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Research Article

Sensitivity Analysis for a System of Generalized Nonlinear Mixed Quasi Variational Inclusions with *H*-Monotone Operators

Han-Wen Cao

Department of Science, Nanchang Institute of Technology, Nanchang 330099, China

Correspondence should be addressed to Han-Wen Cao, chwhappy@163.com

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The existence of the solution for a new system of generalized nonlinear mixed quasi variational inclusions with H-monotone operators is proved by using implicit resolvent technique, and the sensitivity analysis of solution in Hilbert spaces is given. Our results improve and generalize some results of the recent ones.

1. Introduction

Sensitivity analysis of solution for variational inequalities and variational inclusions has been studied by many authors via quite different technique. (See [1–7] and the reference therein).

In 2004, Agarwal et al. [1] introduced and studied the following problem which is called the system of parametric generalized nonlinear mixed quasi variational inclusions.

Let $\mathscr{H}$ be a real Hilbert space endowed with the product $\langle\cdot,\cdot\rangle$ and norm $\|\cdot\|$, respectively. Let Ω and Λ be two nonempty open subsets of $\mathscr{H}$ in which the parametric ω and λ take values. Let $M : \mathscr{H}\times\Omega \to 2^{\mathscr{H}}$ and $N : \mathscr{H}\times\Lambda \to 2^{\mathscr{H}}$ be two maximal monotone mappings with respect to the first argument. $H_1, S : \mathscr{H}\times\Omega \to \mathscr{H}$ and $H_2, T : \mathscr{H}\times\Lambda \to \mathscr{H}$ be nonlinear single-valued mappings. The system of parametric generalized nonlinear mixed quasi variational inclusions problem [1] is to find $(x,y)\in\mathscr{H}\times\mathscr{H}$ such that

$$\begin{aligned} 0 &\in x - y + \rho\big(H_1(y,\omega), S(y,\omega)\big) + \rho M(x,\omega), \\ 0 &\in y - x + \gamma(H_2(x,\lambda) + T(x,\lambda)) + \gamma N(y,\lambda), \end{aligned} \tag{1.1}$$

where $\rho > 0$ and $\gamma > 0$ are two constants.

In this paper, we introduce a new system of parametric generalized nonlinear mixed quasi variational inclusions problem.

For each $\omega \in \Omega$, $\lambda \in \Lambda$, find $x = x(\omega, \lambda)$, $y = y(\omega, \lambda)$ such that

$$\begin{aligned} &0 \in f(x, y, \omega) + \rho_1(F(x, y, \omega) + M(x, \omega)), \\ &0 \in g(x, y, \lambda) + \rho_2(G(x, y, \lambda) + N(y, \lambda)), \end{aligned} \tag{1.2}$$

where $f, F : \mathscr{H} \times \mathscr{H} \times \Omega \to \mathscr{H}$, $g, G : \mathscr{H} \times \mathscr{H} \times \Lambda \to \mathscr{H}$ are nonlinear single-valued mappings, $M : \mathscr{H} \times \Omega \to 2^{\mathscr{H}}$, $N : \mathscr{H} \times \Lambda \to 2^{\mathscr{H}}$ are multivalued mappings, $\rho_1 > 0$, $\rho_2 > 0$ are constants. By using implicit resolvent equations technique of H-monotone operator, the existence of solution is proved and the sensitivity analysis of solution for the problem (1.2) is given. Our results improve and generalize the known results of [1, 2, 8, 9].

2. Preliminaries

Let $\mathscr{H}$ be a real Hilbert space, Ω, Λ be two nonempty open subsets of $\mathscr{H}$.

Definition 2.1 (see [1]). (i) A mapping $T : \mathscr{H} \times \Omega \to \mathscr{H}$ is said to be monotone with respect to the first argument if

$$\langle T(x, \omega) - T(y, \omega), x - y \rangle \geq 0, \quad \forall (x, \omega), (y, \omega) \in \mathscr{H} \times \Omega. \tag{2.1}$$

(ii) T is said to be κ-strongly monotone with respect to the first argument if there exists a constant $\kappa > 0$ such that

$$\langle T(x, \omega) - T(y, \omega), x - y \rangle \geq \kappa \|x - y\|^2, \quad \forall (x, \omega), (y, \omega) \in \mathscr{H} \times \Omega. \tag{2.2}$$

(iii) T is said to be (ξ, η)-Lipschitz continuous if there exist $\xi > 0, \eta > 0$ such that

$$\|T(x, \omega_1) - T(y, \omega_2)\| \leq \xi \|x - y\| + \eta \|\omega_1 - \omega_2\|, \quad \forall (x, \omega_1), (y, \omega_2) \in \mathscr{H} \times \Omega. \tag{2.3}$$

Definition 2.2 (see [1]). A mapping $F : \mathscr{H} \times \mathscr{H} \times \Omega \to \mathscr{H}$ is said to be (ξ, η, ζ)-Lipschitz continuous if there exist $\xi > 0$, $\eta > 0$, $\zeta > 0$ such that

$$\begin{aligned} &\|F(x_1, y_1, \omega_1) - F(x_2, y_2, \omega_2)\| \\ &\leq \xi \|x_1 - x_2\| + \eta \|y_1 - y_2\| + \zeta \|\omega_1 - \omega_2\|, \quad \forall (x_1, y_1, \omega_1), (x_2, y_2, \omega_2) \in \mathscr{H} \times \mathscr{H} \times \Omega. \end{aligned} \tag{2.4}$$

Definition 2.3 (see [1]). A mapping $F : \mathscr{H} \times \mathscr{H} \times \Omega \to \mathscr{H}$ is said to be α-strongly monotone with respect to H in the first argument if there exists $\alpha > 0$ such that

$$\langle F(x_1, y, \omega) - F(x_2, y, \omega), Hx_1 - Hx_2 \rangle \geq \alpha \|x_1 - x_2\|^2, \; \forall (x_1, y, \omega), (x_2, y, \omega) \in \mathscr{H} \times \mathscr{H} \times \Omega. \tag{2.5}$$

In a similar way, we can define the strong monotonicity of F with respect to H in the second argument.

Definition 2.4 (see [9]). Let $H : \mathcal{H} \times \Omega \to \mathcal{H}$ be a single-valued mapping and $M : \mathcal{H} \times \Omega \to 2^{\mathcal{H}}$ be a multi-valued mapping. M is said to be H-monotone if M is monotone with respect to the first argument and $(H(\cdot,\omega) + \rho M(\cdot,\omega))(\mathcal{H}) = \mathcal{H}$ holds for all $\rho > 0$ and $\omega \in \Omega$.

Definition 2.5 (see [9]). Let $H : \mathcal{H}\times\Omega \to \mathcal{H}$ be a strictly monotone mapping and $M : \mathcal{H}\times\Omega \to 2^{\mathcal{H}}$ be an H-monotone mapping. The resolvent operator $J^H_{M(\cdot,\omega),\rho} : \mathcal{H} \to \mathcal{H}$ is defined by

$$J^H_{M(\cdot,\omega),\rho}(u) = \left(H(\cdot,\omega) + \rho M(\cdot,\omega)\right)^{-1}(u), \quad \forall u \in \mathcal{H}. \tag{2.6}$$

Lemma 2.6 (see [9]). *Let $H_1 : \mathcal{H} \times \Omega \to \mathcal{H}$ be γ_1-strongly monotone with respect to the first argument, $H_2 : \mathcal{H} \times \Lambda \to \mathcal{H}$ be γ_2-strongly monotone with respect to the first argument, and $M : \mathcal{H} \times \Omega \to 2^{\mathcal{H}}$ be H_1-monotone, andN $: \mathcal{H} \times \Lambda \to 2^{\mathcal{H}}$ be H_2-monotone. Then for any fixed $\omega \in \Omega, \lambda \in \Lambda$, the resolvent operator $J^{H_1}_{M(\cdot,\omega),\rho_1}$ and $J^{H_2}_{N(\cdot,\lambda),\rho_2}$ are Lipschitz continuous:*

$$\begin{aligned} \left\| J^{H_1}_{M(\cdot,\omega),\rho_1}(u) - J^{H_1}_{M(\cdot,\omega),\rho_1}(v) \right\| &\le \frac{1}{\gamma_1}\|u - v\|, \quad \forall u, v \in \mathcal{H}, \\ \left\| J^{H_2}_{N(\cdot,\lambda),\rho_2}(u) - J^{H_2}_{N(\cdot,\lambda),\rho_2}(v) \right\| &\le \frac{1}{\gamma_2}\|u - v\|, \quad \forall u, v \in \mathcal{H}, \end{aligned} \tag{2.7}$$

where $\gamma_1 > 0$, $\gamma_2 > 0$ are constants.

Lemma 2.7. *Let $M : \mathcal{H} \times \Omega \to 2^{\mathcal{H}}$ be H_1-monotone and $N : \mathcal{H} \times \Lambda \to 2^{\mathcal{H}}$ be H_2-monotone. For any fixed $(\omega,\lambda) \in \Omega \times \Lambda$. $(x(\omega,\lambda), y(\omega,\lambda))$ is a solution of (1.2) if and only if*

$$\begin{aligned} x(\omega,\lambda) &= J^{H_1}_{M(\cdot,\omega),\rho_1}\left(H_1(x(\omega,\lambda),\omega) - f(x(\omega,\lambda), y(\omega,\lambda),\omega) - \rho_1 F(x(\omega,\lambda), y(\omega,\lambda),\omega)\right) \\ &\triangleq T(x, y,\omega,\lambda), \end{aligned} \tag{2.8$_1$}$$

$$\begin{aligned} y(\omega,\lambda) &= J^{H_2}_{N(\cdot,\lambda),\rho_2}\left(H_2(y(\omega,\lambda),\lambda) - g(x(\omega,\lambda), y(\omega,\lambda),\lambda) - \rho_2 G(x(\omega,\lambda), y(\omega,\lambda),\lambda)\right) \\ &\triangleq S(x, y,\omega,\lambda), \end{aligned} \tag{2.8$_2$}$$

here $x = x(\omega,\lambda)$, $y = y(\omega,\lambda)$.

Proof. Assume that $(x(\omega,\lambda), y(\omega,\lambda))$ satisfies relations (2.8$_1$) and (2.8$_2$), since $J^{H_1}_{M(\cdot,\omega),\rho_1} = (H_1(\cdot,\omega) + \rho_1 M(\cdot,\omega))^{-1}$, $J^{H_2}_{M(\cdot,\omega),\rho_2} = (H_2(\cdot,\omega) + \rho_2 N(\cdot,\omega))^{-1}$, then (2.8$_1$) and (2.8$_2$) holds if and only if

$$\begin{aligned} 0 &\in f(x(\omega,\lambda), y(\omega,\lambda),\omega) + \rho_1 F(x(\omega,\lambda), y(\omega,\lambda),\omega) + \rho_1 M(x(\omega,\lambda),\omega), \\ 0 &\in g(x(\omega,\lambda), y(\omega,\lambda),\lambda) + \rho_2 G(x(\omega,\lambda), y(\omega,\lambda),\lambda) + \rho_2 N(y(\omega,\lambda),\lambda), \end{aligned} \tag{2.9}$$

and hence $(x(\omega,\lambda))$, $y(\omega,\lambda)$ is a solution of (2.8$_1$) and (2.8$_2$). □

3. Main Results

Lemma 3.1. *Let* $T : \mathscr{H} \times \mathscr{H} \to \mathscr{H}$, $S : \mathscr{H} \times \mathscr{H} \to \mathscr{H}$ *be two continuous mappings. If there exist* Θ_1, Θ_2, $0 < \Theta_1,\ \Theta_2 < 1$, *such that*

$$\begin{aligned} &\|T(x_1, y_1) - T(x_2, y_2)\| + \|S(x_1, y_1) - S(x_2, y_2)\| \\ &\quad \le \Theta_1\|x_1 - x_2\| + \Theta_2\|y_1 - y_2\|, \quad \forall x_1, x_2, y_1, y_2 \in \mathscr{H}. \end{aligned} \tag{3.1}$$

then there exist $x^*, y^* \in \mathscr{H}$ *such that*

$$x^* = T(x^*, y^*), \quad y^* = S(x^*, y^*). \tag{3.2}$$

Proof. For any $x_0, y_0 \in \mathscr{H}$, let $x_{n+1} = T(x_x, y_n)$, $y_{n+1} = S(x_n, y_n)$, $n = 0, 1, 2, \ldots$, then by (3.1), for all $x_1, x_2, \ldots, x_n, x_{n+1}, y_1, y_2, \ldots, y_n, y_{n+1} \in \mathscr{H}$,we have

$$\begin{aligned} \|x_{n+1} - x_n\| + \|y_{n+1} - y_n\| &\le \Theta_1\|x_n - x_{n-1}\| + \Theta_2\|y_n - y_{n-1}\| \\ &\le \Theta(\|x_n - x_{n-1}\| + \|y_n - y_{n-1}\|), \end{aligned} \tag{3.3}$$

where $\Theta = \max\{\Theta_1, \Theta_2\}$. Let $a = \|x_1 - x_0\| + \|y_1 - y_0\|$, then

$$\begin{aligned} \|x_{n+1} - x_n\| + \|y_{n+1} - y_n\| &\le \Theta(\|x_n - x_{n-1}\| + \|y_n - y_{n-1}\|) \\ &\le \Theta^2(\|x_{n-1} - x_{n-2}\| + \|y_{n-1} - y_{n-2}\|) \\ &\le \cdots \le \Theta^n(\|x_1 - x_0\| + \|y_1 - y_0\|) = \Theta^n a, \end{aligned} \tag{3.4}$$

and hence,

$$\begin{aligned} 0 \le \|x_{n+1} - x_n\| \le \Theta^n a, \\ 0 \le \|y_{n+1} - y_n\| \le \Theta^n a. \end{aligned} \tag{3.5}$$

Since $0 < \Theta = \max\{\Theta_1, \Theta_2\} < 1$, (3.5) implies that $\{x_n\}$ and $\{y_n\}$ are both cauchy sequences. Therefore, there exist $x^*, y^* \in \mathscr{H}$ such that $x_n \to x^*,\ y_n \to y^*(n \to \infty)$. By continuity of T and S, $x^* = T(x^*, y^*),\ y^* = S(x^*, y^*)$. □

Theorem 3.2. *Let* $H_1 : \mathscr{H} \times \Omega \to \mathscr{H}$ *be* (ξ_1, η_1)*-Lipschitz continuous and* $H_2 : \mathscr{H} \times \Lambda \to \mathscr{H}$ *be* (ξ_2, η_2)*-Lipschitz continuous. Let* $f : \mathscr{H} \times \mathscr{H} \times \Omega \to \mathscr{H}$ *be* (ξ_f, η_f, ζ_f)*-Lipschitz continuous and* γ_f*-strongly monotone with respect to* H *in the first argument,* $F : \mathscr{H} \times \mathscr{H} \times \Omega \to \mathscr{H}$ *be* (ξ_F, η_F, ζ_F)*-Lipschitz continuous,* $g : \mathscr{H} \times \mathscr{H} \times \Lambda \to \mathscr{H}$ *be* (ξ_g, η_g, ζ_g)*-Lipschitz continuous and* γ_g*-strongly monotone with respect to* H_2 *in the second argument,* $G : \mathscr{H} \times \mathscr{H} \times \Lambda \to \mathscr{H}$ *be* (ξ_G, η_G, ζ_G)*-Lipschitz* *continuous. Suppose that* $M : \mathscr{H} \times \Omega \to 2^{\mathscr{H}}$ *is* H_1*-monotone and* $N : \mathscr{H} \times \Lambda \to 2^{\mathscr{H}}$ *is* H_2*-monotone If*

$$\Theta_1 = \frac{1}{\gamma_1}\left(\sqrt{\xi_1^2 - 2\gamma_f + \xi_f^2} + \rho_1 \xi_F\right) + \frac{\xi_g + \rho_2 \xi_G}{\gamma_2} < 1,$$
$$\Theta_2 = \frac{1}{\gamma_2}\left(\sqrt{\xi_2^2 - 2\gamma_g + \eta_g^2} + \rho_2 \eta_G\right) + \frac{\eta_f + \rho_1 \eta_F}{\gamma_1} < 1, \tag{3.6}$$

then, for each $\omega, \lambda \in \Omega \times \Lambda$, *the problem* (1.2) *has an unique solution* $(x^*(\omega,\lambda), y^*(\omega,\lambda))$.

Proof. Let $x_1 = x_1(\omega,\lambda)$, $x_2 = x_2(\omega,\lambda)$, $y_1 = y_1(\omega,\lambda)$, $y_2 = y_2(\omega,\lambda)$. By defining (2.8_1) of T and Lemma 2.6, we have

$$\begin{aligned}\|T(x_1,y_1,\omega,\lambda) - T(x_2,y_2,\omega,\lambda)\| &\le \frac{1}{\gamma_1}\|H_1(x_1,\omega) - f(x_1,y_1,\omega) - \rho_1 F(x_1,y_1,\omega) - H_1(x_2,\omega) \\ &\quad + f(x_2,y_2,\omega) + \rho_1 F(x_2,y_2,\omega)\| \\ &\le \frac{1}{\gamma_1}\|H_1(x_1,\omega) - H_1(x_2,\omega) - f(x_1,y_1,\omega) + f(x_2,y_2,\omega)\| \\ &\quad + \frac{1}{\gamma_1}\rho_1\|F(x_1,y_1,\omega) - F(x_2,y_2,\omega)\|.\end{aligned} \tag{3.7}$$

Since f is (ξ_f, η_f, ζ_f)-Lipschitz continuous and γ_f-strongly monotone with respect to H_1 in the first argument, F is (ξ_F, η_F, ζ_F)-Lipschitz continuous, we have

$$\begin{aligned}&\|H_1(x_1,\omega) - H_1(x_2,\omega) - f(x_1,y_1,\omega) + f(x_2,y_2,\omega)\| \\ &\quad \le \|H_1(x_1,\omega) - H_1(x_2,\omega) - f(x_1,y_1,\omega) + f(x_2,y_1,\omega)\| + \|f(x_2,y_1,\omega) - f(x_2,y_2,\omega)\| \\ &\quad \le \sqrt{\xi_1^2 - 2\gamma_f + \xi_f^2}\|x_1 - x_2\| + \eta_f\|y_1 - y_2\|,\end{aligned} \tag{3.8}$$

$$\|F(x_1,y_1,\omega) - F(x_2,y_2,\omega)\| \le \xi_F\|x_1 - x_2\| + \eta_F\|y_1 - y_2\|. \tag{3.9}$$

Combining (3.7)–(3.9), we have

$$\begin{aligned}\|T(x_1,y_1,\omega,\lambda) - T(x_2,y_2,\omega,\lambda)\| &\le \frac{1}{\gamma_1}\left(\sqrt{\xi_1^2 - 2\gamma_f + \xi_f^2} + \rho_1\xi_F\right)\|x_1 - x_2\| \\ &\quad + \frac{\eta_f + \rho_1\eta_F}{\gamma_1}\|y_1 - y_2\|.\end{aligned} \tag{3.10}$$

By defining (2.8_2) of S and Lemma 2.6, we have

$$\begin{aligned}\|S(x_1,y_1,\omega,\lambda)-S(x_2,y_2,\omega,\lambda)\| &\le \frac{1}{\gamma_2}\|H_2(y_1,\lambda)-g(x_1,y_1,\lambda)-\rho_2 G(x_1,y_1,\lambda)\\ &\qquad -H_2(y_2,\lambda)+g(x_2,y_2,\lambda)+\rho_2 G(x_2,y_2,\lambda)\|\\ &\le \frac{1}{\gamma_2}\|H_2(y_1,\lambda)-H_2(y_2,\lambda)-g(x_1,y_1,\lambda)+g(x_2,g_2,\lambda)\|\\ &\quad +\frac{\rho_2}{\gamma_2}\|G(x_1,y_1,\lambda)-G(x_2,y_2,\lambda)\|.\end{aligned}\tag{3.11}$$

Since g is (ξ_g,η_g,ζ_g)-Lipschitz continuous and γ_g-strongly monotone with respect to H_2 in the second argument, G is (ξ_G,η_G,ζ_G)-Lipschitz continuous. We have

$$\begin{aligned}&\|H_2(y_1,\lambda)-H_2(y_2,\lambda)-g(x_1,y_1,\lambda)+g(x_2,y_2,\lambda)\|\\ &\quad\le \|H_2(y_1,\lambda)-H_2(y_2,\lambda)-g(x_1,y_1,\lambda)+g(x_1,y_2,\lambda)\|+\|g(x_1,y_2,\lambda)-g(x_2,y_2,\lambda)\|\\ &\quad\le \sqrt{\xi_2^2-2\gamma_g+\eta_g^2}\|y_1-y_2\|+\xi_g\|x_1-x_2\|,\end{aligned}\tag{3.12}$$

$$\|G(x_1,y_1,\lambda)-G(x_2,y_2,\lambda)\|\le \xi_G\|x_1-x_2\|+\eta_G\|y_1-y_2\|.\tag{3.13}$$

Combining (3.11)–(3.13), we have

$$\begin{aligned}\|S(x_1,y_1,\omega,\lambda)-S(x_2,y_2,\omega,\lambda)\| &\le \frac{1}{\gamma_2}\left(\sqrt{\xi_2^2-2\gamma_g+\eta_g^2}+\rho_2\eta_G\right)\|y_1-y_2\|\\ &\quad+\frac{\xi_g+\rho_2\xi_G}{\gamma_2}\|x_1-x_2\|.\end{aligned}\tag{3.14}$$

By (3.10) and (3.14), we have

$$\begin{aligned}&\|T(x_1,y_1,\omega,\lambda)-T(x_2,y_2,\omega,\lambda)\|+\|S(x_1,y_1,\omega,\lambda)-S(x_2,y_2,\omega,\lambda)\|\\ &\quad\le\left[\frac{1}{\gamma_1}\left(\sqrt{\xi_1^2-2\gamma_f+\xi_f^2}+\rho_1\xi_F\right)+\frac{\xi_g+\rho_2\xi_G}{\gamma_2}\right]\|x_1-x_2\|\\ &\qquad+\left[\frac{1}{\gamma_2}\left(\sqrt{\xi_2^2-2\gamma_g+\eta_g^2}+\rho_2\eta_G\right)+\frac{\eta_f+\rho_1\eta_F}{\gamma_1}\right]\|y_1-y_2\|\\ &\quad=\Theta_1\|x_1-x_2\|+\Theta_2\|y_1-y_2\|.\end{aligned}\tag{3.15}$$

By (3.6) and Lemma 3.1, there exist $x^*=x^*(\omega,\lambda)$, $y^*=y^*(\omega,\lambda)$. (x^*,y^*) such that $x^*=T(x^*,y^*,\omega,\lambda)$, $y^*=S(x^*,y^*,\omega,\lambda)$. By Lemma 2.7, (x^*,y^*) is a solution of (1.2). From (3.15), we easily see that the solution of (1.2) is unique. □

Assumption 1. For implicit resolvent operator $J^{H_1}_{M(\cdot,\omega),\rho_1}$, $J^{H_2}_{N(\cdot,\lambda),\rho_2}$, there are two constants $\xi > 0$ and $\eta > 0$ such that

$$\begin{aligned}
&\left\| J^{H_1}_{M(\cdot,\omega),\rho_1}(u) - J^{H_1}_{M(\cdot,\overline{\omega}),\rho_1}(u) \right\| \le \xi \|\omega - \overline{\omega}\|, \\
&\left\| J^{H_2}_{N(\cdot,\lambda),\rho_2}(v) - J^{H_2}_{N(\cdot,\overline{\lambda}),\rho_2}(v) \right\| \le \eta \left\| \lambda - \overline{\lambda} \right\|, \quad \forall u, v \in \mathscr{H}.
\end{aligned} \tag{3.16}$$

Theorem 3.3. *Suppose that the mappings H_1, f, F, M, H_2, g, G, and N are the same as in Theorem 3.2, and for any fixed $x, y \in \mathscr{H}$, the mappings $\omega \to H_1(x,\omega)$, $\omega \to f(x,y,\omega)$, $\omega \to F(x,y,\omega), \lambda \to H_2(y,\lambda), \lambda \to g(x,y,\lambda), \lambda \to G(x,y,\lambda)$ are continuous (or Lipschitz continuous.) If Assumption 1 and condition* (3.6) *hold, then the solution $(x(\omega,\lambda), y(\omega,\lambda))$ for the problem* (1.2) *is continuous (or Lipschitz continuous).*

Proof. Suppose that $\omega, \overline{\omega} \in \Omega$, $\lambda, \overline{\lambda} \in \Lambda$ such that $\omega \to \overline{\omega}, \lambda \to \overline{\lambda}$. From Theorem 3.2, we know that the problem (1.2) has solution $(x(\omega,\lambda), y(\omega,\lambda))$, $x(\overline{\omega},\overline{\lambda}), y(\overline{\omega},\overline{\lambda}))$, $(x(\overline{\omega},\lambda), y(\overline{\omega},\lambda))$ and $(x(\omega,\overline{\lambda}), y(\omega,\overline{\lambda}))$.

(A) Estimate $\|x(\omega,\lambda) - x(\overline{\omega},\lambda)\|$ and $\|y(\omega,\lambda) - y(\overline{\omega},\lambda)\|$. By Lemma 2.7,we have

$$\begin{aligned}
x(\omega,\lambda) &= J^{H_1}_{M(\cdot,\omega),\rho_1}\left(H_1(x(\omega,\lambda),\omega) - f(x(\omega,\lambda), y(\omega,\lambda),\omega) - \rho_1 F(x(\omega,\lambda), y(\omega,\lambda),\omega)\right) \\
&\triangleq J^{H_1}_{M(\cdot,\omega),\rho_1}(p), \\
x(\overline{\omega},\lambda) &= J^{H_1}_{M(\cdot,\overline{\omega}),\rho_1}\left(H_1(x(\overline{\omega},\lambda),\overline{\omega}) - f(x(\overline{\omega},\lambda), y(\overline{\omega},\lambda),\overline{\omega}) - \rho_1 F(x(\overline{\omega},\lambda), y(\overline{\omega},\lambda),\overline{\omega})\right) \\
&\triangleq J^{H_1}_{M(\cdot,\overline{\omega}),\rho_1}(q).
\end{aligned} \tag{3.17}$$

It follows from Assumption 1, Lemmas 2.6 and 2.7 that

$$\begin{aligned}
\|x(\omega,\lambda) - x(\overline{\omega},\lambda)\| &= \left\| J^{H_1}_{M(\cdot,\omega),\rho_1}(p) - J^{H_1}_{M(\cdot,\overline{\omega}),\rho_1}(q) \right\| \\
&\le \left\| J^{H_1}_{M(\cdot,\omega),\rho_1}(p) - J^{H_1}_{M(\cdot,\omega),\rho_1}(q) \right\| + \left\| J^{H_1}_{M(\cdot,\omega),\rho_1}(q) - J^{H_1}_{M(\cdot,\overline{\omega}),\rho_1}(q) \right\| \le \frac{1}{\gamma_1}\|p - q\| + \xi\|\omega - \overline{\omega}\|.
\end{aligned} \tag{3.18}$$

Since f is (ξ_f, η_f, ζ_f)-Lipschitz continuous, γ_f -strongly monotone with respect to H_1 in the first argument and F is (ξ_F, η_F, ζ_F)-Lipschitz continuous, we conclude

$$\begin{aligned}
\|p - q\| &= \big\| H_1(x(\omega,\lambda),\omega) - f(x(\omega,\lambda), y(\omega,\lambda),\omega) - \rho_1 F(x(\omega,\lambda), y(\omega,\lambda),\omega) \\
&\qquad - H_1(x(\overline{\omega},\lambda),\overline{\omega}) + f(x(\overline{\omega},\lambda), y(\overline{\omega},\lambda),\overline{\omega}) + \rho_1 F(x(\overline{\omega},\lambda), y(\overline{\omega},\lambda),\overline{\omega}) \big\| \\
&\le \big\| H_1(x(\omega,\lambda),\omega) - H_1(x(\overline{\omega},\lambda),\omega) - f(x(\omega,\lambda), y(\omega,\lambda),\omega) + f(x(\overline{\omega},\lambda), y(\omega,\lambda),\omega) \big\| \\
&\quad + \|H_1(x(\overline{\omega},\lambda),\omega) - H_1(x(\overline{\omega},\lambda),\overline{\omega})\| \\
&\quad + \big\| f(x(\overline{\omega},\lambda), y(\omega,\lambda),\omega) - f(x(\overline{\omega},\lambda), y(\overline{\omega},\lambda),\omega) \big\|
\end{aligned}$$

$$
\begin{aligned}
&+ \left\| f\left(x(\overline{\omega},\lambda), y(\overline{\omega},\lambda), \omega\right) - f\left(x(\overline{\omega},\lambda), y(\overline{\omega},\lambda), \overline{\omega}\right) \right\| \\
&+ \rho_1 \left\| F\left(x(\omega,\lambda), y(\omega,\lambda), \omega\right) - F\left(x(\overline{\omega},\lambda), y(\overline{\omega},\lambda), \omega\right) \right\| \\
&+ \rho_1 \left\| F\left(x(\overline{\omega},\lambda), y(\overline{\omega},\lambda), \omega\right) - F\left(x(\overline{\omega},\lambda), y(\overline{\omega},\lambda), \overline{\omega}\right) \right\| \\
\le{}& \sqrt{\xi_1^2 - 2\gamma_f + \xi_f^2} \| x(\omega,\lambda) - x(\overline{\omega},\lambda) \| + \| H_1(x(\overline{\omega},\lambda),\omega) - H_1(x(\overline{\omega},\lambda),\overline{\omega}) \| \\
&+ \eta_f \left\| y(\omega,\lambda) - y(\overline{\omega},\lambda) \right\| + \left\| f\left(x(\overline{\omega},\lambda), y(\overline{\omega},\lambda), \omega\right) - f\left(x(\overline{\omega},\lambda), y(\overline{\omega},\lambda), \overline{\omega}\right) \right\| \\
&+ \rho_1 \xi_F \| x(\omega,\lambda) - x(\overline{\omega},\lambda) \| + \rho_1 \eta_F \left\| y(\omega,\lambda) - y(\overline{\omega},\lambda) \right\| \\
&+ \rho_1 \left\| F\left(x(\overline{\omega},\lambda), y(\overline{\omega},\lambda), \omega\right) - F\left(x(\overline{\omega},\lambda), y(\overline{\omega},\lambda), \overline{\omega}\right) \right\|.
\end{aligned}
\tag{3.19}
$$

It follows from (3.18) and (3.19) that

$$
\begin{aligned}
\| x(\omega,\lambda) - x(\overline{\omega},\lambda) \| \le{}& \theta_1 \| x(\omega,\lambda) - x(\overline{\omega},\lambda) \| + \frac{1}{\gamma_1}\left(\eta_f + \rho_1 \eta_F\right) \left\| y(\omega,\lambda) - y(\overline{\omega},\lambda) \right\| \\
&+ \frac{1}{\gamma_1} \| H_1(x(\overline{\omega},\lambda),\omega) - H_1(x(\overline{\omega},\lambda),\overline{\omega}) \| \\
&+ \frac{1}{\gamma_1} \left\| f\left(x(\overline{\omega},\lambda), y(\overline{\omega},\lambda), \omega\right) - f\left(x(\overline{\omega},\lambda), y(\overline{\omega},\lambda), \overline{\omega}\right) \right\| \\
&+ \frac{\rho_1}{\gamma_1} \left\| F\left(x(\overline{\omega},\lambda), y(\overline{\omega},\lambda), \omega\right) - F\left(x(\overline{\omega},\lambda), y(\overline{\omega},\lambda), \overline{\omega}\right) \right\| + \xi \| \omega - \overline{\omega} \|,
\end{aligned}
\tag{3.20}
$$

that is,

$$
\begin{aligned}
\| x(\omega,\lambda) - x(\overline{\omega},\lambda) \| \le{}& \frac{\eta_f + \rho_1 \eta_F}{(1-\theta_1)\gamma_1} \left\| y(\omega,\lambda) - y(\overline{\omega},\lambda) \right\| \\
&+ \frac{1}{1-\theta_1} \left\{ \frac{1}{\gamma_1} \| H_1(x(\overline{\omega},\lambda),\omega) - H_1(x(\overline{\omega},\lambda),\overline{\omega}) \| \right. \\
&\quad + \frac{1}{\gamma_1} \left\| f\left(x(\overline{\omega},\lambda), y(\overline{\omega},\lambda), \omega\right) - f\left(x(\overline{\omega},\lambda), y(\overline{\omega},\lambda), \overline{\omega}\right) \right\| \\
&\quad + \frac{\rho_1}{\gamma_1} \left\| F\left(x(\overline{\omega},\lambda), y(\overline{\omega},\lambda), \omega\right) - F\left(x(\overline{\omega},\lambda), y(\overline{\omega},\lambda), \overline{\omega}\right) \right\| \\
&\quad \left. + \xi \| \omega - \overline{\omega} \| \right\},
\end{aligned}
\tag{3.21}
$$

where

$$
\theta_1 = \frac{1}{\gamma_1}\left(\sqrt{\xi_1^2 - 2\gamma_f + \xi_f^2} + \rho_1 \xi_F \right), \tag{$*$}
$$

By Lemma 2.7, we have

$$\begin{aligned}
y(\omega,\lambda) &= J^{H_2}_{N(\cdot,\lambda)\rho_2}\big(H_2(y(\omega,\lambda),\lambda) - g(x(\omega,\lambda),y(\omega,\lambda),\lambda) - \rho_2 G(x(\omega,\lambda),y(\omega,\lambda),\lambda)\big) \\
&\triangleq J^{H_2}_{N(\cdot,\lambda),\rho_2}(s), \\
y(\overline{\omega},\lambda) &= J^{H_2}_{N(\cdot,\lambda),\rho_2}\big(H_2(y(\overline{\omega},\lambda),\lambda) - g(x(\overline{\omega},\lambda),y(\overline{\omega},\lambda),\lambda) - \rho_2 G(x(\overline{\omega},\lambda),y(\overline{\omega},\lambda),\lambda)\big) \\
&\triangleq J^{H_2}_{N(\cdot,\lambda),\rho_2}(t).
\end{aligned} \tag{3.22}$$

It follows from Assumption 1, Lemmas 2.6 and 2.7 that

$$\|y(\omega,\lambda) - y(\overline{\omega},\lambda)\| = \left\|J^{H_2}_{N(\cdot,\lambda),\rho_2}(s) - J^{H_2}_{N(\cdot,\lambda),\rho_2}(t)\right\| \le \frac{1}{\gamma_2}\|s-t\|. \tag{3.23}$$

Since g is (ξ_g,η_g,ζ_g)-Lipschitz continuous and γ_g-strongly monotone with respect to H_2 in the second argument and G is (ξ_G,η_G,ζ_G)-Lipschitz continuous,we conclude

$$\begin{aligned}
\|s-t\| = \big\|&H_2(y(\omega,\lambda),\lambda) - g(x(\omega,\lambda),y(\omega,\lambda),\lambda) - \rho_2 G(x(\omega,\lambda),y(\omega,\lambda),\lambda) \\
&-H_2(y(\overline{\omega},\lambda),\lambda) + g(x(\overline{\omega},\lambda),y(\overline{\omega},\lambda),\lambda) + \rho_2 G(x(\overline{\omega},\lambda),y(\overline{\omega},\lambda),\lambda)\big\| \\
\le{}& \big\|H_2(y(\omega,\lambda),\lambda) - H_2(y(\overline{\omega},\lambda),\lambda) - g(x(\omega,\lambda),y(\omega,\lambda),\lambda) + g(x(\omega,\lambda),y(\overline{\omega},\lambda),\lambda)\big\| \\
&+ \big\|g(x(\omega,\lambda),y(\overline{\omega},\lambda),\lambda) - g(x(\overline{\omega},\lambda),y(\overline{\omega},\lambda),\lambda)\big\| \\
&+ \rho_2\big\|G(x(\omega,\lambda),y(\omega,\lambda),\lambda) - G(x(\overline{\omega},\lambda),y(\overline{\omega},\lambda),\lambda)\big\| \\
\le{}& \sqrt{\xi_2^2 - 2\gamma_g + \eta_g^2}\,\|y(\omega,\lambda) - y(\overline{\omega},\lambda)\| + \xi_g\|x(\omega,\lambda) - x(\overline{\omega},\lambda)\| \\
&+ \rho_2\xi_G\|x(\omega,\lambda) - x(\overline{\omega},\lambda)\| + \rho_2\eta_G\|y(\omega,\lambda) - y(\overline{\omega},\lambda)\|.
\end{aligned} \tag{3.24}$$

It follows from (3.23) and (3.24) that

$$\begin{aligned}
\|y(\omega,\lambda) - y(\overline{\omega},\lambda)\| \le{}& \frac{1}{\gamma_2}\left(\sqrt{\xi_2^2 - 2\gamma_g + \eta_g^2} + \rho_2\eta_G\right)\|y(\omega,\lambda) - y(\overline{\omega},\lambda)\| \\
&+ \frac{1}{\gamma_2}(\xi_g + \rho_2\xi_G)\|x(\omega,\lambda) - x(\overline{\omega},\lambda)\|.
\end{aligned} \tag{3.25}$$

That is,

$$\|y(\omega,\lambda) - y(\overline{\omega},\lambda)\| \le \frac{1}{1-\theta_2}\frac{1}{\gamma_2}(\xi_g + \rho_2\xi_G)\|x(\omega,\lambda) - x(\overline{\omega},\lambda)\|, \tag{3.26}$$

where

$$\theta_2 = \frac{1}{\gamma_2}\left(\sqrt{\xi_2^2 - 2\gamma_g + \eta_g^2} + \rho_2\eta_G\right). \tag{**}$$

By combining (3.21) and (3.26), we derive

$$\begin{aligned}\|x(\omega,\lambda) - x(\overline{\omega},\lambda)\| \le{}& \frac{(\eta_f + \rho_1\eta_F)(\xi_g + \rho_2\xi_G)}{(1-\theta_1)(1-\theta_2)\gamma_1\gamma_2}\|x(\omega,\lambda) - x(\overline{\omega},\lambda)\| \\ &+ \frac{1}{1-\theta_1}\left\{\frac{1}{\gamma_1}\|H_1(x(\overline{\omega},\lambda),\omega) - H_1(x(\overline{\omega},\lambda),\overline{\omega})\|\right. \\ &\quad + \frac{1}{\gamma_1}\left\|f\left(x(\overline{\omega},\lambda),y(\overline{\omega},\lambda),\omega\right) - f\left(x(\overline{\omega},\lambda),y(\overline{\omega},\lambda),\overline{\omega}\right)\right\| \\ &\quad + \frac{\rho_1}{\gamma_1}\left\|F\left(x(\overline{\omega},\lambda),y(\overline{\omega},\lambda),\omega\right) - F\left(x(\overline{\omega},\lambda),y(\overline{\omega},\lambda),\overline{\omega}\right)\right\| \\ &\quad \left.+\xi\|\omega - \overline{\omega}\|s\right\}.\end{aligned} \tag{3.27}$$

That is,

$$\begin{aligned}\|x(\omega,\lambda) - x(\overline{\omega},\lambda)\| \le{}& \frac{1}{1-\kappa}\frac{1}{1-\theta_1}\left\{\frac{1}{\gamma_1}\|H_1(x(\overline{\omega},\lambda),\omega) - H_1(x(\overline{\omega},\lambda),\overline{\omega})\|\right. \\ &\quad + \frac{1}{\gamma_1}\left\|f\left(x(\overline{\omega},\lambda),y(\overline{\omega},\lambda),\omega\right) - f\left(x(\overline{\omega},\lambda),y(\overline{\omega},\lambda),\overline{\omega}\right)\right\| \\ &\quad + \frac{\rho_1}{\gamma_1}\left\|F\left(x(\overline{\omega},\lambda),y(\overline{\omega},\lambda),\omega\right) - F\left(x(\overline{\omega},\lambda),y(\overline{\omega},\lambda),\overline{\omega}\right)\right\| \\ &\quad \left.+\xi\|\omega - \overline{\omega}\|\right\},\end{aligned} \tag{3.28}$$

$$\begin{aligned}\left\|y(\omega,\lambda) - y(\overline{\omega},\lambda)\right\| \le{}& \frac{\xi_g + \rho_2\xi_G}{(1-\theta_2)\gamma_2}\frac{1}{1-\kappa}\frac{1}{1-\theta_1}\left\{\frac{1}{\gamma_1}\|H_1(x(\overline{\omega},\lambda),\omega) - H_1(x(\overline{\omega},\lambda),\overline{\omega})\|\right. \\ &\quad + \frac{1}{\gamma_1}\left\|f\left(x(\overline{\omega},\lambda),y(\overline{\omega},\lambda),\omega\right)\right. \\ &\quad \left.-f\left(x(\overline{\omega},\lambda),y(\overline{\omega},\lambda),\overline{\omega}\right)\right\| \\ &\quad + \frac{\rho_1}{\gamma_1}\left\|F\left(x(\overline{\omega},\lambda),y(\overline{\omega},\lambda),\omega\right)\right. \\ &\quad \left.-F\left(x(\overline{\omega},\lambda),y(\overline{\omega},\lambda),\overline{\omega}\right)\right\| \\ &\quad \left.+\|\xi\omega - \overline{\omega}\|\right\},\end{aligned} \tag{3.29}$$

where

$$\kappa = \frac{(\eta_f + \rho_1 \eta_F)(\xi_g + \rho_2 \xi_G)}{(1-\theta_1)(1-\theta_2)\gamma_1\gamma_2}. \tag{$***$}$$

By (3.6),

$$1-\theta_1 > \frac{1}{\gamma_2}(\xi_g + \rho_2\xi_G), \quad 1-\theta_2 > \frac{1}{\gamma_1}(\eta_f + \rho_1\eta_F), \tag{3.30}$$

and hence

$$0 < \frac{(\xi_g + \rho_2\xi_G)(\eta_f + \rho_1\eta_F)}{(1-\theta_1)(1-\theta_2)\gamma_1\gamma_2} < 1, \text{ that is, } 0 < \kappa < 1. \tag{3.31}$$

(B) Estimate $\|y(\overline{\omega},\overline{\lambda}) - y(\overline{\omega},\lambda)\|$ and $\|x(\overline{\omega},\overline{\lambda}) - x(\overline{\omega},\lambda)\|$.

By Lemma 2.7, we have

$$\begin{aligned} y\left(\overline{\omega},\overline{\lambda}\right) &= J^{H_2}_{N(\cdot,\overline{\lambda}),\rho_2}\left(H_2\left(y\left(\overline{\omega},\overline{\lambda}\right),\overline{\lambda}\right) - g\left(x\left(\overline{\omega},\overline{\lambda}\right), y\left(\overline{\omega},\overline{\lambda}\right),\overline{\lambda}\right) - \rho_2 G\left(x\left(\overline{\omega},\overline{\lambda}\right), y\left(\overline{\omega},\overline{\lambda}\right),\overline{\lambda}\right)\right) \\ &= J^{H_2}_{N(\cdot,\overline{\lambda}),\rho_2}(m), \\ y\left(\overline{\omega},\overline{\lambda}\right) &= J^{H_2}_{N(\cdot,\lambda),\rho_2}(H_2(y(\overline{\omega},\lambda),\lambda) - g(x(\overline{\omega},\lambda), y(\overline{\omega},\lambda),\lambda) - \rho_2 G(x(\overline{\omega},\lambda), y(\overline{\omega},\lambda),\lambda)) \\ &= J^{H_2}_{N(\cdot,\lambda),\,\rho_2}(n). \end{aligned} \tag{3.32}$$

It follows from Assumption 1, Lemmas 2.6 and 2.7 that

$$\begin{aligned} \left\|y\left(\overline{\omega},\overline{\lambda}\right) - y(\overline{\omega},\lambda)\right\| &= \left\|J^{H_2}_{N(\cdot,\overline{\lambda}),\rho_2}(m) - J^{H_2}_{N(\cdot,\lambda),\rho_2}(n)\right\| \\ &\le \left\|J^{H_2}_{N(\cdot,\overline{\lambda}),\rho_2}(m) - J^{H_2}_{N(\cdot,\overline{\lambda}),\rho_2}(n)\right\| + \left\|J^{H_2}_{N(\cdot,\overline{\lambda}),\rho_2}(n) - J^{H_2}_{N(\cdot,\lambda),\rho_2}(n)\right\| \\ &\le \frac{1}{\gamma_2}\|m-n\| + \eta\left\|\lambda - \overline{\lambda}\right\|. \end{aligned} \tag{3.33}$$

Since g is (ξ_g, η_g, ζ_g)-Lipschitz continuous and γ_g-strongly monotone with respect to H_2 in the second argument and G is (ξ_G, η_G, ζ_G)-Lipschitz continuous,we conclude

$$\begin{aligned} \|m-n\| = &\left\|H_2\left(y\left(\overline{\omega},\overline{\lambda}\right),\overline{\lambda}\right) - g\left(x\left(\overline{\omega},\overline{\lambda}\right), y\left(\overline{\omega},\overline{\lambda}\right),\overline{\lambda}\right) - \rho_2 G\left(x\left(\overline{\omega},\overline{\lambda}\right), y\left(\overline{\omega},\overline{\lambda}\right),\overline{\lambda}\right)\right. \\ &\left. - H_2(y(\overline{\omega},\lambda),\lambda) + g(x(\overline{\omega},\lambda), y(\overline{\omega},\lambda),\lambda) + \rho_2 G(x(\overline{\omega},\lambda), y(\overline{\omega},\lambda),\lambda)\right\| \\ \le &\left\|H_2\left(y\left(\overline{\omega},\overline{\lambda}\right),\overline{\lambda}\right) - H_2\left(y(\overline{\omega},\lambda),\overline{\lambda}\right)\right. \\ &\left. - g\left(x\left(\overline{\omega},\overline{\lambda}\right), y\left(\overline{\omega},\overline{\lambda}\right),\overline{\lambda}\right) - g\left(x\left(\overline{\omega},\overline{\lambda}\right), y(\overline{\omega},\lambda),\overline{\lambda}\right)\right\| \end{aligned}$$

$$
\begin{aligned}
&+ \left\| H_2\left(y(\overline{\omega},\lambda),\overline{\lambda}\right) - H_2(y(\overline{\omega},\lambda),\lambda)\right\| \\
&+ \left\| g\left(x\left(\overline{\omega},\overline{\lambda}\right), y(\overline{\omega},\lambda),\overline{\lambda}\right) - g\left(x(\overline{\omega},\lambda), y(\overline{\omega},\lambda),\overline{\lambda}\right)\right\| \\
&+ \left\| g\left(x(\overline{\omega},\lambda), y(\overline{\omega},\lambda),\overline{\lambda}\right) - g(x(\overline{\omega},\lambda), y(\overline{\omega},\lambda),\lambda)\right\| \\
&+ \rho_2\left\| G\left(x\left(\overline{\omega},\overline{\lambda}\right), y\left(\overline{\omega},\overline{\lambda}\right),\overline{\lambda}\right) - G\left(x(\overline{\omega},\lambda), y(\overline{\omega},\lambda),\overline{\lambda}\right)\right\| \\
&+ \rho_2\left\| G\left(x(\overline{\omega},\lambda), y(\overline{\omega},\lambda),\overline{\lambda}\right) - G(x(\overline{\omega},\lambda), y(\overline{\omega},\lambda),\lambda)\right\| \\
\leq{}& \sqrt{\xi_2^2 - 2\gamma_g + \eta_g^2}\left\| y\left(\overline{\omega},\overline{\lambda}\right) - y(\overline{\omega},\lambda)\right\| + \xi_g\left\| x\left(\overline{\omega},\overline{\lambda}\right) - x(\overline{\omega},\lambda)\right\| \\
&+ \rho_2\left(\xi_G\left\| x\left(\overline{\omega},\overline{\lambda}\right) - x(\overline{\omega},\lambda)\right\| + \eta_G\left\| y\left(\overline{\omega},\overline{\lambda}\right) - y(\overline{\omega},\lambda)\right\|\right) \\
&+ \left\| g\left(x(\overline{\omega},\lambda), y(\overline{\omega},\lambda),\overline{\lambda}\right) - g(x(\overline{\omega},\lambda), y(\overline{\omega},\lambda),\lambda)\right\| \\
&+ \left\| H_2\left(y(\overline{\omega},\lambda),\overline{\lambda}\right) - H_2(y(\overline{\omega},\lambda),\lambda)\right\| \\
&+ \rho_2\left\| G\left(x(\overline{\omega},\lambda), y(\overline{\omega},\lambda),\overline{\lambda}\right) - G(x(\overline{\omega},\lambda), y(\overline{\omega},\lambda),\lambda)\right\|.
\end{aligned}
\tag{3.34}
$$

It follows from (3.33) and (3.34) that

$$
\begin{aligned}
\left\| y\left(\overline{\omega},\overline{\lambda}\right) - y(\overline{\omega},\lambda)\right\| \leq{}& \frac{\xi_g + \rho_2\xi_G}{(1-\theta_2)\gamma_2}\left\| x\left(\overline{\omega},\overline{\lambda}\right) - x(\overline{\omega},\lambda)\right\| \\
&+ \frac{1}{1-\theta_2}\left\{ \frac{1}{\gamma_2}\left\| H_2\left(y(\overline{\omega},\lambda),\overline{\lambda}\right) - H_2(y(\overline{\omega},\lambda),\lambda)\right\| \right. \\
&\quad + \frac{1}{\gamma_2}\left\| g\left(x(\overline{\omega},\lambda), y(\overline{\omega},\lambda),\overline{\lambda}\right) \right. \\
&\qquad \left. - g(x(\overline{\omega},\lambda), y(\overline{\omega},\lambda),\lambda)\right\| \\
&\quad + \frac{\rho_2}{\gamma_2}\left\| G\left(x(\overline{\omega},\lambda), y(\overline{\omega},\lambda),\overline{\lambda}\right) \right. \\
&\qquad \left. - G(x(\overline{\omega},\lambda), y(\overline{\omega},\lambda),\lambda)\right\| \\
&\quad \left. + \eta\left\| \lambda - \overline{\lambda}\right\| \right\},
\end{aligned}
\tag{3.35}
$$

where θ_2 defined by $(**)$.

By Lemma 2.7, we have

$$x\left(\overline{\omega},\overline{\lambda}\right) = J^{H_1}_{M(\cdot,\overline{\omega}),\rho_1}\left(H_1\left(x\left(\overline{\omega},\overline{\lambda}\right),\overline{\omega}\right)-f\left(x\left(\overline{\omega},\overline{\lambda}\right),y\left(\overline{\omega},\overline{\lambda}\right),\overline{\omega}\right)-\rho_1 F\left(x\left(\overline{\omega},\overline{\lambda}\right),y\left(\overline{\omega},\overline{\lambda}\right),\overline{\omega}\right)\right),$$
$$x(\overline{\omega},\lambda) = J^{H_1}_{M(\cdot,\overline{\omega}),\rho_1}(H_1(x(\overline{\omega},\lambda),\overline{\omega}) - f(x(\overline{\omega},\lambda),y(\overline{\omega},\lambda),\overline{\omega}) - \rho_1 F(x(\overline{\omega},\lambda),y(\overline{\omega},\lambda),\overline{\omega})). \tag{3.36}$$

As the proof of (3.25),we have

$$\begin{aligned}\left\|x\left(\overline{\omega},\overline{\lambda}\right) - x(\overline{\omega},\lambda)\right\| \leq{}& \left(\frac{1}{\gamma_1}\sqrt{\xi_1^2 - 2\gamma_f + \xi_f^2} + \rho_1\xi_F\right)\left\|x\left(\overline{\omega},\overline{\lambda}\right) - x(\overline{\omega},\lambda)\right\| \\ &+ \frac{1}{\gamma_1}(\eta_f + \rho_1\eta_F)\left\|y\left(\overline{\omega},\overline{\lambda}\right) - y(\overline{\omega},\lambda)\right\|,\end{aligned} \tag{3.37}$$

that is,

$$\left\|x\left(\overline{\omega},\overline{\lambda}\right) - x(\overline{\omega},\lambda)\right\| \leq \frac{\eta_f + \rho_1\eta_F}{(1-\theta_1)\gamma_1}\left\|y\left(\overline{\omega},\overline{\lambda}\right) - y(\overline{\omega},\lambda)\right\|, \tag{3.38}$$

where θ_1 is defined by $(*)$.

Therefore

$$\begin{aligned}\left\|y\left(\overline{\omega},\overline{\lambda}\right) - y(\overline{\omega},\lambda)\right\| \leq \frac{1}{1-\kappa}\frac{1}{1-\theta_2}\Bigg\{&\frac{1}{\gamma_2}\left\|H_2\left(y(\overline{\omega},\lambda),\overline{\lambda}\right) - H_2(y(\overline{\omega},\lambda),\lambda)\right\| \\ &+ \frac{1}{\gamma_2}\Big\|g\left(x(\overline{\omega},\lambda),y(\overline{\omega},\lambda),\overline{\lambda}\right) \\ &\qquad -g(x(\overline{\omega},\lambda),y(\overline{\omega},\lambda),\lambda)\Big\| \\ &+ \frac{\rho_2}{\gamma_2}\Big\|G\left(x(\overline{\omega},\lambda),y(\overline{\omega},\lambda),\overline{\lambda}\right) \\ &\qquad -G(x(\overline{\omega},\lambda),y(\overline{\omega},\lambda),\lambda)\Big\| \\ &+\eta\left\|\lambda-\overline{\lambda}\right\|\Bigg\},\end{aligned} \tag{3.39}$$

$$\left\|x\left(\overline{\omega},\overline{\lambda}\right)-x(\overline{\omega},\lambda)\right\| \leq \frac{\eta_f+\rho_1\eta_F}{(1-\theta_1)\gamma_1}\frac{1}{1-\kappa}\frac{1}{1-\theta_2}\left\{\frac{1}{\gamma_2}\left\|H_2\left(y(\overline{\omega},\lambda),\overline{\lambda}\right)-H_2(y(\overline{\omega},\lambda),\lambda)\right\|\right.$$
$$+\frac{1}{\gamma_2}\left\|g\left(x(\overline{\omega},\lambda),y(\overline{\omega},\lambda),\overline{\lambda}\right)\right.$$
$$\left.-g(x(\overline{\omega},\lambda),y(\overline{\omega},\lambda),\lambda)\right\|$$
$$+\frac{\rho_2}{\gamma_2}\left\|G\left(x(\overline{\omega},\lambda),y(\overline{\omega},\lambda),\overline{\lambda}\right)\right.$$
$$\left.-G(x(\overline{\omega},\lambda),y(\overline{\omega},\lambda),\lambda)\right\|$$
$$\left.+\eta\left\|\lambda-\overline{\lambda}\right\|\right\}, \tag{3.40}$$

where κ is defined by $(***)$.

(C) Prove that the conclusion of Theorem 3.3.

From (3.28) and (3.40), by the assumptions for H_1, f, F, H_2, g, and G in Theorem 3.3 and relation

$$\left\|x(\omega,\lambda)-x\left(\overline{\omega},\overline{\lambda}\right)\right\| \leq \|x(\omega,\lambda)-x(\overline{\omega},\lambda)\|+\left\|x(\overline{\omega},\lambda)-x\left(\overline{\omega},\overline{\lambda}\right)\right\|, \tag{3.41}$$

we know that $x(\omega,\lambda)$ is continuous (or Lipschitz continuous).

From (3.29) and (3.39), by the assumptions for H_1, f, F, H_2, g and G in Theorem 3.3 and relation

$$\left\|y(\omega,\lambda)-y\left(\overline{\omega},\overline{\lambda}\right)\right\| \leq \|y(\omega,\lambda)-y(\overline{\omega},\lambda)\|+\left\|y(\overline{\omega},\lambda)-y\left(\overline{\omega},\overline{\lambda}\right)\right\|, \tag{3.42}$$

we know that $y(\omega,\lambda)$ is continuous (or Lipschitz continuous). □

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