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Research Article

Well-Posedness of Generalized Vector Quasivariational Inequality Problems

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We introduce several types of the Levitin-Polyak well-posedness for a generalized vector quasivariational inequality problem with both abstract set constraints and functional constraints. Criteria and characterizations of these types of the Levitin-Polyak well-posednesses with or without gap functions of generalized vector quasivariational inequality problem are given. The results in this paper unify, generalize, and extend some known results in the literature.

1. Introduction

The vector variational inequality in a finite-dimensional Euclidean space has been introduced in [1] and applications have been given. Chen and Cheng [2] studied the vector variational inequality in infinite-dimensional space and applied it to vector optimization problem. Since then, many authors [3–11] have intensively studied the vector variational inequality on different assumptions in infinite-dimensional spaces. Lee et al. [12, 13], Lin et al. [14], Konnov and Yao [15], Daniilidis and Hadjisavvas [16], Yang and Yao [17], and Oettli and Schläger [18] studied the generalized vector variational inequality and obtained some existence results. Chen and Li [19] and Lee et al. [20] introduced and studied the generalized vector quasi-variational inequality and established some existence theorems.

On the other hand, it is well known that the well-posedness is very important for both optimization theory and numerical methods of optimization problems, which guarantees that, for approximating solution sequences, there is a subsequence which converges to a solution. The study of well-posedness originates from Tykhonov [21] in dealing with unconstrained optimization problems. Its extension to the constrained case was developed by Levitin and Polyak [22]. The study of generalized Levitin-Polyak well-posedness for convex scalar optimization problems with functional constraints originates from Konsulova and

Revalski [23]. Recently, this research was extended to nonconvex optimization problems with abstract set constraints and functional constraints (see [24]), nonconvex vector optimization problem with abstract set constraints and functional constraints (see [25]), variational inequality problems with abstract set constraints and functional constraints (see [26]), generalized inequality problems with abstract set constraints and functional constraints [27], generalized quasi-inequality problems with abstract set constraints and functional constraints [28], generalized vector inequality problems with abstract set constraints and functional constraints [29], and vector quasivariational inequality problems with abstract set constraints and functional constraints [30]. For more details on well-posedness on optimizations and related problems, please also see [31–37] and the references therein. It is worthy noting that there is no study on the Levitin-Polyak well-posedness for a generalized vector quasi-variational inequality problem.

In this paper, we will introduce four types of Levitin-Polyak well-posedness for a generalized vector quasivariational inequality problem with an abstract set constraint and a functional constraint. In Section 2, by virtue of a nonlinear scalarization function and a gap function for generalized vector quasi-varitional inequality problems, we show equivalent relations between the Levitin-Polyak well-posedness of the optimization problem and the Levitin-Polyak well-posedness of generalized vector quasi-varitional inequality problems. In Section 3, we derive some various criteria and characterizations for the (generalized) Levitin-Polyak well-posedness of the generalized vector quasi-variational inequality problems. The results in this paper unify, generalize, and extend some known results in [26–30].

2. Preliminaries

Throughout this paper, unless otherwise specified, we use the following notations and assumptions.

Let $(X, \|\cdot\|)$ be a normed space equipped with norm topology, and let (Z, d_1) be a metric space. Let $X_1 \subset X$, $K \subset Z$ be nonempty and closed sets. Let Y be a locally convex space ordered by a nontrivial closed and convex cone C with nonempty interior $\operatorname{int} C$, that is, $y_1 \leq y_2$ if and only if $y_2 - y_1 \in C$ for any $y_1, y_2 \in Y$. Let $L(X, Y)$ be the space of all the linear continuous operators from X to Y. Let $T : X_1 \to 2^{L(X,Y)}$ and $S : X_1 \to 2^{X_1}$ be strict set-valued mappings (i.e., $T(x) \neq \emptyset$ and $S(x) \neq \emptyset$, for all $x \in X_1$), and let $g : X_1 \to Z$ be a continuous vector-valued mapping. We denote by $\langle z, x\rangle$ the value $z(x)$, where $z \in L(X, Y)$, $x \in X_1$. Let $X_0 = \{x \in X_1 : g(x) \in K\}$ be nonempty. We consider the following generalized vector quasi-variational inequality problem with functional constraints and abstract set constraints.

Find $\overline{x} \in X_0$ such that $\overline{x} \in S(\overline{x})$ and there exists $\overline{z} \in T(\overline{x})$ satisfying

$$\langle \overline{z}, x - \overline{x}\rangle \notin -\operatorname{int} C, \quad \forall x \in S(\overline{x}). \tag{GVQVI}$$

Denote by $\overline{X}$ the solution set of (GVQVI).

Let Z_1, Z_2 be two normed spaces. A set-valued map F from Z_1 to 2^{Z_2} is

(i) closed, on $Z_3 \subseteq Z_1$, if for any sequence $\{x_n\} \subseteq Z_3$ with $x_n \to x$ and $y_n \in F(x_n)$ with $y_n \to y$, one has $y \in F(x)$;

(ii) lower semicontinuous (l.s.c. in short) at $x \in Z_1$, if $\{x_n\} \subseteq Z_1$, $x_n \to x$, and $y \in F(x)$ imply that there exists a sequence $\{y_n\} \subseteq Z_2$ satisfying $y_n \to y$ such that $y_n \in F(x_n)$ for n sufficiently large. If F is l.s.c. at each point of Z_1, we say that F is l.s.c. on Z_1;

(iii) upper semicontinuous (u.s.c. in short) at $x \in Z_1$, if for any neighborhood V of $F(x)$, there exists a neighborhood U of x such that $F(x') \subseteq V$, for all $x' \in U$. If F is u.s.c. at each point of Z_1, we say that F is u.s.c. on Z_1.

It is obvious that any u.s.c. nonempty closed-valued map F is closed.

Let (P, d) be a metric space, $P_1 \subset P$, and $x \in P$. We denote by $d_{P_1}(x) = \inf\{d(x, p') : p' \in P_1\}$ the distance from the point x to the set P_1. For a topological vector space V, we denote by V^* its dual space. For any set $\Phi \subset V$, we denote the positive polar cone of Φ by

$$\Phi^* = \{\lambda \in V^* : \lambda(x) \geq 0,\ \forall x \in \Phi\}. \tag{2.1}$$

Let $e \in \operatorname{int} C$ be fixed. Denote

$$C^{*0} = \{\lambda \in C^* : \lambda(e) = 1\}. \tag{2.2}$$

Definition 2.1. (i) A sequence $\{x_n\} \subseteq X_1$ is called a type I Levitin-Polyak (LP in short) approximating solution sequence if there exist $\{\epsilon_n\} \subseteq \mathbf{R}^1_+ = \{r \geq 0 | r \text{ is a real number}\}$ with $\epsilon_n \to 0$ and $z_n \in T(x_n)$ such that

$$d_{X_0}(x_n) \leq \epsilon_n, \tag{2.3}$$

$$x_n \in S(x_n), \tag{2.4}$$

$$\langle z_n, x - x_n \rangle + \epsilon_n e \notin -\operatorname{int} C, \quad \forall x \in S(x_n). \tag{2.5}$$

(ii) $\{x_n\} \subseteq X_1$ is called a type II LP approximating solution sequence if there exist $\{\epsilon_n\} \subseteq \mathbf{R}^1_+$ with $\epsilon_n \to 0$ and $z_n \in T(x_n)$ such that (2.3)–(2.5) hold, and, for any $z \in T(x_n)$, there exists $w(n, z) \in S(x_n)$ satisfying

$$\langle z, w(n, z) - x_n \rangle - \epsilon_n e \in -C. \tag{2.6}$$

(iii) $\{x_n\} \subseteq X_1$ is called a generalized type I LP approximating solution sequence if there exist $\{\epsilon_n\} \subseteq \mathbf{R}^1_+$ with $\epsilon_n \to 0$ and $z_n \in T(x_n)$ such that

$$d_K(g(x_n)) \leq \epsilon_n \tag{2.7}$$

and (2.4), (2.5) hold.

(iv) $\{x_n\} \subseteq X_1$ is called a generalized type II LP approximating solution sequence if there exist $\{\epsilon_n\} \subseteq \mathbf{R}^1_+$ with $\epsilon_n \to 0$, $z_n \in T(x_n)$ such that (2.4), (2.5), and (2.7) hold, and, for any $z \in T(x_n)$, there exists $w(n, z) \in S(x_n)$ such that (2.6) holds.

Definition 2.2. (GVQVI) is said to be type I (resp., type II, generalized type I, generalized type II) LP well-posed if the solution set $\overline{X}$ of (GVQVI) is nonempty, and, for any type I (resp., type II, generalized type I, generalized type II) LP approximating solution sequence $\{x_n\}$, there exists a subsequence $\{x_{n_j}\}$ of $\{x_n\}$ and $\overline{x} \in \overline{X}$ such that $x_{n_j} \to \overline{x}$.

Remark 2.3. (i) It is clear that any (generalized) type II LP approximating solution sequence is a (generalized) type I LP approximating solution sequence. Thus, (generalized) type I LP well-posedness implies (generalized) type II LP well-posedness.

(ii) Each type of LP well-posedness of (GVQVI) implies that the solution set $\overline{X}$ is compact.

(iii) Suppose that g is uniformly continuous functions on a set

$$X_1(\delta_0) = \{x \in X_1 : d_{X_0}(x) \leq \delta_0\}, \tag{2.8}$$

for some $\delta_0 > 0$. Then generalized type I (resp., generalized type II) LP well-posedness of (GVQVI) implies its type I (resp., type II) LP well-posedness.

(iv) If $Y = \mathbf{R}^1$, $C = \mathbf{R}^1_+$, then type I (resp., type II, generalized type I, generalized type II) LP well-posedness of (GVQVI) reduces to type I (resp., type II, generalized type I, generalized type II) LP well-posedness of the generalized quasi-variational inequality problem defined by Jiang et al. [28]. If $Y = \mathbf{R}^1$, $C = \mathbf{R}^1_+$, $S(x) = X_0$ for all $x \in X_1$, then type I (resp., type II, generalized type I, generalized type II) LP well-posedness of (GVQVI) reduces to type I (resp., type II, generalized type I, generalized type II) LP well-posedness of the generalized variational inequality problem defined by Huang, and Yang [27] which contains as special cases for the type I (resp., type II, generalized type I, generalized type II) LP well-posedness of the variational inequality problem in [26].

(v) If $S(x) = X_0$ for all $x \in X_1$, then type I (resp., type II, generalized type I, generalized type II) LP well-posedness of (GVQVI) reduces to type I (resp., type II, generalized type I, generalized type II) LP well-posedness of the generalized vector variational inequality problem defined by Xu et al. [29].

(vi) If the set-valued map T is replaced by a single-valued map F, then type I (resp., type II, generalized type I, generalized type II) LP well-posedness of (GVQVI) reduces to type I (resp., type II, generalized type I, generalized type II) LP well-posedness of the vector quasivariational inequality problems defined by Zhang et al. [30].

Consider the following statement:

$$\begin{aligned}&\Big[\overline{X} \neq \emptyset \text{ and for any type I } \Big(\text{resp., type II, generalized type I, generalized type II}\Big) \\ &\quad \text{LP approximating solution sequence } \{x_n\}, \text{ we have } d_{\overline{X}}(x_n) \longrightarrow 0\Big].\end{aligned} \tag{2.9}$$

Proposition 2.4. *If* (GVQVI) *is type I (resp., type II, generalized type I, generalized type II) LP well-posed, then* (2.9) *holds. Conversely if* (2.9) *holds and* $\overline{X}$ *is compact, then (1) is type I (resp., type II, generalized type I, generalized type II) LP well-posed.*

The proof of Proposition 2.4 is elementary and thus omitted.

To see the various LP well-posednesses of (1) are adaptations of the corresponding LP well-posednesses in minimizing problems by using the Auslender gap function, we consider the following general constrained optimization problem:

$$\begin{aligned} \min \quad & f(x) \\ \text{s.t.} \quad & x \in X_1' \\ & g(x) \in K, \end{aligned} \tag{P}$$

where $X_1' \subseteq X_1$ is nonempty and $\mathrm{f} : X_1 \rightarrow \mathbf{R}^1 \cup \{+\infty\}$ is proper. The feasible set of (P) is X_0', where $X_0' = \{x \in X_1' : g(x) \in K\}$. The optimal set and optimal value of (P) are denoted by $\overline{X'}$ and $\overline{v}$, respectively. Note that if $\mathrm{Dom}(f) \cap X_0' \neq \emptyset$, where

$$\mathrm{Dom}(f) = \{x \in X_1 : f(x) < +\infty\}, \tag{2.10}$$

then $\overline{v} < +\infty$. In this paper, we always assume that $\overline{v} > -\infty$.

Definition 2.5. (i) A sequence $\{x_n\} \subseteq X_1'$ is called a type I LP minimizing sequence for (P) if

$$\limsup_{n \to \infty} f(x_n) \leq \overline{v}, \tag{2.11}$$

$$d_{X_0'}(x_n) \longrightarrow 0. \tag{2.12}$$

(ii) $\{x_n\} \subseteq X_1'$ is called a type II LP minimizing sequence for (P) if

$$\lim_{n \to \infty} f(x_n) = \overline{v} \tag{2.13}$$

and (2.12) hold.

(iii) $\{x_n\} \subseteq X_1'$ is called a generalized type I LP minimizing sequence for (P) if

$$d_K(g(x_n)) \longrightarrow 0 \tag{2.14}$$

and (2.11) hold.

(iv) $\{x_n\} \subseteq X_1'$ is called a generalized type II LP minimizing sequence for (P) if (2.13) and (2.14) hold.

Definition 2.6. (P) is said to be type I (resp., type II, generalized type I, generalized type II) LP well-posed if the solution set $\overline{X}'$ of (P) is nonempty, and for any type I (resp., type II, generalized type I, generalized type II) LP minimizing sequence $\{x_n\}$, there exists a subsequence $\{x_{n_j}\}$ of $\{x_n\}$ and $\overline{x} \in \overline{X}'$ such that $x_{n_j} \rightarrow \overline{x}$.

The Auslender gap function for (GVQVI) is defined as follows:

$$f(x) = \inf_{z \in T(x)} \sup_{x' \in S(x)} \inf_{\lambda \in C^{*0}} \lambda(\langle z, x - x' \rangle), \quad \forall x \in X_1. \tag{2.15}$$

Let $X_2 \subseteq X$ be defined by

$$X_2 = \{x \in X \mid x \in S(x)\}. \tag{2.16}$$

In the rest of this paper, we set X_1' in (P) equal to $X_1 \cap X_2$. Note that if S is closed on X_1, then X_1' is closed.

Recall the following widely used function (see, e.g., [38])

$$\xi : Y \longrightarrow \mathbf{R}^1 : \min\left\{t \in \mathbf{R}^1 : y - te \in -C\right\}. \tag{2.17}$$

It is known that ξ is a continuous, (strictly) monotone (i.e., for any $y_1, y_2 \in Y$, $y_1 - y_2 \in C$ implies that $\xi(y_1) \geq \xi(y_2)$ and $(y_1 - y_2 \in \operatorname{int} C$ implies that $\xi(y_1) > \xi(y_2))$, subadditive and convex function. Moreover, it holds that $\xi(te) = t$, for all $t \in \mathbf{R}^1$ and $\xi(y) = \sup_{\lambda \in C^{*0}} \lambda(y)$, for all $y \in Y$.

Now we given some properties for the function f defined by (2.15).

Lemma 2.7. *Let the function f be defined by* (2.15)*, and let the set-valued map T be compact-valued on X_1. Then*

(i) $f(x) \geq 0$, *for all* $x \in X_1'$;

(ii) *for any* $x \in X_0'$, $f(x) = 0$ *if and only if* $x \in \overline{X}$.

Proof. (i) Let $x \in X_1'$. Suppose to the contrary that $f(x) < 0$. Then, there exists a $\delta > 0$ such that $f(x) < -\delta$. By definition, for $\delta/2 > 0$, there exists a $z \in T(x)$, such that

$$\sup_{x' \in S(x)} \inf_{\lambda \in C^{*0}} \lambda(\langle z, x - x' \rangle) \leq f(x) + \frac{\delta}{2} < -\frac{\delta}{2} < 0. \tag{2.18}$$

Thus, we have

$$\inf_{\lambda \in C^{*0}} \lambda(\langle z, x - x' \rangle) < 0, \quad \forall x' \in S(x), \tag{2.19}$$

which is impossible when $x' = x$. This proves (i).

(ii) Suppose that $x \in X_0'$ such that $f(x) = 0$.

Then, it follows from the definition of X_0' that $x \in S(x)$. And from the definition of $f(x)$ we know that there exist $z_n \in T(x)$ and $0 < \epsilon_n \to 0$ such that

$$\inf_{\lambda \in C^{*0}} \lambda(\langle z_n, x - x' \rangle) \leq f(x) + \epsilon_n = \epsilon_n, \quad \forall x' \in S(x), \tag{2.20}$$

that is,

$$\xi(\langle z_n, x' - x \rangle) \geq -\epsilon_n, \quad \forall x' \in S(x). \tag{2.21}$$

By the compactness of $T(x)$, there exists a sequence $\{z_{n_j}\}$ of $\{z_n\}$ and some $z \in T(x)$ such that

$$z_{n_j} \longrightarrow z. \tag{2.22}$$

This fact, together with the continuity of ξ and (2.21), implies that

$$\xi(\langle z, x' - x \rangle) \geq 0, \quad \forall x' \in S(x). \tag{2.23}$$

It follows that $x \in \overline{X}$.

Conversely, assume that $x \in \overline{X}$. It follows from the definition of $\overline{X}$ that $x \in S(x)$. Suppose to the contrary that $f(x) > 0$. Then, for any $z \in T(x)$,

$$\sup_{x' \in S(x)} \inf_{\lambda \in C^{*0}} \lambda(\langle z, x - x' \rangle) > 0. \tag{2.24}$$

Thus, there exist $\delta > 0$ and $x_0 \in S(x)$ such that

$$\inf_{\lambda \in C^{*0}} \lambda(\langle z, x - x_0 \rangle) \geq \delta. \tag{2.25}$$

It follows that

$$\xi(\langle z, x_0 - x \rangle) \leq -\delta < 0. \tag{2.26}$$

As a result, we have

$$\langle z, x_0 - x \rangle \in -\operatorname{int} C. \tag{2.27}$$

This contradicts the fact that $x \in \overline{X}$. So, $f(x) = 0$. This completes the proof. □

Lemma 2.8. *Let f be defined by* (2.15)*. Assume that the set-valued map T is compact-valued and u.s.c. on X_1 and the set-valued map S is l.s.c. on X_1. Then f is l.s.c. function from X_1 to $\mathbf{R}^1 \cup \{+\infty\}$. Further assume that the solution set $\overline{X}$ of* (GVQVI) *is nonempty, then* $\mathrm{Dom}(f) \neq \emptyset$*.*

Proof. First we show that $f(x) > -\infty$, for all $x \in X_1$. Suppose to the contrary that there exists $x_0 \in X_1$ such that $f(x_0) = -\infty$. Then, there exist $z_n \in T(x_0)$ and $\{M_n\} \subset R^1_+$ with $M_n \to +\infty$ such that

$$\sup_{x' \in S(x_0)} \inf_{\lambda \in C^{*0}} \lambda(\langle z_n, x_0 - x' \rangle) \leq -M_n. \tag{2.28}$$

Thus,

$$\xi(\langle z_n, x' - x_0 \rangle) \geq M_n, \quad \forall x' \in S(x_0). \tag{2.29}$$

By the compactness of $T(x_0)$, there exist a sequence $\{z_{n_j}\} \subset \{z_n\}$ and some $z_0 \in T(x_0)$ such that $z_{n_j} \to z_0$. This fact, together with (2.29) and the continuity of ξ on Y, implies that

$$\xi(\langle z_0, x' - x_0 \rangle) \geq +\infty, \quad \forall x' \in S(x_0) \tag{2.30}$$

which is impossible, since ξ is a finite function on Y.

Second, we show that f is l.s.c. on X_1. Let $a \in \mathbf{R}^1$. Suppose that $\{x_n\} \subset X_1$ satisfies $f(x_n) \leq a$, for all n, and $x_n \to x_0 \in X_1$. It follows that, for each n, there exist $z_n \in T(x_n)$ and $0 < \delta_n \to 0$ such that

$$-\xi(\langle z_n, y - x_n \rangle) \leq a + \delta_n, \quad \forall y \in S(x_n). \tag{2.31}$$

For any $x' \in S(x_0)$, by the l.s.c. of S, we have a sequence $\{y_n\}$ with $\{y_n\} \in S(x_n)$ converging to x' such that

$$-\xi(\langle z_n, y_n - x_n\rangle) \leq a + \delta_n. \tag{2.32}$$

By the u.s.c. of T at x_0 and the compactness of $T(x_0)$, we obtain a subsequence $\{z_{n_j}\}$ of $\{z_n\}$ and some $z_0 \in T(x_0)$ such that $z_{n_j} \to z_0$. Taking the limit in (2.32) (with n replaced by n_j), by the continuity of ξ, we have

$$-\xi(\langle z_0, x' - x_0\rangle) \leq a, \quad \forall x' \in S(x_0). \tag{2.33}$$

It follows that $f(x_0) = \inf_{z\in T(x_0)} \sup_{x'\in S(x_0)} -\xi(\langle z, x' - x_0\rangle) \leq a$. Hence, f is l.s.c. on X_1. Furthermore, if $\overline{X} \neq \emptyset$, by Lemma 2.7, we see that $\mathrm{Dom}(f) \neq \emptyset$. □

Lemma 2.9. *Let the function f be defined by* (2.15)*, and let the set-valued map T be compact-valued on X_1. Then,*

(i) $\{x_n\} \subseteq X_1$ *is a sequence such that there exist* $\{\epsilon_n\} \subseteq \mathbf{R}^1_+$ *with* $\epsilon_n \to 0$ *and* $z_n \in T(x_n)$ *satisfying* (2.4) *and* (2.5) *if and only if* $\{x_n\} \subseteq X'_1$ *and* (2.11) *hold with* $\overline{v} = 0$,

(ii) $\{x_n\} \subseteq X_1$ *is a sequence such that there exist* $\{\epsilon_n\} \subseteq \mathbf{R}^1_+$ *with* $\epsilon_n \to 0$ *and* $z_n \in T(x_n)$ *satisfying* (2.4) *and* (2.5)*, and for any* $z \in T(x_n)$*, there exists* $w(n,z) \in S(x_n)$ *satisfying* (2.6) *if and only if* $\{x_n\} \subseteq X'_1$ *and* (2.13) *hold with* $\overline{v} = 0$.

Proof. (i) Let $\{x_n\} \subseteq X_1$ be any sequence if there exist $\{\epsilon_n\} \subseteq \mathbf{R}^1_+$ with $\epsilon_n \to 0$ and $z_n \in T(x_n)$ satisfying (2.4) and (2.5), then we can easily verify that

$$\{x_n\} \subseteq X'_1, \quad f(x_n) \leq \epsilon_n. \tag{2.34}$$

It follows that (2.11) holds with $\overline{v} = 0$.

For the converse, let $\{x_n\} \subseteq X'_1$ and (2.11) hold with $\overline{v} = 0$. We can see that $\{x_n\} \subseteq X_1$ and (2.4) hold. Furthermore, by (2.11), we have that there exists $\{\epsilon_n\} \subseteq \mathbf{R}^1_+$ with $\epsilon_n \to 0$ such that $f(x_n) \leq \epsilon_n$. By the compactness of $T(x_n)$, we see that for every n there exists $z_n \in T(x_n)$ such that

$$\xi(\langle z_n, x' - x_n\rangle) \geq -\epsilon_n, \quad \forall x' \in S(x_n). \tag{2.35}$$

It follows that for every n there exists $z_n \in T(x_n)$ such that (2.5) holds.

(ii) Let $\{x_n\} \subseteq X_1$ be any sequence we can verify that

$$\liminf_{n\to\infty} f(x_n) \geq 0 \tag{2.36}$$

holds if and only if there exists $\{\alpha_n\} \subseteq \mathbf{R}^1_+$ with $\alpha_n \to 0$ and, for any $z \in T(x_n)$, there exists $w(n,z) \in S(x_n)$ such that

$$\langle z, w(n,z) - x_n\rangle - \alpha_n e \in -C. \tag{2.37}$$

From the proof of (i), we know that $\limsup_{n\to\infty} f(x_n) \le 0$ and $\{x_n\} \subseteq X_1'$ hold if and only if $\{x_n\} \subseteq X_1$ such that there exist $\{\beta_n\} \subseteq \mathbf{R}_+^1$ with $\beta_n \to 0 z_n \in T(x_n)$ satisfying (2.4) and (2.5) (with ϵ_n replaced by β_n). Finally, we let $\epsilon_n = \max\{\alpha_n, \beta_n\}$ and the conclusion follows. □

Proposition 2.10. *Assume that $\overline{X} \neq \emptyset$ and T is compact-valued on X_1. Then*

(i) (GVQVI) *is generalized type I (resp., generalized type II) LP well-posed if and only if* (P) *is generalized type I (resp., generalized type II) LP well-posed with $f(x)$ defined by* (2.15).

(ii) *If* (GVQVI) *is type I (resp., type II) LP well-posed, then* (P) *is type I (resp., type II) LP well-posed with $f(x)$ defined by* (2.15).

Proof. Let $f(x)$ be defined by (2.15). Since $\overline{X} \neq \emptyset$, it follows from Lemma 2.7 that $\overline{x} \in \overline{X}$ is a solution of (GVQVI) if and only if $\overline{x}$ is an optimal solution of (5) with $\overline{v} = f(\overline{x}) = 0$.

(i) Similar to the proof of Lemma 2.9, it is also routine to check that a sequence $\{x_n\}$ is a generalized type I (resp., generalized type II) LP approximating solution sequence if and only if it is a generalized type I (resp., generalized type II) LP minimizing sequence of (P). So (GVQVI) is generalized type I (resp., generalized type II) LP well-posed if and only if (P) is generalized type I (resp., generalized type II) LP well-posed with $f(x)$ defined by (2.15).

(ii) Since $X_0' \subseteq X_0$, $d_{X_0}(x) \le d_{X_0'}(x)$ for any x. This fact together with Lemma 2.9 implies that a type I (resp., type II) LP minimizing sequence of (P) is a type I (resp., type II) LP approximating solution sequence. So the type I (resp., type II) LP well-posedness of (GVQVI) implies the type I (resp., type II) LP well-posedness of (P) with $f(x)$ defined by (2.15). □

3. Criteria and Characterizations for Generalized LP Well-Posedness of (GVQVI)

In this section, we shall present some necessary and/or sufficient conditions for the various types of (generalized) LP well-posedness of (GVQVI) defined in Section 2.

Now consider a real-valued function $c = c(t, s, r)$ defined for $t, s, r \ge 0$ sufficiently small, such that

$$\begin{gathered} c(t,s,r) \ge 0, \quad \forall t,s,r, \quad c(0,0,0) = 0, \\ s_n \longrightarrow 0, \quad t_n \ge 0, \quad r_n = 0, \quad c(t_n, s_n, r_n) \longrightarrow 0 \text{ imply that } t_n \longrightarrow 0, \end{gathered} \tag{3.1}$$

Theorem 3.1. *Let the set-valued map T be compact-valued on X_1. If* (GVQVI) *is type II LP well-posed, the set-valued map S is closed-valued, then there exist a function c satisfying* (3.1) *such that*

$$|f(x)| \ge c\left(d_{\overline{X}}(x), d_{X_0}(x), d_{S(x)}(x)\right), \quad \forall x \in X_1, \tag{3.2}$$

where $f(x)$ is defined by (2.15). *Conversely, suppose that $\overline{X}$ is nonempty and compact and* (3.2) *holds for some c satisfying* (3.1). *Then* (GVQVI) *is type II LP well-posed.*

Proof. Define

$$c(t,s,r) = \inf\{|f(x)| : x \in X_1, d_{\overline{X}}(x) = t, d_{X_0}(x) = s, d_{S(x)}(x) = r\}. \tag{3.3}$$

Since $\overline{X} \neq \emptyset$, it is obvious that $c(0,0,0) = 0$. Moreover, if $s_n \to 0, t_n \geq 0, r_n = 0$, and $c(t_n, s_n, r_n) \to 0$, then there exists a sequence $\{x_n\} \subseteq X_1$ with $d_{\overline{X}}(x_n) = t_n$, $d_{S(x_n)}(x_n) = r_n = 0$,

$$d_{X_0}(x_n) = s_n \longrightarrow 0, \tag{3.4}$$

such that

$$|f(x_n)| \longrightarrow 0. \tag{3.5}$$

Since S is closed-valued, $x_n \in S(x_n)$ for any n. This fact, combined with (3.4) and (3.5) and Lemma 2.9 (ii) implies that $\{x_n\}$ is a type II LP approximating solution sequence of (GVQVI). By Proposition 2.4, we have that $t_n \to 0$.

Conversely, let $\{x_n\}$ be a type II LP approximating solution sequence of (GVQVI). Then, by (3.2), we have

$$|f(x_n)| \geq c(d_{\overline{X}}(x_n), d_{X_0}(x_n), d_{S(x_n)}(x_n)). \tag{3.6}$$

Let

$$t_n = d_{\overline{X}}(x_n), \quad s_n = d_{X_0}(x_n), \quad r_n = d_{S(x_n)}(x_n). \tag{3.7}$$

Then $s_n \to 0$ and $r_n = 0$, for all $n \in N$. Moreover, by Lemma 2.9, we have that $|f(x)| \to 0$. Then, $c(t_n, s_n, r_n) \to 0$. These facts together with the properties of the function c imply that $t_n \to 0$. By Proposition 2.4, we see that (GVQVI) is type II LP well-posed. □

Theorem 3.2. *Let the set-valued map T be compact-valued on X_1. If* (GVQVI) *is generalized type II LP well-posed, the set-valued map S is closed, then there exist a function c satisfying* (3.1) *such that*

$$|f(x)| \geq c(d_{\overline{X}}(x), d_K(g(x)), d_{S(x)}(x)), \quad \forall x \in X_1, \tag{3.8}$$

where $f(x)$ is defined by (2.15). *Conversely, suppose that $\overline{X}$ is nonempty and compact and* (3.8) *holds for some c satisfying* (3.4) *and* (3.5). *Then,* (GVQVI) *is generalized type II LP well-posed.*

Proof. The proof is almost the same as that of Theorem 3.1. The only difference lies in the proof of the first part of Theorem 3.1. Here we define

$$c(t,s,r) = \inf\{|f(x)| : x \in X_1, d_{\overline{X}}(x) = t, d_K(g(x)) = s, d_{S(x)}(x) = r\}. \tag{3.9}$$

Next we give the Furi-Vignoli-type characterizations [39] for the (generalized) type I LP well-posedness of (GVQVI).

Let $(X, \|\cdot\|)$ be a Banach space. Recall that the Kuratowski measure of noncompactness for a subset H of X is defined as

$$\mu(H) = \inf\left\{\epsilon > 0 : H \subseteq \bigcup H_i, \operatorname{diam}(H_i) < \epsilon, i = 1, \ldots, n\right\}, \tag{3.10}$$

where $\operatorname{diam}(H_i)$ is the diameter of H_i defined by

$$\operatorname{diam}(H_i) = \sup\{\|x_1 - x_2\| : x_1, x_2 \in H_i\}. \tag{3.11}$$

Given two nonempty subsets A and B of a Banach space $(X, \|\cdot\|)$, the Hausdorff distance between A and B is defined by

$$h(A, B) = \max\{\sup\{d_B(a) : a \in A\}, \sup\{d_A(b) : b \in B\}\}. \tag{3.12}$$

For any $\epsilon \geq 0$, two types of approximating solution sets for (GVQVI) are defined, respectively, by

$$\begin{aligned}\Omega_1(\epsilon) &= \left\{x \in X_1 : x \in S(x), d_{X_0}(x) \leq \epsilon, \exists z \in T(x), \text{s.t.}\langle z, x' - x\rangle + \epsilon e \notin -\operatorname{int} C,\ \forall x' \in S(x)\right\},\\ \Omega_2(\epsilon) &= \left\{x \in X_1 : x \in S(x), d_K(g(x)) \leq \epsilon, \exists z \in T(x), \text{s.t.}\langle z, x' - x\rangle + \epsilon e \notin -\operatorname{int} C,\ \forall x' \in S(x)\right\}.\end{aligned} \tag{3.13}$$

□

Theorem 3.3. *Assume that T is u.s.c. and compact-valued on $X_1$and S is l.s.c. and closed on X_1. Then*

(a) (GVQVI) *is type I LP well-posed if and only if*

$$\lim_{\epsilon \to 0} \mu(\Omega_1(\epsilon)) = 0, \tag{3.14}$$

(b) (GVQVI) *is generalized type I LP well-posed if and only if*

$$\lim_{\epsilon \to 0} \mu(\Omega_2(\epsilon)) = 0. \tag{3.15}$$

Proof. (a) First we show that, for every $\epsilon > 0$, $\Omega_1(\epsilon)$ is closed. In fact, let $x_n \in \Omega_1(\epsilon)$ and $x_n \to x_0$. Then (2.4) and the following formula hold:

$$\begin{gathered} d_{X_0}(x_n) \leq \epsilon,\\ \exists z_n \in T(x_n), \quad \text{s.t.}\langle z_n, x' - x_n\rangle + \epsilon e \notin -\operatorname{int} C, \quad \forall x' \in S(x_n). \end{gathered} \tag{3.16}$$

Since $x_n \to x_0$, by the closedness of S and (2.4), we have $x_0 \in S(x_0)$. From (3.16), we get

$$d_{X_0}(x_0) \leq \epsilon, \tag{3.17}$$

$$\exists z_n \in T(x_n), \quad \text{s.t. } \xi(\langle z_n, x' - x_n\rangle) \geq -\epsilon, \quad \forall x' \in S(x_n). \tag{3.18}$$

For any $v \in S(x_0)$, by the lower semi-continuity of S and (3.18), we can find $v_n \in S(x_n)$ with $v_n \to v$ such that

$$\xi(\langle z_n, v_n - x_n \rangle) \geq -\epsilon. \tag{3.19}$$

By the u.s.c. of T at x_0 and the compactness of $T(x_0)$, there exist a subsequence $\{z_{n_j}\} \subset \{z_n\}$ and some $z_0 \in T(x_0)$ such that

$$z_{n_j} \longrightarrow z_0. \tag{3.20}$$

This fact, together with the continuity of ξ and (3.19), implies that

$$\xi(\langle z_0, v - x_0 \rangle) \geq -\epsilon \quad \forall v \in S(x_0). \tag{3.21}$$

It follows that

$$\langle z_0, v - x_0 \rangle + \epsilon e \notin -\operatorname{int} C \quad \forall v \in S(x_0). \tag{3.22}$$

Hence, $x_0 \in \Omega_1(\epsilon)$.

Second, we show that $\overline{X} = \bigcap_{\epsilon>0} \Omega_1(\epsilon)$. It is obvious that $\overline{X} \subseteq \bigcap_{\epsilon>0} \Omega_1(\epsilon)$. Now suppose that $\epsilon_n > 0$ with $\epsilon_n \to 0$ and $x^* \in \bigcap_{\epsilon>0} \Omega_1(\epsilon_n)$. Then

$$d_{X_0}(x^*) \leq \epsilon_n, \quad \forall n, \tag{3.23}$$

$$x^* \in S(x^*), \tag{3.24}$$

$$\exists z \in T(x^*), \quad \text{s.t. } \langle z, x' - x^* \rangle + \epsilon_n e \notin -\operatorname{int} C, \quad \forall x' \in S(x^*). \tag{3.25}$$

From (3.23), we have

$$x^* \in X_0. \tag{3.26}$$

From (3.25), we have

$$\langle z, x' - x^* \rangle \notin -\operatorname{int} C, \quad \forall x' \in S(x^*), \tag{3.27}$$

that is $x^* \in \overline{X}$. Hence, $\overline{X} = \bigcap_{\epsilon>0} \Omega_1(\epsilon)$.

Now we assume that (GVQVI) is type I LP well-posed. By Remark 2.3, we know that the solution $\overline{X}$ is nonempty and compact. For every positive real number ϵ, since $\overline{X} \in \Omega_1(\epsilon)$, one gets

$$\Omega_1(\epsilon) \neq \emptyset, \quad h\left(\Omega_1(\epsilon), \overline{X}\right) = \max\left\{ \sup_{u \in \Omega_1(\epsilon)} d_{\overline{X}}(u), \sup_{v \in \overline{X}} d_{\Omega_1(\epsilon)}(v) \right\} = \sup_{u \in \Omega_1(\epsilon)} d_{\overline{X}}(u). \tag{3.28}$$

For every $n \in N$, the following relations hold:

$$\mu(\Omega_1(\epsilon)) \le 2h\left(\Omega_1(\epsilon), \overline{X}\right) + \mu\left(\overline{X}\right) = 2h\left(\Omega_1(\epsilon), \overline{X}\right), \tag{3.29}$$

where $\mu(\overline{X}) = 0$ since $\overline{X}$ is compact. Hence, in order to prove that $\lim_{\epsilon\to 0}\mu(\Omega_1(\epsilon)) = 0$, we only need to prove that

$$\lim_{\epsilon\to 0} h\left(\Omega_1(\epsilon), \overline{X}\right) = \lim_{\epsilon\to 0} \sup_{u\in\Omega_1(\epsilon)} d_{\overline{X}}(u) = 0. \tag{3.30}$$

Suppose that this is not true, then there exist $\beta > 0$, $\epsilon_n \to 0$, and sequence $\{u_n\}$, $u_n \in \Omega_1(\epsilon_n)$, such that

$$d_{\overline{X}}(u_n) > \beta, \tag{3.31}$$

for n sufficiently large.

Since $\{u_n\}$ is type I LP approximating sequence for (GVQVI), it contains a subsequence $\{u_{n_k}\}$ conversing to a point of $\overline{X}$, which contradicts (3.31).

For the converse, we know that, for every $\epsilon > 0$, the set $\Omega_1(\epsilon)$ is closed, $\overline{X} = \bigcap_{\epsilon>0}\Omega_1(\epsilon)$, and $\lim_{\epsilon\to 0}\mu(\Omega_1(\epsilon)) = 0$. The theorem on Page. 412 in [40, 41] can be applied, and one concludes that the set $\overline{X}$ is nonempty, compact, and

$$\lim_{\epsilon\to 0} h\left(\Omega_1(\epsilon), \overline{X}\right) = 0. \tag{3.32}$$

If $\{x_n\}$ is type I LP approximating sequence for (GVQVI), then there exists a sequence $\{\epsilon_n\}$ of positive real numbers decreasing to 0 such that $x_n \in \Omega_1(\epsilon_n)$, for every $n \in N$. Since $\overline{X}$ is compact and

$$\lim_{n\to+\infty} d_{\overline{X}}(x_n) \le \lim_{n\to+\infty} h\left(\Omega_1(\epsilon_n), \overline{X}\right) = 0, \tag{3.33}$$

by Proposition 2.4, (GVQVI) is type I LP well-posed.

(b) The proof is Similar to that of (a), and it is omitted here. This completes the proof. □

Definition 3.4. (i) Let Z be a topological space, and let $Z_1 \subseteq Z$ be nonempty. Suppose that $h : Z \to R^1 \cup \{+\infty\}$ is an extended real-valued function. h is said to be level-compact on Z_1 if, for any $s \in \mathbf{R}^1$, the subset $\{z \in Z_1 : h(z) \le s\}$ is compact.

(ii) Let X be a finite-dimensional normed space, and let $Z_1 \subset Z$ be nonempty. A function $h : Z \to \mathbf{R}^1 \cup \{+\infty\}$ is said to be level-bounded on Z_1 if Z_1 is bounded or

$$\lim_{z\in Z_1, \|z\|\to+\infty} h(z) = +\infty. \tag{3.34}$$

Now we establish some sufficient conditions for type I (resp., generalized I type) LP well-posedness of (GVQVI).

Proposition 3.5. *Suppose that the solution set* $\overline{X}$ *of* (GVQVI) *is nonempty and set-valued map* S *is l.s.c. and closed on* X_1*, the set-valued map* T *is u.s.c. and compact-valued on* X_1*. Suppose that one of the following conditions holds:*

(i) *there exists* $0 < \delta_1 \leq \delta_0$ *such that* $X_1(\delta_1)$ *is compact, where*

$$X_1(\delta_1) = \{x \in X_1 \cap X_2 : d_{X_0}(x) \leq \delta_1\}; \tag{3.35}$$

(ii) *the function* f *defined by* (2.15) *is level-compact on* $X_1 \cap X_2$*;*

(iii) X *is finite-dimensional and*

$$\lim_{x \in X_1 \cap X_2, \|x\| \to +\infty} \max\{f(x), d_{X_0}(x)\} = +\infty, \tag{3.36}$$

where f *is defined by* (2.15)*;*

(iv) *there exists* $0 < \delta_1 \leq \delta_0$ *such that* f *is level-compact on* $X_1(\delta_1)$ *defined by* (3.35)*. Then* (GVQVI) *is type I LP well-posed.*

Proof. First, we show that each of (i), (ii), and (iii) implies (iv). Clearly, either of (i) and (ii) implies (iv). Now we show that (iii) implies (iv). Indeed, we need only to show that, for any $t \in \mathbf{R}^1$, the set

$$A = \{x \in X_1(\delta_1) : f(x) \leq t\} \tag{3.37}$$

is bounded since X is finite-dimensional space and the function f defined by (2.15) is l.s.c. on X_1 and thus A is closed. Suppose to the contrary that there exists $t \in \mathbf{R}^1$ and $\{x'_n\} \subseteq X_1(\delta_1)$ such that $\|x'_n\| \to +\infty$ and $f(x'_n) \leq t$. From $\{x'_n\} \subseteq X_1(\delta_1)$, we have $d_{X_0}(x'_n) \leq \delta_1$.

Thus,

$$\max\{f(x'_n), d_{X_0}(x'_n)\} \leq \max\{t, \delta_1\}, \tag{3.38}$$

which contradicts (3.36).

Therefore, we only need to we show that if (iv) holds, then (GVQVI) is type I LP well-posed. Let $\{x_n\}$ be a type I LP approximating solution sequence for (GVQVI). Then, there exist $\{\epsilon_n\} \subseteq \mathbf{R}^1_+$ with $\epsilon_n \to 0$ and $z_n \in T(x_n)$ such that (2.3), (2.4), and (2.5) hold. From (2.3) and (2.4), we can assume without loss of generality that $\{x_n\} \subseteq X_1(\delta_1)$. By Lemma 2.9, we can assume without loss of generality that $\{x_n\} \subseteq \{x \in X_1(\delta_1) : f(x) \leq 1\}$. By the level-compactness of f on $X_1(\delta_1)$, we can find a subsequence $\{x_{n_j}\}$ of $\{x_n\}$ and $\overline{x} \in X_1(\delta_1)$ such that $x_{n_j} \to \overline{x}$. Taking the limit in (2.3) (with x_n replaced by x_{n_j}), we have $\overline{x} \in X_0$. Since S is closed and (2.4) holds, we also have $\overline{x} \in S(\overline{x})$.

Furthermore, from the u.s.c. of T at $\overline{x}$ and the compactness of $T(\overline{x})$, we deduce that there exist a subsequence $\{z_{n_j}\}$ of $\{z_n\}$ and some $\overline{z} \in T(\overline{x})$ such that $z_{n_j} \to \overline{z}$. From this fact, together with (2.5), we have

$$\langle \overline{z}, x' - \overline{x} \rangle \notin -\operatorname{int} C, \quad \forall x' \in S(\overline{x}). \tag{3.39}$$

Thus, $\overline{x} \in \overline{X}$.

The next proposition can be proved similarly. □

Proposition 3.6. *Suppose that the solution set* $\overline{X}$ *of* (GVQVI) *is nonempty and set-valued map S is l.s.c. and closed on* X_1*, the set-valued map T is u.s.c. and compact-valued on* X_1*. Suppose that one of the following conditions holds:*

(i) *there exists* $0 < \delta_1 \leq \delta_0$ *such that* $X_2(\delta_1)$ *is compact, where*

$$X_2(\delta_1) = \{x \in X_1 \cap X_2 : d_K(g(x)) \leq \delta_1\}; \tag{3.40}$$

(ii) *the function f defined by* (2.15) *is level-compact on* $X_1 \cap X_2$*;*

(iii) *X is finite-dimension and*

$$\lim_{x \in X_1 \cap X_2, \|x\| \to +\infty} \max\{f(x), d_K g(x)\} = +\infty, \tag{3.41}$$

where f is defined by (2.15)*,*

(iv) *there exists* $0 < \delta_1 \leq \delta_0$ *such that f is level-compact on* $X_2(\delta_1)$ *defined by* (3.40)*. Then* (GVQVI) *is generalized type II LP well-posed.*

Remark 3.7. If X is finite-dimensional, then the "level-compactness" condition in Propositions 3.1 and 3.6 can be replaced by "level boundedness" condition.

Remark 3.8. It is easy to see that the results in this paper unify, generalize and extend the main results in [26–30] and the references therein.

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