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SOME PROPERTIES OF THE IDEAL OF CONTINUOUS FUNCTIONS WITH PSEUDOCOMPACT SUPPORT

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Abstract. Let $C(X)$ be the ring of all continuous real-valued functions defined on a completely regular T_1-space. Let $C_\Psi(X)$ and $C_K(X)$ be the ideal of functions with pseudocompact support and compact support, respectively. Further equivalent conditions are given to characterize when an ideal of $C(X)$ is a P-ideal, a concept which was originally defined and characterized by Rudd (1975). We used this new characterization to characterize when $C_\Psi(X)$ is a P-ideal, in particular we proved that $C_K(X)$ is a P-ideal if and only if $C_K(X) = \{f \in C(X) : f = 0$ except on a finite set$\}$. We also used this characterization to prove that for any ideal I contained in $C_\Psi(X)$, I is an injective $C(X)$-module if and only if $\operatorname{coz} I$ is finite. Finally, we showed that $C_\Psi(X)$ cannot be a proper prime ideal while $C_K(X)$ is prime if and only if X is an almost compact noncompact space and ∞ is an F-point. We give concrete examples exemplifying the concepts studied.

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1. Introduction. Let X be a completely regular T_1-space, βX the Stone-Cech compactification of X, υX the Hewitt real compactification of X, $C(X)$ the ring of all continuous real-valued functions defined on X and $C^*(X)$ the subring of all bounded functions in $C(X)$.

For each $f \in C(X)$, let $Z(f) = \{x \in X : f(x) = 0\}$, $\operatorname{coz} f = X - Z(f)$, and $\operatorname{supp} f = \overline{\operatorname{coz} f}$. If I is an ideal of $C(X)$, let $\operatorname{coz} I = \bigcup_{f\in I} \operatorname{coz} f$. For each $A \subseteq \beta X$ let $O^A = \{f \in C(X) : A \subseteq \operatorname{Int}_{\beta X} Z(\bar{f})\}$, where $\bar{f}$ is the continuous extension to βX of the bounded function

$$f^*(x) = \begin{cases} 1, & f(x) \geq 1, \\ f(x), & -1 \leq f(x) \leq 1, \\ -1, & f(x) \leq -1, \end{cases} \tag{1.1}$$

and let $M^A = \{f \in C(X) : A \subseteq \operatorname{cl}_{\beta X} Z(f)\}$. For $p \in X$, $O^p = O_p = \{f \in C(X) : p \in \operatorname{Int}_X Z(f)\}$ and $M^p = M_p = \{f \in C(X) : f(p) = 0\}$. Let $C_K(X)$ denote the ideal of functions with compact support and $C_\Psi(X)$ denote the ideal of functions with pseudocompact support in the ring $C(X)$. It is known that $C_K(X) = O^{\beta X - X}$ and $C_\Psi(X) = O^{\beta X - \upsilon X}$.

For all notation and undefined terms in this paper the reader may consult [4].

It was mentioned in [4] that if X is a P-space, then $\operatorname{supp} f$ is finite for each $f \in C_K(X)$. This raises the question; when does the converse of this hold? At first we thought the converse is always true, but we could not prove it, and we were able later on to find an example showing that the converse is not always true. Until by chance while we were studying properties of P-ideals, we found that the above property is equivalent to $C_K(X)$ being a P-ideal. We were able to give more equivalent conditions to P-ideals which leads us to our result, moreover we used this new characterization for P-ideals to characterize when $C_\Psi(X)$ is an injective $C(X)$-module and to generalize a result which was proved earlier by Vechtomov in [10]. We also study when $C_K(X)$ is a prime ideal and when it is maximal.

2. P-ideals.

Recall that an ideal I of $C(X)$ is called pure if for each $f \in I$ there exists $g \in I$ such that $f = fg$, and in this case $g = 1$ on $\operatorname{supp} f$. The ideal I is called regular if for each $f \in I$ there exists $g \in I$ such that $f = f^2g$. A space X is called a P-space if every G_δ-set in X is open. A point $p \in \beta X$ is called a P-point if $O^p = M^p$, it is called an F-point if O^p is prime. The space X is called an F-space if O^p is prime for each $p \in \beta X$.

The space X is a P-space if and only if every prime ideal of $C(X)$ is maximal if and only if $O_x = M_x$ for each $x \in X$, see [4]. Rudd in [9] extends this concept to ideals in rings of continuous functions.

Definition 2.1 (see Rudd [9])**.** A nonzero ideal I of $C(X)$ is called a P-ideal if every proper prime ideal of I is maximal in I.

If X is a P-space, then every ideal of $C(X)$ is a P-ideal. Later on we will give an example of a nonzero ideal I of $C(X)$ that is a P-ideal while X is not a P-space.

Theorem 2.2 (see Rudd [9])**.** *Let I be a nonzero ideal of $C(X)$. Then the following statements are equivalent:*

(1) *I is a P-ideal.*

(2) *$Z(f)$ is open for each $f \in I$.*

(3) *Every ideal of I is pure.*

(4) *Every prime ideal of $C(X)$ which does not contain I is maximal in $C(X)$.*

Now we prove the following theorem which is the main result in this section, it gives more equivalent conditions to Theorem 2.2 and it will be used to characterize when $C_\Psi(X)$ is a P-ideal and when it is injective.

Theorem 2.3. *Let I be a nonzero ideal of $C(X)$. Then the following statements are equivalent:*

(1) *I is a P-ideal.*

(2) *$O_x = M_x$ for each $x \in \operatorname{coz} I$ and $I \subseteq O_x$ for $x \notin \operatorname{coz} I$.*

(3) *I is a regular ring.*

(4) *$\operatorname{coz} I$ is a P-space and I is pure.*

Proof. (1)⇒(2). Let $x \in \operatorname{coz} I$, then I is not contained in M_x. Hence for each prime ideal $P \subseteq M_x$, P does not contain I, and therefore it is maximal in $C(X)$. Thus $O_x = M_x$, since O_x is the intersection of all prime ideals contained in M_x, see [4].

Now, if $x \notin \operatorname{coz} I$, then $I \subseteq M_x$. Suppose that there exists $f \in I - O_x$. So there exists a prime ideal $P \subseteq M_x$ such that $f \in I - P$. Hence P is a maximal ideal of $C(X)$, and so $P = M_x$ which is a contradiction, since $I \subseteq M_x$. Hence, $I \subseteq O_x$ for each $x \notin \operatorname{coz} I$.

(2)⇒(3). It is clear that for each $f \in I$, $Z(f)$ is open. So it follows by Theorem 2.2 that each ideal of I is pure, and so a z-ideal. Hence $f \in f^2 I$, since $Z(f) = Z(f^3)$. Thus, there exists $g \in I$ such that $f = f^2 g$.

(3)⇒(1). Clear, since the condition implies that each ideal of I is pure.

(2)⇒(4). The condition implies that I is pure. Let $G = \bigcap_{n=1}^{\infty} G_n$, where each G_n is an open set in $\operatorname{coz} I$. Let $y \in G$. For each $n \in \mathbb{N}$ there exists $f_n \in C(X)$ such that $0 \le f_n \le 1$, $f_n(y) = 0$ and $f_n(X - G_n) = 1$. Let $f = \sum_{n=1}^{\infty} f_n^2/2^n$. Then $f \in C(X)$ and $f(y) = 0$. So $f \in M_y = O_y$. Hence, $y \in \operatorname{Int}_X Z(f) \subseteq Z(f) \subseteq \bigcap_{n=1}^{\infty} G_n = G$. Thus G is an open set, and therefore $\operatorname{coz} I$ is a P-space.

(4)⇒(2). Let y $\in \operatorname{coz} I$, and $f \in M_y$. Then $y \in Z(f) \bigcap \operatorname{coz} I = \bigcap_{n=1}^{\infty} \{x \in X : |f(x)| < 1/n\} \bigcap \operatorname{coz} I$. Hence, $y \in \operatorname{Int}_X Z(f)$, and so $f \in O_y$. Now, let $y \notin \operatorname{coz} I$ and $f \in I$. Purity of I implies that there exists $g \in I$ such that $f = fg$. So $\operatorname{supp} f \subseteq \operatorname{coz} g \subseteq \operatorname{coz} I$. Hence $y \in X - \operatorname{supp} f \subseteq Z(f)$. Consequently, $f \in O_y$. □

It will be shown later in Examples 3.6 and 3.7 that the two conditions in statement (4) above are both necessary.

3. When is $C_\Psi(X)$ a P-ideal? If X is a P-space, then $C_K(X)$ is the set of all functions in $C(X)$ which are eventually zero (see [4]). Now our aim is to show when does the converse of this result hold? In fact, we are going to show that $C_K(X)$ is the set of all functions that are eventually zero if and only if $C_K(X)$ is a P-ideal. The point is that we were able to find a space X which is not a P-space, while $C_K(X)$ is a P-ideal. In fact, one can say more, but first we will need the following lemma.

Lemma 3.1. *If I is a pure ideal, then* $\operatorname{supp} f \subseteq \operatorname{coz} I$ *for each $f \in I$.*

Proof. Let $f \in I$, then there exists $g \in I$ such that $f = fg$, which implies that $\operatorname{supp} f \subseteq \operatorname{coz} g \subseteq \operatorname{coz} I$. □

In the following theorem we will use the result in Theorem 2.3 to characterize when $C_\Psi(X)$ is a P-ideal.

Theorem 3.2. *Let I be an ideal contained in $C_\Psi(X)$. Then I is a P-ideal if and only if* $\operatorname{coz} I$ *is discrete and I is pure.*

Proof. Suppose I is a P-ideal. Then $\operatorname{coz} I$ is a P-space, and I is pure. So $\operatorname{supp} f \subseteq \operatorname{coz} I$ for each $f \in I$. Hence $\operatorname{supp} f$ is a pseudocompact P-space. Thus $\operatorname{coz} f$ is a finite open set, since a pseudocompact P-space is finite (see [4]). So it follows that for each $x \in \operatorname{coz} f, \{x\}$ is clopen in X. Hence, $\operatorname{coz} I$ is a discrete space of X. The converse is clear. □

The following corollary is the main result in this paper, it gives a concrete description of the ideal $C_K(X)$ when it is a P-ideal.

Corollary 3.3. *The following statements are equivalent:*

(1) *$C_K(X)$ is a P-ideal.*

(2) $\operatorname{coz} C_K(X)$ *is discrete and $C_K(X)$ is pure.*

(3) *$C_K(X) = \{f \in C(X) : f = 0$ except on a finite set$\}$.*

Proof. The equivalence of (1) and (2) follows from Theorem 3.2, since $C_K(X) \subseteq C_\Psi(X)$.

(1)⇒(3). Let $f \in C_K$(X). Then $\operatorname{supp} f$ is finite, since it is a compact P-space. So $\operatorname{coz} f$ is finite. Hence $f = 0$ except on a finite set.

(3)⇒(1). Let $f \in C_K(X)$. Then $\operatorname{coz} f$ is finite. So $Z(f)$ is clopen, since $\operatorname{coz} f$ is. Hence, $C_K(X)$ is a P-ideal. □

In [6, 8] several generalizations of real compactness are studied, from them the concept of Ψ-compactness. A space X is called Ψ-compact if $C_K(X) = C_\Psi(X)$. The following corollary relates Ψ-compactness of the space X and $C_\Psi(X)$ being a P-ideal.

Corollary 3.4. *If $C_\Psi(X)$ is a P-ideal, then X is Ψ-compact.*

Proof. If $f \in C_\Psi(X)$, then $\operatorname{supp} f$ is finite. Hence $f = 0$ except on a finite set. Thus $C_\Psi(X) \subseteq \{f \in C(X) : f = 0 \text{ except on a finite set}\} \subseteq C_K(X) \subseteq C_\Psi(X)$. Thus $C_\Psi(X) = C_K(X)$, and so X is Ψ-compact. □

The following is an example of a non P-space X such that $C_K(X)$ is a P-ideal. This shows that the converse of the result in [4] needs not be true.

Example 3.5. Let X be the set of all rational numbers with $\{0\}$ clopen and all other points have their usual neighborhoods. Then $\operatorname{coz} C_\Psi(X) = \{0\}$ is discrete and $C_\Psi(X) = \{f \in C(X) : f = 0 \text{ except for } x = 0\}$ is pure. The space X is not a P-space, since $\{2\} = \bigcap_{n=1}^{\infty}(2-1/n, 2+1/n) \bigcap X$ is not open.

The following two examples show that the two conditions in Theorem 2.3(4) and Theorem 3.2 are both necessary.

Example 3.6. Let $X = [-1,1]$ with all points isolated and $\{0\}$ has its usual neighborhoods. Then $\operatorname{coz} C_\Psi(X) = X - \{0\}$ is discrete.

Now, define

$$f(X) = \begin{cases} x, & x = \dfrac{1}{n} \text{ for } n \in \mathbb{Z}^*, \\ 0, & \text{otherwise}. \end{cases} \tag{3.1}$$

Then $\operatorname{supp} f = \{1/n : n \in \mathbb{Z}^*\} \bigcup \{0\}$ is compact, but $Z(f)$ is not open. So $C_\Psi(X)$ is not a P-ideal, although $\operatorname{coz} C_\Psi(X)$ is discrete.

Example 3.7. Let $\mathbb{R}$ be the space of all real numbers, then $C_K(\mathbb{R})$ is a pure ideal, since $\mathbb{R}$ is locally compact (see [2]), while $C_K(\mathbb{R})$ is not a P-ideal.

4. Injectivity of $C_\Psi(X)$.

Recall that an ideal I is called an injective ideal if it is an injective $C(X)$-module. Vechtomov in [10] mentioned that if X is locally compact, then $C_K(X)$ is an injective $C(X)$-module if and only if X is finite. We now use the result in Theorem 2.3 to extend the result of Vechtomov to any ideal contained in $C_\Psi(X)$ for any space X. In fact we will show that the concept of P-ideals and injective modules are equivalent for principal ideals contained in $C_\Psi(X)$.

Theorem 4.1. *Let I be any ideal contained in $C_\Psi(X)$. Then I is an injective $C(X)$-module if and only if $\operatorname{coz} I$ is finite.*

Proof. Suppose I is an injective $C(X)$-module, then I is a regular ring and so it follows by Theorems 2.3 and 3.2 that $\operatorname{coz} I$ is discrete. Moreover, since I is a direct summand of any module of which it is a submodule (see [5]), it follows that $I = (e)$

with $e^2 = e$. So $\operatorname{coz} I = \operatorname{coz} e = \operatorname{supp} e$, since $Z(e)$ is clopen (see [1]). Hence $\operatorname{coz} I$ is a pseudocompact discrete space. Thus $\operatorname{coz} I$ is finite.

Conversely, suppose $\operatorname{coz} I$ is finite. Then I is a P-ideal, since $\operatorname{coz} f$ is clopen for each $f \in I$, and so every ideal of I is a z-ideal in $C(X)$. Let J be any ideal of $C(X)$, and let $\varphi \in \operatorname{Hom}_{C(X)}(J, I)$. Define

$$e_1(x) = \begin{cases} 1, & x \in \operatorname{coz} \varphi(J), \\ 0, & \text{otherwise}. \end{cases} \tag{4.1}$$

Then $\varphi(J) = (e_1)$, since it is a z-ideal contained in I, and $\operatorname{coz} \varphi(J)$ is compact. Let $f_1 \in J$ such that $\varphi(f_1) = e_1$. Let $A = \operatorname{coz} \varphi(J) \bigcap \operatorname{coz} f_1$. Define

$$g(x) = \begin{cases} 1, & x \in A, \\ 0, & \text{otherwise}. \end{cases} \tag{4.2}$$

Then $g \in I$, since $Z(e_1) \subseteq Z(g)$, and I is a z-ideal. Also there exists $h \in J$ such that $Z(h) = Z(g)$. Define

$$k(x) = \begin{cases} \dfrac{1}{h}(x), & x \in A, \\ 0, & \text{otherwise}. \end{cases} \tag{4.3}$$

Then $g = kh \in J$. Moreover, $f_1 e_1 = f_1 g$. Now, $e_1 = \varphi(f_1) = \varphi(f_1)e_1 = \varphi(f_1 e_1) = \varphi(f_1 g) = g\varphi(f_1) \in J$. So for each $f \in J$, $\varphi(f) = \varphi(f)e_1 = \varphi(f e_1) = f\varphi(e_1)$. Define $\Psi : C(X) \to I$, such that $\Psi(f) = f\varphi(e_1)$ for each $f \in C(X)$. Then $\Psi \in \operatorname{Hom}_{C(X)}(C(X), I)$, and $\Psi|_J = \varphi$. Thus I is an injective $C(X)$-module. □

A commutative ring R with identity is called hereditary if every ideal of R is a projective R-module. Brookshear in [3] showed that $C(X)$ is hereditary if and only if X is finite and Vechtomov in [10] showed that if X is locally compact, then $C_K(X)$ is an injective $C(X)$-module if and only if X is finite. The following corollary gives more equivalent conditions in case that X is locally compact.

Corollary 4.2. *If X is a locally compact space, then the following statements are equivalent:*

(1) *$C(X)$ is hereditary.*
(2) *X is finite.*
(3) *$C_K(X)$ is an injective $C(X)$-module.*
(4) *$C_\Psi(X)$ is an injective $C(X)$-module.*

The following corollary gives more equivalent conditions to Theorem 3.2 and relates P-ideals and injective modules.

Corollary 4.3. *Let I be an ideal contained in $C_\Psi(X)$. Then I is a P-ideal if and only if for every $f \in I$, the principal ideal (f) is an injective $C(X)$-module.*

Proof. Suppose I is a P-ideal, then for each $f \in I$, $\operatorname{coz} f$ is finite. So it follows by Theorem 4.1 that the ideal (f) is an injective $C(X)$-module.

Conversely, let $f \in I$. Then $\operatorname{coz} f$ is finite, and therefore $Z(f)$ is clopen for each $f \in I$. Hence I is a P-ideal. □

Example 4.4. Let X be the space defined in Example 3.5, then $C_\Psi(X)$ is an injective $C(X)$-module, since $\operatorname{coz} C_\Psi(X) = \{0\}$ is finite.

The following example shows that there exists a P-ideal which is not injective.

Example 4.5. Let $X = \mathbb{N}^*$, the one point compactification of the natural numbers. Let $I = \{f \in C(X) : f = 0$ except on a finite set$\}$. Then I is a P-ideal, since $Z(f)$ is open for each $f \in I$, but I is not injective since $\operatorname{coz} I = \mathbb{N}$ is an infinite set.

5. When is $C_\Psi(X)$ prime?

Johnson and Mandelker in [6] used some ideals in $C(X)$ to study various generalizations of real compact spaces as μ-compactness, η-compactness, and Ψ-compactness. A space X is called μ-compact if $C_K(X) = I(X)$, the intersection of all free maximal ideals in $C(X)$, it is called η-compact if $C_\Psi(X) = I(X)$ and it is called Ψ-compact if $C_\Psi(X) = C_K(X)$. In this section we will prove that $C_K(X)$ is maximal if and only if $I(X)$ is maximal and X is μ-compact. We will show that $C_\Psi(X)$ could not be a proper prime ideal. We will also show that if $I(X)$ is prime, then it must be maximal.

Recall that a space X is called an almost compact space if $|\beta X - X| \leq 1$. That is, X is either compact or $\beta X = X \bigcup \{\infty\} = X^*$, the one-point compactification of X.

It is known that if X is an almost compact space, then it is pseudocompact (see [4]). For more properties of this space, see [7, 11].

Theorem 5.1. *The set $C_\Psi(X)$ cannot be a proper prime ideal in $C(X)$.*

Proof. Suppose that $C_\Psi(X)$ is a proper prime ideal, then it is contained in a unique maximal ideal. So $\beta X - \upsilon X$ is a one-point set since $C_\Psi(X) = O^{\beta X - \upsilon X} \subseteq M^x$ for each $x \in \beta X - \upsilon X$. Thus υX is an almost compact space and so it is pseudocompact.

Hence υX is compact and therefore $\upsilon X = \beta(\upsilon X) = \beta X$. This is a contradiction.

Also, since υX is compact, it follows that $C_K(\upsilon X) = C(\upsilon X)$ which implies that $C_\Psi(X) = C(X)$. □

Theorem 5.2. *The ideal $I(X)$ is prime if and only if X is an almost compact noncompact space.*

Proof. Suppose that $I(X)$ is prime, then $|\beta X - X| = 1$, since $I(X) = M^{\beta X - X} \subseteq M^x$ for each $x \in \beta X - X$. Hence X is an almost compact noncompact space.

The converse is clear. □

The following corollary follows easily from Theorem 5.2.

Corollary 5.3. *If $I(X)$ is prime, then it is maximal.*

Example 5.4. Let $\mathbf{T}$ be the Tychonoff plank, then $\beta\mathbf{T} = \mathbf{T}^* = \mathbf{T} \cup \{t_0\}$ (see [4]). So $I(\mathbf{T}) = M^{t_0}$ is maximal.

Theorem 5.5. *The ideal $C_K(X)$ is prime if and only if X is an almost compact noncompact space and ∞ is an F-point.*

Proof. If $C_K(X)$ is prime, then $|\beta X - X| = 1$. So $C_K(X) = O^\infty$ is prime and ∞ is an F-point.

The converse is clear. □

The following example shows that the two conditions in Theorem 5.5 are both necessary.

Example 5.6. $C_K(\mathbf{T}) = O^{t_0}$ is not prime (see [4]), although $\mathbf{T}$ is an almost compact space. On the other hand, every point in $\beta\mathbb{N}$ is an F-point but $C_K(\mathbb{N})$ is not prime.

Theorem 5.7. *The following statements are equivalent:*
(1) $C_K(X)$ *is maximal.*
(2) $C(X)$ *contains only one proper free ideal.*
(3) X *is an almost compact noncompact space and* ∞ *is a* P*-point.*
(4) X *is* μ*-compact and* $I(X)$ *is maximal.*

Proof. (1)⇒(2). Clear, since $C_K(X)$ is the intersection of all free ideals.

(2)⇒(3). X is noncompact, since $C(X)$ contains a proper free ideal. Also since $C_K(X)$ is maximal it follows that X is an almost compact space and $C_K(X) = O^\infty = M^\infty$, and so ∞ is a P-point.

(3)⇒(4). $C_K(X) = O^\infty = M^\infty = I(X)$, so X is μ-compact and $I(X)$ is maximal.

(4)⇒(1). Clear. □

Example 5.8. Let $\mathbf{W}$ denote the space of all ordinals less than the first uncountable ordinal number ω_1. Then $C_K(\mathbf{W}) = M^{\omega_1}$ (see [4]). So $C_K(\mathbf{W})$ is maximal and $\mathbf{W}$ is an almost compact space.

The following is an example of a space X such that $C_K(X)$ is prime but not maximal.

Example 5.9. Let $\mathbb{N}$ be the set of all natural numbers and let $p \in \beta\mathbb{N} - \mathbb{N}$, such that p is not a P-point. Such a point exists since $\beta\mathbb{N}$ is not a P-space and $\mathbb{N}$ is discrete. So there exists $f \in C^*(\mathbb{N})$, such that $p \in \mathrm{cl}_{\beta\mathbb{N}} Z(f)$, but $p \notin \mathrm{Int}_{\beta\mathbb{N}} Z(\bar{f})$ where $\bar{f}$ is the continuous extension of f to $\beta\mathbb{N}$. Since p is not a P-point, it is not isolated, so the space $X = \beta\mathbb{N} - \{p\}$ is an almost compact space. Let $f_1 = \bar{f}|_X$, then $f_1 \in C^*(X)$, since X is pseudocompact. Moreover, $p \in \mathrm{cl}_{\beta\mathbb{N}} Z(f) \subseteq \mathrm{cl}_{\beta\mathbb{N}} Z(f_1) = \mathrm{cl}_{\beta X} Z(f_1)$, since $Z(f) \subseteq Z(f_1)$. Hence $f_1 \in M^p$. Also, since $\bar{f}_1|_X = f_1 = \bar{f}|x$, it follows that $\bar{f}_1 = \bar{f}$, because X is dense in βX. So $p \notin \mathrm{Int}_{\beta\mathbb{N}} Z(\bar{f}) = \mathrm{Int}_{\beta\mathbb{N}} Z(\bar{f}_1) = \mathrm{Int}_{\beta X} Z(\bar{f}_1)$. Hence $f_1 \notin O^p$. That is, $f_1 \in M^p - O^p$. Now, since $\mathbb{N}$ is an F-space, it follows that $\beta\mathbb{N}$ is an F-space. So βX is an F-space, which implies that X is an F-space. (This is because Y is an F-space if and only if βY is an F-space (see [4]).) Thus p is an F-point. So $C_K(X) = O^p$ is a prime ideal. Hence $C_K(X)$ is a prime ideal which is not maximal.

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