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Research Article

Mixed Boundary-Value Analysis of Rocking Vibrations of an Elastic Strip Foundation on Elastic Soil with Saturated Substrata

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The dynamic response of an elastic strip foundation lying on elastic soil with saturated substrata is greatly affected by pore pressure induced by a rocking moment. In this paper, we explore the mixed boundary-value problem of the rocking vibration of an elastic strip foundation on elastic soil with saturated substrata via Biot dynamic equations. First, the wave equations concerning both the single-phase elastic layer and the saturated half-space are solved using a Fourier integral transform technique. The dual integral equations of the rocking vibration of an elastic strip foundation are established according to the mixed boundary conditions. Finally, the relationship of the dynamic compliance coefficient with the dimensionless frequency is obtained by applying Simpson's rule to conduct numerical calculation. We also analyse the influences of the elastic layer's thickness and elastic characteristic parameters of the foundation on the rocking vibration.

1. Introduction

Dynamic interaction between structural foundations and the underlying soil, both in theory and in practice, is widely studied in the field of geotechnical engineering and has important implications in power machine design and fundamental analysis of foundations under seismic loads. To simplify the boundary-value problem, in the early theoretical studies, the soil under the forced vibration of the foundation was often assumed to be a single-phase linear elastic medium [1–6].

However, soil is generally a two-phase material consisting of a solid skeleton and pores, which are filled with fluid. Such materials are commonly known as poroelastic materials in mechanics literature. After Biot established a theory of propagation of elastic waves in a fluid-saturated porous solid [7, 8] in 1956, the research significance on vibration characteristics of foundation on saturated soil became apparent. Lin [9] studied the vertical and rocking vibrations of an elastic circular plate lying on a single-phase viscoelastic medium. Iguchi and Luco [10] studied the dynamic response of a massless flexible circular plate supported on a layered viscoelastic half-space, obtaining the vertical and rocking impedance of the flexible plate and the numerical solution of contact stress beneath the plate. Halpern and Christiano [11, 12] evaluated compliance functions for the harmonic rocking and vertical motions of rigid permeable and impermeable plates bearing on a poroelastic half-space. Kassir and Xu [13] studied the mixed boundary-value problem of the vibration of a rigid strip foundation on a fluid-saturated porous half-space. Jin and Liu [14, 15] analysed the dynamic response of a rigid disk on a saturated half-space subjected to harmonic horizontal and rocking excitation. Li [16] studied the vertical vibration of a rigid strip foundation on saturated soil. Finally, a parametric study by Ma et al. [17] examined the influences of dimensionless frequency, dynamic permeability, and Poisson's ratio on saturated soil under a rocking rigid strip footing.

Most of the results reported previously concern the dynamic interaction between the rigid structural foundation and the underlying saturated half-space. As research advances in this field of mechanics, a more realistically analytical model becomes increasingly necessary. In fact, the soil of the earth's surface, because of differences in structure and sedimentation, usually has an apparent stratification, formed naturally over the course of history. During foundation construction, underlying soil is routinely reinforced via a variety of methods, inevitably leading to some degree of soil stratification. In practice, the underlying soil will have different physical properties (porosity, permeability, etc.), which have a layered distribution in depth. In researching dynamic interactions of soil and a structural foundation, considering the underlying soil as a homogeneous elastic or saturated medium is not sufficiently accurate. Taking into account the presence of groundwater, the soil below the groundwater level should be considered as saturated soil and the soil above the groundwater level may be regarded as an ideal, single-phase elastic layer. As for the structural foundation, assuming it to be an elastic body is more accurate than assuming it to be a rigid body.

Based on the Biot theory of elastic waves in fluid-saturated porous medium, Philippacopoulos [18] studied the vertical vibration of a rigid circular disk resting on a saturated layered half-space. Bougacha et al. [19, 20] analysed the dynamic stiffness coefficients of rigid strip and circular foundations on a saturated layered half-space using spatially semi-discrete finite element technology. Rajapakse and Senjuntichai [21] presented an exact stiffness matrix method to evaluate the dynamic response of a multilayered poroelastic medium due to time-harmonic loads and fluid sources applied in the interior of the layered medium. Yang et al. [22] neglected the fluid inertia force exerted on the soil skeleton as proposed in the works of Zienkiewicz et al. [23] and studied the steady state response of an elastic soil layer and a saturated layered half-space. Chen [24] explored the characteristics of vertical vibration of both rigid and elastic circular plates on elastic soils with saturated substrata, utilising the Hankel transform to solve the wave equations. Furthermore, the torsional and rocking vibration characteristics of a rigid circular plate on elastic soil with saturated substrata were studied by Wang [25] and Fu [26], respectively, and the effects of the thickness of the elastic layer and the vibration frequency on the plate's dynamics were analysed. The previously listed literature reviews do not present a study of the dynamics between a vibrating elastic strip foundation and elastic soil with saturated substrata.

In this paper, a novel study is presented to make up a deficiency. The wave equations concerning both the single-phase elastic layer and the saturated half-space are solved using a Fourier integral transform technique. Then, the dual integral equations of the rocking vibration of an elastic strip foundation are established according to mixed boundary conditions. The dynamic compliance coefficient's variation curve with the dimensionless frequency is obtained by applying Simpson's rule to conduct numerical calculation, and the effects of the elastic layer's thickness and the elastic characteristic parameters of the foundation on the rocking vibration are analysed.

2. The Dynamic Equations and Their Solutions

Soil and water weight are ignored; the soil is considered to be isotropic and the water incompressible. This paper studies the plane strain problem for an infinite-length strip with a footing

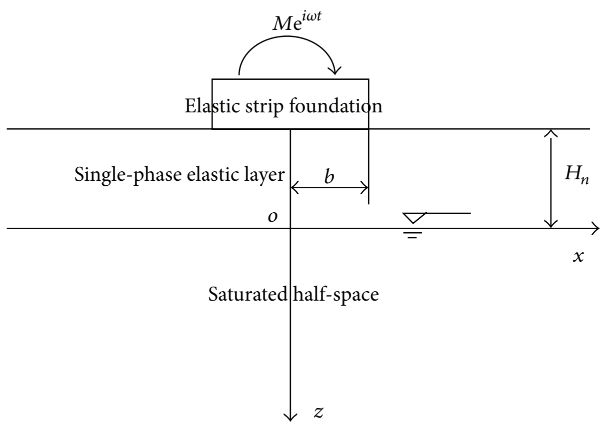


FIGURE 1: Description of the model and coordinate system.

of width $2b$ and an elastic layer thickness of H_n. The centre of the footing is subjected to a harmonic moment force, $Me^{i\omega t}$, with ω denoting circular frequency. The horizontal direction is established as the x-axis and the vertical direction as the z-axis, and the origin of the coordinate is placed at the interface of the elastic soil and the saturated soil. The model is shown in Figure 1.

2.1. The Dynamic Equations of Elastic Layer under Plane Strain. The wave equations of the elastic layer are

$$\frac{\partial \sigma_{xL}}{\partial x} + \frac{\partial \tau_{xzL}}{\partial z} + \rho_L \frac{\partial^2 u}{\partial t^2} = 0, \tag{1a}$$

$$\frac{\partial \tau_{xzL}}{\partial x} + \frac{\partial \sigma_{zL}}{\partial z} + \rho_L \frac{\partial^2 w}{\partial t^2} = 0, \tag{1b}$$

where u and w are the horizontal and vertical displacements of the soil skeleton, respectively; σ_{xL} and σ_{zL} are the horizontal and vertical normal stresses, respectively; and ρ_L is single-phase elastic soil density.

The relationship between the stress and the displacement is

$$\sigma_{xL} = -\frac{2G_L(1-\mu_L)}{1-2\mu_L}\frac{\partial u}{\partial x} - \frac{2G_L\mu_L}{1-2\mu_L}\frac{\partial w}{\partial z}, \tag{2a}$$

$$\sigma_{zL} = -\frac{2G_L\mu_L}{1-2\mu_L}\frac{\partial u}{\partial x} - \frac{2G_L(1-\mu_L)}{1-2\mu_L}\frac{\partial w}{\partial z}, \tag{2b}$$

$$\tau_{xzL} = -G_L\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right), \tag{2c}$$

where G_L and μ_L are the shear modulus and Poisson's ratio of single-phase elastic soil, respectively.

2.2. The Dynamic Equations of Saturated Half-Space under Plane Strain. The basic dynamic equations of saturated half-space are:

$$\frac{\partial \sigma'_x}{\partial x}+\frac{\partial \tau_{xz}}{\partial z}+\frac{\partial p_f}{\partial x}+\rho\frac{\partial^2 u}{\partial t^2}+\rho_f\frac{\partial^2 w_x}{\partial t^2}=0, \quad (3a)$$

$$\frac{\partial \tau_{xz}}{\partial x}+\frac{\partial \sigma'_z}{\partial z}+\frac{\partial p_f}{\partial z}+\rho\frac{\partial^2 w}{\partial t^2}+\rho_f\frac{\partial^2 w_z}{\partial t^2}=0, \quad (3b)$$

$$-\frac{\partial p_f}{\partial x}=\frac{\rho_f g}{k_d}\frac{\partial w_x}{\partial t}+\rho_f\frac{\partial^2 u}{\partial t^2}+\frac{\rho_f}{n}\frac{\partial^2 w_x}{\partial t^2}, \quad (3c)$$

$$-\frac{\partial p_f}{\partial z}=\frac{\rho_f g}{k_d}\frac{\partial w_z}{\partial t}+\rho_f\frac{\partial^2 w}{\partial t^2}+\frac{\rho_f}{n}\frac{\partial^2 w_z}{\partial t^2}, \quad (3d)$$

$$-\frac{\partial^2 w_x}{\partial x\,\partial t}-\frac{\partial^2 w_z}{\partial z\,\partial t}=\frac{\partial^2 u}{\partial x\,\partial t}+\frac{\partial^2 w}{\partial z\,\partial t}, \quad (3e)$$

where u and w are the horizontal and vertical displacements of the soil skeleton, respectively; w_x and w_z are the horizontal and vertical displacements of water relative to the soil skeleton; σ'_x and σ'_z are the horizontal and vertical effective normal stresses, respectively; p_f is the excess pore pressure; ρ is the mass density of the saturated soil with $\rho=(1-n)\rho_s+n\rho_f$; ρ_s and ρ_f are the densities of the soil and water, respectively; and n is the porosity of the saturated soil.

The equations of stress and displacement are

$$\partial\sigma'_x=-\frac{2G(1-\mu)}{1-2\mu}\frac{\partial u}{\partial x}-\frac{2G\mu}{1-2\mu}\frac{\partial w}{\partial z}, \quad (4a)$$

$$\partial\sigma'_z=-\frac{2G\mu}{1-2\mu}\frac{\partial u}{\partial x}-\frac{2G(1-\mu)}{1-2\mu}\frac{\partial w}{\partial z}, \quad (4b)$$

$$\tau_{xz}=-G\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right), \quad (4c)$$

where G and μ are the shear modulus and Poisson's ratio of the saturated soil, respectively.

2.3. The Solutions of the Dynamic Equations. For a simple harmonic load, the displacement, stress, and excess pore pressure may be expressed as

$$\begin{aligned} u&=\overline{u}e^{i\omega t},\\ w&=\overline{w}e^{i\omega t},\\ w_x&=\overline{w}_x e^{i\omega t},\\ w_z&=\overline{w}_z e^{i\omega t},\\ \sigma_{xL}&=\overline{\sigma}_{xL}e^{i\omega t},\\ \tau_{xzL}&=\overline{\tau}_{xzL}e^{i\omega t},\\ \sigma_{zL}&=\overline{\sigma}_{zL}e^{i\omega t},\\ \sigma'_x&=\overline{\sigma}_x e^{i\omega t},\\ \tau_{xz}&=\overline{\tau}_{xz}e^{i\omega t},\\ \sigma'_z&=\overline{\sigma}'_z e^{i\omega t},\\ p_f&=\overline{p}_f e^{i\omega t}. \end{aligned} \quad (5)$$

Here, ω is the circular frequency. The Fourier transform can be written as:

$$\phi^*=\Re\phi=\int_{-\infty}^{\infty}\phi e^{i\zeta x}dx. \quad (6)$$

Combining (1a)-(1b) and (2a)–(2c) and utilising the Fourier transform, we can obtain the solutions of the single-phase elastic layer as follows:

$$i\overline{u}^*(\zeta,z)=-A_1\zeta H_L e^{q_L z}-\frac{A_2e^{F_L z}}{\zeta}-B_1\zeta H_L e^{-q_L z}-\frac{B_2e^{-F_L z}}{\zeta}, \quad (7a)$$

$$\overline{w}^*(\zeta,z)=A_1q_LH_Le^{q_L z}+\frac{A_2e^{F_L z}}{F_L}-B_1q_LH_Le^{-q_L z}-\frac{B_2e^{-F_L z}}{F_L}, \quad (7b)$$

$$\overline{e}^*(\zeta,z)=A_1e^{q_L z}+B_1e^{-q_L z}, \quad (7c)$$

$$i\overline{\tau}^*_{xzL}(\zeta,z)=2A_1\zeta q_LH_LG_Le^{q_L z}-2B_1\zeta q_LH_LG_Le^{-q_L z}+A_2\left(\frac{F_L}{\zeta}+\frac{\zeta}{F_L}\right)G_Le^{F_L z}-B_2\left(\frac{F_L}{\zeta}+\frac{\zeta}{F_L}\right)G_Le^{-F_L z}, \quad (7d)$$

$$\sigma^*_{zL}(\zeta,z)=-2G_L\left[A_1\left(q_L^2H_L+\frac{\mu_L}{1-2\mu_L}\right)e^{q_L z}+A_2e^{F_L z}+B_1\left(q_L^2H_L+\frac{\mu_L}{1-2\mu_L}\right)e^{-q_L z}+B_2e^{-F_L z}\right], \quad (7e)$$

where

$$\bar{e} = \frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{w}}{\partial z},$$
$$F_L^2 = \zeta^2 - \frac{\omega^2}{G_L}\rho_L,$$
$$D_L = -\frac{\omega^2 (1-2\mu_L)\rho_L}{2G_L(1-\mu_L)}, \quad (8)$$
$$q_L^2 = \zeta^2 + D_L,$$
$$H_L = \frac{1}{q_L^2 - \zeta^2} = \frac{1}{D_L}.$$

Similarly, combining (3a)–(3e) and (4a)–(4c) and utilising the Fourier transform, we obtain the solutions of the saturated half-space as follows:

$$i\bar{u}^*(\zeta, z) = -A_0\zeta H e^{-qz} - B_0\zeta N e^{-\zeta z} - \frac{C_0 e^{-Fz}}{\zeta}, \quad (9a)$$

$$\bar{w}^*(\zeta, z) = -A_0 q H e^{-qz} - B_0\zeta N e^{-\zeta z} - \frac{C_0 e^{-Fz}}{F}, \quad (9b)$$

$$\bar{p}_f^*(\zeta, z) = A_0 E e^{-qz} + B_0 e^{-\zeta z}, \quad (9c)$$

$$\bar{e}^*(\zeta, z) = A_0 e^{-qz}, \quad (9d)$$

$$i\bar{\tau}_{xz}{}^*(\zeta, z) = -2A_0\zeta q H G e^{-qz} - 2B_0\zeta^2 N G e^{-\zeta z} - C_0\left(\frac{F}{\zeta} + \frac{\zeta}{F}\right) G e^{-Fz}, \quad (9e)$$

$$\bar{\sigma}_z^*(\zeta, z) = -2G\left[A_0\left(q^2 H + \frac{\mu}{1-2\mu}\right)e^{-qz} + B_0\zeta^2 N e^{-\zeta z} + C_0 e^{-Fz}\right] + A_0 E e^{-qz} + B_0 e^{-\zeta z}, \quad (9f)$$

where

$$c = \frac{n k_d \omega}{gni - k_d\omega},$$
$$\bar{e} = \frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{w}}{\partial z},$$
$$F^2 = \zeta^2 - \frac{\omega^2}{G}(c\rho_f + \rho),$$
$$E = \frac{\omega^2 \rho_f (c+1)}{Dc},$$
$$D = \frac{\omega^2 (1-2\mu)(2c\rho_f + \rho_f - c\rho)}{2cG(1-\mu)},$$
$$q^2 = \zeta^2 + D,$$
$$H = \frac{1}{q^2 - \zeta^2} = \frac{1}{D},$$
$$N = \frac{c+1}{G(\zeta^2 - F^2)} = \frac{c+1}{\omega^2(c\rho_f + \rho)}. \quad (10)$$

The transformation as shown in the following is introduced:

$$A_1 = \frac{1}{2}\left(\bar{A}_1 + \bar{B}_1\right), \quad B_1 = \frac{1}{2}\left(\bar{A}_1 - \bar{B}_1\right),$$
$$A_2 = \frac{1}{2}\left(\bar{A}_2 + \bar{B}_2\right), \quad B_2 = \frac{1}{2}\left(\bar{A}_2 - \bar{B}_2\right). \quad (11)$$

Equations (7a)–(7e) can be further transformed as below:

$$i\bar{u}^*(\zeta, z) = -\bar{A}_1\zeta H_L \mathrm{ch} q_L z - \frac{\bar{A}_2 \mathrm{ch} F_L z}{\zeta} - \bar{B}_1\zeta H_L \mathrm{sh} q_L z - \frac{\bar{B}_2 \mathrm{sh} F_L z}{\zeta}, \quad (12a)$$

$$\bar{w}^*(\zeta, z) = \bar{A}_1 q_L H_L \mathrm{sh} q_L z + \frac{\bar{A}_2 \mathrm{sh} F_L z}{F_L} + \bar{B}_1 q_L H_L \mathrm{ch} q_L z + \frac{\bar{B}_2 \mathrm{ch} F_L z}{F_L}, \quad (12b)$$

$$\bar{e}^*(\zeta, z) = \bar{A}_1 \mathrm{ch} q_L z + \bar{B}_1 \mathrm{sh} q_L z, \quad (12c)$$

$$i\bar{\tau}_{xzL}^*(\zeta, z) = 2\bar{A}_1\zeta q_L H_L G_L \mathrm{sh} q_L z + \bar{A}_2\left(\frac{F_L}{\zeta} + \frac{\zeta}{F_L}\right) G_L \mathrm{sh} F_L z + 2\bar{B}_1\zeta q_L H_L G_L \mathrm{ch} q_L z + \bar{B}_2\left(\frac{F_L}{\zeta} + \frac{\zeta}{F_L}\right) G_L \mathrm{ch} F_L z, \quad (12d)$$

$$\sigma_{zL}^*(\zeta, z) = -2G_L\left[\bar{A}_1\left(q_L^2 H_L + \frac{\mu_L}{1-2\mu_L}\right)\mathrm{ch} q_L z + \bar{A}_2 \mathrm{ch} F_L z + \bar{B}_1 \times \left(q_L^2 H_L + \frac{\mu_L}{1-2\mu_L}\right)\mathrm{sh} q_L z + \bar{B}_2 \mathrm{sh} F_L z\right]. \quad (12e)$$

3. The Mixed Boundary-Value Problem of the Rocking Vibration of an Elastic Strip Foundation on Elastic Soil with Saturated Substrata and Boundary Conditions

It is assumed that the contact between the elastic strip foundation and the saturated soil is smooth and that the surface of the saturated soil is pervious. The boundary conditions are expressed as

$$\overline{w}(x,0) = x\phi - \overline{\Delta}(x) \quad |x| \le b, \tag{13a}$$

$$\overline{\tau}_{xzL}(x,-H_n) = 0 \quad -\infty < x < \infty, \tag{13b}$$

$$[\overline{w}(x,0)]_L = \overline{w}(x,0) \quad -\infty < x < \infty, \tag{13c}$$

$$[\overline{u}(x,0)]_L = \overline{u}(x,0) \quad -\infty < x < \infty, \tag{13d}$$

$$\overline{\tau}_{xzL}(\zeta,0) = \overline{\tau}_{xz}(\zeta,0) \quad -\infty < x < \infty, \tag{13e}$$

$$\overline{\sigma}_{zL}(\zeta,0) = \overline{\sigma}_{z}(\zeta,0) \quad -\infty < x < \infty, \tag{13f}$$

$$\overline{p}_f(\zeta,0) = 0 \quad -\infty < x < \infty, \tag{13g}$$

$$\overline{\sigma}_{zL}(x,-H_n) = 0 \quad |x| > b, \tag{13h}$$

where $\overline{\sigma}_z$ and $\overline{\tau}_{xz}$ are the normal stress and the shear stress of the soil skeleton, respectively; $\overline{p}_f$ is the pore pressure; $\overline{w}$ is the contact surface displacement between the strip foundation and underlying soil; ϕ is the rotation of the centre of the strip foundation; and $\overline{\Delta}(x)$ is the deflection of the strip foundation relative to the centre.

Combining (9a)–(9f) and (12a)–(12e) and applying the Fourier transform to (13b)–(13h), we can obtain the following relationships:

$$\begin{aligned}&2\overline{A}_1\zeta q_L H_L G_L \text{sh}(-q_L H_n) + \overline{A}_2\left(\frac{F_L}{\zeta}+\frac{\zeta}{F_L}\right)G_L\text{sh}(-F_L H_n)\\&+2\overline{B}_1\zeta q_L H_L G_L\text{ch}(-q_L H_n)\\&+\overline{B}_2\left(\frac{F_L}{\zeta}+\frac{\zeta}{F_L}\right)G_L\text{ch}(-F_L H_n) = 0,\end{aligned} \tag{14a}$$

$$\overline{B}_1 q_L H_L + \frac{\overline{B}_2}{F_L} + A_0 qH + B_0\zeta N + \frac{C_0}{F} = 0, \tag{14b}$$

$$-\overline{A}_1\zeta H_L - \frac{\overline{A}_2}{\zeta} + A_0\zeta H + B_0\zeta N + \frac{C_0}{\zeta} = 0, \tag{14c}$$

$$\begin{aligned}&2\overline{B}_1\zeta q_L H_L G_L + \overline{B}_2\left(\frac{F_L}{\zeta}+\frac{\zeta}{F_L}\right)G_L + 2A_0\zeta qHG\\&+2B_0\zeta^2 NG + C_0\left(\frac{F}{\zeta}+\frac{\zeta}{F}\right)G = 0,\end{aligned} \tag{14d}$$

$$\begin{aligned}&-2G_L\left[\overline{A}_1\left(q_L^2 H_L + \frac{\mu_L}{1-2\mu_L}\right)+\overline{A}_2\right]\\&+2G\left[A_0\left(q^2 H + \frac{\mu}{1-2\mu}\right)+B_0\zeta^2 N + C_0\right] - A_0 E - B_0 = 0\end{aligned} \tag{14e}$$

$$A_0 E + B_0 = 0. \tag{14f}$$

From (14f) we can obtain

$$B_0 = -A_0 E. \tag{15}$$

Utilising the Fourier transform on (12b) and (12e) gives

$$\begin{aligned}-2G_L\Bigg[&\overline{A}_1\left(q_L^2 H_L + \frac{\mu_L}{1-2\mu_L}\right)\text{ch}(-q_L H_n) + \overline{A}_2\text{ch}(-F_L H_n)\\&+\overline{B}_1\left(q_L^2 H_L + \frac{\mu_L}{1-2\mu_L}\right)\text{sh}(-q_L H_n)\\&+\overline{B}_2\text{sh}(-F_L H_n)\Bigg] = \overline{\sigma}_z^*(\zeta,0),\end{aligned} \tag{16a}$$

$$\begin{aligned}&\overline{A}_1 q_L H_L\text{sh}(-q_L H_n) + \frac{\overline{A}_2\text{sh}(-F_L H_n)}{F_L} +\\&\overline{B}_1 q_L H_L\text{ch}(-q_L H_n) + \frac{\overline{B}_2\text{ch}(-F_L H_n)}{F_L} = \overline{w}^*(\zeta,0).\end{aligned} \tag{16b}$$

Substituting (15) into (14b), (14c), (14d), and (14e), then (16a), (14a), (14b), (14c), (14d), and (14e) may be transformed into the following matrix form:

$$\begin{bmatrix}T_{11}&T_{12}&T_{13}&T_{14}&T_{15}&T_{16}\\T_{21}&T_{22}&T_{23}&T_{24}&T_{25}&T_{26}\\T_{31}&T_{32}&T_{33}&T_{34}&T_{35}&T_{36}\\T_{41}&T_{42}&T_{43}&T_{44}&T_{45}&T_{46}\\T_{51}&T_{52}&T_{53}&T_{54}&T_{55}&T_{56}\\T_{61}&T_{62}&T_{63}&T_{64}&T_{65}&T_{66}\end{bmatrix}\cdot\begin{bmatrix}\overline{A}_1\\\overline{B}_1\\\overline{A}_2\\\overline{B}_2\\A_0\\C_0\end{bmatrix} = \begin{bmatrix}\overline{\sigma}_z^*(\zeta,0)\\0\\0\\0\\0\\0\end{bmatrix}. \tag{17}$$

The expression of each element of matrix T can be seen in the appendix.

The displacement of the strip foundation surface can be expressed in the following matrix form:

$$\begin{aligned}&[\Gamma_1\ \ \Gamma_2\ \ \Gamma_3\ \ \Gamma_4\ \ \Gamma_5\ \ \Gamma_6]\left[\overline{A}_1\ \ \overline{B}_1\ \ \overline{A}_2\ \ \overline{B}_2\ \ A_0\ \ C_0\right]^T\\&= \overline{w}^*(\zeta,0),\end{aligned} \tag{18}$$

where

$$\begin{aligned}\Gamma_1 &= q_L H_L \mathrm{sh}\left(-q_L H_n\right),\\ \Gamma_2 &= q_L H_L \mathrm{ch}\left(-q_L H_n\right),\\ \Gamma_3 &= \frac{\mathrm{sh}\left(-F_L H_n\right)}{F_L},\\ \Gamma_4 &= \frac{\mathrm{ch}\left(-F_L H_n\right)}{F_L},\\ \Gamma_5 &= \Gamma_6 = 0.\end{aligned} \tag{19}$$

Establishing the following equations by the matrix T and the matrix Γ gives

$$T \cdot X = \Gamma, \tag{20}$$

where X is a 6×1 matrix.

We find from (17) and (18) that the element X_1 in the first row of the matrix X denotes the relationship between the displacement and stress on the strip foundation surface. Thus, when

$$\overline{w}^*(\zeta, 0) = f(\zeta) \cdot \overline{\sigma}_z^*(\zeta, 0) \tag{21}$$

we obtain $f(\zeta) = X_1$, thereby expressing every element in matrix X by solving (20). We can obtain $f(\zeta)$ and find that $f(\zeta)$ and $1/\zeta$ are infinitesimal of the same order when $\zeta \to \infty$.

Using the Fourier inverse transform and combining (21), we obtain

$$\begin{aligned}\overline{w}(x, 0) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \overline{w}^*(\zeta, 0) e^{-i\zeta x} d\zeta\\ &= \frac{1}{\pi} \int_0^{\infty} f(\zeta) \overline{\sigma}_z^*(\zeta, 0) \sin(\zeta x) d\zeta.\end{aligned} \tag{22}$$

Additionally,

$$\begin{aligned}\overline{\sigma}_z(x, 0) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \overline{\sigma}_z^*(\zeta, 0) e^{-i\zeta x} d\zeta\\ &= \frac{1}{\pi} \int_0^{\infty} \overline{\sigma}_z^*(\zeta, 0) \sin(\zeta x) d\zeta.\end{aligned} \tag{23}$$

The dual integral equations of the rocking vibration of an elastic strip foundation on elastic soil with saturated substrata are as follows:

$$\begin{aligned}&\int_0^{\infty} \tilde{f}(\tilde{\zeta}) \tilde{\overline{\sigma}}_z^*(\tilde{\zeta}, 0) \sin(\tilde{\zeta}\tilde{x}) d\tilde{\zeta}\\ &\quad + \delta b^2 \left(6\tilde{x}^3 - \tilde{x}^2\right)\\ &\quad \times \int_0^{\infty} \frac{\left(1 + \tilde{\rho}\tilde{h}\omega^2 \tilde{f}(\tilde{\zeta})/G\right)}{\tilde{\zeta}} \tilde{\overline{\sigma}}_z^*(\tilde{\zeta}, 0) \cos\tilde{\zeta} d\tilde{\zeta}\\ &\quad - \delta b^2 \tilde{x}^2 \int_0^{\infty} \frac{\left(1 + \tilde{\rho}\tilde{h}\omega^2 \tilde{f}(\tilde{\zeta})/G\right)}{\tilde{\zeta}^2} \tilde{\overline{\sigma}}_z^*(\tilde{\zeta}, 0) \sin\tilde{\zeta} d\tilde{\zeta}\\ &\quad - \delta b^2 \tilde{x} \int_0^{\infty} \frac{\left(1 + \tilde{\rho}\tilde{h}\omega^2 \tilde{f}(\tilde{\zeta})/G\right)}{\tilde{\zeta}^3} \tilde{\overline{\sigma}}_z^*(\tilde{\zeta}, 0) d\tilde{\zeta}\\ &\quad - b^2 \delta \int_0^{\infty} \frac{\left(1 + \tilde{\rho}\tilde{h}\omega^2 \tilde{f}(\tilde{\zeta})/G\right)}{\tilde{\zeta}^4} \tilde{\overline{\sigma}}_z^*(\tilde{\zeta}, 0) \left(\sin\tilde{\zeta}\tilde{x} - 2\right) d\tilde{\zeta}\\ &= \pi x \phi b^2 \quad 0 \le |\tilde{x}| \le 1,\\ &\int_0^{\infty} \tilde{\overline{\sigma}}_z^*(\tilde{\zeta}, 0) \sin(\tilde{\zeta}\tilde{x}) d\tilde{\zeta} = 0 \quad |\tilde{x}| > 1,\end{aligned} \tag{24}$$

where ϕ is the rotation of the centre of the strip foundation and D_f is the flexural stiffness of the foundation.

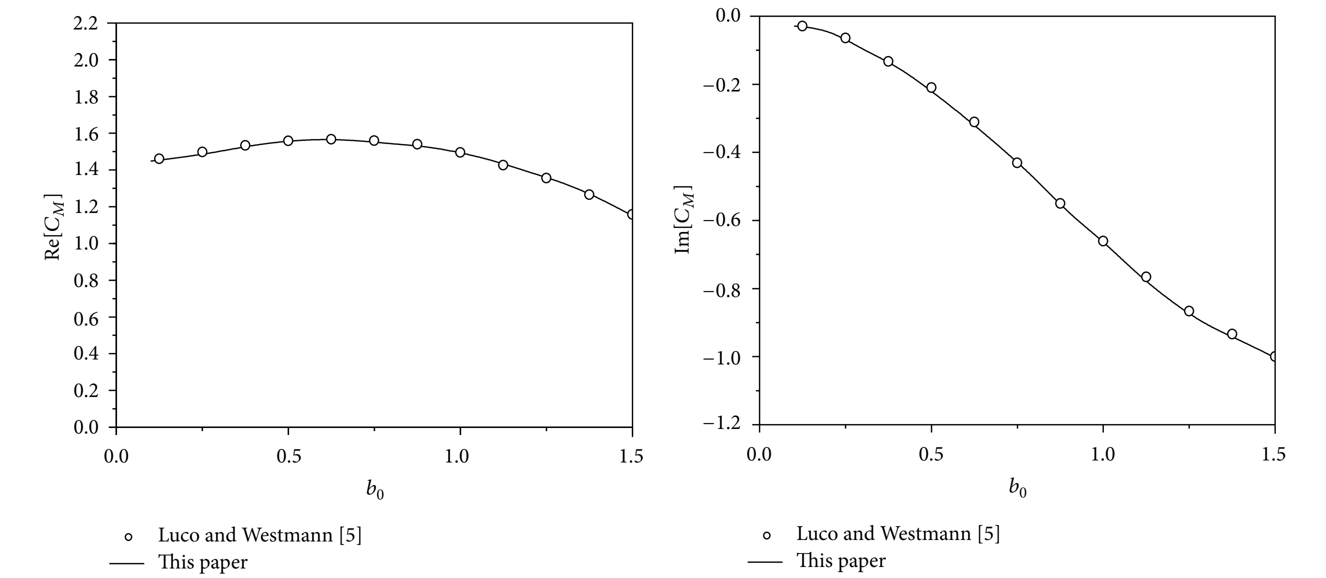
Simpson's rule is used to conduct numerical calculation. The dynamic compliance coefficient, C_M, of the rocking vibration of a strip foundation can be expressed as follows [27]:

$$C_M = \frac{1}{ba_0}. \tag{25}$$

Defining $f_1 = \mathrm{Re}[C_M]$ and $f_2 = \mathrm{Im}[C_M]$, we can obtain foundation stiffness $K = f_1/(f_1^2 + f_2^2)$ and the damping coefficient of the foundation $C = -f_2/(f_1^2 + f_2^2)b_0$. Here, $b_0 = b\omega\sqrt{\rho/G}$ is the dimensionless frequency.

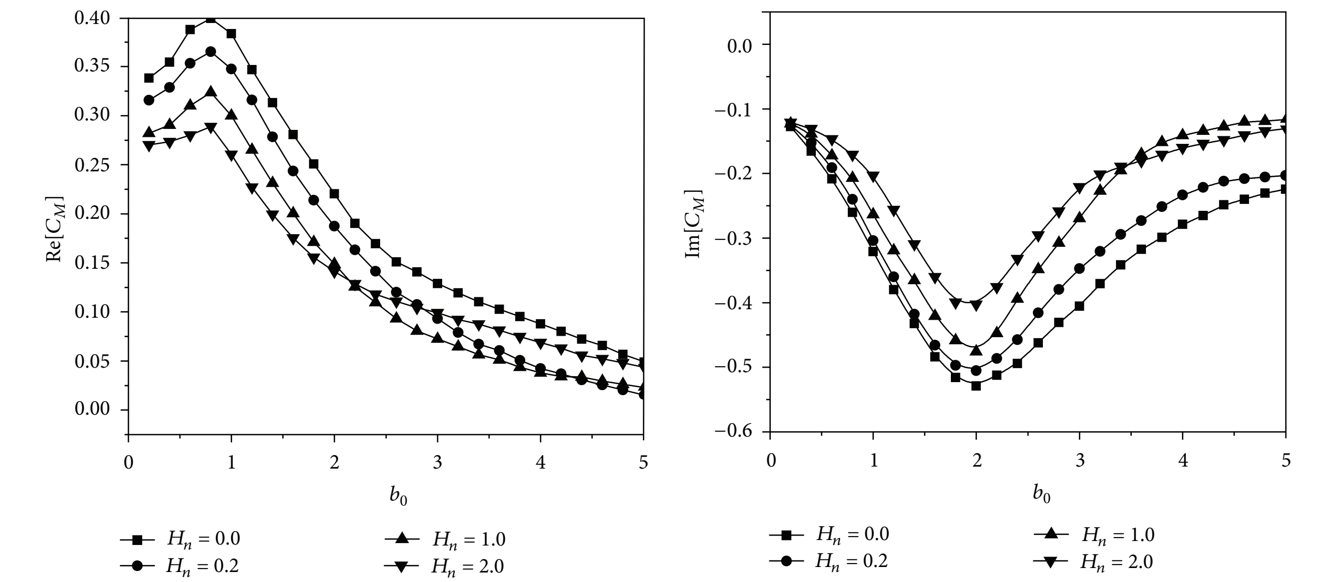
4. Verifications and Numerical Example Analysis

The rocking vibration solution of an elastic strip foundation on elastic soil with saturated substrata can be degenerated to the single-phase elastic half-space case by defining $\rho_f = 0$, $\delta = 0$, and $H_n = 0$. The foundation parameters for the degenerated case are $b = 2$ m, $G = 35$ MPa, $n = 0.35$, and $\rho_s = 2650\,\mathrm{kg/m^3}$. The variation of the dynamic compliance coefficient C_M with dimensionless frequency b_0 is analysed and is then compared with the numerical results by Luco and Westmann [5] when μ is 0.25. In Figure 2, "∘" represents the numerical results obtained by Luco and Westmann [5] when μ is 0.25. Both of the results derived from Figure 2 are consistent and verify the feasibility and accuracy of the calculating methods described in this paper. Meanwhile, the rigid foundation is considered as a special case of the elastic foundation when δ equals zero.



(a) (b)

Figure 2: The dynamic compliance coefficient versus the dimensionless frequency. (a) Real part of C_M versus the dimensionless frequency and (b) imaginary part of C_M versus the dimensionless frequency.

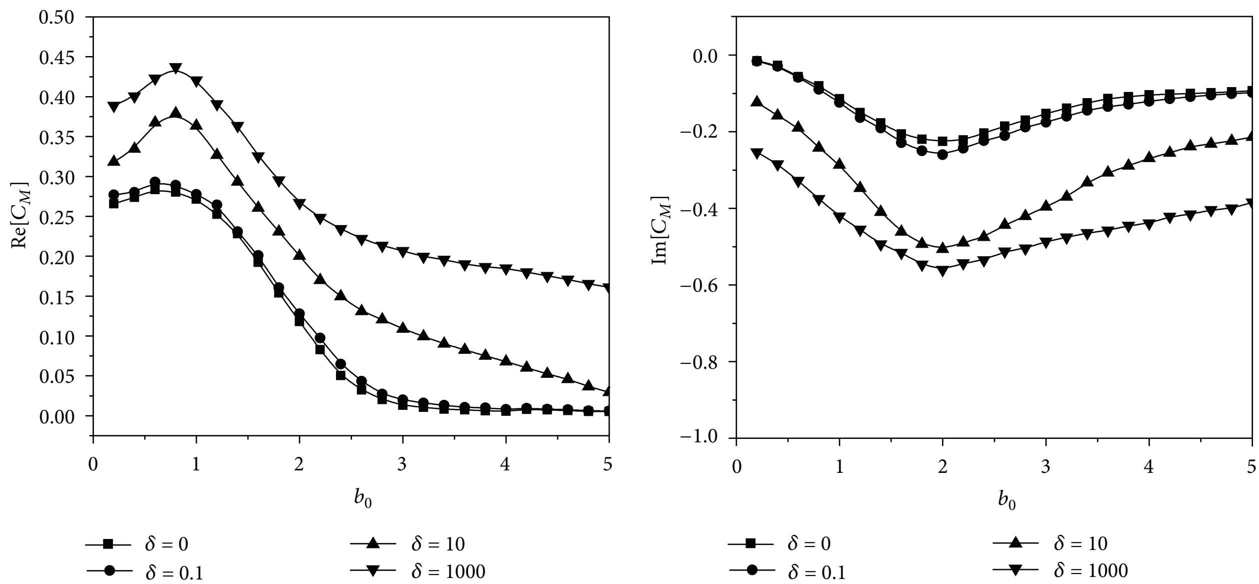


(a) (b)

Figure 3: The dynamic compliance coefficient C_M of different single-phase elastic layer thicknesses (H_n). (a) Real part of C_M versus the dimensionless frequency and (b) imaginary part of C_M versus the dimensionless frequency.

Concerning the rocking vibration of an elastic strip foundation on elastic soil with saturated substrata, the physical and mechanical parameters of a single-phase elastic layer are $G_L = 35\,\text{MPa}$, $\mu_L = 0.45$, and $\rho_L = 1722.5\,\text{kg/m}^3$; for a saturated half-space, $n = 0.35$, $G = 35\,\text{MPa}$, $k_d = 10^{-5}\,\text{m/s}$, $\rho_f = 1000\,\text{kg/m}^3$, $\rho_s = 2650\,\text{kg/m}^3$, and $\mu = 0.25$. The dynamic compliance coefficient C_M changes over the dimensionless frequency, the state of which is calculated when $H_n = 0$, 0.2, 1.0, and 2.0 m for $\delta = 10$, and the calculation results are shown in Figure 3. Meanwhile, the dynamic compliance coefficient C_M changes over the dimensionless frequency, the state of which is calculated when $\delta = 0.0$, 0.1, 10, and 1000 for $H_n = 0.2$, and the calculation results are shown in Figure 4.

We can see from Figure 3 that C_M decreases with an increase in the elastic layer thickness, which indicates that



(a) (b)

Figure 4: The dynamic compliance coefficient C_M for different δ. (a) Real part of C_M versus the dimensionless frequency and (b) imaginary part of C_M versus the dimensionless frequency.

the presence of elastic soil can reduce the vibration to the foundation. Meanwhile, in the given parameters for the real part of C_M, as b_0 increases, the real part of C_M decreases, but the curve tends to flatten. For the imaginary part of C_M, as b_0 increases, the imaginary part of C_M first decreases and then increases, finally becoming smooth.

For the rocking vibration of an elastic strip foundation on elastic soil with saturated substrata, the curves of C_M are essentially coincident when $\delta = 0.0$ and 0.1, which can be seen from Figure 4. The curve for $\delta = 0.0$ is the one of the dynamic compliance coefficients of the rocking vibration of a rigid strip foundation on elastic soil with saturated substrata. Therefore, it can be inferred that the rocking vibration of an elastic and a rigid strip foundation on elastic soil with saturated substrata has the similar dynamic characteristics in variations of b_0 when $\delta \leq 0.1$. It is also demonstrated that the real part of C_M is greatly influenced by variations in b_0 when $b_0 < 2.8$ and that the imaginary part of C_M is greatly influenced by variations in b_0 when $b_0 < 3.6$. However, when b_0 exceeds the critical values (2.8 for the real part of C_M and 3.6 for the imaginary part of C_M), the curve of C_M tends to flatten and C_M is only slightly influenced by variations in the dimensionless frequency b_0. Figure 4 also shows that the dynamic compliance coefficient curve obtained when $\delta = 10$ and 1000 is remarkably different from the one obtained when $\delta = 0.0$ and 0.1, and the absolute values concerning both the real parts and imaginary parts of the dynamic compliance coefficient when $\delta = 10$ and 1000 are larger than the ones when $\delta = 0.0$ and 0.1. Moreover, it can be seen from Figure 4 that when δ is large, the variation of the dynamic compliance coefficient curve with the dimensionless frequency b_0 tends to be smooth. However, for an average quantity of δ, the variation of the absolute values of the real parts and imaginary parts of C_M with b_0 is significant. Thus, under ordinary circumstances ($\delta = 10$), we must consider effects exerted by the dimensionless frequency b_0.

5. Conclusions

In this paper, an analytical solution for the rocking vibration of an elastic strip foundation on elastic soil with saturated substrata is developed. The solution is based on dual integral equations, which are formulated from Biot's equations of dynamic poroelasticity by means of the Fourier transform in combination with mixed boundary conditions. Validation of the analytical solution for dry soil is based on the solution presented by Luco and Westmann [5].

Our conclusions of this study are as follows. (1) The dynamic compliance coefficient C_M decreases with an increase in the elastic layer thickness, which indicates that the presence of elastic soil can reduce the vibration to the foundation. (2) The real part of dynamic compliance coefficient C_M is greatly influenced by variations in the dimensionless frequency b_0 when $b_0 < 2.8$, and the imaginary part of C_M is greatly influenced by variations in the dimensionless frequency b_0 when $b_0 < 3.6$. (3) When the flexural stiffness of the elastic foundation is comparatively large or when $\delta \leq 0.1$, the influence of δ on the rocking vibration can be ignored. (4) When $\delta > 0.1$ and as δ increases, the rocking vibration of the elastic foundation changes significantly, and the absolute values of both the real parts and imaginary parts of C_M increase.

Appendix

Expressions for each element of the matrix T appearing previously are given by

$$T_{11} = -2G_L\left(q_L^2 H_L + \frac{\mu_L}{1-2\mu_L}\right)\mathrm{ch}\left(-q_L H_n\right),$$
$$T_{12} = -2G_L\left(q_L^2 H_L + \frac{\mu_L}{1-2\mu_L}\right)\mathrm{sh}\left(-q_L H_n\right),$$
$$T_{13} = -2G_L \mathrm{ch}\left(-F_L H_n\right),$$
$$T_{14} = -2G_L \mathrm{sh}\left(-F_L H_n\right),$$
$$T_{15} = 0,$$
$$T_{16} = 0,$$
$$T_{21} = 2\zeta q_L H_L G_L \mathrm{sh}\left(-q_L H_n\right),$$
$$T_{22} = 2\zeta q_L H_L G_L \mathrm{ch}\left(-q_L H_n\right),$$
$$T_{23} = \left(\frac{F_L}{\zeta} + \frac{\zeta}{F_L}\right) G_L \mathrm{sh}\left(-F_L H_n\right),$$
$$T_{24} = \left(\frac{F_L}{\zeta} + \frac{\zeta}{F_L}\right) G_L \mathrm{ch}\left(-F_L H_n\right),$$
$$T_{25} = 0,$$
$$T_{26} = 0,$$
$$T_{31} = 0,$$
$$T_{32} = q_L H_L,$$
$$T_{33} = 0,$$
$$T_{34} = \frac{1}{F_L},$$
$$T_{35} = qH - \zeta NE,$$
$$T_{36} = \frac{1}{F},$$
$$T_{41} = -\zeta H_L,$$
$$T_{42} = 0,$$
$$T_{43} = -\frac{1}{\zeta},$$
$$T_{44} = 0,$$
$$T_{45} = \zeta H - \zeta NE,$$
$$T_{46} = \frac{1}{\zeta},$$
$$T_{51} = 0,$$
$$T_{52} = 2\zeta q_L H_L G_L,$$
$$T_{53} = 0,$$
$$T_{54} = \left(\frac{F_L}{\zeta} + \frac{\zeta}{F_L}\right) G_L,$$
$$T_{55} = 2\zeta qHG - 2\zeta^2 NGE,$$
$$T_{56} = \left(\frac{F}{\zeta} + \frac{\zeta}{F}\right) G,$$
$$T_{61} = -2G_L\left(q_L^2 H_L + \frac{\mu_L}{1-2\mu_L}\right),$$
$$T_{62} = 0,$$
$$T_{63} = -2G_L,$$
$$T_{64} = 0,$$
$$T_{65} = 2G\left[\left(q^2 H + \frac{\mu}{1-2\mu}\right) - \zeta^2 NE\right],$$
$$T_{66} = 2G. \tag{A.1}$$

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