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ANALYSIS OF SECTORIAL PLATES NORMALLY LOADED OVER CIRCULAR DOMAINS. II

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ABSTRACT. The method of images is applied to derive exact expressions for the deflections of unlimited wedge-shaped plates the central parts of which are cut, when the plate is simply supported on the radial edges, elastically restrained or free along the circular edge and is acted upon by one of three types of normal loading distributed over the surface of a circular domain. Formulae for the bending and twisting moments along the circular edge are given. Limiting forms of the obtained solutions are considered.

KEY WORDS AND PHRASES. *Deflections of wedge-shaped plates, the method of images, the bending and twisting moments.*
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1. INTRODUCTION.

The deflections of an infinite isotropic plate which has its outer edge free, is elastically restrained along an inner circular boundary and is subject to certain normal symmetrical or linearly varying loadings on a circular patch were obtained by the author [1,2], through the use of complex variable methods. When the inner circular edge is free and the plate is under the same normal loadings and is supported at any number of points the appropriate solutions were also derived by the author [3,4], references are to be found at the ends of these papers. The bending of a thin plate bounded by two arcs of concentric circles and two radii, normally loaded by an isolated force at any point, simply supported along the radial edges and subject to various combinations of boundary conditions along the circular edges was investigated by Bassali and Gorgui [5]. In a recent paper by the author [6] sectorial plates bounded by two radii and a circular boundary are analysed when the radial edges are simply supported, the circular edge is elastically restrained or free and the plate is under one of three types of normal loading on a circular patch. In this paper the foregoing solutions of the author [1 - 4] are used to establish the corresponding solutions for unlimited wedge-shaped plates the central parts of which are cut when the plates are simply supported along the radial edges, elastically restrained or free along the circular edge and are acted upon by the same loadings on a circular patch. As the radius of the loaded patch tends to zero these loadings reduce to a concentrated load or a concentrated couple acting at any point of the plate. Solutions for complete wedge-shaped plates are obtained by letting the radius of the removed part tend to zero. Expressions for the bending and twisting moments along the circular boundary are provided.

2. FORMULATION OF THE PROBLEMS AND BOUNDARY CONDITIONS.

We consider a thin isotropic sectorial plate bounded by the infinite extensions of the two radii OA, OB and the circular edge Γ of radius c. See Fig.1. Let $A\hat{O}B = \gamma = \pi/k$ where k is an integer > 1. It is assumed that the plate is simply supported along the radial edges and elastically restrained or free along Γ. Let $z = re^{i\theta} = c\zeta$ be the complex variable of any point P in the mid-plane of the plate referred to O as origin and Q_o ($z_o = r_o e^{i\beta} = c\zeta_o$) be the centre of a circle of radius $b = cq$ and boundary C. It is clear that b must be $\le$ the least of the three quantities $r_o \sin\beta$, $r_o \sin(\gamma - \beta)$ and $r_o - c$. Let $p_1(r,\theta)$ and $p_2 = 0$ be the load intensities at points of regions 1 and 2 of the plate which lie inside and outside C, respectively. It is known that the small deflections w_1 and w_2, measured positively downwards, satisfy the two partial differential equations

$$\nabla^4 w_1 = p_1(r,\theta)/D \quad , \tag{2.1a}$$

$$\nabla^4 w_2 = 0 \quad , \tag{2.1b}$$

where D is the flexural rigidity of the plate. If (R_o,Θ_o) are the polar coordinates of P referred to Q_o as pole and OQ_o as initial line then the three cases of loading which we are going to discuss are defined by

$$p_1 = p_o R_o^{s-2} \qquad (s \ge 2) \qquad \text{(Case I)} \quad , \tag{2.2a}$$

$$p_1 = p_o R_o^{s-3} \cos\Theta_o \qquad (s \ge 3) \qquad \text{(Case II)} \quad , \tag{2.2b}$$

$$p_1 = p_o R_o^{s-3} \sin\Theta_o \qquad (s \ge 3) \qquad \text{(Case III)}. \tag{2.2c}$$

We have now to find w_1 and w_2 which fulfil the following conditions:

(i) w_1 satisfies (2.1a) ,

(ii) w_2 satisfies (2.1b) together with the boundary conditions along the radial edges and Γ ,

(iii) w_1 and w_2 must satisfy the necessary and sufficient conditions for the physical continuity along C [1, p.250].

Introducing the notations

$$|\zeta| = r/c = \rho, \quad |\zeta_o| = r_o/c = \rho_o, \quad d = \partial/\partial\rho, \quad d' = \partial/\partial\theta \ , \tag{2.3}$$

we assume in the first set of boundary conditions that the radial edges are simply supported and the circular edge Γ is elastically restrained according to

$$(w_2)_{\rho=1} = 0, \quad (d^2 w_2 + \nu d w_2)_{\rho=1} = 0 \ , \tag{2.4}$$

where

$$\nu = (1 + \lambda)/(1 - \lambda) \ , \tag{2.5}$$

λ is the restraining parameter. It was shown that Γ is rigidly clamped if $\nu = \infty$,

$\lambda = 1$ and is simply supported if $\nu = \eta$, $\lambda = (\eta - 1)/(\eta + 1)$, η being Poisson's ratio. In the general case we have

$$\eta \geq \nu \geq -\infty \quad \text{and} \quad \infty \geq \lambda \geq 1 \quad \text{or} \quad (\eta - 1)/(\eta + 1) \geq \lambda \geq -\infty \ . \tag{2.6}$$

The boundary values of the bending and twisting moments along the elastically restrained edge Γ are given by

$$M_r/(\eta - \nu) = M_\theta/(1 - \eta\nu) = -(D/c^2)(dw_2)_{\rho=1} \ , \tag{2.7a}$$

$$M_{r\theta} = (1 - \eta)(D/c^2)(d'dw_2)_{\rho=1} \ . \tag{2.7b}$$

Along a clamped boundary we have

$$M_r = M_\theta/\eta = -(D/c^2)(d^2w_2)_{\rho=1} \ , \quad M_{r\theta} = 0 \ . \tag{2.7c}$$

In the second set of boundary conditions the radial edges are simply supported and Γ is free. Along the free boundary we have [7, p.283] ,

$$(M_r)_{r=c} = -\frac{D}{c^2}[(d^2 + \eta d + \eta d'^2)w_2]_{\rho=1} = 0 \ , \tag{2.8a}$$

$$(M_\theta)_{r=c} = -\frac{D}{c^2}[(\eta d^2 + d + d'^2)w_2]_{\rho=1}$$

$$= (\eta^{-1} - \eta)\frac{D}{c^2}(d^2w_2)_{\rho=1} = \frac{(\eta^2 - 1)D}{c^2}[(d + d'^2)w_2]_{\rho=1}, \tag{2.8b}$$

$$(M_{r\theta})_{r=c} = \frac{(1 - \eta)D}{c^2}[d'(d-1)w_2]_{\rho=1} \ , \tag{2.8c}$$

$$\left(Q_r - \frac{1}{c}\frac{\partial M_{r\theta}}{\partial\theta}\right)_{r=c} = 0 \ . \tag{2.9}$$

3. SECTORIAL PLATE UNDER THE FIRST TYPE OF NORMAL LOADING.

In this section we treat the case of the infinite plate bounded by the two infinite simply supported radial edges $\theta = 0$, $\theta = \gamma$ $(\infty \geq r \geq c)$ and the circular arc Γ which is either elastically restrained or free (Fig.1), the plate being normally loaded by the pressure (2.2a) on the circular patch of centre Q_o and radius b. The deflected form of the infinite sectorial plate is the same as that of an infinitely large plate having an inner circular boundary with the same boundary conditions along Γ and subject to normal loadings of intensities $(-1)^t p_o R_t^{s-2}$ on the circular domains with centres Q_t $(z_t = c\rho_o e^{i\alpha_t})$ and equal radii b, where $R_t = PQ_t$ and

$$\alpha_t = t\gamma + \beta \quad (t=0,2,\ldots,2k-2), \quad \alpha_t = (t+1)\gamma - \beta \quad (t=1,3,\ldots,2k-1). \tag{3.1}$$

If n is a positive integer then it is easily proved that

$$\left.\begin{aligned}\sum_{t=0}^{2k-1} (-1)^t \sin n\alpha_t &= 2k \sin n\beta \ \text{ if } n \text{ is a multiple of } k, \\ &= 0 \ \text{ otherwise;}\end{aligned}\right\} \tag{3.2a}$$

$$\left.\begin{aligned}\sum_{t=0}^{2k-1} \cos n\alpha_t &= 2k \cos n\beta \ \text{ if } n \text{ is a multiple of } k, \\ &= 0 \ \text{ otherwise;}\end{aligned}\right\} \tag{3.2b}$$

$$\sum_{t=0}^{2k-1} \sin n\alpha_t = 0 , \quad \sum_{t=0}^{2k-1} (-1)^t \cos n\alpha_t = 0 . \tag{3.2c}$$

Assuming that $\theta_t = \theta - \alpha_t$ and applying equations (3.2) we find

$$\left.\begin{aligned}\sum_{t=0}^{2k-1} (-1)^t \cos n\theta_t &= 2k \sin n\theta \sin n\beta \quad (n=k,2k,3k,\ldots)\\ &= 0 \quad \text{otherwise;}\end{aligned}\right\} \tag{3.3a}$$

$$\left.\begin{aligned}\sum_{t=0}^{2k-1} \sin n\theta_t &= 2k \sin n\theta \cos n\beta \quad (n=k,2k,3k,\ldots)\\ &= 0 \quad \text{otherwise .}\end{aligned}\right\} \tag{3.3b}$$

If Q'_t $(z'_t = c\zeta'_t)$ is the inverse of Q_t $(z_t = c\zeta_t)$ with respect to Γ then $\bar{\zeta}_t\zeta'_t = 1$. Introducing the notation

$$u_t = \zeta - \zeta_t, \quad v_t = \bar{\zeta}_t\zeta - 1, \quad |u_t| = U_t, \quad |v_t| = V_t, \tag{3.4}$$

it is easily seen that

$$R_t = PQ_t = c|u_t| = cU_t = c\sqrt{(\rho^2 + \rho_o^2 - 2\rho_o\rho \cos \theta_t)}, \tag{3.5a}$$

$$R'_t = PQ'_t = \frac{c}{\rho_o}|v_t| = \frac{c}{\rho_o}V_t = \frac{c}{\rho_o}\sqrt{(1 + \rho_o^2\rho^2 - 2\rho_o\rho \cos \theta_t)}, \tag{3.5b}$$

$$\mathrm{Re}(1/v_t) + \tfrac{1}{2} = \mathrm{Re}(\bar{\zeta}_t\zeta/v_t) - \tfrac{1}{2} = (\rho_o^2\rho^2 - 1)c^2/2\rho_o^2R_t'^2 , \tag{3.5c}$$

$$\mathrm{Re}(\zeta_t/u_t) + \tfrac{1}{2} = \mathrm{Re}(\zeta/u_t) - \tfrac{1}{2} = (\rho^2 - \rho_o^2)c^2/2R_t^2 , \tag{3.5d}$$

$$\mathrm{Re}(\bar{\zeta}_t\zeta/v_t^2) = (\rho_o^2\rho^2 - 1)^2c^4/2\rho_o^4R_t'^4 - (\rho_o^2\rho^2 + 1)c^2/2\rho_o^2R_t'^2 , \tag{3.5e}$$

$$\mathrm{Im}(1/v_t) = \mathrm{Im}(\bar{\zeta}_t\zeta/v_t) = -\rho\rho_o c^2 \sin \theta_t/\rho_o^2R_t'^2 , \tag{3.5f}$$

$$\mathrm{Im}(\zeta/u_t) = \mathrm{Im}(\zeta_t/u_t) = -\rho\rho_o c^2 \sin \theta_t/R_t^2 , \tag{3.5g}$$

$$\mathrm{Im}(\bar{\zeta}_t\zeta/v_t^2) = -\rho(\rho^2\rho_o^2 - 1)c^4 \sin \theta_t/\rho_o^3R_t'^4 . \tag{3.5h}$$

The two cases of the different boundary conditions along Γ will now be dealt with separately.

CASE (a). THE CIRCULAR EDGE Γ IS ELASTICALLY RESTRAINED.

We assume that w_{er}^1 and w_{er}^2 denote the deflections in regions 1 and 2 of the entire infinite plate with an inner circular edge and subject to the normal loading (2.2a) over the circular domain with centre Q_o and radius b. With the notations adopted here and measuring the deflection positively downwards it was proved by the author [1, p.253] that

$$w_{er}^2 = \frac{P_o}{8\pi D}\Big[(R_o^2+b'^2)\ln \frac{R_o}{\rho_oR'_o} + \{r_o^2 + b'^2 - c^2 - \frac{2c^2}{\lambda}\}\ln \rho_o \ln \rho$$
$$+ (1 - \rho^2)\{-c^2 \ln \rho_o + (c^2-r_o^2+m'b'^2)\mathrm{Re}\ S_m(\bar{\zeta}_o\zeta)\}\Big] , \tag{3.6}$$

$$[w]_2^1 = \frac{P_o}{8\pi D}\left[(1-\frac{1}{s})(R_o^2-b^2) + \frac{4(R_o^{s'}-b^{s'})}{ss'^2b^s} + (R_o^2+b'^2)\ln\frac{b}{R_o}\right], \qquad (3.7)$$

where $P_o = 2\pi p_o b^s/s$ is the total load on the plate and

$$s' = s+2,\quad b'^2 = sb^2/s',\quad q'^2 = b'^2/c^2,\quad m = (1-\lambda)^{-1},\quad m' = m-1, \qquad (3.8a)$$

$$S_m(x) = \sum_{n=1}^{\infty}\frac{x^n}{n-m} = x^m\int_0^x \frac{\xi^{-m}}{1-\xi}\,d\xi \qquad (|x|<1). \qquad (3.8b)$$

The corresponding solution w'^2_{er} for the sectorial plate is obtained by summing for all the loads on the $2k$ circular domains in the figure. Thus we get

$$w'^2_{er} = \frac{P_o}{8\pi D}\sum_{t=0}^{2k-1}(-1)^t\left[(R_t^2+b'^2)\ln\frac{R_t}{\rho_o R'_t}\right.$$
$$\left. + (1-\rho^2)(c^2-r_o^2+m'b'^2)\sum_{n=1}^{\infty}\frac{(\rho\rho_o)^{-n}}{n-m}\cos n\theta_t\right]. \qquad (3.9)$$

For a clamped boundary we have the closed form

$$w'^2_{cl} = \frac{P_o}{8\pi D}\sum_{t=0}^{2k-1}(-1)^t\,\mathrm{Re}\left[(R_t^2+b'^2)\ln\frac{u_t}{v_t} + \frac{(\rho^2-1)b'^2}{v_t}\right]. \qquad (3.10)$$

Subtracting (3.10) form (3.9) gives

$$w'^2_{er} - w'^2_{cl} = \frac{P_o c^2(1-\rho^2)}{8\pi D}\sum_{t=0}^{2k-1}(-1)^t\sum_{n=1}^{\infty}\left\{1-\rho_o^2+(n-1)q'^2\right\}\frac{(\rho\rho_o)^{-n}\cos n\theta_t}{n-m}. \qquad (3.11)$$

Applying (3.3a) we get

$$w'^2_{er} - w'^2_{cl} = \frac{kP_o c^2(1-\rho^2)}{4\pi D}\sum{}'\frac{1-\rho_o^2+(n-1)q'^2}{n-m}(\rho\rho_o)^{-n}\sin n\theta\sin n\beta, \qquad (3.12)$$

where the accent on Σ means that the summation is taken only over integral multiples of k. Using (3.5c) we find

$$w'^2_{cl} = \frac{P_o}{8\pi D}\sum_{t=0}^{2k-1}(-1)^t\left[(R_t^2+b'^2)\ln\frac{R_t}{\rho_o R'_t} + \frac{(\rho^2-1)(\rho^2\rho_o^2-1)b'^2c^2}{2\rho_o^2 R'^2_t}\right]. \qquad (3.13)$$

By expanding $\ln\frac{u_t}{v_t}$ and v_t^{-1} in (3.10) in powers of ζ, taking the real parts and using (3.3a) we obtain

$$w'^2_{cl} = \frac{kP_o c^2}{4\pi D}\sum{}' f_n(\rho,\rho_o)\sin n\theta\sin n\beta, \qquad (3.14a)$$

$$w'^2_{er} = \frac{kP_o c^2}{4\pi D}\sum{}' g_n(\rho,\rho_o)\sin n\theta\sin n\beta, \qquad (3.14b)$$

where

$$g_n(\rho,\rho_o) = f_n(\rho,\rho_o) + \frac{(\rho\rho_o)^{-n}}{n-m}(\rho^2-1)\{\rho_o^2-1-(n-1)q'^2\}, \qquad (3.15a)$$

$$f_n(\rho,\rho_o) = \left[t_{n+1}(\rho)+\rho^2\rho_o^2 t_{n-1}(\rho)-(\rho^2+\rho_o^2)t_n(\rho)+\{\rho^2-1-t_n(\rho)\}q'^2\right](\rho\rho_o)^{-n}$$

$$(\rho < \rho_o) \qquad (3.15b)$$

$$f_n(\rho,\rho_o) = \left[t_{n+1}(\rho_o)+\rho^2\rho_o^2 t_{n-1}(\rho_o)-(\rho^2+\rho_o^2)t_n(\rho_o)+\{\rho^2-1-t_n(\rho_o)\}q'^2\right](\rho\rho_o)^{-n}$$

$$(\rho > \rho_o) \qquad (3.15c)$$

$$t_n(\rho) = (\rho^{2n} - 1)/n \ . \qquad (3.16)$$

Setting $q' = 0$ in (3.15a,b) we obtain expressions which agree with equations (75) and (77) derived by Bassali and Gorgui [5] , on noting the difference in notation and that the deflection there is measured positively upwards. Equations (3.6) and (3.7) show that at any point of region 1 the deflection w'^1_{er} is given by

$$w'^1_{er} = \frac{P_o}{8\pi D}\left[(R_o^2+b'^2)\ln\frac{b}{\rho_o R'_o} + \sum_{t=1}^{2k-1}(-1)^t(R_t^2+b'^2)\ln\frac{R_t}{\rho_o R'_t} + (1-\frac{1}{s})(R_o^2-b^2)\right.$$

$$\left. + \frac{4(R_o^{s'}-b^{s'})}{ss'^2 b^s} + 2kc^2(\rho^2-1)(\rho_o^2-1-m'q'^2)\sum{}' \frac{(\rho\rho_o)^{-n}}{n-m}\sin n\theta \sin n\beta\right]. \quad (3.17)$$

The boundary values of the bending and twisting moments along the circular edge Γ are determined by substituting from (3.14b), (3.15a,b) and (3.16) in (2.7a,b). This leads to

$$\frac{M_r}{\eta-\nu} = \frac{M_\theta}{1-\eta\nu} = \frac{kP_o}{2\pi}\sum{}' \frac{\rho_o^{-n}}{n-m}\{1-\rho_o^2+(n-1)q'^2\}\sin n\theta \sin n\beta, \qquad (3.18a)$$

$$\frac{M_{r\theta}}{1-\eta} = \frac{kP_o}{2\pi}\sum{}' \frac{n\rho_o^{-n}}{n-m}\{\rho_o^2-1-(n-1)q'^2\}\cos n\theta \sin n\beta \ . \qquad (3.18b)$$

For a clamped circular edge Γ we have

$$M_r = \frac{M_\theta}{\eta} = \frac{kP_o}{\pi}\sum{}' \{1-\rho_o^2+(n-1)q'^2\}\rho_o^{-n}\sin n\theta \sin n\beta \ . \qquad (3.19)$$

CASE (b). THE CIRCULAR EDGE Γ IS FREE.

Let w^1_{fr} and w^2_{fr} denote the deflections in regions 1 and 2 of the infinite perforated plate having free inner circular boundary, subject to the normal loading (2.2a) on the circular domain with centre Q_o and supported at the N points $z_j = c\zeta_j$ $(j=1,2,\ldots,N)$. It was shown by the author [3, p.749] that

$$w^2_{fr} = \frac{c^2}{8\pi\kappa D}\left[\sum_{j=0}^{N} P_j\left\{U_j^2(\kappa \ln R_j - \ln\frac{R'_j}{r}) - \kappa(1+\kappa)\ln\rho \ln\rho_j\right\} + (1-\kappa^2)\mathrm{Re}\ I_N(\zeta)\right.$$

$$\left. + P_o q'^2\left\{\kappa \ln R_o - \ln\frac{R'_o}{r} + \mathrm{Re}\frac{u_o}{\zeta_o\bar{v}_o}\right\}\right] + \gamma_1 x + \gamma_2 y + \gamma_3 \ , \qquad (3.20)$$

where P_j $(j=1,2,\dots,N)$ are the concentrated reactions, measured positively downwards at the N points of support, γ_1, γ_2, γ_3 are real constants and

$$\kappa = (3+\eta)/(\eta-1)\,, \tag{3.21}$$

$$I_N(\zeta) = \sum_{j=0}^{N} P_j L_j(\zeta)\,, \qquad L_j(\zeta) = \int_{\infty}^{1} \left(\frac{1}{\xi} - \bar{\zeta}_j\zeta\right) \ln\left(1 - \frac{1}{\xi\bar{\zeta}_j\zeta}\right) d\xi\,. \tag{3.22}$$

It must be noticed that only the real part in the coefficient of $\ln\zeta$ in equation (2.36b), p.749 is taken. See the footnote p.113 of Bassali [4].

We now proceed to obtain the appropriate expression for the deflection w'^2_{fr} of the sectorial plate. The vanishing of the deflection on the radial edges yields $\gamma_1 = \gamma_2 = \gamma_3 = 0$. Putting $P_j = 0$ $(j=1,2,\dots,N)$ and summing for all the loadings on the $2k$ circular patches we get

$$w'^2_{fr} = \frac{P_o}{8\pi\kappa D} \sum_{t=0}^{2k-1} (-1)^t \mathrm{Re}\Big[(R_t^2 + b'^2)(\kappa \ln R_t - \ln R'_t) + \frac{b'^2\bar{u}_t}{\bar{\zeta}_t v_t} + (1-\kappa^2)c^2 L_t(\zeta)\Big]. \tag{3.23}$$

Noting that

$$\frac{\bar{u}_t}{\bar{\zeta}_t v_t} = \frac{\rho^2 - 1}{v_t} - \frac{\rho^2}{\bar{\zeta}_t\zeta}\,,$$

and subtracting (3.10) from (3.23) we find

$$w'^2_{fr} - w'^2_{cl} = \frac{P_o(\kappa-1)c^2}{8\pi\kappa D} \sum_{t=0}^{2k-1} (-1)^t \mathrm{Re}\Big[(U_t^2 + q'^2)\ln v_t - (1+\kappa)L_t(\zeta) + (\rho^2-1)q'^2 v_t^{-1}\Big]\,. \tag{3.24}$$

Expanding $\ln v_t$, $L_t(\zeta)$, v_t^{-1} in powers of ζ, taking the real parts and using (3.3a), we obtain, after some algebraic manipulation

$$w'^2_{fr} = \frac{kP_o c^2}{4\pi D} \Sigma' \, h_n(\rho,\rho_o) \sin n\theta \sin n\beta, \tag{3.25}$$

where

$$h_n(\rho,\rho_o) = f_n(\rho,\rho_o) + \frac{4(\rho\rho_o)^{-n}}{3+\eta}\left[(1-\rho^2-\frac{1}{n})\left(\frac{1}{n} - \frac{\rho_o^2}{n-1} + q'^2\right) - \frac{\kappa}{n^2(n+1)}\right]. \tag{3.26}$$

For $q' = 0$ this is in agreement with equation (80), p.93 of Bassali and Gorgui [5].

Substitution from (3.15b) and (3.26) in (2.8b,c) yields

$$(M_\theta)_{\rho=1} = \frac{kP_o(1+\eta)}{\pi(3+\eta)} \Sigma' \, \rho_o^{-n}\left[\frac{4}{n} + (\eta-1)\{1-\rho_o^2 + (n-1)q'^2\}\right] \sin n\theta \sin n\beta, \tag{3.27}$$

$$(M_{r\theta})_{\rho=1} = \frac{kP_o}{\pi\kappa} \quad \Sigma' \ \rho_o^{-n}\left[1-\rho_o^2- \frac{\kappa+1}{n} + (n-1)q'^2\right] \cos n\theta \sin n\beta \ . \tag{3.28}$$

The solution of the problem of a wedge-shaped plate simply supported along the radial edges and carrying the lateral load (2.2a) on a circular domain is found by letting c tend to 0 in any of the solutions w'^2_{cl}, w'^2_{er} and w'^2_{fr}. We therefore find

$$w'^2 = \frac{P_o}{8\pi D} \sum_{t=0}^{2k-1} (-1)^t (R_t^2+b'^2) \ln \frac{R_t}{b} \ , \tag{3.29}$$

$$w'^1 = \frac{P_o}{8\pi D} \left[\sum_{t=1}^{2k-1} (-1)^t (R_t^2+b'^2) \ln \frac{R_t}{b} + (1-\frac{1}{s})(R_o^2-b^2) + \frac{4(R_o^{s'}-b^{s'})}{ss'^2b^s}\right] . \tag{3.30}$$

The solution (3.29) can be expanded in the forms

$$w'^2 = \frac{kP_o}{4\pi D} \ \Sigma' \ \frac{1}{n} \left(\frac{r_o}{r}\right)^n \left(\frac{r^2}{n-1} - \frac{r_o^2}{n+1} - b'^2\right) \sin n\theta \sin n\beta \quad (r > r_o), \tag{3.31a}$$

$$w'^2 = \frac{kP_o}{4\pi D} \ \Sigma' \ \frac{1}{n} \left(\frac{r}{r_o}\right)^n \left(\frac{r_o^2}{n-1} - \frac{r^2}{n+1} - b'^2\right) \sin n\theta \sin n\beta \quad (r < r_o). \tag{3.31b}$$

4. SECTORIAL PLATE UNDER THE SECOND TYPE OF NORMAL LOADING.

We now consider the infinite sectorial plate bounded by the circular arc Γ and the infinite extensions of the two radii OA, OB when it is under the same sets of boundary conditions but subject to the normal loading (2.2b) on the circular patch with centre Q_o and radius b.

CASE (a). THE CIRCULAR EDGE Γ IS ELASTICALLY RESTRAINED.

The deflections w^2_{er} and w^1_{er} of the infinitely large plate which is elastically restrained along an inner circular boundary and is acted upon normally by the loading (2.2b) on the circular domain with centre Q_o and radius b (Fig.1) were found by Bassali [2]. Measuring the deflection positively downwards and making obvious changes in notation we have

$$w^2_{er} = \frac{Mc}{16\pi\rho_o D} \mathrm{Re}\left[2(\bar{\zeta}_o u_o+\zeta_o\bar{u}_o)\ln \frac{v_o}{u_o} - \left(\frac{\bar{\zeta}_o\zeta}{v_o} + \frac{\zeta_o}{u_o}\right) q'^2 + 4(\rho_o^2-\lambda^{-1})\ln \rho \right.$$
$$\left. + 2(1-\rho^2) \sum_1^\infty \left\{m(\rho_o^2-1)-2\rho_o^2-\frac{1}{2} m'nq'^2\right\} \frac{(\bar{\zeta}_o\zeta)^{-n}}{n-m} \right] \ , \tag{4.1}$$

where $M = \pi p_o b^s/s$ is the moment of the loading (2.2b) about the diameter perpendicular to OQ_o. Taking the real part of the expression between the square brackets gives

$$w^2_{er} = \frac{Mc}{16\pi\rho_o D}\left[\rho_o(\rho\cos\theta_o - \rho_o)\left(4\ln\frac{\rho_o R'_o}{R_o} - \frac{b'^2}{R_o^2}\right) + \frac{\rho(\cos\theta_o-\rho\rho_o)b'^2}{\rho_o R'^2_o}\right.$$
$$\left. + 4(\rho_o^2-\lambda^{-1})\ln\rho - 2(\rho^2-1)\sum_{n=1}^\infty \frac{(\rho\rho_o)^{-n}}{n-m}\{m(\rho_o^2-1)-2\rho_o^2-\frac{1}{2}m'nq'^2\}\cos n\theta_o\right] . \tag{4.2}$$

The formula for w^1_{er} is obtained by adding (4.2) to the expression

$$[w]^1_2 = \frac{Mc}{16\pi D}(\rho\cos\theta_o - \rho_o)\left[4\ln\frac{R_o}{b} + \frac{4}{s} + \frac{b'^2}{R_o^2} - \frac{R_o^2}{b_1^2} + \frac{16(R_o/b)^s}{s(s^2-4)}\right], \tag{4.3a}$$

where

$$b_1^2 = (s-2)b^2/s\ . \tag{4.3b}$$

The corresponding solution for the sectorial plate is

$$w'^2_{er} = \frac{Mc}{16\pi\rho_o D}\sum_{t=0}^{2k-1}(-1)^t \mathrm{Re}\left[2(\bar\zeta_t u_t + \zeta_t \bar u_t)\ln\frac{v_t}{u_t} - \left(\frac{\zeta_t}{u_t} + \frac{\bar\zeta_t\zeta}{v_t}\right)q'^2\right.$$

$$\left. - 2(\rho^2-1)\sum_{n=1}^{\infty}\left\{m(\rho_o^2-1) - 2\rho_o^2 - \frac{1}{2}m'nq'^2\right\}\frac{(\bar\zeta_t\zeta)^{-n}}{n-m}\right]\ . \tag{4.4}$$

Letting m tend to ∞ in (4.4) yields

$$w'^2_{cl} = \frac{Mc}{16\pi\rho_o D}\sum_{t=0}^{2k-1}(-1)^t \mathrm{Re}\left[2(\bar\zeta_t u_t + \zeta_t \bar u_t)\ln\frac{v_t}{u_t} - \left(\frac{\zeta_t}{u_t} + \frac{\bar\zeta_t\zeta}{v_t}\right)q'^2\right.$$

$$\left. + 2(\rho^2-1)\sum_{n=1}^{\infty}(\rho_o^2 - 1 - \frac{1}{2}nq'^2)(\bar\zeta_t\zeta)^{-n}\right]\ . \tag{4.5}$$

Summing the infinite series we obtain

$$w'^2_{cl} = \frac{Mc}{8\pi\rho_o D}\sum_{t=0}^{2k-1}(-1)^t \mathrm{Re}\left[\rho_o(\rho\cos\theta_t - \rho_o)\left(2\ln\frac{R'_t}{R_t} - \frac{b'^2}{2R_t^2}\right)\right.$$

$$\left. + \left\{(\rho_o^2-1)(\rho^2-1) - \frac{1}{2}\left(1 + \frac{\rho^2-1}{v_t}\right)q'^2\right\}\frac{\bar\zeta_t\zeta}{v_t}\right]\ . \tag{4.6}$$

Subtracting (4.5) from (4.4) we get

$$w'^2_{er} - w'^2_{cl} = \frac{Mc(\rho^2-1)}{8\pi\rho_o D}\sum_{t=0}^{2k-1}(-1)^t\sum_{n=1}^{\infty}\frac{(\rho\rho_o)^{-n}}{n-m}\left\{2\rho_o^2 - n(\rho_o^2-1) + \frac{1}{2}n(n-1)q'^2\right\}\cos n\theta_t\ . \tag{4.7}$$

Applying (3.3a) leads to

$$w'^2_{er} - w'^2_{cl} = \frac{kMc(\rho^2-1)}{4\pi\rho_o D}\Sigma'\frac{(\rho\rho_o)^{-n}}{n-m}\left\{2\rho_o^2 - n(\rho_o^2-1) + \frac{1}{2}n(n-1)q'^2\right\}\sin n\theta\sin n\beta\ . \tag{4.8}$$

Expanding all the terms in (4.6) in powers of ζ, taking the real parts and using (3.3a) we arrive at

$$w'^2_{cl} = \frac{kMc}{4\pi\rho_o D}\Sigma' f_n(\rho,\rho_o)\sin n\theta\sin n\beta, \tag{4.9a}$$

$$w'^2_{er} = \frac{kMc}{4\pi\rho_o D}\Sigma' g_n(\rho,\rho_o)\sin n\theta\sin n\beta, \tag{4.9b}$$

where

$$g_n(\rho,\rho_o) = f_n(\rho,\rho_o) + \frac{(\rho^2-1)(\rho\rho_o)^{-n}}{n-m}\left\{2\rho_o^2 - n(\rho_o^2-1) + \frac{1}{2}n(n-1)q'^2\right\}, \tag{4.10a}$$

$$f_n(\rho,\rho_o) = \left[t_{n+1}(\rho) + \rho^2\rho_o^2 t_{n-1}(\rho) - 2\rho_o^2 t_n(\rho) + (\rho^2-1)(\rho_o^2-1) + \frac{1}{2}nq'^2\{1-\rho^2+t_n(\rho)\}\right](\rho\rho_o)^{-n} \quad (\rho < \rho_o), \tag{4.10b}$$

$$f_n(\rho,\rho_o) = \left[t_{n+1}(\rho_o) + \rho^2\rho_o^2 t_{n-1}(\rho_o) - 2\rho_o^2 t_n(\rho_o) + (\rho^2-1)(\rho_o^2-1) + \frac{1}{2}nq'^2\{1-\rho^2-t_n(\rho_o)\} - q'^2\right](\rho\rho_o)^{-n} \quad (\rho > \rho_o), \tag{4.10c}$$

and $t_n(\rho)$ is defined by (3.16). Equations (4.2) and (4.3a) show that w'^1_{er} is given by

$$w'^1_{er} = \frac{Mc}{4\pi\rho_o D}\left[\rho_o(\rho\cos\theta_o - \rho_o)\left\{\ln\frac{\rho_o R'_o}{b} + \frac{1}{s} - \frac{R_o^2}{4b_1^2} + \frac{4(R_o/b)^s}{s(s^2-4)}\right\} + \frac{(1-\rho_o^2\rho^2)b'^2}{8\rho_o^2 R'^2_o}\right.$$

$$+ \sum_{t=1}^{2k-1}(-1)^t\left\{\rho_o(\rho\cos\theta_t - \rho_o)\left(\ln\frac{\rho_o R'_t}{R_t} - \frac{b'^2}{4R_t^2}\right) + \frac{(1-\rho_o^2\rho^2)b'^2}{8\rho_o^2 R'^2_t}\right\}$$

$$\left. - k(\rho^2-1)\,\Sigma'\,\frac{(\rho\rho_o)^{-n}}{n-m}\left\{m(\rho_o^2-1) - 2\rho_o^2 - \frac{1}{2}m'nq'^2\right\}\sin n\theta\sin n\beta\right]. \tag{4.11}$$

Introducing (4.9b) in (2.7a,b) yields

$$\frac{M_r}{\eta-\nu} = \frac{M_\theta}{1-\eta\nu} = \frac{kM}{2\pi r_o}\,\Sigma'\,\frac{\rho_o^{-n}}{n-m}\left\{n(\rho_o^2-1) - 2\rho_o^2 - \frac{1}{2}n(n-1)q'^2\right\}\sin n\theta\sin n\beta, \tag{4.12a}$$

$$\frac{M_{r\theta}}{1-\eta} = \frac{kM}{2\pi r_o}\,\Sigma'\,\frac{n\rho_o^{-n}}{n-m}\left\{n(1-\rho_o^2) + 2\rho_o^2 + \frac{1}{2}n(n-1)q'^2\right\}\cos n\theta\sin n\beta, \tag{4.12b}$$

and for a clamped boundary we have

$$M_r = \frac{M_\theta}{\eta} = \frac{kM}{\pi r_o}\,\Sigma'\,\rho_o^{-n}\left\{n(\rho_o^2-1) - 2\rho_o^2 - \frac{1}{2}n(n-1)q'^2\right\}\sin n\theta\sin n\beta. \tag{4.13}$$

CASE (b). THE CIRCULAR EDGE Γ IS FREE.

The problem of an infinite thin elastic plate with an outer free edge and an inner free circular edge was solved by the author [4] when the plate is subject to a general loading including (2.2b), (2.2c) on a circular domain and is supported at any number N of points. The deflection w^2_{fr} corresponding to the loading (2.2b) on the circular patch with centre Q_o and radius b is obtained by putting

$$\lambda = e^{i\beta} = \zeta_o/\rho_o$$

in the solution given by equations (20) and (41) of Bassali [4] . Thus we get

$$w_{fr}^2 = \frac{c^2}{8\pi\kappa D}\Big[\sum_{j=1}^{N} P_j\Big\{U_j^2(\kappa \ln R_j - \ln \frac{R_j'}{r}) - \kappa(1+\kappa)\ln\rho \ln\rho_j + (1-\kappa^2)\operatorname{Re} L_j(\zeta)\Big\}$$

$$+ \frac{M}{c\rho_o}\operatorname{Re}\Big\{2\bar{\zeta}_o u_o \ln\Big(\frac{R_o'}{r} - \kappa \ln R_o - \frac{\kappa b'^2}{4R_o^2}\Big) + (\kappa^2-1)v_o \ln\frac{v_o}{\bar{\zeta}_o\zeta} - \kappa(1+\kappa)\ln\rho$$

$$- \frac{\bar{u}_o}{\bar{\zeta}_o v_o}\Big(1-\rho_o^2 + \frac{q'^2}{2v_o}\Big) - \frac{\bar{\zeta} q'^2}{\bar{\zeta}_o v_o}\Big\}\Big] + \gamma_1 x + \gamma_2 y + \gamma_3 , \tag{4.14}$$

where P_j is the concentrated reaction, measured positively downwards at the points of support $z_j = c\zeta_j$ $(j=1,2,\dots,N)$ and $\gamma_1, \gamma_2, \gamma_3$ are real constants.

The expression for $w_{fr}'^2$ corresponding to the sectorial plate can now be found by putting $\gamma_1 = \gamma_2 = \gamma_3 = 0$, $P_j = 0$ $(j=1,2,\dots,N)$ and summing for all pressures on the 2k circular domains. We therefore get

$$w_{fr}'^2 = \frac{Mc}{8\pi\kappa\rho_o D}\sum_{t=0}^{2k-1}(-1)^t\operatorname{Re}\Big[2\bar{\zeta}_t u_t\Big(\ln R_t' - \kappa \ln R_t - \frac{\kappa b'^2}{4R_t^2}\Big) + (\kappa^2-1)v_t \ln\frac{v_t}{\bar{\zeta}_t\zeta}$$

$$- \frac{\bar{u}_t}{\bar{\zeta}_t v_t}\Big(1-\rho_o^2 + \frac{q'^2}{2v_t}\Big) - \frac{\bar{\zeta} q'^2}{\bar{\zeta}_t v_t}\Big] . \tag{4.15a}$$

Noting that

$$\frac{\bar{u}_t}{\bar{\zeta}_t v_t} = \frac{\rho^2-1}{v_t} - \frac{\rho^2}{\bar{\zeta}_t\zeta}, \quad \frac{\bar{u}_t}{\bar{\zeta}_t v_t^2} = \frac{\rho^2-1}{v_t^2} - \frac{\rho^2}{v_t} + \frac{\rho^2}{\bar{\zeta}_t\zeta}, \quad \frac{\bar{\zeta}}{\bar{\zeta}_t v_t} = \frac{\rho^2}{v_t} - \frac{\rho^2}{\bar{\zeta}_t\zeta} , \tag{4.15b}$$

we easily see that

$$w_{fr}'^2 = \frac{Mc}{8\pi\kappa\rho_o D}\sum_{t=0}^{2k-1}(-1)^t\operatorname{Re}\Big[2\bar{\zeta}_t u_t\Big(\ln R_t' - \kappa \ln R_t - \frac{\kappa b'^2}{4R_t^2}\Big) + (\kappa^2-1)v_t \ln\frac{v_t}{\bar{\zeta}_t\zeta}$$

$$+ \Big\{(\rho_o^2-1)(\rho^2-1) - \frac{1}{2}\Big(1+\frac{\rho^2-1}{v_t}\Big)q'^2\Big\}\frac{\bar{\zeta}_t\zeta}{v_t}\Big] . \tag{4.16}$$

Subtracting (4.6) from (4.16) we arrive at

$$w_{fr}'^2 - w_{cl}'^2 = \frac{Mc(1-\kappa)}{8\pi\kappa\rho_o D}\sum_{t=0}^{2k-1}(-1)^t\operatorname{Re}\Big[2\bar{\zeta}_t u_t \ln R_t' - (1+\kappa)v_t \ln\frac{v_t}{\bar{\zeta}_t\zeta}$$

$$+ \Big\{(\rho_o^2-1)(\rho^2-1) - \frac{1}{2}\Big(1+\frac{\rho^2-1}{v_t}\Big)q'^2\Big\}\frac{\bar{\zeta}_t\zeta}{v_t}\Big] . \tag{4.17}$$

Expanding in powers of ζ, taking the real parts and using (3.3a) we get

$$w_{fr}'^2 = \frac{kMc}{4\pi\rho_o D}\sum{}' h_n(\rho,\rho_o)\sin n\theta \sin n\beta, \tag{4.18}$$

where

$$h_n(\rho,\rho_o) = f_n(\rho,\rho_o) + \frac{4(\rho\rho_o)^{-n}}{3+n}\Big[(\rho^2-1+\frac{1}{n})\Big(1-\frac{n-2}{n-1}\rho_o^2 + \frac{1}{2}nq'^2\Big) + \frac{\kappa}{n(n+1)}\Big] \quad (1\le\rho\le\infty). \tag{4.19}$$

Applying equations (2.8b,c), we obtain the boundary values

$$(M_\theta)_{\rho=1} = \frac{kM(1+\eta)}{\pi(3+\eta)r_o} \Sigma' \rho_o^{-n}\left[(1-\eta)\left\{n-(n-2)\rho_o^2+\frac{1}{2}n(n-1)q'^2\right\}-4\right]\sin n\theta \sin n\beta, \quad (4.20)$$

$$(M_{r\theta})_{\rho=1} = \frac{kM}{\pi\kappa r_o} \Sigma' \rho_o^{-n}\left[\kappa-n+1+(n-2)\rho_o^2-\frac{1}{2}n(n-1)q'^2\right]\cos n\theta \sin n\beta. \quad (4.21)$$

The deflection at any point of a complete wedge-shaped plate under the normal loading (2.2b) on any circular domain can now be found by letting c tend to 0 in (4.6). Thus we get

$$w'^2 = \frac{M}{4\pi D}\sum_{t=0}^{2k-1}(-1)^t(r_o-r\cos\theta_t)\left(\ln\frac{R_t}{b}+\frac{b'^2}{4R_t^2}\right), \quad (4.22a)$$

$$w'^1 = \frac{M}{4\pi D}\left[(r\cos\theta_o-r_o)\left\{\frac{1}{s}-\frac{R_o^2}{4b_1^2}+\frac{4(R_o/b)^s}{s(s^2-4)}\right\}\right.$$

$$\left.+\sum_{t=1}^{2k-1}(-1)^t(r_o-r\cos\theta_t)\left(\ln\frac{R_t}{b}+\frac{b'^2}{4R_t^2}\right)\right]. \quad (4.22b)$$

The expansions of (4.22a) are

$$w'^2 = \frac{M}{4\pi r_o D}\Sigma'\left(\frac{r_o}{r}\right)^n\left[\frac{r^2}{n-1}-\frac{n+2}{n(n+1)}r_o^2-\frac{1}{2}b'^2\right]\sin n\theta\sin n\beta \quad (r>r_o), \quad (4.23a)$$

$$w'^2 = \frac{M}{4\pi r_o D}\Sigma'\left(\frac{r}{r_o}\right)^n\left[\frac{r^2}{n+1}-\frac{n-2}{n(n-1)}r_o^2+\frac{1}{2}b'^2\right]\sin n\theta\sin n\beta \quad (r<r_o). \quad (4.23b)$$

Setting $q'=0$ $(b'=0)$ in the results of this section yields the solutions corresponding to a couple nucleus M operating at Q_o in a plane normal to the plate through OQ_o .

5. SECTORIAL PLATE UNDER THE THIRD TYPE OF NORMAL LOADING.

Finally we deal with the infinite sectorial plate under the two sets of boundary conditions and the normal loading (2.2c) on the circular patch with centre Q_o and radius b. For this type of loading the deflected form of the sectorial plate is the same as that of the infinite plate with a circular hole when this plate is acted upon by the normal pressures of intensities $p_o R_t^{s-3}\sin\theta_t$ on the $2k$ circular patches.

CASE (a). THE CIRCULAR EDGE Γ IS ELASTICALLY RESTRAINED.

Putting $\alpha=\pi/2$ in equation(3.23), p.124 of Bassali [2] and measuring the deflection positively downwards we find that the deflections w_{er}^1 and w_{er}^2 of the infinite perforated plate which is elastically restrained along the inner circular edge and is acted upon by the loading (2.2c) on the circular patch with centre Q_o are furnished by

$$w_{er}^2 = \frac{Mc}{8\pi\rho_o D} \operatorname{Im}\left[(\bar{\zeta}_o\zeta - \zeta_o\bar{\zeta}) \ln \frac{v_o}{u_o} + \frac{1}{2}\left(\frac{\zeta_o}{u_o} - \frac{1}{v_o}\right) q'^2 \right.$$

$$\left. + (\rho^2 - 1) \sum_{n=1}^{\infty} \{m(1-\rho_o^2) + \frac{1}{2} m' n q'^2\} \frac{(\bar{\zeta}_o\zeta)^{-n}}{n-m} \right], \tag{5.1}$$

$$[w]_2^1 = \frac{Mr \sin\theta_o}{4\pi D} \left[\ln \frac{R_o}{b} + \frac{1}{s} + \frac{b'^2}{4R_o^2} - \frac{R_o^2}{4b_1^2} + \frac{4(R_o/b)^s}{s(s^2-4)} \right]. \tag{5.2}$$

Letting m tend to ∞ in (5.1) gives

$$w_{cl}^2 = \frac{Mc}{8\pi\rho_o D} \operatorname{Im}\left[(\bar{\zeta}_o\zeta - \zeta_o\bar{\zeta}) \ln \frac{v_o}{u_o} + \frac{1}{2}\left(\frac{\zeta_o}{u_o} - \frac{1}{v_o}\right) q'^2 \right.$$

$$\left. + (\rho^2 - 1) \sum_{n=1}^{\infty} (\rho_o^2 - 1 - \frac{1}{2} n q'^2)(\bar{\zeta}_o\zeta)^{-n} \right], \tag{5.3}$$

which can be written in the closed form

$$w_{cl}^2 = \frac{Mc}{8\pi\rho_o D} \operatorname{Im}\left[(\bar{\zeta}_o\zeta - \zeta_o\bar{\zeta}) \ln \frac{v_o}{u_o} + \frac{1}{2}\left(\frac{\zeta_o}{u_o} - \frac{1}{v_o}\right) q'^2 \right.$$

$$\left. + \frac{\rho^2 - 1}{v_o}\left\{\rho_o^2 - 1 - \frac{\bar{\zeta}_o\zeta}{2v_o} q'^2\right\}\right]. \tag{5.4}$$

Subtracting (5.3) from (5.1) yields

$$w_{er}^2 - w_{cl}^2 = \frac{Mc(\rho^2-1)}{8\pi\rho_o D} \operatorname{Im} \sum_{n=1}^{\infty} n\left\{1 - \rho_o^2 + \frac{1}{2}(n-1)q'^2\right\} \frac{(\bar{\zeta}_o\zeta)^{-n}}{n-m}. \tag{5.5}$$

The corresponding results for the sectorial plate are

$$w'^2_{cl} = \frac{Mc}{8\pi\rho_o D} \sum_{t=0}^{2k-1} \operatorname{Im}\left[(\bar{\zeta}_t\zeta - \zeta_t\bar{\zeta}) \ln \frac{v_t}{u_t} + \frac{1}{2}\left(\frac{\zeta_t}{u_t} - \frac{1}{v_t}\right) q'^2 \right.$$

$$\left. + \frac{\rho^2 - 1}{v_t}\left\{\rho_o^2 - 1 - \frac{\bar{\zeta}_t\zeta}{2v_t} q'^2\right\}\right], \tag{5.6}$$

$$w'^2_{er} - w'^2_{cl} = \frac{Mc(\rho^2-1)}{8\pi\rho_o D} \sum_{t=0}^{2k-1} \sum_{n=1}^{\infty} n\left\{1 - \rho_o^2 + \frac{1}{2}(n-1)q'^2\right\} \frac{(\rho\rho_o)^{-n}}{n-m} \sin n\theta_t, \tag{5.7}$$

where the imaginary parts of the terms in (5.6) are given by equations (3.5f,g,h). Expanding these terms in powers of ζ, taking the imaginary parts and using (3.3b) we obtain

$$w'^2_{cl} = \frac{kMc}{4\pi\rho_o D} \Sigma' f_n(\rho, \rho_o) \sin n\theta \cos n\beta, \tag{5.8a}$$

$$w'^2_{er} = \frac{kMc}{4\pi\rho_o D} \Sigma' g_n(\rho,\rho_o) \sin n\theta \cos n\beta, \tag{5.8b}$$

where

$$g_n(\rho,\rho_o) = f_n(\rho,\rho_o) + \frac{n(\rho^2-1)(\rho\rho_o)^{-n}}{n-m} \left\{1 - \rho_o^2 + \frac{1}{2}(n-1)q'^2\right\}, \tag{5.9a}$$

$$f_n(\rho,\rho_o) = \left[\rho_o^2\rho^2 t_{n-1}(\rho) - t_{n+1}(\rho) - (\rho_o^2-1)(\rho^2-1) - \frac{1}{2}n\{1-\rho^2+t_n(\rho)\}q'^2\right](\rho\rho_o)^{-n}$$

$$(\rho < \rho_o), \tag{5.9b}$$

$$f_n(\rho,\rho_o) = \left[\rho_o^2\rho^2 t_{n-1}(\rho_o) - t_{n+1}(\rho_o) - (\rho_o^2-1)(\rho^2-1) - \frac{1}{2}n\{1-\rho^2+t_n(\rho_o)\}q'^2\right](\rho\rho_o)^{-n}$$

$$(\rho > \rho_o). \tag{5.9c}$$

The boundary values of the bending and twisting moments along the circular edge Γ are determined by

$$\frac{M_r}{\eta-\nu} = \frac{M_\theta}{1-\eta\nu} = \frac{kM}{2\pi r_o} \Sigma' \frac{n\rho_o^{-n}}{n-m}\left\{\rho_o^2 - 1 - \frac{1}{2}(n-1)q'^2\right\} \sin n\theta \cos n\beta, \tag{5.10a}$$

$$\frac{M_{r\theta}}{1-\eta} = \frac{kM}{2\pi r_o} \Sigma' \frac{n^2\rho_o^{-n}}{n-m}\left\{1-\rho_o^2 + \frac{1}{2}(n-1)q'^2\right\}\cos n\theta \cos n\beta, \tag{5.10b}$$

and for a clamped boundary we have

$$M_r = \frac{M_\theta}{\eta} = \frac{kM}{\pi r_o} \Sigma' n\rho_o^{-n}\left\{\rho_o^2 - 1 - \frac{1}{2}(n-1)q'^2\right\} \sin n\theta \cos n\beta. \tag{5.11}$$

CASE (b). THE CIRCULAR EDGE Γ IS FREE.

The deflection w^2_{fr} of the infinite plate having a free inner circular edge, supported at the N points $z_j = c\zeta_j$ $(j=1,2,\ldots,N)$ and subject to the normal loading (2.2c) on the circular patch with centre Q_o is now obtained by setting

$$\lambda = e^{i(\beta + \pi/2)} = ie^{i\beta} = i\zeta_o/\rho_o$$

in the general solution given by equations (20) and (41) of Bassali [4]. This yields

$$w^2_{fr} = \frac{c^2}{8\pi\kappa D}\left[\sum_{j=1}^{N} P_j\left\{U_j^2\left(\kappa \ln R_j - \ln \frac{R'_j}{r}\right) - \kappa(1+\kappa)\ln\rho \ln\rho_j + (1-\kappa^2)\mathrm{Re}\, L_j(\zeta)\right\}\right.$$

$$+ \frac{M}{c\rho_o} \mathrm{Im}\left\{2\bar{\zeta}_o\zeta\left(\ln\frac{R'_o}{r} - \kappa\ln R_o - \frac{\kappa b'^2}{4R_o^2}\right) + (\kappa^2-1)v_o \ln\frac{v_o}{\bar{\zeta}_o\zeta}\right.$$

$$\left.\left. - \frac{\bar{u}_o}{\bar{\zeta}_o v_o}\left(1-\rho_o^2 + \frac{q'^2}{2v_o}\right) - \frac{\bar{\zeta}q'^2}{\bar{\zeta}_o v_o}\right\}\right] + \gamma_1 x + \gamma_2 y + \gamma_3 . \tag{5.12}$$

Setting $\gamma_1 = \gamma_2 = \gamma_3 = 0$, $P_j = 0$ $(j=1,2,\ldots,N)$, summing for all the loadings on the $2k$ circular patches and using (4.15b) we arrive at the following solution for the sectorial plate under consideration:

$$w'^2_{fr} = \frac{Mc}{8\pi\kappa\rho_o D}\sum_{t=0}^{2k-1}(-1)^t \operatorname{Im}\left[2\bar{\zeta}_t\zeta\left(\ln R'_t - \kappa\ln R_t - \frac{\kappa b'^2}{4R_t^2}\right) + (\kappa^2-1)v_t \ln\frac{v_t}{\bar{\zeta}_t\zeta}\right.$$
$$\left. + \frac{2(\rho_o^2-1)(\rho^2-1)-q'^2}{2v_t} + \frac{(1-\rho^2)\bar{\zeta}_t\zeta q'^2}{2v_t^2}\right] . \quad (5.13)$$

Subtracting (5.6) from (5.13) gives

$$w'^2_{fr} - w'^2_{cl} = \frac{Mc(1-\kappa)}{8\pi\kappa\rho_o D}\sum_{t=0}^{2k-1}(-1)^t \operatorname{Im}\left[2\bar{\zeta}_t\zeta\ln V_t - (1+\kappa)v_t\ln\frac{v_t}{\bar{\zeta}_t\zeta} + \frac{2(\rho_o^2-1)(\rho^2-1)-q'^2}{2v_t}\right.$$
$$\left. + \frac{(1-\rho^2)\bar{\zeta}_t\zeta q'^2}{2v_t^2}\right] . \quad (5.14)$$

When this expression is expanded in powers of ζ , the imaginary parts are taken and equation (3.3b) is used, the following formula is obtained:

$$w'^2_{fr} = \frac{kMc}{4\pi\rho_o D}\sum{}' h_n(\rho,\rho_o)\sin n\theta\cos n\beta, \quad (5.15)$$

where

$$h_n(\rho,\rho_o) = f_n(\rho,\rho_o) + \frac{4(\rho\rho_o)^{-n}}{3+\eta}\left[(1-\rho^2-\frac{1}{n})\left(1-\frac{n\rho_o^2}{n-1}+\frac{1}{2}nq'^2\right) - \frac{\kappa}{n(n+1)}\right] . \quad (5.16)$$

Applying equations (2.8a,b) yields

$$(M_\theta)_{\rho=1} = \frac{kM(1+\eta)}{\pi(3+\eta)r_o}\sum{}'\rho_o^{-n}\left[4+(\eta-1)n\{1-\rho_o^2+\frac{1}{2}(n-1)q'^2\}\right]\sin n\theta\cos n\beta, \quad (5.17)$$

$$(M_{r\theta})_{\rho=1} = \frac{kM}{\pi\kappa r_o}\sum{}'\rho_o^{-n}\left[n-\kappa-1-n\rho_o^2+\frac{1}{2}n(n-1)q'^2\right]\cos n\theta\cos n\beta. \quad (5.18)$$

The solution for a complete wedge-shaped plate which is simply supported along the radial edges and normally loaded by the pressure (2.2c) on a circular domain is now derived by letting c tend to 0 in (5.6) and adding the result to (5.2). We therefore obtain

$$w'^2 = -\frac{Mr}{4\pi D}\sum_{t=0}^{2k-1}\left(\ln\frac{R_t}{b}+\frac{b'^2}{4R_t^2}\right)\sin\theta_t \quad , \quad (5.19a)$$

$$w'^1 = \frac{Mr}{4\pi D}\left[\left\{\frac{1}{s}-\frac{R_o^2}{4b_1^2}+\frac{4(R_o/b)^s}{s(s^2-4)}\right\}\sin\theta_o - \sum_{t=1}^{2k-1}\left(\ln\frac{R_t}{b}+\frac{b'^2}{4R_t^2}\right)\sin\theta_t\right] . \quad (5.19b)$$

Expanding (5.19a) and using (3.3b) give

$$w'^2 = \frac{kM}{4\pi r_o D}\sum{}'\left(\frac{r_o}{r}\right)^n\left(\frac{r^2}{n-1}-\frac{r_o^2}{n+1}-\frac{1}{2}b'^2\right)\sin n\theta\cos n\beta \quad (r > r_o), \quad (5.20a)$$

$$w'^2 = \frac{kM}{4\pi r_o D}\sum{}'\left(\frac{r}{r_o}\right)^n\left(\frac{r_o^2}{n-1}-\frac{r^2}{n+1}-\frac{1}{2}b'^2\right)\sin n\theta\cos n\beta \quad (r < r_o). \quad (5.20b)$$

Putting $q' = 0$ in the results of this section we obtain the solutions when the sectorial plate is acted upon at any point Q_o by an isolated couple M with axis along OQ_o .

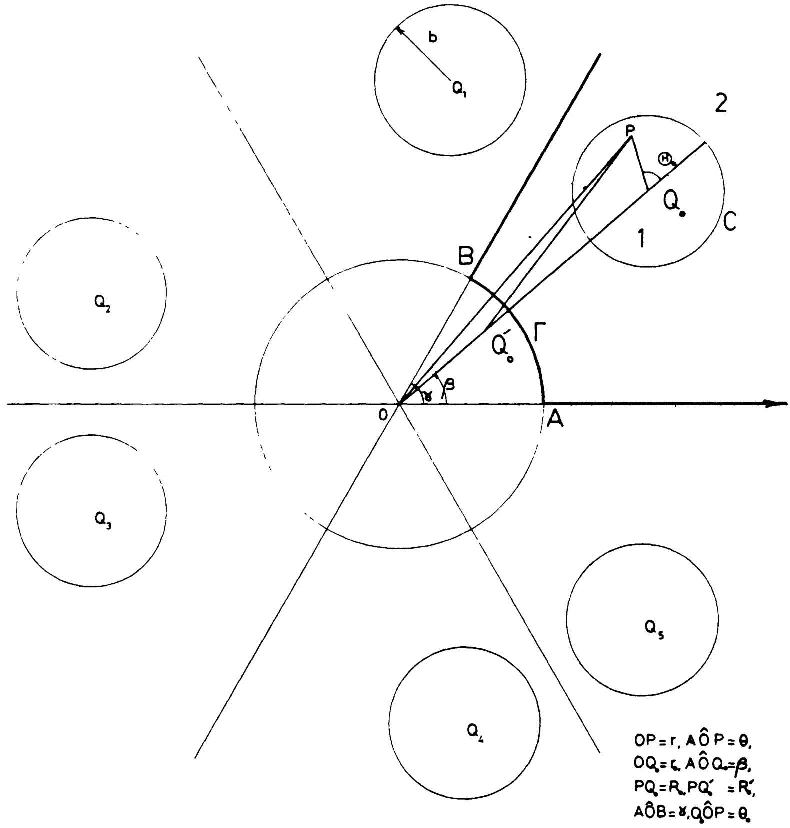

FIG.1

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