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FUZZY NEIGHBORHOOD STRUCTURES ON PARTIALLY ORDERED GROUPS

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Ahsanullah (1988) showed the compatibility between group structures and I-fuzzy neighborhood systems. In this paper, we require not only that the I-fuzzy neighborhood systems be compatible with the group structures, but also compatible with the order relation, in one sense or another.

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1. Introductions. In [8], Katsaras combine the concepts of $[0,1]$-topology and order structure to bring out the so-called ordered fuzzy topological spaces. Several authors have continued on the work of Katsaras in the area of $[0,1]$-topology and order [3, 4, 10].

In [2] Ahsanullah introduced the notion of I-fuzzy neighborhood groups. In this paper, we aim to introduce and study the concept of I-fuzzy neighborhood structures on ordered groups.

2. Preliminaries. Let X be a nonempty set. A relation $\le$ on X is said to be preorder if it is reflexive and transitive. An antisymmetric preorder is said to be a partially order. By a preordered (resp., an ordered) set, we mean a set X with a preorder (resp., a partially order) relation on it and we denote it by $(X,\le)$. Every set can be considered as a partially ordered set equipped with the discrete order ($x \le y$ if and only if $x = y$).

A function f from a preordered set $(X,\le)$ to a preordered set $(X',\le')$ is called isotone or order-preserving (resp., antitone or order-inverting) if $x \le y$ in X implies $f(x) \le' f(y)$ (resp., $f(y) \le' f(x)$) in X'. The function f is said to be order isomorphism if it is bijection and $(\forall x,y \in X)\ x \le y \Leftrightarrow f(x) \le' f(y)$.

Suppose that $(G,*)$ is a semigroup and that G is endowed with an order $\le$. We say that $(G,*,\le)$ is an ordered semigroup if the low of composition and the order are related by the property: for all $x,y \in G$

$$x \le y \Longrightarrow (\forall z \in G)\ x*z \le y*z, \quad z*x \le z*y. \tag{2.1}$$

If $(G_1,T_1,\le_1)$ and $(G_2,T_2,\le_2)$ are ordered semigroups. A mapping $f: G_1 \to G_2$ is said to be order-homomorphism if it is both isotone and semigroup homomorphism. By an ordered group we mean an ordered semigroup which is a group.

In this paper, we use the multiplicative ordered group $(G,\cdot,\le)$ which is sometimes written as $(G,\le)$.

Combining the notion of order-isomorphism and group isomorphism, we say that an ordered group $(G_1,\leq_1)$ is OG-isomorphic to an ordered group $(G_2,\leq_2)$ if there is a mapping $f: G_1 \to G_2$ which is both order isomorphism and group isomorphism.

An I-fuzzy set μ, in a preordered set $(X,\leq)$, is called increasing (resp., decreasing) if $x \leq y$ implies $\mu(x) \leq \mu(y)$ (resp., $\mu(y) \leq \mu(x)$) [8].

A Chang-Goguen L-topology (cf. [5, 6, 7]) on a set X is a subset $\tau \subset L^X$, closed under finite infs and arbitrary sups. A pair (X,τ) is called a Chang-Goguen L-topological space; (X,τ) is called stratified L-topological space if τ contains all the constant L-fuzzy sets. The category of Chang-Goguen L-topological spaces (resp., stratified Chang-Goguen L-topological spaces) is denoted by $|L\text{-Top}|$ (resp., $|SL\text{-Top}|$). Both $|L\text{-Top}|$ and $|SL\text{-Top}|$ are topological categories. If $L = I = [0,1]$, the above categories are denoted by $|I\text{-Top}|$ and $|SI\text{-Top}|$, respectively.

By an I-topological (resp., stratified I-topological) ordered space are we mean a triplet $(X,\leq,\tau)$, consisting of a partially ordered set $(X,\leq)$ and an I-topology (resp., stratified I-topology) τ on X.

By $|I\text{-TopOS}|$ (resp., $|SI\text{-TopOS}|$), we mean the category of all I-topological (resp., stratified I-topological) ordered spaces as object and all order-preserving continuous mappings between them as morphisms.

The order $\leq$, in an I-topological ordered space $(X,\leq,\tau)$, is said to be closed [8] if and only if the following condition holds: if $x \not\leq y$, then there are neighborhoods μ, ρ of x, y, respectively, such that $i(\mu) \wedge d(\rho) = 0$.

Let $(X,\leq,\tau)$ be an L-topological ordered space. If the order is closed, then X is Hausdorff [8].

An I-fuzzy quasi-uniformity [9] is a subset $\mathbf{U}$ of $I^{X\times X}$ which is prefilter and has the following three properties:

(1) $\alpha(x,x) = 1\ \forall \alpha \in \mathbf{U}$ and $\forall x \in X$,

(2) $\forall \alpha \in \mathbf{U}$, $\forall \varepsilon > 0$, $\exists \alpha_1 \in \mathbf{U}$ such that $\alpha_1 \circ \alpha_1 - \varepsilon \leq \alpha$,

(3) $\mathbf{U} = \mathbf{U}$, that is, for every family $\{\alpha_\varepsilon \in \mathbf{U}, \varepsilon \in I_0\}$ we have $\sup_{\varepsilon \in I(\alpha_\varepsilon - \varepsilon) \in \mathbf{U}}$.

The family $\mathbf{U}^{-1} = \{\alpha^{-1} : \alpha \in \mathbf{U},\ \alpha^{-1}(x,y) = \alpha(y,x)\}$ is an I-fuzzy quasi-uniformity on X called the conjugate of $\mathbf{U}$. We denote by $\mathbf{U}^*$ the I-fuzzy uniformity which generated by $\mathbf{U}$, that is, $\mathbf{U}^* = \mathbf{U} \vee \mathbf{U}^{-1} = \{\alpha \wedge \alpha^{-1} : \alpha \in \mathbf{U},\ \alpha^{-1} \in \mathbf{U}^{-1}\}$. The I-fuzzy quasi-uniformity $\mathbf{U}$ can generate an order, say $\leq_u$, by setting

$$x \leq_u y \Longleftrightarrow \begin{cases} \alpha(x,z) \leq \alpha(y,z) & \forall z \geq x,y, \\ \alpha(x,z) \geq \alpha(y,z) & \forall z \leq x,y. \end{cases} \tag{2.2}$$

A triplet $(X,\leq,\mathbf{U}^*)$, consisting of an ordered set $(X,\leq)$ and an I-fuzzy uniformity $\mathbf{U}^*$, is called an I-fuzzy uniform ordered space [10] if there exists an I-fuzzy quasi-uniformity $\mathbf{U}$ on X such that $\mathbf{U}^* = \mathbf{U} \vee \mathbf{U}^{-1}$ and $G(\leq) = G(\leq_u)$.

Definition 2.1 [10]. Let $(X_1,\mathbf{U}_1)$ and $(X_2,\mathbf{U}_2)$ be I-fuzzy quasi-uniform spaces. A mapping $f:(X_1,\mathbf{U}_1) \to (X_2,\mathbf{U}_2)$ is said to be quasi-uniformly continuous if and only if $\forall \alpha_2 \in \mathbf{U}_2$, $\exists \alpha_1 \in \mathbf{U}_1$ such that $\alpha_1 \in (f\times f)^{-1}(\alpha_2)$. Where f is called quasi-uniform equivalence if f is bijective and both f and f^{-1} are quasi-uniformly continuous.

Definition 2.2 [10]. A mapping $f : (X, \le, \mathbf{U}^*) \to (X_1, \le_1, \mathbf{U}^*)$ is said to be uniformly order-mapping if there exist I-fuzzy quasi-uniformities u and u_1 on X and X_1, respectively such that

(i) $\mathbf{U}^* = \mathbf{U} \vee \mathbf{U}^{-1}$ and $G(\le) = G(\le_u)$;
(ii) $\mathbf{U}_1^* = \mathbf{U}_1 \vee \mathbf{U}_1^{-1}$ and $G(\le_1) = G(\le_{u_1})$;
(iii) $f : (X, \mathbf{U}) \to (X_1, \mathbf{U}_1)$ is quasi-uniformly continuous.

Definition 2.3 [2]. Let $(G, \cdot)$ be a group and let $\aleph$ be an I-fuzzy neighborhood system on G. Then, the triplet $(G, \cdot, t(\aleph))$ is called I-fuzzy neighborhood group if and only if the following conditions are fulfilled:

(1) the mapping $m : (G \times G, t(\aleph) \times t(\aleph)) \to (G, t(\aleph)) : (x, y) \to xy$ is continuous;
(2) the mapping $r : (G, t(\aleph)) \to (G, t(\aleph)) : x \to x^{-1}$ is continuous.

Proposition 2.4 [2]. *Let $(G, \cdot)$ be a group and let $\aleph$ be an I-fuzzy neighborhood system on G. Then, $(G, \cdot, t(\aleph))$ is an I-fuzzy neighborhood group if and only if the mapping*

$$h : (G \times G, t(\aleph) \times t(\aleph)) \longrightarrow (G, t(\aleph)) : (x, y) \longrightarrow xy^{-1} \tag{2.3}$$

is continuous

3. Fuzzy neighborhood ordered groups

Definition 3.1. A triplet $(G, \le, t(\aleph))$ is called I-fuzzy neighborhood ordered groups if the following statements hold:

(1) $(G, \le)$ is a partially ordered group;
(2) $(G, t(\aleph))$ is an I-fuzzy neighborhood group;
(3) the order $\le$ is closed.

By $|I - \text{FNOGr}|$, we mean the category of all I-fuzzy neighborhood ordered groups as objects and all order-preserving homeomorphisms between them as morphisms.

In agreement with [1], a faithful functor $T : A \to \text{Set}$ is said to be topological (mono-topological) if and only if, given any index class $((X_j, \xi_j) : j \in J)$ of A-objects indexed by a class J and any source (resp., mono-source) $(f_j : X \to X_j)$ in Set, there exists a unique A-structure ξ on X which is initial with respect to $(f_j : X \to (X_j, \xi_j))_{j \in J}$, that is, such that for any A-object (Y, ζ), a mapping $h : (Y, \zeta) \to (X, \xi)$ is an A-morphism if and only if for every $j \in J$, the composition $f_j \circ h : (Y, \zeta) \to (X_j, \xi_j)$ is an A-morphism. Also, we have that the constant function lift to morphism in A and the A-fibre $T^{-1}(S)$ for any set S is small.

Proposition 3.2. *The category $|I - \text{FNOGr}|$ is mono-topological.*

Proof. The forgetful functor $T : |I - \text{FNOGr}| \to |\text{Group}|$ is given by $T(G, \le, t(\aleph)) = G$. For some index class J, let $(G_\alpha, \le_\alpha, t(\aleph_\alpha)) \in |I - \text{FNOGr}|$ and $(f_\alpha : G \to G_\alpha)_{\alpha \in J}$ be a monosource in $|\text{Group}|$. Let $\aleph$ be the I-fuzzy neighborhood system making the monosource

$$(f_\alpha : (G, t(\aleph)) \longrightarrow (G_\alpha, t(\aleph_\alpha)))_{\alpha \in J} \tag{3.1}$$

initial and let $\leq$ be the order defined by $x \leq y$ if and only if $f_\alpha(x) \leq_\alpha f_\alpha(y)$ for all $\alpha \in J$. Then $(G, \leq, t(\aleph)) \in |I-\text{FNOGr}|$. Initiality of the mono-source

$$(f_\alpha : (G, \leq, t(\aleph)) \longrightarrow (G_\alpha, \leq_\alpha, t(\aleph_\alpha)))_{\alpha \in J} \tag{3.2}$$

can easily be checked; thus T is mono-topological. The other conditions for a mono-topological category are clearly met. □

Proposition 3.3. *Let $(G, \leq, t(\aleph)) \in |I-\text{FNOGr}|$. Then, for $x, a \in G$,*

(i) *the mapping $L_a : G \to G$ (resp., $R_a : G \to G$) defined by $x \to ax$ (resp., $x \to xa$) is an order-preserving homeomorphism;*

(ii) *the mapping $r : (G, t(\aleph)) \to (G, t(\aleph)) : x \to x^{-1}$ is an order-inverting homeomorphism.*

Proof. The proof follows from Definition 2.3. □

Lemma 3.4. *Let $(G, \leq, t(\aleph)) \in |I-\text{FNOGr}|$ and μ be an increasing (resp., decreasing) I-fuzzy set in G, then*

(i) *$R_a^{-1}(\mu)$ is increasing (resp., decreasing);*

(ii) *$r^{-1}(\mu)$ is decreasing (resp., increasing).*

Proof. Let μ be an increasing I-fuzzy set.

(i) We have

$$\begin{aligned} R_a^{-1}(\mu)(x) &= \mu(R_a(x)) = \mu(xa) \leq \mu(ya) = \mu(R_a(y)), \\ R_a^{-1}(\mu)(x) &= \mu(R_a(x)) \leq \mu(R_a(y)) = R_a^{-1}(\mu)(y), \end{aligned} \tag{3.3}$$

that is, $R_a^{-1}(\mu)(x) \leq R_a^{-1}(\mu)(y)$ whenever $x \leq y$.

(ii) The mapping $r : G \to G$ is decreasing, then

$$r^{-1}(\mu)(x) = \mu(r(x)) = \mu(x^{-1}) \geq \mu(y^{-1}) = \mu(r(y)) = r^{-1}(\mu)(y), \tag{3.4}$$

that is, $r^{-1}(\mu)$ is decreasing. □

Proposition 3.5. *If $(G, \leq, t(\aleph)) \in |I-\text{FNOGr}|$ and μ is an increasing (resp., decreasing) open I-fuzzy set in G and $\rho \in I^G$, then the I-fuzzy set $(\mu \cdot \rho)$ is an increasing (resp., decreasing) open I-fuzzy set in G.*

Proof. By [2, Proposition 1.10], an I-fuzzy set $(\mu \cdot \rho)$ is open. To prove the second part, let $x, y \in G$ with $x \leq y$ and μ be increasing I-fuzzy, then

$$\begin{aligned} \mu \cdot \rho(x) &= \sup_{x = s \cdot t} \mu(s) \wedge \rho(t) = \sup_{t \in G} \mu(xt^{-1}) \wedge \rho(t) \\ &= \sup_{t \in G} \mu(R_t^{-1}(x)) \wedge \rho(t) = \sup_{t \in G} R_t(\mu)(x) \wedge \rho(t). \end{aligned} \tag{3.5}$$

But the mapping $R_t : G \to G : x \to xt$ is increasing, then, by fixing $t \in G$, it follows that

$$\mu \cdot \rho(x) = \sup_{t \in G} R_t(\mu)(x) \wedge \rho(t) \leq \sup_{t \in G} R_t(\mu)(y) \wedge \rho(t) = \mu \cdot \rho(y), \tag{3.6}$$

that is, I-fuzzy set $\mu \cdot \rho$ is increasing. □

Proposition 3.6. *Let $(G,\leq,t(\aleph)) \in |I-\mathrm{FNOGr}|$, then for all increasing (resp., decreasing) I-fuzzy set $\mu \in \aleph(e)$ and for all $\varepsilon \in I_0$, there exists $\rho \in \aleph(e)$ such that $i(\rho\cdot\rho)-\varepsilon \leq \mu$ (resp., $d(\rho\cdot\rho)-\varepsilon \leq \mu$).*

Proof. Since $(G,t(\aleph))$ is an I-fuzzy neighborhood group, then the continuity of the mapping $m:(G\times G,t(\aleph)\times t(\aleph)) \to (G,t(\aleph)):(x,y)\to xy$ is equivalent to the fact that $\forall\mu \in \aleph(e)$ and $\forall\varepsilon \in I_0$, there exists $\rho \in \aleph(e)$ (see [2, Proposition 2.5]) such that $\rho\cdot\rho-\varepsilon \leq \mu$. If we choose μ to be increasing then

$$\rho\cdot\rho \leq i(\rho\cdot\rho) \leq \mu+\varepsilon, \tag{3.7}$$

where $i(\rho\cdot\rho)$ is the smallest increasing I-fuzzy set containing $(\rho\cdot\rho)$ and it follows that $i(\rho\cdot\rho)-\varepsilon \leq \mu$ and this completes the proof. □

4. Fuzzy quasi-uniformity on I-fuzzy neighborhood ordered groups.

As given in [2], if $(G,\cdot)$ is a group, then we define

$$\begin{aligned} &\mu_L : G\times G \longrightarrow I, \quad \text{where } \mu_L(x,y)=\mu(x^{-1}y),\\ &\mu_R : G\times G \longrightarrow I, \quad \text{where } \mu_R(x,y)=\mu(yx^{-1}). \end{aligned} \tag{4.1}$$

If $(G,\cdot,t(\aleph))$ is an I-fuzzy neighborhood group and $\mu \in \aleph(e)$, then μ_L (resp., μ_R) is called the left (resp., right) I-fuzzy entourages associated with μ. We can easily note that the left (resp., right) I-fuzzy entourages μ_L (resp., μ_R) is not symmetric, if $x \neq y$, then $y^{-1}x \neq e \neq x^{-1}y$ and this implies that $\mu_L(x,y)=\mu(x^{-1}y)\neq\mu(y^{-1}x)=\mu_L(y,x)$. Also, $\mu_R(x,y)\neq\mu_R(y,x)$.

In the sequel, we use $\aleph^i(e)$ (resp., $\aleph^d(e)$) to denote the system of all increasing (resp., decreasing) I-fuzzy neighborhoods of e. From the above discussion we have the following easily established result.

Theorem 4.1. *Let $(G,\leq,t(\aleph)) \in |I-\mathrm{FNOGr}|$ and $\aleph^i(e)$ (resp., $\aleph^d(e)$) denote the system of all increasing (resp., decreasing) I-fuzzy neighborhoods of e. Then,*

(i) *the family β_L (resp., β_R) $=\{\mu_L$ (resp., μ_R) $:\mu \in \aleph^i(e)\}$ is a basis for the left (resp., right) I-fuzzy quasi-uniformity u_L (resp., u_R) on G;*

(ii) *the family β_L^{-1} (resp., β_R^{-1}) $=\{\mu_L^{-1}$ (resp., μ_R^{-1}) $:\mu \in \aleph^d(e)\}$ is a basis for the conjugate left (resp., right) I-fuzzy quasi-uniformity $\mathbf{U}_L^{-1}$ (resp., $\mathbf{U}_R^{-1}$) on G;*

(iii) *the family $\beta_s=\{\mu_L\wedge\mu_R:\mu \in \aleph^i(e)\}$ is a basis for the two-sided I-fuzzy quasi-uniformity $(u_R\vee u_L)$ on G.*

We denote $\mathbf{U}_L\vee\mathbf{U}_L^{-1}$ (resp., $\mathbf{U}_R\vee\mathbf{U}_R^{-1}$) by $\mathbf{U}_L^*$ (resp., $\mathbf{U}_R^*$). It is clear that $\mathbf{U}_L^*$ (resp., $\mathbf{U}_R^*$) is an I-fuzzy uniformity on G called the left (resp., right) I-fuzzy uniformity generated by $\mathbf{U}_L$ (resp., $\mathbf{U}_R$). Also, the two-sided I-fuzzy uniformity $\mathbf{U}^*=\mathbf{U}_R^*\vee\mathbf{U}_L^*$ can be generated by the two-sided I-fuzzy quasi-uniformity $(\mathbf{U}_R\vee\mathbf{U}_L)$.

It is known that the entourages of the above I-fuzzy quasi-uniformities can generate an order on G by setting

$$x \leq^* y \Longleftrightarrow (\forall z \in G)\ \mu_L(y,z) \leq \mu_L(x,z). \tag{4.2}$$

The partial order $\leq^*$ is said to be generated by the left I-fuzzy quasi-uniformity $\mathbf{U}_L$.

Definition 4.2. Let G_1, G_2 be groups and $\mathbf{U}_2$, $\mathbf{U}_2$ be quasi-uniformities on G_1 and G_2, respectively. A mapping $f : G_1 \to G_2$ is called a quasi-uniform isomorphism if it is a quasi-uniform equivalence (see Definition 2.1) and group isomorphism.

Proposition 4.3. *Let $(G, \leq, t(\aleph)) \in |I - \mathrm{FNOGr}|$ and let $\mathbf{U}_L$ be the associated left I-fuzzy quasi-uniformity on G, then*

(i) *L_x (resp., R_x) : $(G, \mathbf{U}_L) \to (G, \mathbf{U}_L)$ is a quasi-uniform isomorphism;*
(ii) *L_x (resp., R_x) : $(G, \leq, \mathbf{U}_L^*) \to (G, \leq, \mathbf{U}_L^*)$ is a uniformly order isomorphism.*

Proof. (i) It follows immediately from the formulas

$$(L_x \times L_x)^{-1}(\mu_L) = \mu_L. \tag{4.3}$$

(ii) The existence of the associated left I-fuzzy quasi-uniformity $\mathbf{U}_L$ which generate the I-fuzzy uniformity $\mathbf{U}_L^*$ and the order $\leq^*$ with $G(\leq^*) = G(\leq)$ and from (i) the proof becomes clear. □

Proposition 4.4. *Let $(G, \leq, t(\aleph)) \in |I - \mathrm{FNOGr}|$ and $\mathbf{U}_L$ (resp., $\mathbf{U}_R$) be the associated left (resp., right) I-fuzzy quasi-uniformity on G, then*

(i) *the mapping $r : (G, \mathbf{U}_L) \to (G, \mathbf{U}_R)$ is a quasi-uniform isomorphism;*
(ii) *the mapping $r : (G, \leq, \mathbf{U}_L^*) \to (G, \leq, \mathbf{U}_R^*)$ is a uniform order-isomorphism.*

Proof. (i) The mapping $r : G \to G$ is a group isomorphism. But for $\mu_R \in \mathbf{U}_R$, we have that $(r \times r)^{-1}(\mu_R)(x, y) = \mu_R(r(x), r(y)) = \mu_R(X^{-1}, y^{-1}) = \mu(y^{-1}x)$, that is, $(r \times r)^{-1}(\mu_R) = \tilde{\mu}_L$. And this means that $r : (G, u_L) \to (G, u_R)$ is a quasi-uniform equivalence and so it is quasi-uniform isomorphism.

(ii) This can be proven by Definition 2.1 and part (i) and this completes the proof. □

We omit the proof of the following easily established proposition.

Proposition 4.5. *Let $(G, \leq, t(\aleph))$ and $(G', \leq', t(\aleph')) \in |I - \mathrm{FNOGr}|$ and let $\mathbf{U}_L$, $\mathbf{U}_L'$ be the associated left I-fuzzy quasi-uniformities on G and G', respectively. Then, the order-preserving homeomorphism $f : G \to G'$ is uniformly order-mapping.*

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