{\infty} x_n$ converges and $FS(\langle x_n \rangle_{n=1}^{\infty}) \subseteq A$. This implies that $\sum_{n=1}^{\infty} (s \cdot x_n)$ is also convergent and $FS(\langle s x_n \rangle_{n=1}^{\infty}) \subseteq sA$. This proves that $sA$ is also an IP* set near 0.

For the second let $A$ be a an IP* set near 0 and $s \in S \cap (0, 1)$. To prove that $s^{-1}A$ is a an IP* set near 0 it is sufficient to show that if $B$ is any IP set near 0 then $B \cap s^{-1}A \neq \emptyset$. Since $B$

is an IP set near 0, $sB$ is also an IP set near 0 by the first part of the proof, so that $A \cap sB \neq \emptyset$. Choose $t \in sB \cap A$ and $k \in B$ such that $t = sk$. Therefore $k \in s^{-1}A$ so that $B \cap s^{-1}A \neq \emptyset$. □

Given $A \subseteq S$ and $s \in S$, $s^{-1}A = \{t \in S\colon st \in A\}$, and $-s + A = \{t \in S\colon s + t \in A\}$.

**Theorem 2.3.** *Let $S$ be a dense subsemigroup of $((0,\infty),+)$ such that $S \cap (0,1)$ is a subsemigroup of $((0,1),\cdot)$. Also let $\langle x_n\rangle_{n=1}^{\infty}$ be a sequence in $S$ such that $\sum_{n=1}^{\infty} x_n$ converges and let $A$ be a IP* set near 0 in $S$. Then there exists a sum subsystem $\langle y_n\rangle_{n=1}^{\infty}$ of $\langle x_n\rangle_{n=1}^{\infty}$ such that*

$$FS(\langle y_n\rangle_{n=1}^{\infty}) \cup FP(\langle y_n\rangle_{n=1}^{\infty}) \subseteq A. \tag{2.1}$$

*Proof.* Since $\sum_{n=1}^{\infty} x_n$ converges, from [6, Theorem 3.1] it follows that we can find some idempotent $p \in 0^+(S)$ for which $FS(\langle x_n\rangle_{n=1}^{\infty}) \in p$. In fact $T = \bigcap_{m=1}^{\infty} c\ell_{\beta S_d} FS(\langle y_n\rangle_{n=m}^{\infty}) \subseteq 0^+(S)$ and $p \in T$. Again, since $A$ is a IP* set near 0 in $S$, by Lemma 2.2 for every $s \in S \cap (0,1)$, $s^{-1}A \in p$. Let $A^{\star} = \{s \in A\colon -s + A \in p\}$. Then by [1, Lemma 4.14] $A^{\star} \in p$. We can choose $y_1 \in A^{\star} \cap FS(\langle x_n\rangle_{n=1}^{\infty})$. Inductively let $m \in \mathbb{N}$ and $\langle y_i\rangle_{i=1}^{m}$, $\langle H_i\rangle_{i=1}^{m}$ in $\mathcal{P}_f(\mathbb{N})$ be chosen with the following properties:

(1) $i \in \{1,2,\ldots,m-1\}$ $\max H_i < \min H_{i+1}$;

(2) if $y_i = \sum_{t\in H_i} x_t$ then $\sum_{t\in H_m} x_t \in A^{\star}$ and $FP(\langle y_i\rangle_{i=1}^{m}) \subseteq A$.

We observe that $\{\sum_{t\in H} x_t : H \in \mathcal{P}_f(\mathbb{N}),\ \min H > \max H_m\} \in p$. Let $B = \{\sum_{t\in H} x_t : H \in \mathcal{P}_f(\mathbb{N}),\ \min H > \max H_m\}$, let $E_1 = FS(\langle y_i\rangle_{i=1}^{m})$ and $E_2 = FP(\langle y_i\rangle_{i=1}^{m})$. Now consider

$$D = B \cap A^{\star} \cap \bigcap_{s\in E_1} (-s + A^{\star}) \cap \bigcap_{s\in E_2} \left(s^{-1}A^{\star}\right). \tag{2.2}$$

Then $D \in p$. Now choose $y_{m+1} \in D$ and $H_{m+1} \in \mathcal{P}_f(\mathbb{N})$ such that $\min H_{m+1} > \max H_m$. Putting $y_{m+1} = \sum_{t\in H_{m+1}} x_t$ shows that the induction can be continued and proves the theorem. □

If we turn our attention to central* sets then the above result holds for a restricted class of sequences which we call minimal sequence near 0.

*Definition 2.4.* Let $S$ be a dense subsemigroup of $((0,\infty),+)$. A sequence $\langle x_n\rangle_{n=1}^{\infty}$ in $S$ is said to be a *minimal sequence near* 0 if

$$\bigcap_{m=1}^{\infty} \overline{FS(\langle x_n\rangle_{n=m}^{\infty})} \cap K(0^+(S)) \neq \emptyset. \tag{2.3}$$

The notion of piecewise syndetic set near 0 was first introduced in [6].

*Definition 2.5.* For a dense subsemigroup $S$ of $((0,\infty),+)$, a subset $A$ of $S$ is *piecewise syndetic near* 0 if and only if $c\ell_{\beta S_d} A \cap K(0^+(S)) \neq \emptyset$.

The following theorem characterizes minimal sequences near 0 in terms of piecewise syndetic set near 0.

**Theorem 2.6.** *Let $S$ be a dense subsemigroup of $((0,\infty),+)$. Then the following conditions are equivalent:*

(a) $\langle x_n\rangle_{n=1}^{\infty}$ *is a minimal sequence near 0.*

(b) $FS(\langle x_n\rangle_{n=1}^{\infty})$ *is piecewise syndetic near 0.*

(c) *There is an idempotent in* $\bigcap_{m=1}^{\infty}\overline{FS(\langle x_n\rangle_{n=m}^{\infty})}\cap K(0^+(S))\neq\emptyset$.

*Proof.* $(a)\Rightarrow(b)$ follows from (see [6, Theorem 3.5]).

To prove that (b) implies (a) let us consider that $FS(\langle x_n\rangle_{n=1}^{\infty})$ be a piecewise syndetic near 0. Then there exists a minimal left ideal $L$ of $0^+(S)$ such that $L\cap\overline{FS(\langle x_n\rangle_{n=1}^{\infty})}\neq\emptyset$. We choose $q\in L\cap\overline{FS(\langle x_n\rangle_{n=1}^{\infty})}$. By [6, Theorem 3.1], $\bigcap_{m=1}^{\infty}c\ell_{\beta S_d}FS(\langle x_n\rangle_{n=m}^{\infty})$ is a subsemigroup of $0^+(S)$, so it suffices to show that for each $m\in\mathbb{N}$, $L\cap\overline{FS(\langle x_n\rangle_{n=m}^{\infty})}\neq\emptyset$. In fact minimal left ideals being closed, we can conclude that $L\cap\bigcap_{n=m}^{\infty}\overline{FS(\langle x_n\rangle_{n=m}^{\infty})}\neq\emptyset$ and so $L\cap\bigcap_{n=m}^{\infty}\overline{FS(\langle x_n\rangle_{n=m}^{\infty})}$ is a compact right topological semigroup so that it contains idempotents. To this end, let $m\in\mathbb{N}$ with $m>1$. Then $FS(\langle x_n\rangle_{n=1}^{\infty}) = FS(\langle x_n\rangle_{n=m}^{\infty})\cup FS(\langle x_n\rangle_{n=1}^{m-1})\cup\bigcup\{t+FS(\langle x_n\rangle_{n=m}^{\infty}) : t\in FS(\langle x_n\rangle_{n=1}^{m-1})\}$. So we must have one of the following:

(i) $FS(\langle x_n\rangle_{n=m}^{\infty})\in q$,

(ii) $FS(\langle x_n\rangle_{n=1}^{m-1})\in q$,

(iii) $t+FS(\langle x_n\rangle_{n=m}^{\infty})\in q$ for some $t\in FS(\langle x_n\rangle_{n=1}^{m-1})$.

Clearly (ii) does not hold, because in that case $q$ becomes a member of $S$ while it is a member of minimal left ideal. If (iii) holds then we have $t+FS(\langle x_n\rangle_{n=m}^{\infty})\in q$ for some $t\in FS(\langle x_n\rangle_{n=1}^{m-1})$. Since $q\in 0^+(S)$, we have $(0,t)\cap S\in q$. But $(0,t)\cap(t+FS(\langle x_n\rangle_{n=m}^{\infty}))=\emptyset$, a contradiction. Hence (i) must hold so that $q\in L\cap\overline{FS(\langle x_n\rangle_{n=m}^{\infty})}$.

$(a)\Leftrightarrow(c)$ is obvious. □

Let us recall following lemma for our purpose.

**Lemma 2.7.** *Let $S$ be a dense subsemigroup of $((0,\infty),+)$ such that $S\cap(0,1)$ is a subsemigroup of $((0,1),\cdot)$ and assume that for each $y\in S\cap(0,1)$ and each $x\in S$, $x/y\in S$ and $yx\in S$. If $A\subseteq S$ and $y^{-1}A$ is a central set near 0, then $A$ is also a central set near 0.*

*Proof.* See [6, Lemma 4.8]. □

**Lemma 2.8.** *Let $S$ be a dense subsemigroup of $((0,\infty),+)$ such that $S\cap(0,1)$ is a subsemigroup of $((0,1),\cdot)$ and assume that for each $s\in S\cap(0,1)$ and each $t\in S$, $t/s\in S$ and $st\in S$. If $A$ is central set near 0 in $S$ then $sA$ is also central set near 0.*

*Proof.* Since $s^{-1}(sA)=A$ and $A$ is central set near 0 then by Lemma 2.7, $sA$ is central set near 0. □

**Lemma 2.9.** *Let $S$ be a dense subsemigroup of $((0,\infty),+)$ such that $S\cap(0,1)$ is a subsemigroup of $((0,1),\cdot)$ and assume that for each $s\in S\cap(0,1)$ and each $t\in S$, $t/s\in S$ and $st\in S$. If $A$ is a central* set near 0 in $(S,+)$ then $s^{-1}A$ is also central* set near 0.*

*Proof.* Let $A$ be a central* set near 0 and $s\in S\cap(0,1)$. To prove that $s^{-1}A$ is a central* set near 0 it is sufficient to show that for any central set near 0 $C$, $C\cap s^{-1}A\neq\emptyset$. Since $C$ is central set near 0, $sC$ is also central set near 0 so that $A \cap sC \neq \emptyset$. Choose $t \in sC \cap A$ and $k \in C$ such that $t = sk$. Therefore $k \in s^{-1}A$ so that $C \cap s^{-1}A \neq \emptyset$. □

We end this paper by following generalization of Theorem 2.3, whose proof is also straight forward generalization of Theorem 2.3 and hence omitted.

**Theorem 2.10.** *Let $S$ be a dense subsemigroup of $((0,\infty),+)$ such that $S \cap (0,1)$ is a subsemigroup of $((0,1),\cdot)$ and assume that for each $s \in S \cap (0,1)$ and each $t \in S$, $t/s \in S$ and $st \in S$. Also let $\langle x_n \rangle_{n=1}^{\infty}$ be a minimal sequence near 0 and let $A$ be a central* set near 0 in $S$. Then there exists a sum subsystem $\langle y_n \rangle_{n=1}^{\infty}$ of $\langle x_n \rangle_{n=1}^{\infty}$ such that*

$$FS(\langle y_n \rangle_{n=1}^{\infty}) \cup FP(\langle y_n \rangle_{n=1}^{\infty}) \subseteq A. \tag{2.4}$$

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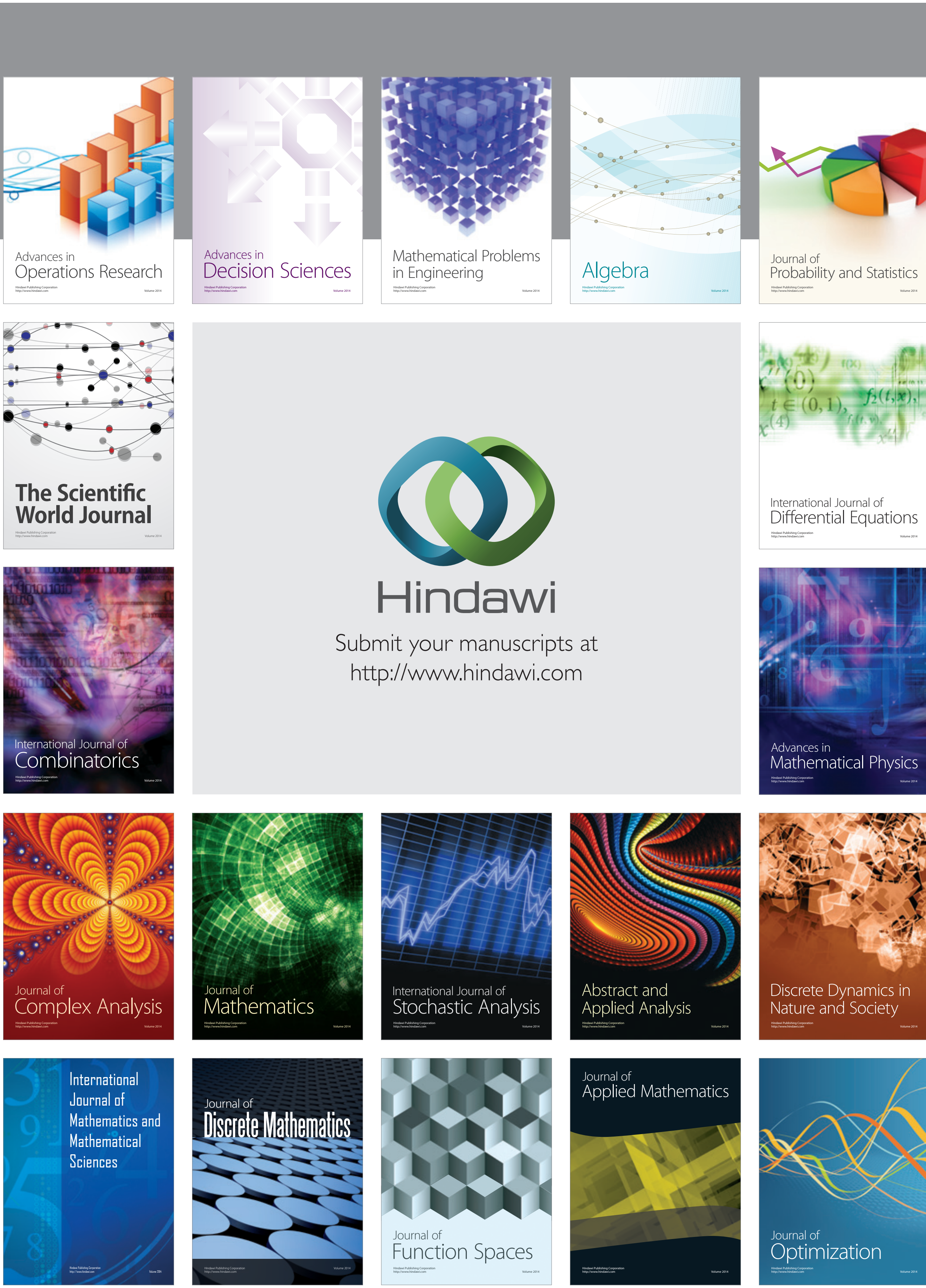
## References

[1] N. Hindman and D. Strauss, *Algebra in the Stone-Čech Compactification: Theory and Applications*, vol. 27, Walter de Gruyter, Berlin, Germany, 1998.
[2] V. Bergelson and N. Hindman, "On IP* sets and central sets," *Combinatorica*, vol. 14, no. 3, pp. 269–277, 1994.
[3] D. De, "Combined algebraic properties of central* sets," *Integers*, vol. 7, p. A37, 2007.
[4] N. Hindman, "Partitions and pairwise sums and products," *Journal of Combinatorial Theory A*, vol. 37, no. 1, pp. 46–60, 1984.
[5] V. Bergelson, N. Hindman, and I. Leader, "Additive and multiplicative Ramsey theory in the reals and the rationals," *Journal of Combinatorial Theory A*, vol. 85, no. 1, pp. 41–68, 1999.
[6] N. Hindman and I. Leader, "The semigroup of ultrafilters near 0," *Semigroup Forum*, vol. 59, no. 1, pp. 33–55, 1999.
[7] D. De and N. Hindman, "Image partition regularity near zero," *Discrete Mathematics*, vol. 309, no. 10, pp. 3219–3232, 2009.
[8] D. De and R. K. Paul, "Image partition regularity of matrices near 0 with real entries," *New York Journal of Mathematics*, vol. 17, pp. 149–161, 2011.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