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Research Article

A Part-Metric-Related Inequality Chain and Application to the Stability Analysis of Difference Equation

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We find a new part-metric-related inequality of the form $\min\{a_i, 1/a_i : 1 \le i \le 5\} \le ((1+w)a_1a_2a_3 + a_4 + a_5)/(a_1a_2 + a_1a_3 + a_2a_3 + wa_4a_5) \le \max\{a_i, 1/a_i : 1 \le i \le 5\}$, where $1 \le w \le 2$. We then apply this result to show that $\hat{c} = 1$ is a globally asymptotically stable equilibrium of the rational difference equation $x_n = (x_{n-1} + x_{n-2} + (1+w)x_{n-3}x_{n-4}x_{n-5})/(wx_{n-1}x_{n-2} + x_{n-3}x_{n-4} + x_{n-3}x_{n-5} + x_{n-4}x_{n-5})$, $n = 1, 2, \ldots$, $a_0, a_{-1}, a_{-2}, a_{-3}, a_{-4} > 0$.

1. Introduction

Let $f(x_1, \ldots, x_r)$ and $g(x_1, \ldots, x_r)$ be polynomial functions with nonnegative coefficients and nonnegative constant terms. Suppose that, for all possible positive combinations of a_1 through a_r, the following inequality chain holds:

$$\min\left\{a_i, \frac{1}{a_i} : 1 \le i \le r\right\} \le \frac{f(a_1, \ldots, a_r)}{g(a_1, \ldots, a_r)} \le \max\left\{a_i, \frac{1}{a_i} : 1 \le i \le r\right\}. \tag{1.1}$$

In this paper, we refer to such an elegant inequality chain as a *part-metric-related* (PMR) *inequality chain* because it is closely related to the well-known part-metric p, which is defined on $(\mathbb{R}_+)^r$ (where $\mathbb{R}_+$ stands for the whole set of positive reals) in this way: for $\mathbf{X} = (x_1, \ldots, x_r)^T \in (\mathbb{R}_+)^r$, $\mathbf{Y} = (y_1, \ldots, y_r)^T \in (\mathbb{R}_+)^r$,

$$p(\mathbf{X}, \mathbf{Y}) = -\log_2 \min\left\{\frac{x_i}{y_i}, \frac{y_i}{x_i} : 1 \le i \le r\right\}. \tag{1.2}$$

Below, there are some known PMR inequality chains [1–3]:

$$\min\left\{a_i,\frac{1}{a_i}:1\le i\le 4\right\}\le\frac{a_1+a_2+a_3a_4}{a_1a_2+a_3+a_4}\le\max\left\{a_i,\frac{1}{a_i}:1\le i\le 4\right\},$$
$$\min\left\{a_i,\frac{1}{a_i}:1\le i\le k\right\}\le\frac{a_1+\cdots+a_{k-2}+a_{k-1}a_k}{a_1a_2+a_3+\cdots+a_k}\le\max\left\{a_i,\frac{1}{a_i}:1\le i\le k\right\},$$
$$\min\left\{a_i,\frac{1}{a_i}:1\le i\le 4\right\}\le\frac{A_1a_1+A_2a_2+A_3a_3a_4+A_4}{B_1a_1a_2+B_2a_3+B_3a_4+B_4}\le\max\left\{a_i,\frac{1}{a_i}:1\le i\le 4\right\},\tag{1.3}$$

where A_1, A_2, A_3, A_4, B_1, B_2, B_3, B_4 are positive numbers, $A_1+A_2+A_3+A_4=B_1+B_2+B_3+B_4$, $A_1+A_2>B_1$, $A_3<B_2+B_3<A_3+A_4$.

To our knowledge, all of the previously known PMR inequality chains were established provided that both the numerator polynomial and the denominator polynomial have a degree ≤ 2.

In this paper, we find a new PMR inequality chain of the form

$$\min\left\{a_i,\frac{1}{a_i}:1\le i\le 5\right\}\le\frac{(1+w)a_1a_2a_3+a_4+a_5}{a_1a_2+a_1a_3+a_2a_3+wa_4a_5}\le\max\left\{a_i,\frac{1}{a_i}:1\le i\le 5\right\},\tag{1.4}$$

where $1\le w\le 2$. Unlike previous PMR inequality chains, this PMR inequality chain has a numerator polynomial of degree $=3$.

PMR inequality chains are very useful in establishing the stability results of some rational difference equations. For instance, Kruse and Nesemann [1] proved that $\hat{c}=1$ is a globally asymptotically stable equilibrium of the following well-known Putnam equation:

$$x_n=\frac{x_{n-1}+x_{n-2}+x_{n-3}x_{n-4}}{x_{n-1}x_{n-2}+x_{n-3}+x_{n-4}},\quad n=1,2,\ldots,\qquad a_0,a_{-1},a_{-2},a_{-3}>0.\tag{1.5}$$

For more information on this topic the reader is referred to [1–7].

With the aid of PMR inequality chain (1.4) and provided that $1\le w\le 2$, we prove that $\hat{c}=1$ is a globally asymptotically stable equilibrium of the rational difference equation

$$x_n=\frac{x_{n-1}+x_{n-2}+(1+w)x_{n-3}x_{n-4}x_{n-5}}{wx_{n-1}x_{n-2}+x_{n-3}x_{n-4}+x_{n-3}x_{n-5}+x_{n-4}x_{n-5}},\quad n=1,2,\ldots,\qquad a_0,a_{-1},a_{-2},a_{-3},a_{-4}>0.\tag{1.6}$$

Equation (1.6) can be viewed as a higher-degree extension of the Putnam equation.

2. A new PMR inequality chain

Instead of merely giving a new PMR inequality chain, we present a more general result as follows.

Theorem 2.1. *Let a_1, a_2, a_3, a_4, a_5 be positive numbers. Let $1 \leq w \leq 2$. Let*

$$a_i = \frac{(1+w)a_{i-5}a_{i-4}a_{i-3} + a_{i-2} + a_{i-1}}{a_{i-5}a_{i-4} + a_{i-5}a_{i-3} + a_{i-4}a_{i-3} + wa_{i-2}a_{i-1}}, \quad i = 6,7,\dots. \tag{2.1}$$

Then,

$$\min\left\{a_i, \frac{1}{a_i} : 1 \leq i \leq 5\right\} \leq a_k \leq \max\left\{a_i, \frac{1}{a_i} : 1 \leq i \leq 5\right\}, \quad k = 6,7,\dots. \tag{2.2}$$

In the case $k \geq 7$, one of the two equalities holds if and only if $(a_1,a_2,a_3,a_4,a_5) = (1,1,1,1,1)$.

In order to prove Theorem 2.1, we need three lemmas, which are presented as follows.

Lemma 2.2 [8, page 1]. *Let $a_1,\dots,a_n$, $b_1,\dots,b_n$ be positive numbers. Then,*

$$\min\left\{\frac{a_i}{b_i} : 1 \leq i \leq n\right\} \leq \frac{a_1 + \cdots + a_n}{b_1 + \cdots + b_n} \leq \max\left\{\frac{a_i}{b_i} : 1 \leq i \leq n\right\}. \tag{2.3}$$

Moreover, at least one equality holds if and only if $a_1/b_1 = \cdots = a_n/b_n$.

Lemma 2.3. *Let a_1, a_2, a_3, a_4, a_5 be positive numbers. Let*

$$a_6 = \frac{2a_1a_2a_3 + a_4 + a_5}{a_1a_2 + a_1a_3 + a_2a_3 + a_4a_5}. \tag{2.4}$$

Then,

$$\min\left\{a_i, \frac{1}{a_i} : 1 \leq i \leq 5\right\} \leq a_6 \leq \max\left\{a_i, \frac{1}{a_i} : 1 \leq i \leq 5\right\}. \tag{2.5}$$

Moreover, at least one equality holds if and only if $(a_1,a_2,a_3,a_4,a_5) = (1,1,1,1,1)$.

Proof. We consider only the second inequality of this chain because the first one can be treated in a similar way. We distinguish among three possibilities.

Case 1 ($\min\{a_4,a_5\} < \max\{a_1,a_2,a_3\}$). We may, without loss of generality, assume that $a_4 < a_1$. By Lemma 2.2, we get

$$a_6 < \frac{a_1 + a_1a_2a_3 + a_1a_2a_3 + a_5}{a_1a_2 + a_1a_3 + a_2a_3 + a_4a_5} \leq \max\left\{\frac{1}{a_2}, a_2, a_1, \frac{1}{a_4}\right\} \leq \max\left\{a_i, \frac{1}{a_i} : 1 \leq i \leq 5\right\}. \tag{2.6}$$

Case 2 ($\max\{a_4,a_5\} > \min\{a_1,a_2,a_3\}$). Without loss of generality, assume that $a_4 > a_1$. Define an auxiliary function in this way:

$$f(x) = \frac{2a_1a_2a_3 + x + a_5}{a_1a_2 + a_1a_3 + a_2a_3 + a_5x}, \quad x \in [a_1, +\infty). \tag{2.7}$$

Then, $df(x)/dx = (a_1a_2 + a_1a_3 + a_2a_3 - a_5(2a_1a_2a_3 + a_5))/(a_1a_2 + a_1a_3 + a_2a_3 + a_5x)^2$. Let

$$\Delta = a_1a_2 + a_1a_3 + a_2a_3 - a_5(2a_1a_2a_3 + a_5). \tag{2.8}$$

Then, there are two possible cases.

Subcase 2.1. $\Delta \neq 0$. Then, $f(x)$ is strictly increasing or strictly decreasing and hence,

$$a_6 = f(a_4) < \max\left\{\lim_{x\to+\infty} f(x), f(a_1)\right\}. \tag{2.9}$$

As $\lim_{x\to+\infty} f(x) = 1/a_5 \leq \max\{a_i, 1/a_i : 1 \leq i \leq 5\}$ and

$$f(a_1) = \frac{a_1 + a_1a_2a_3 + a_1a_2a_3 + a_5}{a_1a_2 + a_1a_3 + a_2a_3 + a_1a_5} \leq \max\left\{\frac{1}{a_2}, a_2, a_1, \frac{1}{a_1}\right\} \leq \max\left\{a_i, \frac{1}{a_i} : 1 \leq i \leq 5\right\}, \tag{2.10}$$

it follows from (2.9) that $a_6 < \max\{a_i, 1/a_i : 1 \leq i \leq 5\}$.

Subcase 2.2. $\Delta = 0$. Then, $f(x)$ is a fixed-valued function and hence,

$$\begin{aligned}
a_6 = f(a_4) &= \frac{1}{a_5} \leq \max\left\{a_i, \frac{1}{a_i} : 1 \leq i \leq 5\right\},\\
a_6 = f(a_1) &= \frac{a_1 + a_1a_2a_3 + a_1a_2a_3 + a_5}{a_1a_2 + a_1a_3 + a_2a_3 + a_1a_5} \leq \max\left\{\frac{1}{a_2}, a_2, a_1, \frac{1}{a_1}\right\} \leq \max\left\{a_i, \frac{1}{a_i} : 1 \leq i \leq 5\right\},\\
a_6 = f(a_3) &= \frac{a_1a_2a_3 + a_1a_2a_3 + a_3 + a_5}{a_1a_2 + a_1a_3 + a_2a_3 + a_3a_5} \leq \max\left\{a_3, a_2, \frac{1}{a_2}, \frac{1}{a_3}\right\} \leq \max\left\{a_i, \frac{1}{a_i} : 1 \leq i \leq 5\right\}.
\end{aligned} \tag{2.11}$$

Suppose that $a_6 = \max\{a_i, 1/a_i : 1 \leq i \leq 5\}$. Then, all of the equalities in (2.11) hold and, by Lemma 2.2, we have $(a_1, a_2, a_3, a_4, a_5) = (1, 1, 1, 1, 1)$. This, however, contradicts the assumption that $a_4 > a_1$. So, $a_6 < \max\{a_i, 1/a_i : 1 \leq i \leq 5\}$.

Case 3 ($\max\{a_4, a_5\} \leq \min\{a_1, a_2, a_3\} \leq \max\{a_1, a_2, a_3\} \leq \min\{a_4, a_5\}$). This is equivalent to $a_1 = a_2 = a_3 = a_4 = a_5$. By Lemma 2.2, we get

$$a_6 = \frac{a_1^3 + a_1}{a_1^2 + a_1^2} \leq \max\left\{a_1, \frac{1}{a_1}\right\} = \max\left\{a_i, \frac{1}{a_i} : 1 \leq i \leq 5\right\}. \tag{2.12}$$

Suppose $a_6 = \max\{a_i, 1/a_i : 1 \leq i \leq 5\}$. Then the equality in (2.12) holds and, by Lemma 2.2, we get $a_1 = 1$. Hence, $(a_1, a_2, a_3, a_4, a_5) = (1, 1, 1, 1, 1)$.

The proof is complete. □

Lemma 2.4. *Let a_1, a_2, a_3, a_4, a_5 be positive numbers. Let*

$$a_6 = \frac{3a_1a_2a_3 + a_4 + a_5}{a_1a_2 + a_1a_3 + a_2a_3 + 2a_4a_5}. \tag{2.13}$$

Then,

$$\min\left\{a_1, a_2, a_3, \frac{1}{a_4}, \frac{1}{a_5}\right\} \leq a_6 \leq \max\left\{a_1, a_2, a_3, \frac{1}{a_4}, \frac{1}{a_5}\right\}. \tag{2.14}$$

Moreover, one of the equalities holds if and only if $a_1 = a_2 = a_3 = 1/a_4 = 1/a_5$.

Proof. The claimed results follow from Lemma 2.2 and the inspection that

$$a_6 = \frac{a_1a_2a_3 + a_1a_2a_3 + a_1a_2a_3 + a_4 + a_5}{a_1a_2 + a_1a_3 + a_2a_3 + a_4a_5 + a_4a_5}. \tag{2.15}$$

□

We are now in a position to prove Theorem 2.1.

Proof of Theorem 2.1. Define two auxiliary functions in this way:

$$\begin{aligned} f_1(w) &= \frac{(1+w)a_1a_2a_3 + a_4 + a_5}{a_1a_2 + a_1a_3 + a_2a_3 + wa_4a_5}, \quad w \in [1,2]; \\ f_2(w) &= \frac{(1+w)a_2a_3a_4 + a_5 + a_6}{a_2a_3 + a_2a_4 + a_3a_4 + wa_5a_6}, \quad w \in [1,2]. \end{aligned} \tag{2.16}$$

Then,

$$\begin{aligned} \frac{df_1(w)}{dw} &= \frac{a_1a_2a_3(a_1a_2 + a_1a_3 + a_2a_3) - a_4a_5(a_1a_2a_3 + a_4 + a_5)}{(a_1a_2 + a_1a_3 + a_2a_3 + wa_4a_5)^2}, \\ \frac{df_2(w)}{dw} &= \frac{a_2a_3a_4(a_2a_3 + a_2a_4 + a_3a_4) - a_5a_6(a_2a_3a_4 + a_5 + a_6)}{(a_2a_3 + a_2a_4 + a_3a_4 + wa_5a_6)^2}. \end{aligned} \tag{2.17}$$

Let

$$\begin{aligned} \Delta_1 &= a_1a_2a_3(a_1a_2 + a_1a_3 + a_2a_3) - a_4a_5(a_1a_2a_3 + a_4 + a_5), \\ \Delta_2 &= a_2a_3a_4(a_2a_3 + a_2a_4 + a_3a_4) - a_5a_6(a_2a_3a_4 + a_5 + a_6). \end{aligned} \tag{2.18}$$

Notice that $f_1(w)$ is nondecreasing or is strictly decreasing according as $\Delta_1 \geq 0$ or $\Delta_1 < 0$. This and Lemmas 2.3-2.4 yield

$$\begin{aligned} \min\left\{a_i, \frac{1}{a_i} : 1 \leq i \leq 5\right\} &\leq \min\{f_1(1), f_1(2)\} \leq a_6 = f_1(w) \\ &\leq \max\{f_1(1), f_1(2)\} \leq \max\left\{a_i, \frac{1}{a_i} : 1 \leq i \leq 5\right\}. \end{aligned} \tag{2.19}$$

Notice that $f_2(w)$ is nondecreasing or is strictly decreasing according as $\Delta_2 \geq 0$ or $\Delta_2 < 0$. This and Lemmas 2.3–2.4 lead to

$$\begin{aligned} \min\left\{a_i, \frac{1}{a_i} : 2 \leq i \leq 6\right\} &\leq \min\{f_2(1), f_2(2)\} \leq a_7 = f_2(w) \\ &\leq \max\{f_2(1), f_2(2)\} \leq \max\left\{a_i, \frac{1}{a_i} : 2 \leq i \leq 6\right\}. \end{aligned} \tag{2.20}$$

By (2.19), we have

$$\begin{aligned}\max\left\{a_i,\frac{1}{a_i}:2\le i\le 6\right\}&=\max\left\{\max\left\{a_i,\frac{1}{a_i}:2\le i\le 5\right\},a_6,\frac{1}{a_6}\right\}\\&\le\max\left\{a_i,\frac{1}{a_i}:1\le i\le 5\right\},\\\min\left\{a_i,\frac{1}{a_i}:2\le i\le 6\right\}&=\min\left\{\min\left\{a_i,\frac{1}{a_i}:2\le i\le 5\right\},a_6,\frac{1}{a_6}\right\}\\&\ge\min\left\{a_i,\frac{1}{a_i}:1\le i\le 5\right\}.\end{aligned}\tag{2.21}$$

Plugging (2.21) into (2.20), we get

$$\min\left\{a_i,\frac{1}{a_i}:1\le i\le 5\right\}\le a_7\le\max\left\{a_i,\frac{1}{a_i}:1\le i\le 5\right\}.\tag{2.22}$$

Working inductively, we can prove that

$$\min\left\{a_i,\frac{1}{a_i}:1\le i\le 5\right\}\le a_k\le\max\left\{a_i,\frac{1}{a_i}:1\le i\le 5\right\},\quad k=6,7,\dots.\tag{2.23}$$

Suppose that

$$a_7=\max\left\{a_i,\frac{1}{a_i}:1\le i\le 5\right\}.\tag{2.24}$$

Equations (2.20)–(2.24) imply that $\max\{f_2(1),f_2(2)\}=\max\{a_i,1/a_i:2\le i\le 6\}$. So, we are confronted with two possibilities.

Case 1 ($f_2(1)=\max\{a_i,1/a_i:2\le i\le 6\}$). By Lemma 2.3, we get $(a_2,a_3,a_4,a_5,a_6)=(1,1,1,1,1)$, implying $a_7=1$. So, (2.24) reduces to $1=\max\{1,a_1,1/a_1\}$, implying $a_1=1$. Hence, $(a_1,a_2,a_3,a_4,a_5)=(1,1,1,1,1)$.

Case 2 ($f_2(2)=\max\{a_i,1/a_i:2\le i\le 6\}$). By Lemma 2.4, we get

$$a_2=a_3=a_4=\frac{1}{a_5}=\frac{1}{a_6},\qquad f_2(2)=\frac{1}{a_6}.\tag{2.25}$$

By (2.19), (2.20), (2.24), and (2.25), we derive

$$\begin{aligned}\max\left\{a_i,\frac{1}{a_i}:1\le i\le 5\right\}=a_7\le f_2(2)=\frac{1}{a_6}&\le\frac{1}{\min\{f_1(1),f_1(2)\}}\\&\le\max\left\{a_i,\frac{1}{a_i}:1\le i\le 5\right\}.\end{aligned}\tag{2.26}$$

So, all of the equalities in (2.26) hold. In particular, we have

$$\min\{f_1(1),f_1(2)\}=\min\left\{a_i,\frac{1}{a_i}:1\le i\le 5\right\}.\tag{2.27}$$

In the case $f_1(1) = \min\{a_i, 1/a_i : 1 \le i \le 5\}$, it follows from Lemma 2.3 that $a_1 = a_2 = a_3 = a_4 = a_5 = 1$, and the claimed result is proven. Now, suppose that $f_1(2) = \min\{a_i, 1/a_i : 1 \le i \le 5\}$. By Lemma 2.4, we get

$$a_1 = a_2 = a_3 = \frac{1}{a_4} = \frac{1}{a_5}. \tag{2.28}$$

Then, (2.25) and (2.28) yield $a_1 = a_2 = a_3 = a_4 = a_5 = 1$.

The proof is complete. □

3. Application to difference equation

For fundamental knowledge concerning the stability of difference equations, refer to [9, 10]. In what follows, $\mathbb{R}_+$ stands for the whole set of positive reals, p for the part-metric defined on $(\mathbb{R}_+)^r$.

LEMMA 3.1 [1]. *Let $((\mathbb{R}_+)^r, d)$ be a metric space, T a continuous mapping defined on this space and with an equilibrium $\mathbf{C} \in (\mathbb{R}_+)^r$. Consider the first-order difference equation system*

$$\mathbf{X}_n = T(\mathbf{X}_{n-1}), \quad n = 1, 2, \ldots. \tag{3.1}$$

Suppose there is a positive integer k such that $d(T^k(\mathbf{X}), \mathbf{C}) < d(\mathbf{X}, \mathbf{C})$ holds for each $\mathbf{X} \neq \mathbf{C}$. Then $\mathbf{C}$ is globally asymptotically stable.

Now, let us establish the following result with the aid of Theorem 2.1.

THEOREM 3.2. *$\hat{c} = 1$ is a globally asymptotically stable equilibrium point of the rational difference equation*

$$x_n = \frac{x_{n-1} + x_{n-2} + (1+w)x_{n-3}x_{n-4}x_{n-5}}{wx_{n-1}x_{n-2} + x_{n-3}x_{n-4} + x_{n-3}x_{n-5} + x_{n-4}x_{n-5}}, \quad n = 1, 2, \ldots; \quad x_0, x_{-1}, x_{-2}, x_{-3}, x_{-4} > 0. \tag{3.2}$$

Proof. The first-order difference equation system associated with (3.2) is

$$\mathbf{X}_n = T(\mathbf{X}_{n-1}), \quad n = 1, 2, \ldots, \tag{3.3}$$

where T is a continuous mapping defined on the metric space $((\mathbb{R}_+)^5, p)$ by

$$T\big((a_1, a_2, a_3, a_4, a_5)^T\big) = (a_2, a_3, a_4, a_5, a_6)^T, \qquad a_6 = \frac{(1+w)a_1a_2a_3 + a_4 + a_5}{a_1a_2 + a_1a_3 + a_2a_3 + wa_4a_5}. \tag{3.4}$$

For our purpose, it suffices to show that $\mathbf{C} = (1,1,1,1,1)^T$ is a globally asymptotically stable equilibrium of system (3.3). Consider an arbitrary point $\mathbf{X} = (a_1, a_2, a_3, a_4, a_5)^T \in (\mathbb{R}_+)^5$, $\mathbf{X} \neq (1,1,1,1,1)^T$. Let

$$T^6(\mathbf{X}) = (a_7, a_8, a_9, a_{10}, a_{11})^T. \tag{3.5}$$

Then,

$$a_k = \frac{(1+w)a_{k-5}a_{k-4}a_{k-3} + a_{k-2} + a_{k-1}}{a_{k-5}a_{k-4} + a_{k-5}a_{k-3} + a_{k-4}a_{k-3} + wa_{k-2}a_{k-1}}, \quad 6 \le k \le 11. \tag{3.6}$$

By Theorem 2.1, we have

$$\min\left\{a_i, \frac{1}{a_i} : 1 \le i \le 5\right\} < a_k < \max\left\{a_i, \frac{1}{a_i} : 1 \le i \le 5\right\}, \quad 7 \le k \le 11, \tag{3.7}$$

which implies

$$\min\left\{a_i, \frac{1}{a_i} : 7 \le i \le 11\right\} > \min\left\{a_i, \frac{1}{a_i} : 1 \le i \le 5\right\}. \tag{3.8}$$

So,

$$\begin{aligned} p(T^6(\mathbf{X}), \mathbf{C}) &= -\log_2 \min\left\{a_i, \frac{1}{a_i} : 7 \le i \le 11\right\} \\ &< -\log_2 \min\left\{a_i, \frac{1}{a_i} : 1 \le i \le 5\right\} = p(\mathbf{X}, \mathbf{C}). \end{aligned} \tag{3.9}$$

The claimed result then follows from Lemma 3.1. The proof is complete. □

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