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MODEL TRACKING FOR RISK PROBLEMS

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We assume that we have M candidate insurance models for describing a process. The models considered consist of a risk process driven by right-constant, finite-state spaces, jump processes. Based on observing the history of the risk process, we propose dynamics whose solutions indicate the likelihoods of each candidate model.

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1. Introduction. Risk theory deals with stochastic models in insurance business, see, for example, Grandell [2]. Usually, in such models claims are described by point processes and the amounts claimed by policies holders are sequences of random variables. The profit, or the loss, of the company is the difference between premiums income and the claims. In this paper, we assume that we have M competing models, denoted by $\{H_1,\dots,H_M\}$, describing the risk process, see Section 2. We are interested in ranking the candidate models based on their likelihood of being most appropriate for describing the risk process and some other processes driving the risk process. This problem as well as others fall within the category of Model Tracking or Detection problems as we are interested in tracking (or detecting) the most appropriate model for describing the proposed risk model, see, for example, Poor [5] and Snyder [6].

In the next section, we present the model of the paper. The main result of the paper is found in Section 3 where the likelihood that our model is best described by a certain candidate model is derived. In Section 4, a filtering problem is discussed.

2. The model. Assume initially that all processes are defined on a probability space $(\Omega,\mathcal{F},P)$.

Consider an insurance "risk process" R which at time t is the sum of an initial capital R_0, an integrated premiums process with integrand a nonnegative, bounded, and measurable real-valued function $P(\cdot)$, a new premiums process, a lost premiums process, and a claims process. We also assume that we have M candidate models denoted by $\{H_1,\dots,H_M\}$ representing the dynamics of the risk process. Then, under the hypothesis that model H_h is used, $h=1,\dots,M$, we have

$$\begin{aligned} R_t^h = R_0 &+ \int_0^t P_{H_h}(s)\,ds + \int_0^t\int_{\mathbb{R}^+} Y_{H_h}^1(\tilde{Z}_{r-}^1)\nu^1(dr,dx) \\ &- \int_0^t\int_{\mathbb{R}^+} Y_{H_h}^2(\tilde{Z}_{r-}^2)\nu^2(dr,dx) - \int_0^t\int_{\mathbb{R}^+} Y_{H_h}^3(\tilde{Z}_{r-}^3)\nu^3(dr,dx), \end{aligned} \tag{2.1}$$

where $Y_{H_h}^i(\cdot)$, $i=1,2,3$, are bounded nonnegative functions and each ν^i, $i=1,2,3$, is an integer-valued random measure which, under probability measure P, has predictable compensator (see Jacod [3]) $\bar{\nu}^i$ function of Z_t^i.

Here $\tilde{Z}_t^i$, $i = 1,2,3$, $t \in \mathbb{R}_+$, are finite-state spaces processes with right-constant sample paths on the state spaces $\tilde{S}^i = \{s_1^i,\dots,s_{n_i}^i\}$; s^i will denote the (column) vector $(s_1^i,\dots,s_{n_i}^i)'$.

Suppose $1 \le \ell \le N$, and for $j \ne \ell$

$$\pi_\ell^i(x) = \prod_{j=1}^{n_i}(x - s_j), \tag{2.2}$$

and $\phi_\ell^i(x) = \pi_\ell^i(x)/\pi_\ell^i(s_\ell)$; then $\phi_\ell^i(s_j) = \delta_{\ell j}$ and $\phi^i = (\phi_1^i,\dots,\phi_{n_i}^i)$ is a bijection of the set $\tilde{S}^i = \{s_1^i,\dots,s_{n_i}^i\}$ with the set $S^i = \{e_1^i,e_2^i,\dots,e_{n_i}^i\}$; e_j^i is the standard basis (column) vector in $\mathbb{R}^{n_i}$ with unity in the jth position and zero elsewhere. Consequently, without loss of generality, we consider processes Z_t^i on S^i for $i = 1,2,3$. If $Z_t^i \in S^i$ denotes the state of this process at time $t \ge 0$, then the corresponding value of $\tilde{Z}_t^i$ is $\langle Z_t^i, s^i\rangle$, where $\langle\,,\rangle$ denotes the inner product in $\mathbb{R}^{n_i}$.

Let $T_k^i(\omega)$ be the kth jump time of Z^i, $\delta_{T_k^i(\omega)}(dr)$ the unit mass at time $T_k^i(\omega)$ and $\delta_{Z^i_{T_k^i(\omega)}}(e_j^i)$ is the unit mass at $Z^i_{T_k^i(\omega)}(\omega)$.

Since Z_t^i is a jump process taking values in the vector space $\mathbb{R}^{n_i}$ we can write

$$Z_t^i = Z_0^i + \sum_{0<r\le t}\Delta Z_r^i. \tag{2.3}$$

Here

$$\begin{aligned}\Delta Z_r^i &= Z_r^i - Z_{r-}^i\\ &= \sum_{j=1}^{n_i}\left(e_j^i - Z_{r-}^i\right)\sum_{k=1}^{\infty}\delta_{T_k^i(\omega)}(dr)\delta_{Z^i_{T_k^i(\omega)}}\left(e_j^i\right)\\ &\triangleq \sum_{j=1}^{n_i}\left(e_j^i - Z_{r-}^i\right)\mu^{Z^i}\left(dr,e_j^i\right).\end{aligned} \tag{2.4}$$

We assume that each Z_t^i has almost surely finitely many jumps in any finite interval so that the random measure μ^{Z^i} is σ-finite. Let $\tilde{\mu}^{Z^i}(dr,e_j^i)$ be the predictable compensator of μ^{Z^i} so that (2.5) leads to

$$Z_t^i = Z_0^i + \sum_{j=1}^{n_i}\int_0^t\left(e_j^i - Z_{r-}^i\right)\tilde{\mu}^{Z^i}\left(dr,e_j^i\right) + W_t, \tag{2.5}$$

where

$$W_t \triangleq \sum_{j=1}^{n_i}\int_0^t\left(e_j^i - Z_{r-}^i\right)\left(\mu^{Z^i}\left(dr,e_j^i\right) - \tilde{\mu}^{Z^i}\left(dr,e_j^i\right)\right). \tag{2.6}$$

Now $\tilde{\mu}^{Z^i}$ factors into its Lévy system

$$\tilde{\mu}^{Z^i}\left(dr,e_j^i\right) = \beta\left(e_j^i, Z_{r-}^i, r\right)dF(Z_{r-}^i, r), \tag{2.7}$$

where $dF(Z^i_{r-},r)$ represents the conditional probability that the next jump occurs at time r given the previous history of the process.

Assume that the nonnegative measure $dF(Z^i_{r-},r)$ is absolutely continuous with respect to Lebesgue measure so that

$$dF(Z^i_{r-},r) = f(Z^i_{r-},r)dr \tag{2.8}$$

for some nonnegative function $f(\cdot)$.

On the set $[Z^i_{r-} \neq e^i_j]$ we have, from (2.8) and (2.9), that

$$\tilde{\mu}^{Z^i}\left(dr,e^i_j\right) = \beta\left(e^i_j,Z^i_{r-},r\right)f(Z^i_{r-},r)dr \triangleq a^i_{jZ^i_{r-}}(r,\omega)dr. \tag{2.9}$$

For $1 \leq j \leq n_i$ put

$$a^i_{jj}(r,\omega) = -\sum_{k\neq j} a^i_{jk}(r,\omega). \tag{2.10}$$

Define the matrix $A^i(r,\omega) = \{a^i_{jk}(r,\omega)\}$. Then we have the representation

$$Z^i_t = Z^i_0 + \int_0^t A^i(r,\omega)Z^i_r dr + W^i_t. \tag{2.11}$$

We assume here that the Z^i's have no common jumps, that is, with $\Delta Z^i_u = Z^i_u - Z^i_{u-}$ and for $i \neq j$

$$\sum_{0<u\leq t} \Delta Z^i_u \Delta Z^j_u = 0, \quad \forall t > 0 \text{ a.s.} \tag{2.12}$$

Let

$$\mathcal{R}_t = \sigma\{R_u, 0 \leq u \leq t\} \tag{2.13}$$

denote the complete filtration generated by the risk process and let

$$\mathcal{G}_t = \sigma\{R_s, Z^i_s; 1 \leq i \leq 3; s \leq t\} \tag{2.14}$$

be the complete filtration generated by the risk process R and the processes Z^i, $i = 1,2,3$.

Now, given the filtration $\mathcal{R}$, and, a set of competing hypotheses $\{H_1,\dots,H_M\}$, where $H_h = \{P_{H_h}(\cdot);\ Y^i_{H_h},\ i = 1,2,3\}$, we want to determine the dynamics to compute the posterior probabilities

$$P(H_h \mid \mathcal{R}_t), \quad 1 \leq h \leq M. \tag{2.15}$$

Consider a simple random variable α, where $\alpha \in \{f_1,\dots,f_M\}$ and $f_h = (0,\dots,1,\dots,0)'$ $\in \mathbb{R}^M$. The "1" here is in position h. We suppose α is an indicator function such that $\alpha = f_h$, that is, $\langle \alpha, f_h\rangle = 1$ if and only if hypothesis H_h holds. Then (2.15) may be rewritten as

$$P(H_h \mid \mathcal{R}_t) = E[\langle \alpha, f_h\rangle \mid \mathcal{R}_t], \tag{2.16}$$

where the expectation is taken under probability measure P.

In Section 3, we propose dynamics to (2.15) whose solution is a solution of some stochastic differential equation. Section 4 is concerned with a filtering problem.

3. M-ary detection filters. Suppose $\bar{P}$ is a reference probability, under which ν^i, $i = 1,2,3$, have deterministic compensators $H^i(dx)dt$ independent of Z^i, $i = 1,2,3$. In order to recover the "real world" probability measure P under which the model dynamics introduced in Section 2 hold, define the Radon-Nikodym derivative Λ such that

$$\left.\frac{dP}{d\bar{P}}\right|_{\mathcal{G}_t} = \Lambda_t, \tag{3.1}$$

where (see Jacod and Shiryaev [4])

$$\Lambda_t = 1 + \sum_{i=1}^{3} \int_0^t \int_{\mathbb{R}^+} \Lambda_{s-}\left(\tilde{\nu}^i(s, Z^i_{s-}, x) - 1\right)\left[\nu^i(dr, dx) - H(dx)dt\right]. \tag{3.2}$$

However, in this section, we will be working under the "reference probability" $\bar{P}$. By an abstract version of Bayes' rule (see [1])

$$P(\alpha = f_h \mid \mathcal{R}_t) = E\left[\langle \alpha, f_h\rangle \mid \mathcal{R}_t\right] = \frac{\bar{E}\left[\Lambda_t \langle \alpha, f_h\rangle \mid \mathcal{R}_t\right]}{\bar{E}\left[\Lambda_t \mid \mathcal{R}_t\right]}. \tag{3.3}$$

Theorem 3.1. *Let*

$$q_t^h \triangleq \bar{E}\left[\langle \alpha, f_h\rangle \Lambda_t \mid \mathcal{R}_t\right]. \tag{3.4}$$

The unnormalized probability q_t^h is given by the equation

$$q_t^h = q_0^h + \sum_{i=1}^{3} \int_0^t \int_{\mathbb{R}^+} \left(\sum_{J=1}^{3} E\left[\left\langle Z^i_{u-}, e^i_j \right\rangle \mid \mathcal{R}_{u-}\right] \tilde{\nu}^i(u, e^i_j, x) - 1 \right) q^h_{u-}\left[\nu^i(du, dx) - H(dx)du\right]. \tag{3.5}$$

Here $E[\langle Z^i_{u-}, e^i_j\rangle \mid \mathcal{R}_{u-}]$ is evaluated under the probability measure P, given that the hypothesis H_h holds.

Proof. Using (3.2), we have

$$\langle \alpha, f_h\rangle \Lambda_t = \langle \alpha, f_h\rangle + \sum_{i=1}^{3} \int_0^t \int_{\mathbb{R}^+} \langle \alpha, f_h\rangle \Lambda_{s-}\left(\tilde{\nu}^i(s, Z^i_{s-}, x) - 1\right)\left[\nu^i(ds, dx) - H(dx)ds\right], \tag{3.6}$$

with optional projection on the σ-field $\mathcal{R}_t$

$$\begin{aligned} &\bar{E}\left[\Lambda_t \langle \alpha, f_h\rangle \mid \mathcal{R}_t\right] \\ &= \langle \alpha, f_h\rangle + \bar{E}\left[\sum_{i=1}^{3} \int_0^t \int_{\mathbb{R}^+} \langle \alpha, f_h\rangle \Lambda_{s-}\left(\tilde{\nu}^i(s, Z^i_{s-}, x) - 1\right)\left[\nu^i(ds, dx) - H(dx)ds\right] \mid \mathcal{R}_t \right]. \end{aligned} \tag{3.7}$$

Using (3.4) and [7, Chapter 7, Lemma 3.2] to exchange stochastic integration and conditional expectation under $\bar{P}$, we have

$$q_t^h = q_0^h + \sum_{i=1}^{3} \int_0^t \int_{\mathbb{R}^+} \bar{E}\left[\langle \alpha, f_h\rangle \Lambda_{s-}\left(\tilde{\nu}^i(s, Z^i_{s-}, x) - 1\right) \mid \mathcal{R}_{s-}\right]\left[\nu^i(ds, dx) - H(dx)dt\right]. \tag{3.8}$$

Now

$$\frac{\bar{E}[\langle\alpha,f_h\rangle\Lambda_{s-}(\tilde{\nu}^i(s,Z^i_{s-},x)-1)\mid\mathcal{R}_{s-}]}{\bar{E}[\Lambda_{s-}\mid\mathcal{R}_{s-}]}=E[\langle\alpha,f_h\rangle(\tilde{\nu}^i(s,Z^i_{s-},x)-1)\mid\mathcal{R}_{s-}], \tag{3.9}$$

or

$$\begin{aligned}\bar{E}[\langle\alpha,f_h\rangle\Lambda_{s-}(\tilde{\nu}^i(s,Z^i_{s-},x)-1)\mid\mathcal{R}_{s-}]\\=E[\langle\alpha,f_h\rangle(\tilde{\nu}^i(s,Z^i_{s-},x)-1)\mid\mathcal{R}_{s-}]\bar{E}[\Lambda_{s-}\mid\mathcal{R}_{s-}],\end{aligned} \tag{3.10}$$

which, using elementary rules for conditional probabilities and Bayes rule, is

$$\begin{aligned}&=E[(\tilde{\nu}^i(s,Z^i_{s-},x)-1)\mid\alpha=f_h,\mathcal{R}_{s-}]E[\langle\alpha,f_h\rangle\mid\mathcal{R}_{s-}]\bar{E}[\Lambda_t\mid\mathcal{R}_{s-}]\\&=E[(\tilde{\nu}^i(s,Z^i_{s-},x)-1)\mid\alpha=f_h,\mathcal{R}_{s-}]\frac{\bar{E}[\langle\alpha,f_h\rangle\Lambda_{s-}\mid\mathcal{R}_{s-}]}{\bar{E}[\Lambda_{s-}\mid\mathcal{R}_{s-}]}\bar{E}[\Lambda_{s-}\mid\mathcal{R}_{s-}]\\&=E[(\tilde{\nu}^i(s,Z^i_{s-},x)-1)\mid\alpha=f_h,\mathcal{R}_{s-}]q^h_{s-}\quad\text{(using (3.4))}\\&=\Big(E[\tilde{\nu}^i(s,Z^i_{s-},x)\mid\alpha=f_h,\mathcal{R}_{s-}]-1\Big)q^h_{s-}.\end{aligned} \tag{3.11}$$

Using the notation $Z^i_t=\sum_{J=1}^3\langle Z^i_t,e^i_j\rangle e^i_j$ gives (3.5). □

Note that the normalized form of (3.5) is given by

$$p^h_t=\frac{q^h_t}{\sum_{l=1}^M q^l_t}. \tag{3.12}$$

As an example: suppose that the set of candidate models consists of two models, that is, $\alpha\in\{(1,0),(0,1)\}$ and $p^1_t=P(\alpha=(1.0)\mid\mathcal{R}_t)=E[\langle\alpha,(1,0)\rangle\mid\mathcal{R}_t]$ and $p^2_t=P(\alpha=(0,1)\mid\mathcal{R}_t)=E[\langle\alpha,(0,1)\rangle\mid\mathcal{R}_t]$. Define the log-likelihood or test statistic process,

$$l_t=\ln\left(\frac{p^1_t}{p^2_t}\right)=\ln\left(\frac{q^1_t}{q^2_t}\right). \tag{3.13}$$

Large values of l are in favor of model 1 whereas, small values of l are in favor of model 2.

4. The filtering problem. Equation (3.5) contains $E[\langle Z^i_{u-},e^i_j\rangle\mid\mathcal{R}_{u-}]$. The following result gives the dynamics of the unnormalized version of this filter. Here we assume that the random matrix A is adapted to the filtration $\mathcal{R}$. Again we work under the "reference probability" $\bar{P}$, under which ν^i, $i=1,2,3$, have deterministic compensators $H^i(dx)dt$ independent of Z^i, $i=1,2,3$.

Theorem 4.1. *Let*

$$\sigma_t(\ell,m,n)\triangleq\bar{E}\Big[\Big\langle Z^1_t,e^1_\ell\Big\rangle\langle Z^2_t,e^2_m\rangle\langle Z^3_t,e^3_n\rangle\Lambda_t\mid\mathcal{R}_t\Big]. \tag{4.1}$$

The unnormalized probability process $\sigma_t(\ell,m,n)$ satisfies the stochastic integral equation

$$\begin{aligned}\sigma_t(\ell,m,n) &= \sigma_0(\ell,m,n)\\ &\quad+\sum_{i=1}^{3}\int_0^t\int_{\mathbb{R}^+}\sigma_{u-}(\ell,m,n)\left(\tilde{\nu}^i\left(u,e_j^i,x\right)-1\right)\left[\nu^i(dr,dx)-H(dx)dt\right]\\ &\quad+\sum_{k_1}\int_0^t a^1_{\ell k_1}(u,\omega)\sigma_u(k_1,m,n)du\\ &\quad+\sum_{k_2}\int_0^t a^2_{mk_2}(u,\omega)\sigma_u(\ell,k_2,n)du\\ &\quad+\sum_{k_3}\int_0^t a^3_{nk_3}(u,\omega)\sigma_u(\ell,m,k_3)du.\end{aligned} \tag{4.2}$$

Proof. Note that (2.11) gives

$$\begin{aligned}\left\langle Z_t^1,e_\ell^1\right\rangle &= \left\langle Z_0^1,e_\ell^1\right\rangle+\int_0^t\left\langle A^1(u,\omega)Z_u^1,e_\ell^1\right\rangle du+\left\langle W_t^1,e_\ell^1\right\rangle\\ &= \left\langle Z_0^1,e_\ell^1\right\rangle+\sum_{k_1}\int_0^t a^1_{\ell k_1}(u,\omega)\left\langle Z_{u-}^1,e_{k_1}^1\right\rangle du+\left\langle W_t^1,e_\ell^1\right\rangle.\end{aligned} \tag{4.3}$$

Since the processes Z_t^1 and Z_t^2 share no common jumps,

$$\begin{aligned}\left\langle Z_t^1,e_\ell^1\right\rangle\left\langle Z_t^2,e_m^2\right\rangle &= \left\langle Z_0^1,e_\ell^1\right\rangle\left\langle Z_0^2,e_m^2\right\rangle+\int_0^t\left\langle Z_{u-}^1,e_\ell^1\right\rangle\left\langle dZ_u^2,e_m^2\right\rangle\\ &\quad+\int_0^t\left\langle Z_{u-}^2,e_m^2\right\rangle\left\langle dZ_u^1,e_\ell^1\right\rangle\\ &= \left\langle Z_0^1,e_\ell^1\right\rangle\left\langle Z_0^2,e_m^2\right\rangle+\sum_{k_2}\int_0^t a^2_{mk_2}(u,\omega)\left\langle Z_u^1,e_\ell^1\right\rangle\left\langle Z_{u-}^2,e_{k_2}^2\right\rangle du\\ &\quad+\sum_{k_1}\int_0^t a^1_{\ell k_1}(u,\omega)\left\langle Z_u^2,e_m^2\right\rangle\left\langle Z_{u-}^1,e_{k_1}^1\right\rangle du\\ &\quad+\int_0^t\left\langle Z_{u-}^2,e_m^2\right\rangle\left\langle dV_u^1,e_\ell^1\right\rangle+\int_0^t\left\langle Z_{u-}^1,e_\ell^1\right\rangle\left\langle dW_u^2,e_m^2\right\rangle,\end{aligned}$$

$$\begin{aligned}\left\langle Z_t^1,e_\ell^1\right\rangle\left\langle Z_t^2,e_m^2\right\rangle\left\langle Z_t^3,e_n^3\right\rangle &= \left\langle Z_0^1,e_\ell^1\right\rangle\left\langle Z_0^2,e_m^2\right\rangle\left\langle Z_0^3,e_n^3\right\rangle\\ &\quad+\int_0^t\left\langle Z_{u-}^1,e_\ell^1\right\rangle\left\langle Z_{u-}^2,e_m^2\right\rangle\left\langle dZ_u^3,e_n^3\right\rangle\\ &\quad+\int_0^t\left\langle Z_{u-}^3,e_n^3\right\rangle d\left(\left\langle Z_u^1,e_\ell^1\right\rangle\left\langle Z_u^2,e_m^2\right\rangle\right)\\ &= \left\langle Z_0^1,e_\ell^1\right\rangle\left\langle Z_0^2,e_m^2\right\rangle\left\langle Z_0^3,e_n^3\right\rangle\\ &\quad+\sum_{k_3}\int_0^t a^3_{nk_3}(u,\omega)\left\langle Z_u^1,e_\ell^1\right\rangle\left\langle Z_u^2,e_m^2\right\rangle\left\langle Z_u^3,e_{k_3}^3\right\rangle du\\ &\quad+\sum_{k_1}\int_0^t a^1_{\ell k_1}(u,\omega)\left\langle Z_u^3,e_n^3\right\rangle\left\langle Z_u^2,e_m^2\right\rangle\left\langle Z_u^1,e_{k_1}^1\right\rangle du\end{aligned}$$

$$\begin{aligned}
&+\sum_{k_2}\int_0^t a^2_{mk_2}(u,\omega)\left\langle Z^3_u,e^3_n\right\rangle\left\langle Z^2_u,e^2_m\right\rangle\left\langle Z^1_u,e^1_{k_1}\right\rangle du\\
&+\int_0^t\left\langle Z^3_{u-},e^3_n\right\rangle\left\langle Z^2_{u-},e^2_m\right\rangle\left\langle dW^1_u,e^1_\ell\right\rangle\\
&+\int_0^t\left\langle Z^3_{u-},e^3_n\right\rangle\left\langle Z^1_{u-},e^1_\ell\right\rangle\left\langle dW^2_u,e^2_m\right\rangle\\
&+\int_0^t\left\langle Z^1_{u-},e^1_\ell\right\rangle\left\langle Z^2_{u-},e^2_m\right\rangle\left\langle dW^3_u,e^3_n\right\rangle.
\end{aligned}\tag{4.4}$$

Using (3.2) and recalling that the processes Z^1_t, Z^2_t, Z^3_t, and Λ_t share no common jumps under $\bar{P}$

$$\begin{aligned}
d\left(\Lambda_t\left\langle Z^1_t,e^1_\ell\right\rangle\left\langle Z^2_t,e^2_m\right\rangle\left\langle Z^3_t,e^3_n\right\rangle\right)&=\left\langle Z^1_t,e^1_\ell\right\rangle\left\langle Z^2_t,e^2_m\right\rangle\left\langle Z^3_t,e^3_n\right\rangle d\Lambda_t\\
&\quad+\Lambda_t d\left(\left\langle Z^1_t,e^1_\ell\right\rangle\left\langle Z^2_t,e^2_m\right\rangle\left\langle Z^3_t,e^3_n\right\rangle\right),
\end{aligned}\tag{4.5}$$

so

$$\begin{aligned}
&\Lambda_t\left\langle Z^1_t,e^1_\ell\right\rangle\left\langle Z^2_t,e^2_m\right\rangle\left\langle Z^3_t,e^3_n\right\rangle\\
&\quad=\left\langle Z^1_0,e^1_\ell\right\rangle\left\langle Z^2_0,e^2_m\right\rangle\left\langle Z^3_0,e^3_n\right\rangle\\
&\qquad+\sum_{i=1}^3\int_0^t\int_{\mathbb{R}^+}\Lambda_{u-}\left\langle Z^1_{u-},e^1_\ell\right\rangle\left\langle Z^2_{u-},e^2_m\right\rangle\left\langle Z^3_{u-},e^3_n\right\rangle\\
&\qquad\qquad\times\left(\sum_j\left\langle Z^i_{u-},e^i_j\right\rangle\tilde{\nu}^i\left(u,e^i_j,x\right)-1\right)\left[\nu^i(du,dx)-H(dx)du\right]\\
&\qquad+\sum_{k_3}\int_0^t a^3_{nk_3}(u,\omega)\Lambda_u\left\langle Z^1_u,e^1_\ell\right\rangle\left\langle Z^2_u,e^2_m\right\rangle\left\langle Z^3_u,e^3_{k_3}\right\rangle du\\
&\qquad+\sum_{k_1}\int_0^t a^1_{\ell k_1}(u,\omega)\Lambda_u\left\langle Z^3_u,e^3_n\right\rangle\left\langle Z^2_u,e^2_m\right\rangle\left\langle Z^1_u,e^1_{k_1}\right\rangle du\\
&\qquad+\sum_{k_2}\int_0^t a^2_{mk_2}(u,\omega)\Lambda_u\left\langle Z^3_u,e^3_n\right\rangle\left\langle Z^2_u,e^2_{k_1}\right\rangle\left\langle Z^1_u,e^1_\ell\right\rangle du\\
&\qquad+\text{martingales.}
\end{aligned}\tag{4.6}$$

Simplifying the integrand in the stochastic integral in (4.6) gives

$$\begin{aligned}
&\sum_{i=1}^3\int_0^t\int_{\mathbb{R}^+}\Lambda_{u-}\left\langle Z^1_{u-},e^1_\ell\right\rangle\left\langle Z^2_{u-},e^2_m\right\rangle\left\langle Z^3_{u-},e^3_n\right\rangle\\
&\qquad\times\left(\sum_j\left\langle Z^i_{u-},e^i_j\right\rangle\tilde{\nu}^i(u,e^i_j,x)-1\right)\left[\nu^i(du,dx)-H(dx)du\right]\\
&\quad=\sum_{i=1}^3\int_0^t\int_{\mathbb{R}^+}\Lambda_{u-}\left\langle Z^1_{u-},e^1_\ell\right\rangle\left\langle Z^2_{u-},e^2_m\right\rangle\left\langle Z^3_{u-},e^3_n\right\rangle\\
&\qquad\qquad\times\left(\tilde{\nu}^i(u,e^i_j,x)-1\right)\left[\nu^i(du,dx)-H(dx)du\right].
\end{aligned}\tag{4.7}$$

Conditioning each side of (4.6) on $\mathcal{R}_t$, under the measure $\bar{P}$, and using again [7, Chapter 7, Lemma 3.2] to exchange stochastic integration and conditional expectation establishes the result. □

In this paper, a risk model described by M candidate models was discussed. Detection filters were derived using measure change techniques.

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