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THE REFLECTION PHENOMENA OF SV-WAVES IN A GENERALIZED THERMOELASTIC MEDIUM

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ABSTRACT. We discuss the reflection of thermoelastic plane waves at a solid half-space nearby a vacuum. We use the generalized thermoelastic waves to study the effects of one or two thermal relaxation times on the reflection plane harmonic waves. The study considered the thermal and the elastic waves of small amplitudes in a homogeneous, isotropic, and thermally conducting elastic solid. The expressions for the reflection coefficients, which are the ratio of the amplitudes of the reflected waves to the amplitude of the incident waves are obtained. It has been shown, analytically, that the elastic waves are modified due to the thermal effect. The reflection coefficients of a shear wave that incident from within the solid on its boundary, which depend on the thermoelastic coupling factor and included the thermal relaxation times, have been found in the general case. The numerical values of reflection coefficients against the angle of incidence for different values of thermal relaxation times have been calculated and the results are given in the form of graphs. Some special cases of reflection have also been discussed, for example, in the absence of thermal effect our results reduce to the ordinary pure elastic case.

Keywords and phrases. Generalized thermoelastic waves, reflection phenomena, thermal relaxation times.

2000 Mathematics Subject Classification. Primary 74J10.

1. Introduction. Since the early 1960's there has been an increased usage of composite materials in a variety of commercial, aerospace, and military structural configurations involving extreme temperature environments. Therefore, during the past three decades, wide spread attention has been given to thermoelasticity theories which admit a finite speed for the propagation of thermal signals. In contrast to the conventional theories based on parabolic-type heat equation, these theories involve a hyperbolic-type heat equation and are referred to as generalized theories. Various authors have formulated these generalized theories on different grounds. For example, Lord and Shulman [11] have developed a theory based on a modified heat conduction law which involves heat flux rate. This thermoelastic theory is including the finite velocity of thermal wave by correcting the Fourier thermal conduction law by introducing one relaxation time of thermoelastic process. Green and Lindsay [8] formulated a more rigorous theory by including a temperature rate among the constitutive variables; they are considered the finite velocity of the thermal wave by correcting the energy equation and Duhamel-Neumann relation, by introducing two relaxation times of the thermal process. These theories are considered to be more realistic than the conventional theories in dealing with problems involving high heat fluxes and/or small time intervals, like those occurring in laser units and energy channels. Various problems characterizing these two theories are investigated, and some interesting phenomena have been revealed. These nonclassical theories are often regarded as the generalized dynamic theory of thermoelasticity. Brief reviews of this topic have been reported by Chandrasekharaiah [4]. The phenomenon of reflection of pure elastic waves may be found in many references [1, 2, 5, 6, 10, 13]. Also an extensive literature on the development of the interaction of two fields, namely the thermal field and the elastic field, and the phenomenon of reflection of elastic waves, is available in many works such as [3, 9, 12].

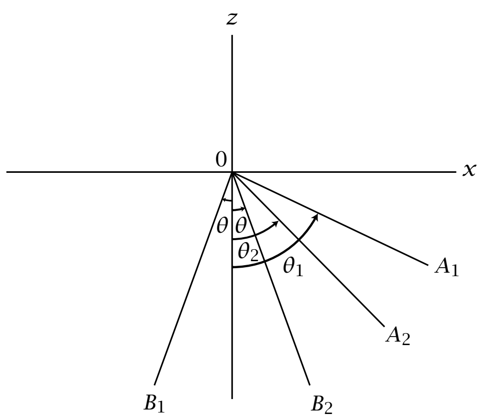


FIGURE 1.

The object of the present paper is to discuss the reflection of thermoelastic plane waves at a solid half-space nearby a vacuum. Generalized thermoelastic waves is used to study the effects of one or two thermal relaxation times on the reflection plane harmonic waves. The study considered the thermal and elastic waves of small amplitude in a homogeneous, isotropic, and thermally conducting elastic solid. The expressions for the reflection coefficients, which are the ratios of the amplitudes of the reflected waves to the amplitude of the incident wave are obtained. The thermal relaxation times and the thermal effect on the reflection coefficients are studied by comparing the results with their counterparts in the following cases:

(i) approximate expressions for reflection coefficients and

(ii) pure elastic case.

Finally, we find a numerical solution in the case of metal Aluminium, and present the results graphically.

2. Formulation of the problem and fundamental equations. We assume that the elastic medium is an isotropic, homogeneous, and undergoing with small temperature variations, i.e., the whole body is at a constant temperature T_0. The problem is to investigate thermoelastic waves occupying the Cartesian space where a semi-infinite elastic solid bounded by the plane $z = 0$ extends in the negative direction of x-axis. A rotational wave propagating from infinity within the solid is assumed to be incident on the boundary $z = 0$, making an angle θ with the negative direction of z-axis Figure 1. We also assume that the body is thermally conducting and the thermal wave velocity is small in compared with the dilatational elastic wave velocity.

The equation of motion in the elastic medium in terms of the elastic displacement in generalized thermoelasticity in its linearized form is given as

$$(\lambda+2\mu)\,\mathrm{grad}\,(\mathrm{div}\,\vec{u}) - \mu\,\mathrm{curl}\,\mathrm{curl}\,\vec{u} - \gamma\Big(\mathrm{grad}\,T + t_1\frac{\partial}{\partial t}\,\mathrm{grad}\,T\Big) = \rho\frac{\partial^2\vec{u}}{\partial t^2}. \tag{2.1}$$

The modified heat conduction equation is

$$K\nabla^2 T = \rho c_e\Big(\frac{\partial T}{\partial t} + t_0\frac{\partial^2 T}{\partial t^2}\Big) + T_0\gamma\Big(\frac{\partial}{\partial t}(\mathrm{div}\,\vec{u}) + t_0\delta\frac{\partial^2}{\partial t^2}(\mathrm{div}\,\vec{u})\Big), \tag{2.2}$$

where

$\nabla^2 = (\partial^2/\partial x^2) + (\partial^2/\partial z^2)$,
$\vec{u}$ denotes the displacement vector,
λ and μ are the Lamé constants,
T is the perturbed temperature over the constant temperature T_0,
γ is equal to $\alpha_0(3\lambda+2\mu)$,
α_0 is the thermal expansion coefficient,
K is the thermal conductivity,
c_e is the specific heat per unit mass at constant strain, and
ρ is the density of the medium.

Moreover, the use of the relaxation times t_1, t_0 and Kronecker δ makes the above fundamental equations of possible validity for the three different theories:

(i) Classical Dynamical Coupled theory (1956) (C-D), where $t_0 = t_1 = 0$, $\delta = 0$,
(ii) Lord-Shulman theory (1967) (L-S), where $t_1 = 0$, $t_0 > 0$, $\delta = 1$,
(iii) Green-Lindsay theory (1972) (G-L), where $t_1 \geqslant t_0 \geqslant 0$, $\delta = 0$.

To separate the dilatational and rotational components of strain, we introduce the elastic displacement potentials ϕ and ψ in the following relations:

$$\begin{gathered} u_i = \phi_i + e_{irs}A_{s,r}, \quad i,r,s = 1,2,3, \\ \vec{A} = \psi\vec{e_2}, \end{gathered} \tag{2.3}$$

where $\vec{e_2}$ is a unit vector in y-direction, the potential ϕ and the vector potentials $\vec{A}$ are Lame's potentials, and e_{irs} is the permutation symbol. Taking divergence of each term of (2.1) and using (2.3), we get the equation for dilatation waves as

$$c_1^2\Big(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\Big)\phi - \frac{\gamma}{\rho}\Big(T + t_1\frac{\partial T}{\partial t}\Big) = \frac{\partial^2\phi}{\partial t^2}. \tag{2.4}$$

Taking curl of each term in (2.1) and using some well-known vector identities, we get in a similar way, the equation for shear waves as

$$c_2^2\Big(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\Big)\psi = \frac{\partial^2\psi}{\partial t^2}, \tag{2.5}$$

with

$$c_1 = \sqrt{\frac{\lambda+2\mu}{\rho}}, \qquad c_2 = \sqrt{\frac{\mu}{\rho}}, \tag{2.6}$$

where c_1 and c_2 are the isothermal dilatational and shear elastic wave velocities which sometimes called the velocities of P and SV waves. The vector $\vec{u}$ has y-component assumed to be zero. We also assume that all the variables are functions of x- and z- and independent of the y-coordinate.

The heat conduction equation (2.2), after using (2.3), becomes

$$K\nabla^2 T = \rho c_e\left(\frac{\partial T}{\partial t} + t_0\frac{\partial^2 T}{\partial t^2}\right) + T_0\gamma\left(\frac{\partial}{\partial t}\nabla^2\phi + t_0\delta\frac{\partial^2}{\partial t^2}\nabla^2\phi\right). \tag{2.7}$$

It is obvious from (2.3), (2.4), (2.5), and (2.7) that the P-wave is affected due to the presence of the thermal field, while the SV-wave remains unaffected.

3. Solution of the problem. For studying plane wave motion, assume that the wave normal lies in the xz-plane and take solutions of the system of equations (2.4) through (2.7) in the form [1]

$$\begin{gathered}(\phi, T) = (\phi_1, T_1)\exp\left[i(k(x\sin\theta + z\cos\theta) - \omega t)\right],\\ \psi = \psi_1\exp\left[i(l(x\sin\theta + z\cos\theta) - \omega t)\right],\end{gathered} \tag{3.1}$$

where ω is the frequency, and k and l are of the dilatational and the rotational wave numbers, respectively.

Substitution of the relevant equations of (3.1) in (2.4) and (2.7), gives a system of two homogeneous equations. Then, we obtain the following system for the amplitudes ϕ_1 and T_1:

$$\begin{bmatrix} c_1^2\left(\dfrac{\omega^2}{c_1^2} - k^2\right) & -\dfrac{\gamma}{\rho}\tau_1 \\ -iT_0\gamma\omega\tau_0^* k^2 & (-Kk^2 + i\rho c_e\omega\tau_0)\end{bmatrix}\begin{bmatrix}\phi_1 \\ T_1\end{bmatrix} = [0]. \tag{3.2}$$

This system has nontrivial solutions if only if the determinant of the factor matrix vanishes. This yields

$$v^4 - (1 + \epsilon_T^* - i\chi^*)v^2 - i\chi^* = 0, \tag{3.3}$$

where we have introduced the following notation:

$$\begin{gathered} v = \frac{\omega}{kc_1}, \qquad \chi = \frac{\omega K}{\rho c_e c_1^2}, \qquad \epsilon_T = \frac{T_0\gamma^2}{\rho^2 c_e c_1^2}, \qquad \chi^* = \frac{\chi}{\tau_0}, \qquad \epsilon_T^* = \frac{\epsilon_T\tau_0^*\tau_1}{\tau_0}, \\ \tau_1 = 1 - it_1\omega, \qquad \tau_0 = 1 - it_0\omega, \qquad \tau_0^* = 1 - it_0\omega\delta, \end{gathered} \tag{3.4}$$

where ϵ_T is the usual thermoelastic coupling factor [12].

Since (3.3) is a quadratic in v^2, there are dilatational waves travelling with two different velocities. Therefore, if a rotational wave falls on the boundary $z = 0$ from the solid, we have one reflected rotational wave and two reflected dilatational waves, assuming that the radiation into the vacuum is neglected. Accordingly, if the wave normal of the incident rotational wave makes angle θ with the positive direction of z-axis, and those of reflected dilatational waves make angles θ_1, θ_2 with the same direction, the displacement potentials ϕ and ψ may be taken in the forms

$$\begin{aligned}\phi = {} & A_1 \exp\left[i(k_1(x\sin\theta_1 - z\cos\theta_1) - wt)\right] \\ & + A_2 \exp\left[i(k_2(x\sin\theta_2 - z\cos\theta_2) - wt)\right],\end{aligned} \tag{3.5}$$

$$\begin{aligned}\psi = {} & B_1 \exp\left[i(l(x\sin\theta + z\cos\theta) - wt)\right] \\ & + B_2 \exp\left[i(l(x\sin\theta - z\cos\theta) - wt)\right].\end{aligned} \tag{3.6}$$

The ratios of the amplitudes of the reflected waves to the amplitude of the incident wave, namely B_2/B_1, A_1/B_1, and A_2/B_1 give the corresponding reflection coefficients. Figure 1 shows the wave normal of the incident and reflected waves denoted by their respective amplitudes. It may be noted that the angles $\theta, \theta_1, \theta_2$ and the corresponding wave numbers l, k_1, k_2 are connected by the relations

$$k_1 \sin\theta_1 = k_2 \sin\theta_2 = l \sin\theta, \tag{3.7}$$

on the interface $z = 0$ of the mediums, relations (3.7) may also be written in order to satisfy the boundary conditions given in Section 4 as

$$\frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2} = \frac{\sin\theta}{\nu^{1/2}}, \tag{3.8}$$

where

$$v_1 = \frac{\omega}{k_1 c_1}, \qquad v_2 = \frac{\omega}{k_2 c_1}, \qquad \nu = \left(\frac{c_2}{c_1}\right)^2, \tag{3.9}$$

the squares of the former two are the roots of (3.3).

4. Boundary conditions. Since the boundary $z = 0$ is adjacent to the vacuum, it is free from surface tractions. This boundary condition may be expressed as

$$T_{zj} = 0, \quad (j = x, y, z) \text{ on } z = 0. \tag{4.1}$$

Here T_{zj} is the mechanical stress [12] given by

$$T_{zj} = \mu(u_{z,j} + u_{j,z}) + \left(\lambda \operatorname{div}\vec{u} - \gamma\left(T + t_1 \frac{\partial T}{\partial t}\right)\right)\delta_{zj}, \tag{4.2}$$

where $\delta_{zj} = 1$ or 0 according to whether $j = z$ or $j \neq z$. Writing in explicit forms, we have the components of T_{zj} as

$$T_{zx} = \mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right), \qquad T_{zy} = 0, \qquad T_{zz} = (\lambda + 2\mu)\frac{\partial w}{\partial z} + \lambda\frac{\partial u}{\partial x} - \gamma\left(T + t_1\frac{\partial T}{\partial t}\right). \tag{4.3}$$

We also assume that the boundary $z = 0$ is thermally insulated, so that there is no variation of temperature on it. This means that

$$\frac{\partial T}{\partial z} = 0 \quad \text{on } z = 0. \tag{4.4}$$

5. Expressions for the reflection coefficients. For the boundary conditions expressed by (4.1), (4.2), and (4.4) and with the help of (3.5) and (3.6), after rearrangement, we obtain, for SV-wave, the following relations:

$$\begin{gathered}
X_1 \cos 2\theta + X_2 \frac{\nu}{v_1^2} \sin 2\theta_1 + X_3 \frac{\nu}{v_2^2} \sin 2\theta_2 + \cos 2\theta = 0, \\
-X_1 \sin 2\theta + \frac{X_2}{v_1^2}\left(1 - 2\nu \sin^2\theta_1 + \frac{\epsilon_T^* v_1^2 \tau_1}{v^2 + i\chi^*}\right) \\
+ \frac{X_3}{v_2^2}\left(1 - 2\nu \sin^2\theta_2 + \frac{\epsilon_T^* v_2^2 \tau_1}{v^2 + i\chi^*}\right) + \sin 2\theta = 0, \\
X_2 \frac{\epsilon_T^* \cos\theta_1}{v_1(v^2 + i\chi^*)} + X_3 \frac{\epsilon_T^* \cos\theta_2}{v_2(v^2 + i\chi^*)} = 0,
\end{gathered} \tag{5.1}$$

where

$$X_1 = \frac{B_2}{B_1}, \qquad X_2 = \frac{A_1}{B_1}, \qquad X_3 = \frac{A_2}{B_1}. \tag{5.2}$$

The solutions of this system of equations for the reflection coefficient of rotational waves X_1 and the reflection coefficients of dilatational waves X_2 and X_3 are

$$X_1 = -\frac{P_1}{Q}, \qquad X_2 = \frac{P_2}{Q}, \qquad X_3 = -\frac{P_3}{Q}, \tag{5.3}$$

where

$$\begin{aligned}
P_1 = {} & v_2 \cos\theta_2 \left[(v_1^2 + i\chi^*)(\nu \cos 2(\theta + \theta_1) + (1-\nu)\cos 2\theta) + \epsilon_T^* v_1^2 \cos 2\theta\right] \\
& - v_1 \cos\theta_1 \left[(v_2^2 + i\chi^*)(\nu \cos 2(\theta + \theta_2) + (1+\nu)\cos 2\theta) + \epsilon_T^* v_2^2 \cos 2\theta\right],
\end{aligned} \tag{5.4}$$

$$P_2 = -2 v_1^2 v_2 (v_1^2 + i\chi^*) \cos\theta_2 \cos 2\theta \sin 2\theta, \tag{5.5}$$

$$P_3 = -2 v_2^2 v_1 (v_2^2 + i\chi^*) \cos\theta_1 \cos 2\theta \sin 2\theta, \tag{5.6}$$

and

$$\begin{aligned}
Q = {} & v_2 \cos\theta_2 \left[(v_1^2 + i\chi^*)(\nu \cos 2(\theta - \theta_1) + (1-\nu)\cos 2\theta) + \epsilon_T^* v_1^2 \cos 2\theta\right] \\
& - v_1 \cos\theta_1 \left[(v_2^2 + i\chi^*)(\nu \cos 2(\theta - \theta_2) + (1-\nu)\cos 2\theta) + \epsilon_T^* v_2^2 \cos 2\theta\right].
\end{aligned} \tag{5.7}$$

The absolute values of the reflection coefficients X_1, X_2, and X_3 for this general case are plotted versus the angle of incidence θ for the three different cases:

(i) Green-Lindsay model, i.e., the variation of the second relaxation time while the first one is fixed.

(ii) Lord-Shulman model, i.e., the variation of the first relaxation time when neglecting the second one.

(iii) Classical-Dynamical Coupled model when neglecting the two relaxation times and remaining the thermal effect.

Equations (5.3) contain a number of particular cases which we now proceed to examine.

6. Special cases

6.1. Approximate expressions for reflection coefficients. For most elastic materials, it is known that $\epsilon_T^* \ll 1$ and $\chi^* \ll 1$. Therefore, retaining only the first degree terms in ϵ_T^* and χ^*, the roots of (3.3) are

$$v_1^2 = 1+\epsilon_T^*, \qquad v_2^2 = -i\chi^*. \tag{6.1}$$

Their square roots are given by

$$v_1 = 1+\frac{1}{2}\epsilon_T^*, \qquad v_2 = i^{3/2}\chi^{*(1/2)}. \tag{6.2}$$

Substitution of these values in the expressions for X_1, X_2, and X_3 given by (5.3) together with relations (5.4), (5.5), (5.6), and (5.7), and simplification after using relations (3.5) and (3.6), give

$$X_1 = \frac{m_1}{M}, \qquad X_2 = -\frac{m_2}{M}, \qquad X_3 = 0. \tag{6.3}$$

In these relations

$$m_1 = a_1a_2 - b, \qquad m_2 = 2\cos 2\theta \sin 2\theta (1+\epsilon_T^*), \qquad M = a_1a_2+b, \tag{6.4}$$

where

$$\begin{gathered} a_1 = 4\nu^{1/2}\left[1+\frac{1}{2}\epsilon_T^*\right], \qquad a_2 = \left[1-\frac{1}{\nu}(1+\epsilon_T^*)\sin^2\theta\right]^{1/2}\sin^2\theta\cos\theta, \\ b = \cos^2 2\theta - 2\epsilon_T^*\cos 2\theta \sin^2\theta + \epsilon_T^*\cos 2\theta\left[1-i\chi^* - i^{3/2}\chi^{*(1/2)}a_2\right]. \end{gathered} \tag{6.5}$$

Now, it is easy to see that in this case the incoming SV-wave is split into two waves at the flat boundary, one reflected P-wave (dilatational wave) X_1 and the second reflected SV-wave X_2. This is presented in Figure 11 for $\epsilon_T^* = 0.01, 0.02, 0.03, 0.4$.

6.2. Pure elastic case. When the thermal effect is neglected, i.e., $\epsilon_T^* = 0$ and $\chi^* = 0$, we get the pure elastic case. Therefore, we have $v_1 = 1$, $v_2 = 0$, $\theta_1 = \alpha$, say, and $\theta_2 = 0$. Then the expressions for X_1 and X_2 simplify to

$$X_1 = \frac{\nu^{1/2}\cos\alpha\tan^2 2\theta - \cos\theta}{\nu^{1/2}\cos\alpha\tan^2 2\theta + \cos\theta}, \qquad X_2 = \frac{2\tan 2\theta\cos\theta}{\nu^{1/2}\cos\alpha\tan^2 2\theta + \cos\theta}. \tag{6.6}$$

These equations are the same as those given by Brekhoviskikh [2] if slight changes in notation are introduced there.

7. Numerical results and conclusions. With a view to illustrating the advantage of this study, we consider now a numerical example. The results describe the variation for reflection coefficients for an SV-wave with the various values of the angle of incidence. For this purpose, metal Aluminium is taken as the thermoelastic material body for which we have the physical constants at $T_0 = 27\,^\circ\mathrm{C}$ as follows [7].

$$\begin{aligned} &\rho = 2.70\,\mathrm{g/(cm)^3}, && \alpha_0 = 0.23\times 10^{-4}\,\mathrm{cm/(cm\,deg\,C)}, \\ &\lambda = 5.775\times 10^{11}\,\mathrm{dyne/(cm)^2}, && K = 0.480\,\mathrm{cal/(g\,deg\,C)}, \\ &\mu = 2.646\times 10^{11}\,\mathrm{dyne/(cm)^2}, && c_e = 0.216\,\mathrm{cal/(g\,deg\,C)}, \end{aligned}$$

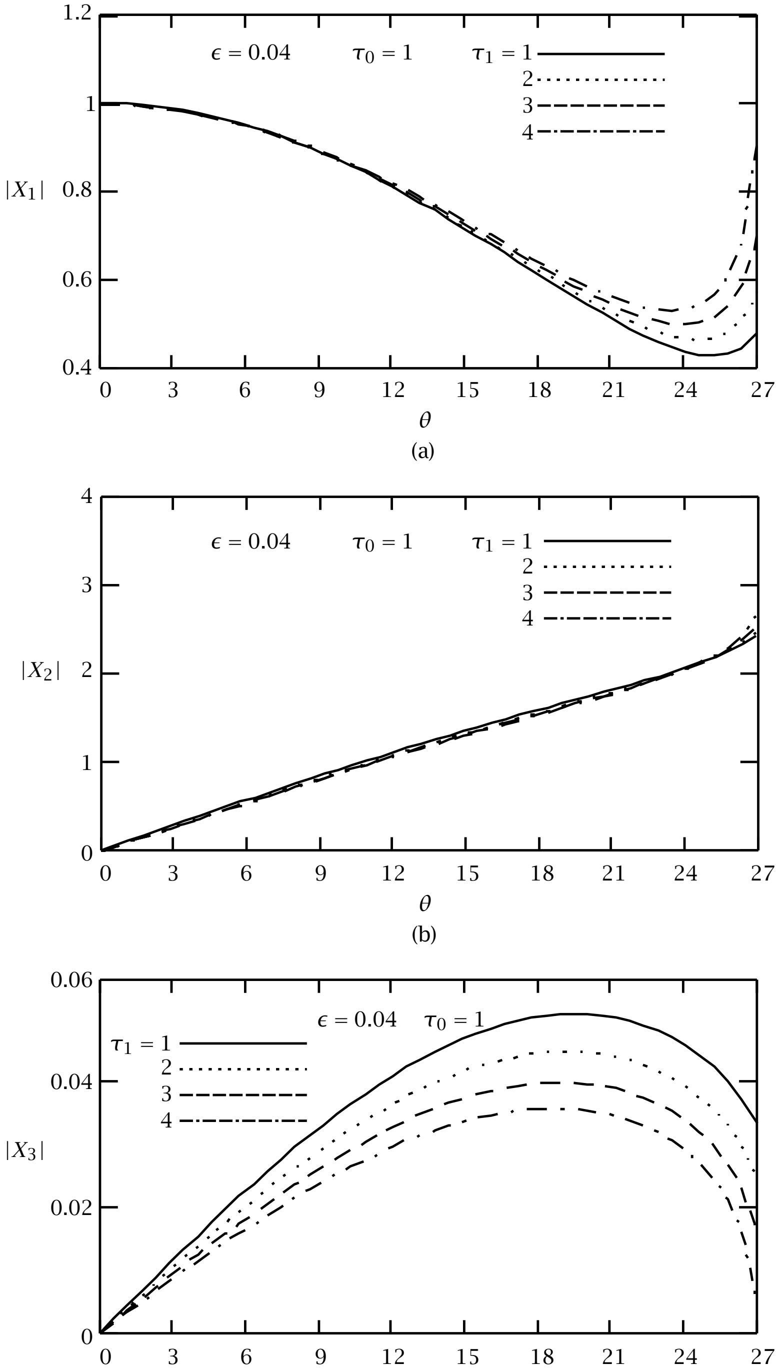


FIGURE 2. (The effect of the thermal relaxation times in G-L theory) $|X_1|, |X_2|, |X_3|$ versus the angle of incidence θ.

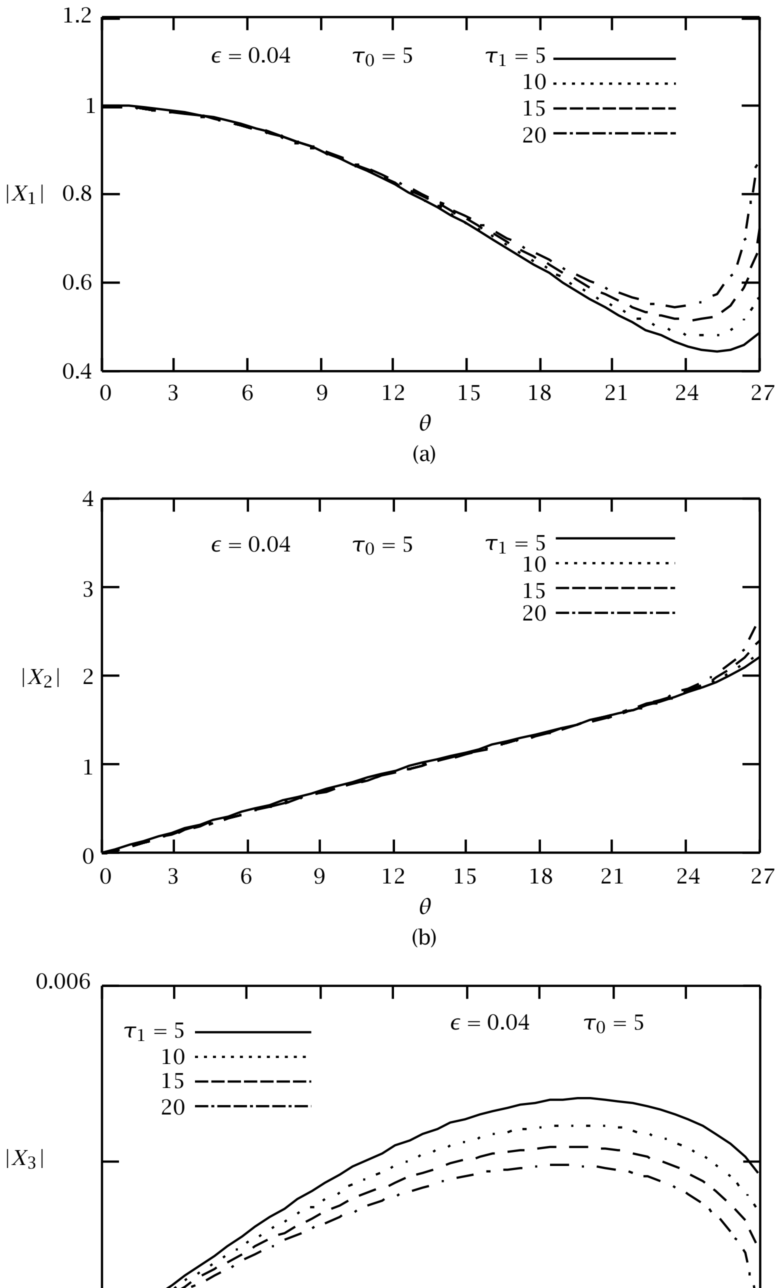


FIGURE 3. (The effect of the thermal relaxation times in G-L theory) $|X_1|, |X_2|, |X_3|$ versus the angle of incidence θ.

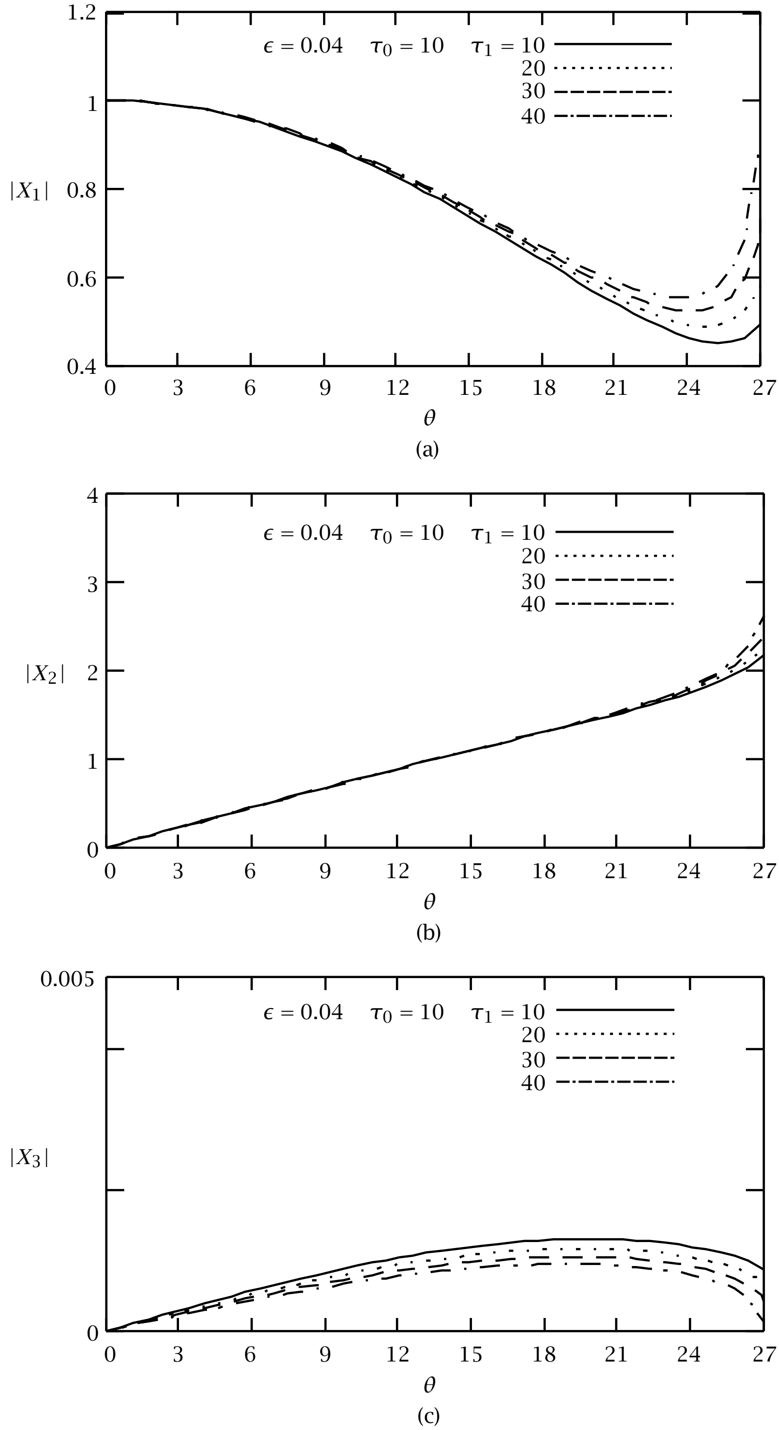


FIGURE 4. (The effect of the thermal relaxation times in G-L theory) $|X_1|, |X_2|, |X_3|$ versus the angle of incidence θ.

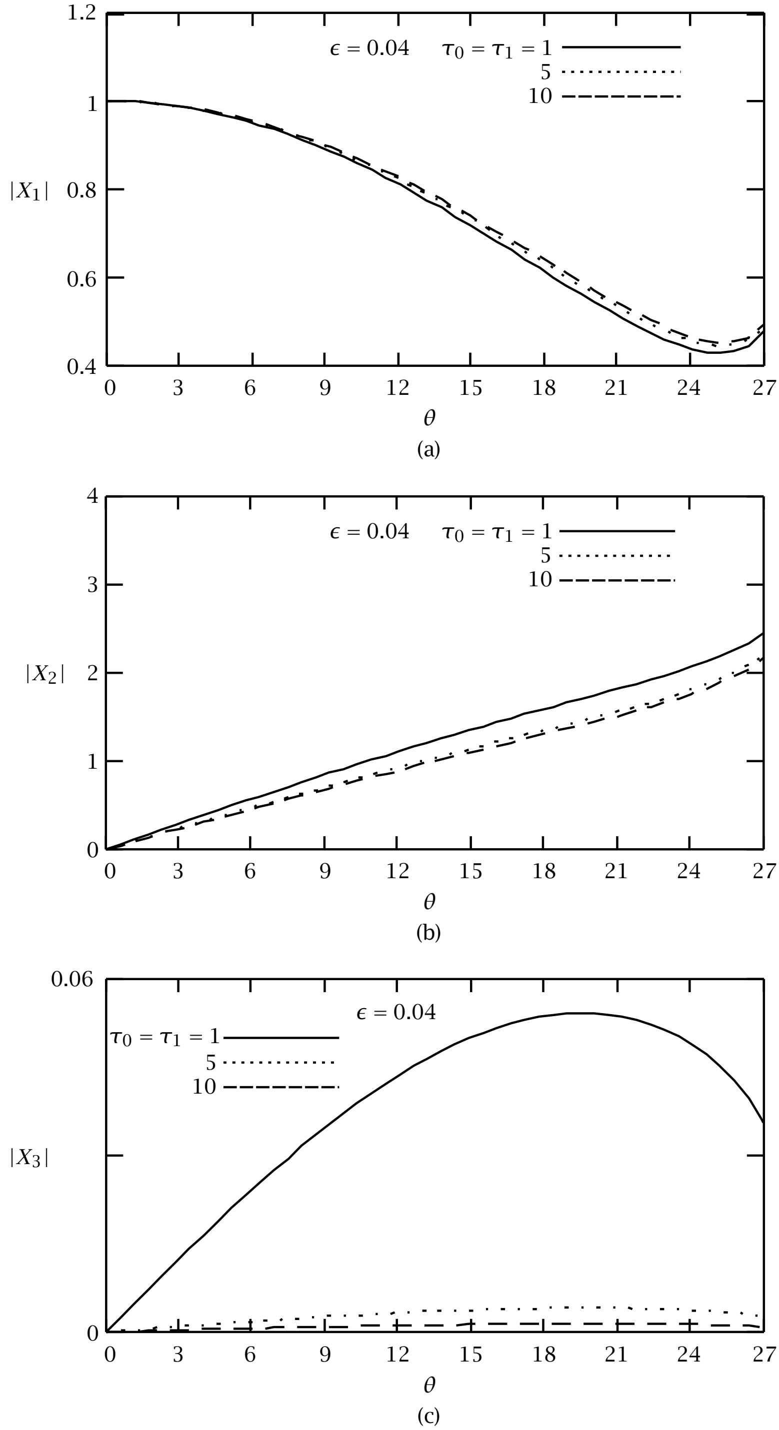


FIGURE 5. (The effect of the thermal relaxation times in G-L theory) $|X_1|, |X_2|, |X_3|$ versus the angle of incidence θ.

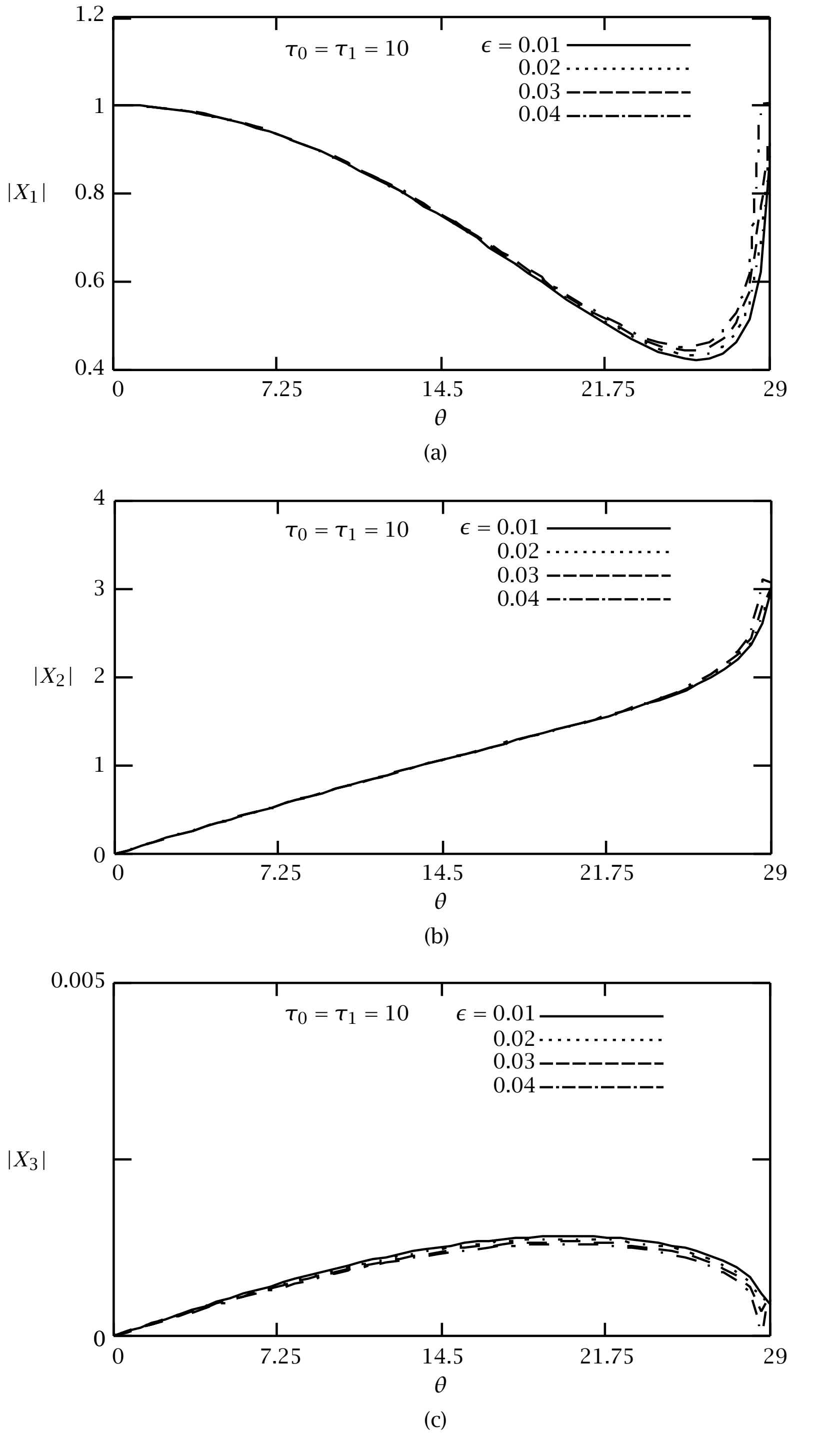


FIGURE 6. (The effect of the thermal coefficient ϵ in G-L theory) $|X_1|, |X_2|, |X_3|$ versus the angle of incidence θ.

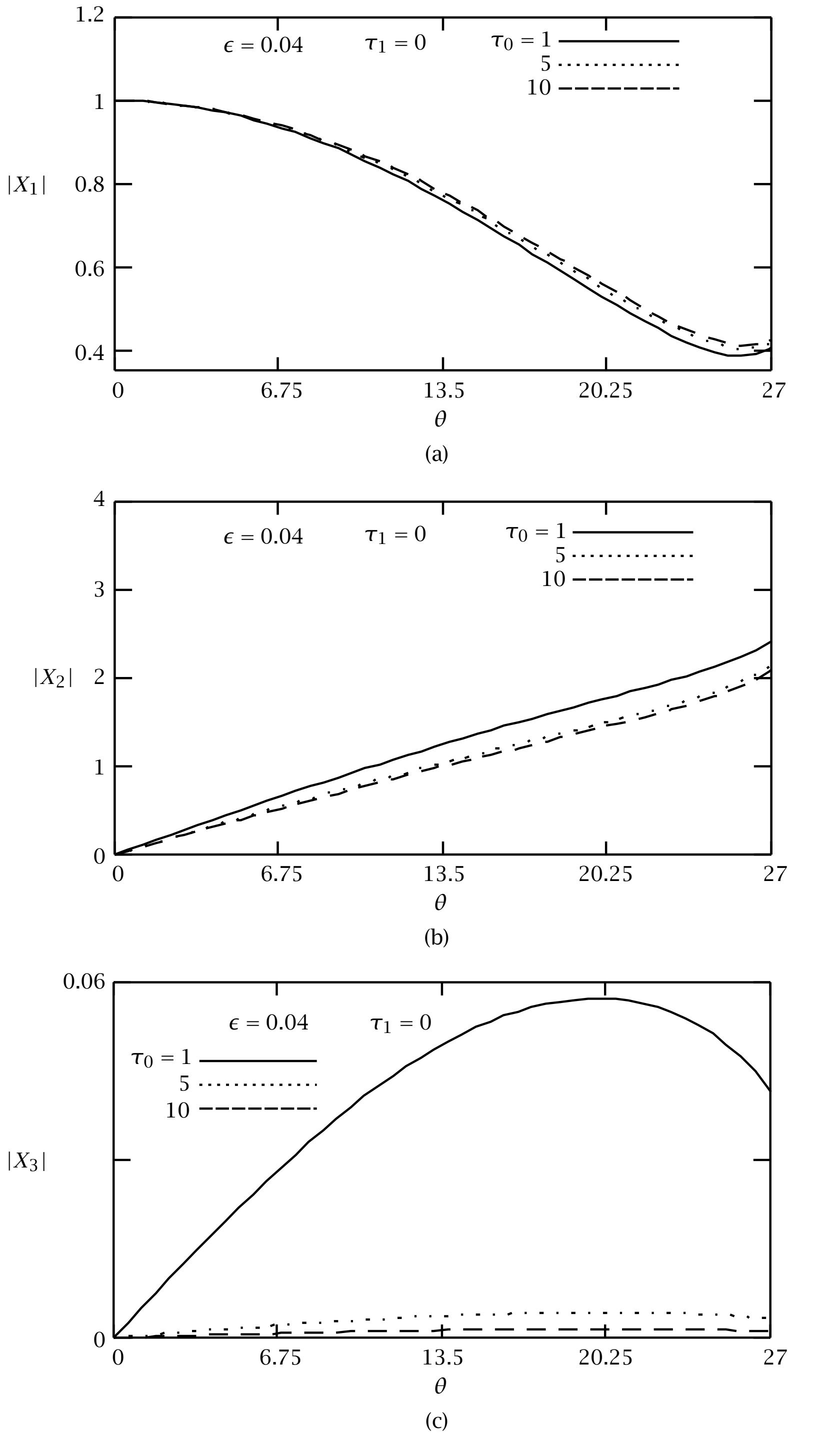


FIGURE 7. (The effect of the thermal relaxation time in L-S theory) $|X_1|, |X_2|, |X_3|$ versus the angle of incidence θ.

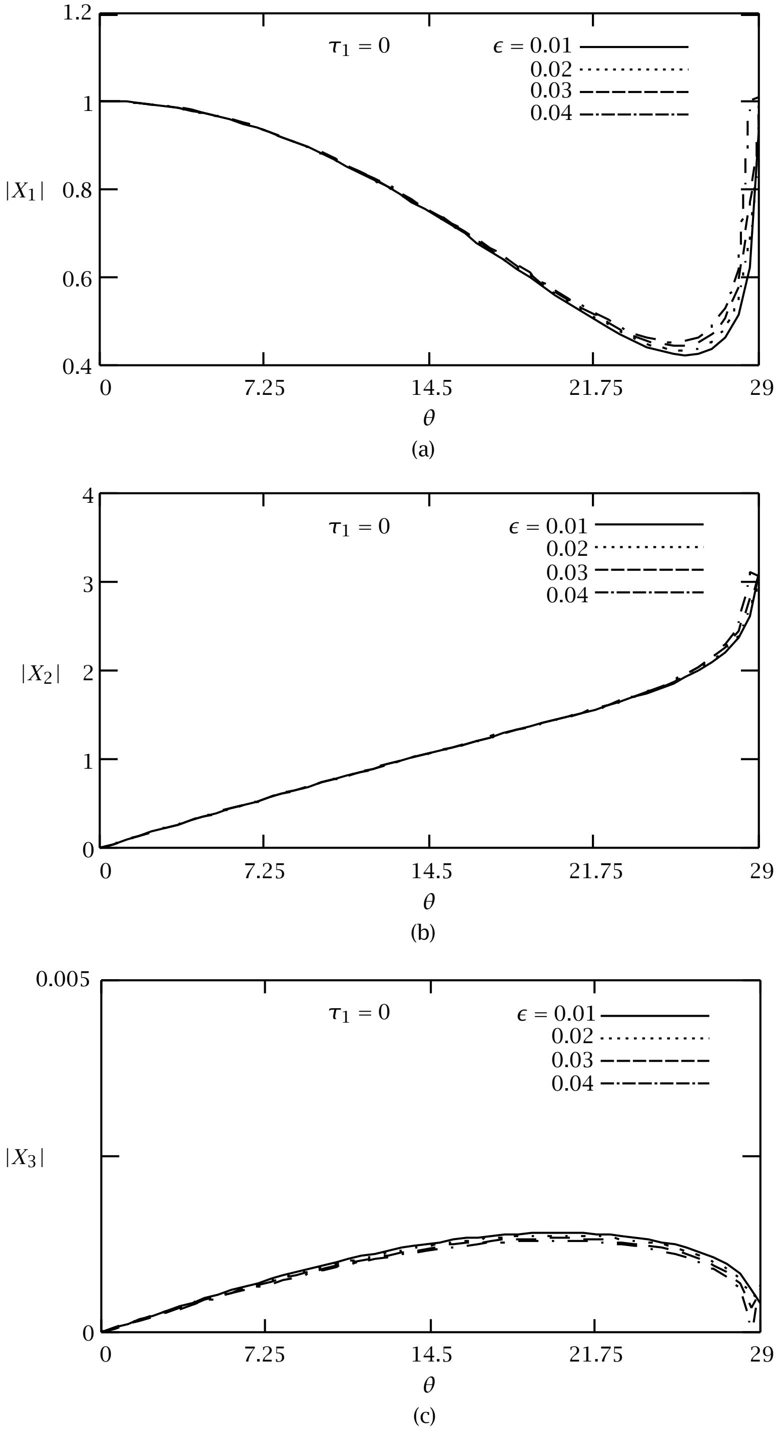


FIGURE 8. (The effect of the thermal coefficient ϵ in L-S theory) $|X_1|, |X_2|, |X_3|$ versus the angle of incidence θ.

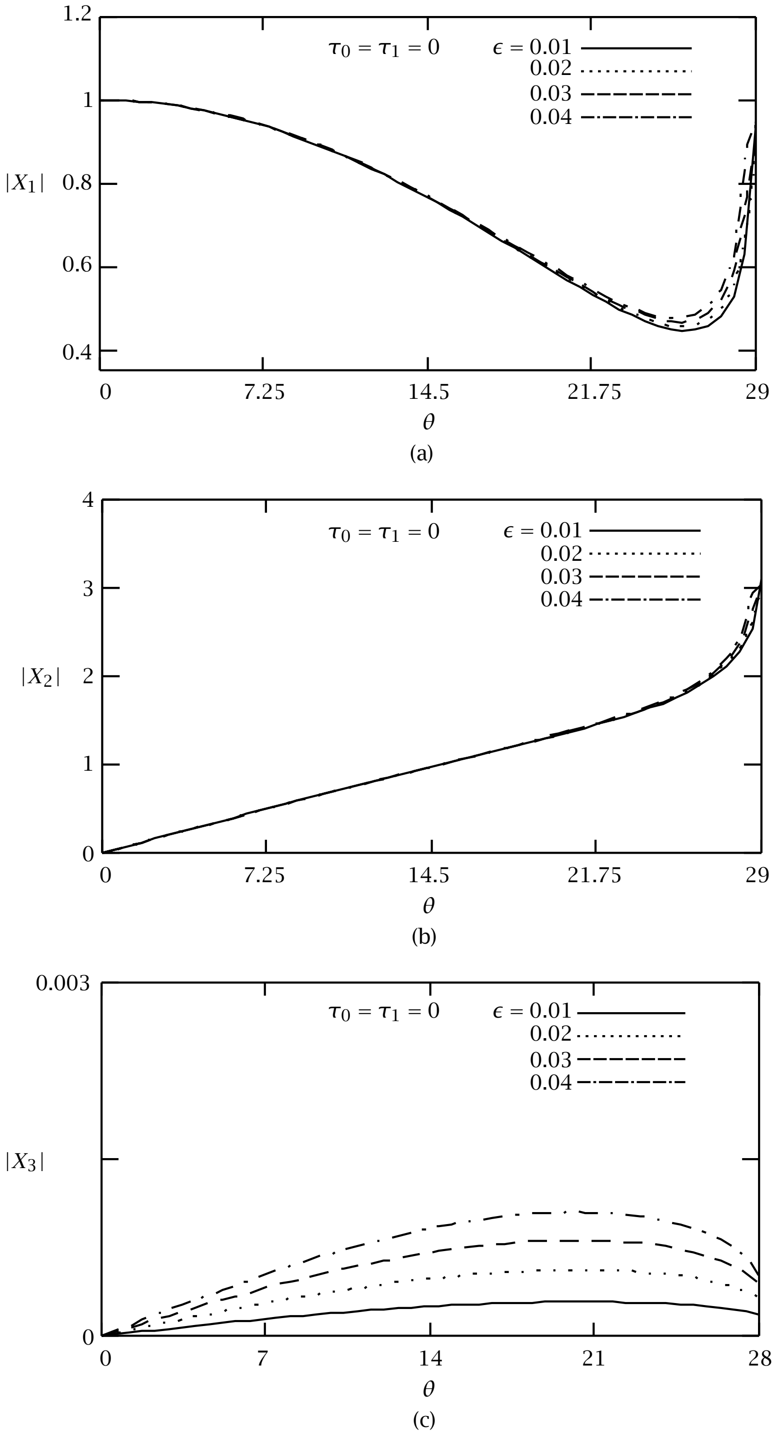


FIGURE 9. (The thermal effect in C-D theory) $|X_1|, |X_2|, |X_3|$ versus the angle of incidence θ.

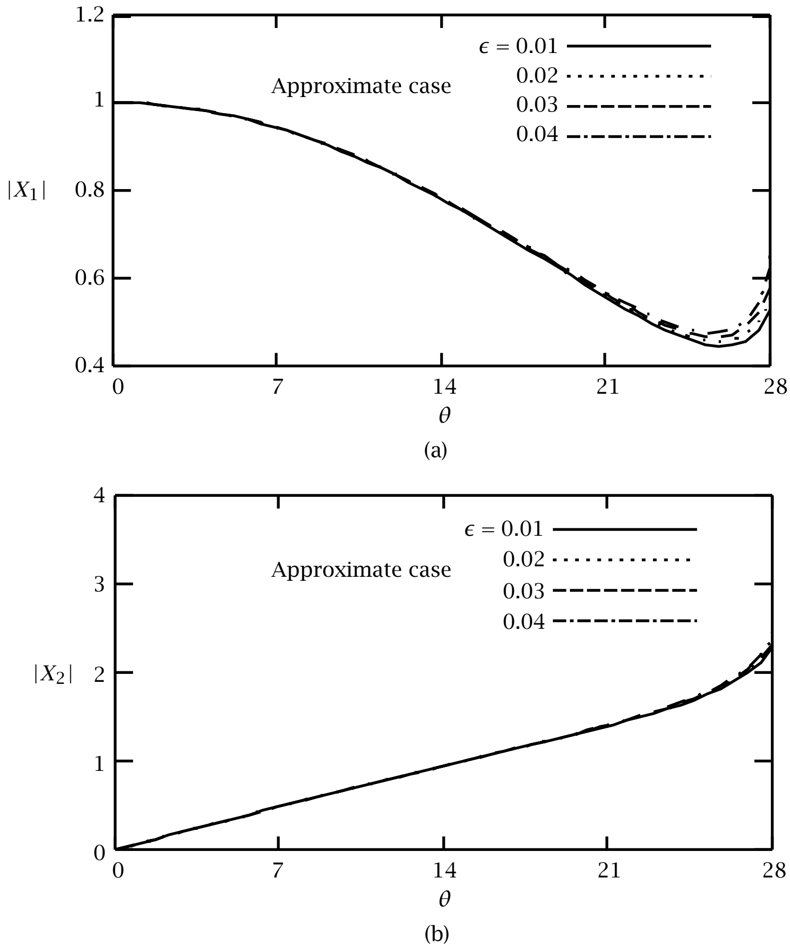


FIGURE 10. (The thermal effect in the approximate case) $|X_1|, |X_2|$ versus the angle of incidence θ.

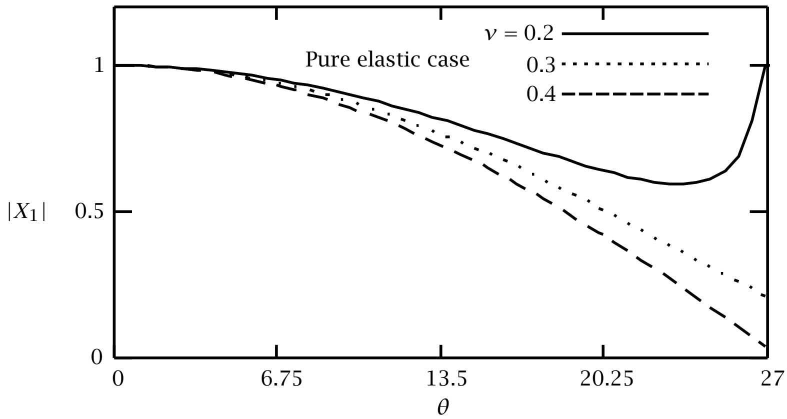
According to these values, when θ tends to $\pi/2$, we obtain, in the approximate case,

$$X_1 \longrightarrow -1, \qquad X_2 \longrightarrow 0. \tag{7.1}$$

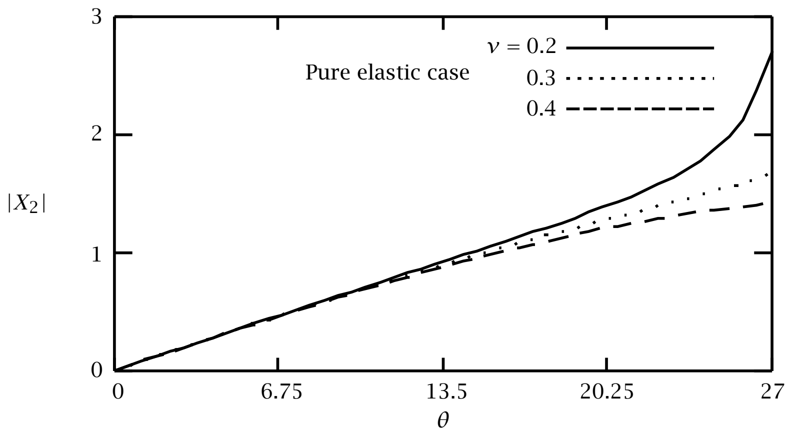
Thus, this case is so-called the grazing incidence which has the incidence and reflected rotational waves cancel on the boundary, and there will be no dilatational wave. From this, we infer the impossibility of existence of plane waves on the boundary $z = 0$. This result is the same as that in pure elastic case [2].

Taking $t_0, t_1 \simeq 0(10^{-13}\,\mathrm{s})$, the corresponding dimensionless values of them are: τ_0 which is of ordered 0(1) to 0(5), while τ_1 which assume to be given by $\tau_1 = n\tau_0$ $(n = 1, 2, 3, 4)$. Now, it is easy to see from the graphs the following:

(i) Figures 2, 3, and 4 exhibit the variation of the angle of incidence with the reflection coefficients ratios for SV-wave under the consideration of the fixed $\epsilon = 0.04$



(a)



(b)

FIGURE 11. Pure elastic case. $|X_1|, |X_2|$ versus the angle of incidence θ.

whereas $\tau_0 = 1, 5, 10$, respectively and $\tau_1 = n\tau_0$ $(n = 1, 2, 3, 4)$. Moreover, Figure 5 consider $\epsilon = 0.01, 0.02, 0.03, 0.04$ and $\tau_0 = \tau_1 = 10$ all of them for (G-L) model.

(ii) Figures 2, 3, and 4, display the increasing of the second relaxation time which has a sensitive influence on the absolute values of the reflection coefficients X_1, X_3 while X_2 is not affected.

(iii) Figures 5, 8, 9, and 10 show the variation of thermal effect ϵ on $|X_1|, |X_2|$, and $|X_3|$ according to (G-L), (L-S), (C-D) models and the approximate case, respectively. It is clear that ϵ has appreciated effect on $|X_1|$ and $|X_3|$ while $|X_2|$ is not affected. Also, the influence of poison's ratio ν can seen in Figure 11 which display the pure elastic case.

(vi) Figure 7, shows that, in the (L-S) model, the absolute value of X_1, X_2, and X_3 remarkably changes with the increasing of the relaxation time τ_0.

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