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Research Article

Exact Solutions for the Generalized BBM Equation with Variable Coefficients

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The variational iteration algorithm combined with the exp-function method is suggested to solve the generalized Benjamin-Bona-Mahony equation (BBM) with variable coefficients. Periodic and soliton solutions are formally derived in a general form. Some particular cases are considered.

1. Introduction

The BBM equation

$$u_t + uu_x + u_x - \mu u_{xxt} = 0, \tag{1.1}$$

which describes approximately the unidirectional propagation of long waves in certain nonlinear dispersive systems, has been proposed by Benjamin et al. in 1972 [1] as a more satisfactory model than the KdV equation [2]

$$u_t + uu_x + u_{xxx} = 0. \tag{1.2}$$

It is easy to see that (1.1) can be derived from the equal width EW-equation [3]:

$$u_t + uu_x - \mu u_{xxt} = 0, \tag{1.3}$$

by means of the change of variable $u = u + 1$, that is, by replacing u with $u + 1$. This last equation is considered as an equally valid and accurate model for the same wave phenomena simulated by (1.1) and (1.2). On the other hand, some researches analyzed the generalized KdV equation with variable coefficients

$$u_t + \sigma(t)u^p u_x + \mu(t)u_{xxx} = 0, \tag{1.4}$$

because this model has important applications in several fields of science [4–7].

Motivated by these facts, we will consider here the generalized EW-equation with variable coefficients

$$u_t + \sigma(t)u^p u_x - \mu(t)u_{xxt} = 0. \tag{1.5}$$

Using the solutions of (1.5) we obtain exact solutions to the generalized BBM equation

$$u_t + \sigma(t)(u+1)^p u_x - \mu(t)u_{xxt} = 0, \tag{1.6}$$

of order $p > 0$.

2. Exact Solutions to Generalized BBM Equation

2.1. The Variational Iteration Method

Consider the following nonlinear equation:

$$Lu(x,t) + Nu(x,t) = g(x,t), \tag{2.1}$$

where L and N are linear and nonlinear operators, respectively, and $g(x,t)$ is an inhomogeneous term. According to the variational iteration method (VIM) [8–14], a functional correction to (2.1) is given by

$$u_{n+1}(x,t) = u_n(x,t) + \int_0^t \theta(\tau)\left(Lu_n(x,\tau) + N\tilde{u}_n(x,\tau) - g(x,\tau)\right)d\tau, \tag{2.2}$$

where $\theta(\tau)$ is a general Lagrange's multiplier, which can be identified via the variational theory; the subscript $n \geq 0$ denotes the nth order approximation and $\tilde{u}$ is a restricted variation which means $\delta\tilde{u} = 0$. In this method, we first determine the Lagrange multiplier $\theta(\tau)$ that will be identified optimally via integration by parts. The successive approximation u_{n+1} of the solution u will be readily obtained upon using the determined Lagrangian multiplier and any selective function u_0. One of the advantages of the VIM, is the free choice of the initial solution $u_0(x,t)$. If we consider a special form to u_0 with arbitrary parameters, using the relations

$$u_n(x,t) = u_{n+1}(x,t), \qquad \frac{\partial^k}{\partial t^k}u_n(x,t) = \frac{\partial^k}{\partial t^k}u_{n+1}(x,t), \tag{2.3}$$

we can obtain a set of algebraic equations in the unknowns given by the parameters that appear in u_0. Solving this system, we have exact solutions to (2.1). To solve (1.5), we construct the following functional equation

$$u_{n+1}(x,t) = u_n(x,t) + \int_0^t \theta(\tau)(Lu_n(x,\tau) + N\tilde{u}_n(x,\tau))d\tau, \tag{2.4}$$

where

$$\begin{aligned} Lu_n(x,\tau) &= (u_n)_\tau(x,\tau), \\ N\tilde{u}_n(x,\tau) &= \sigma(\tau)(\tilde{u}+1)^p \tilde{u}_x(x,\tau) - \mu(\tau)\tilde{u}_{xx\tau}(x,\tau). \end{aligned} \tag{2.5}$$

Taking in (2.4) variation with respect to the independent variable u_n, and noticing that $\delta N\tilde{u}_n = 0$ we have

$$\begin{aligned} \delta u_{n+1}(x,t) &= \delta u_n(x,t) + \delta \int_0^t \theta(\tau)(Lu_n(x,\tau) + N\tilde{u}_n(x,\tau))d\tau \\ &= \delta u_n(x,t) + \theta(t)\delta u_n(x,t) - \int_0^t \theta'(\tau)\delta u_n(x,\tau)d\tau = 0. \end{aligned} \tag{2.6}$$

This yields the stationary conditions

$$\begin{aligned} 1 + \theta(t) &= 0, \\ \theta'(t) &= 0. \end{aligned} \tag{2.7}$$

Therefore,

$$\theta(t) = -1. \tag{2.8}$$

Substituting this value into (2.4) we obtain the formula

$$u_{n+1}(x,t) = u_n(x,t) - \int_0^t (Lu_n(x,\tau) + N\tilde{u}_n(x,\tau))d\tau. \tag{2.9}$$

Using the wave transformation

$$\xi = x + \lambda t + \xi_0, \tag{2.10}$$

setting

$$\frac{\partial}{\partial t}u_1(\xi) = \frac{\partial}{\partial t}u_0(\xi), \tag{2.11}$$

and performing one integration, (2.9) reduces to

$$\lambda u_0(\xi) + \frac{\sigma(t)}{p+1}u_0^{p+1}(\xi) - \lambda\mu(t)u_0''(\xi) = 0, \tag{2.12}$$

where for sake of simplicity we set the constant of integration equal to zero. With the change of variable

$$u_0(\xi) = v^{2/p}(\xi), \tag{2.13}$$

equation (2.12) converts to

$$\lambda v^2(\xi) - \frac{2\mu(t)(2-p)}{p^2}\lambda\left(v'\right)^2 - \frac{2\mu(t)}{p}\lambda v(\xi)v''(\xi) + \frac{\sigma(t)}{p+1}v(\xi)^4 = 0. \tag{2.14}$$

Observe that if $v(\xi)$ is a solution to (2.14), then $-v(\xi)$ is also a solution to this equation.

2.2. *The Exp-Function Method*

Recently, He and Wu [15] have introduced the Exp-function method to solve nonlinear differential equations. In particular, the Exp-function method is an effective method for solving nonlinear equations with high nonlinearity. The method has been used in a satisfactory way by other authors to solve a great variety of nonlinear wave equations [15–21]. The Exp-function method is very simple and straightforward, and can be briefly revised as follows: Given the nonlinear partial differential equation

$$F(u, u_x, u_t, u_{xx}, u_{xt}, u_{tt}, \ldots) = 0, \tag{2.15}$$

it is transformed to ordinary differential equation

$$F\left(u, u', u'', u''', u_{xt}, \ldots\right) = 0, \tag{2.16}$$

by mean of wave transformation $\xi = x + \lambda t + \xi_0$. Solutions to (2.16) can then be found using the expression

$$u(\xi) = \frac{\sum_{n=-c}^{d} a_n \exp(n\xi)}{\sum_{n=-p}^{q} b_n \exp\ (n\xi)}, \tag{2.17}$$

where c, d, p, and q are positive integers which are unknown to be determined later, a_n and b_n are unknown constants.

After balancing, we substitute (2.17) into (2.16) to obtain an algebraic systems in the variable $\zeta = \exp(n\xi)$. Solving the algebraic system we can obtain exact solutions to (2.16) and reversing, solutions to (2.15) in the original variables.

3. Solutions to (2.14) by the Exp-Function Method

Using the Exp-function method, we suppose that solutions to (2.14) can be expressed in the form

$$v(\xi) = \frac{\sum_{n=-1}^{1} a_n \exp(nr\xi)}{\sum_{m=-1}^{1} b_m \exp(mr\xi)} = \frac{a_{-1}\exp(-r\xi) + a_0 + a_1\exp(r\xi)}{b_{-1}\exp(-r\xi) + b_0 + b_1\exp(r\xi)}. \tag{3.1}$$

We obtain following solutions to (2.14):

$$\begin{aligned} v_1 &= \pm\frac{2\lambda k(p+1)(p+2)}{2(p+1)(p+2)\lambda\exp\left(\left(p/2\sqrt{\mu(t)}\right)\xi\right) - k^2\sigma(t)\exp\left(-\left(p/2\sqrt{\mu(t)}\right)\xi\right)}, \qquad \lambda = \lambda(t), \\ v_2 &= \pm\frac{2\lambda k(p+1)(p+2)}{\sigma(t)k^2\exp\left(\left(p/2\sqrt{\mu(t)}\right)\xi\right) - 2\lambda(p+1)(p+2)\exp\left(-\left(p/2\sqrt{\mu(t)}\right)\xi\right)}, \qquad \lambda = \lambda(t). \end{aligned} \tag{3.2}$$

Some special solutions are obtained if

$$\lambda = \lambda(t) = \pm\frac{k^2}{2(p^2+3p+2)}\sigma(t). \tag{3.3}$$

This choice gives solutions

$$v_3 = \pm\frac{k}{2}\operatorname{csch}\left(\frac{p}{2\sqrt{\mu(t)}}\xi\right), \qquad \lambda = \frac{k^2}{2(p^2+3p+2)}\sigma(t), \tag{3.4}$$

$$v_4 = \frac{k}{2}\operatorname{sech}\left(\frac{p}{2\sqrt{\mu(t)}}\xi\right), \qquad \lambda = -\frac{k^2}{2(p^2+3p+2)}\sigma(t), \tag{3.5}$$

$$v_5 = \pm\frac{k}{2}\csc\left(\frac{p}{2\sqrt{\mu(t)}}\xi\right), \qquad \lambda = -\frac{k^2}{2(p^2+3p+2)}\sigma(t). \tag{3.6}$$

Solution (3.6) follows from (3.4) with the identifications $\mu(t) \to -\mu(t)$ and $k \to -k\sqrt{-1}$.

$$v_6 = -\frac{k}{2}\sec\left(\frac{p}{2\sqrt{\mu(t)}}\xi\right), \qquad \lambda = -\frac{k^2}{2(p^2+3p+2)}\sigma(t). \tag{3.7}$$

Solution (3.7) follows from (3.4) with the identifications $\mu(t) \to -\mu(t)$ and $k \to -k$.

4. Particular Cases

4.1. Case 1: Solutions to (2.14) When $p = 2$

Equation (2.14) takes the form

$$\lambda v^2(\xi) - \lambda\mu(t)v(\xi)v''(\xi) + \frac{1}{3}\sigma(t)v(\xi)^4 = 0. \tag{4.1}$$

From (3.2) with $p = 2$:

$$\begin{aligned} v_7 &= \pm\frac{24\lambda k}{24\lambda\exp\left(\left(1/\sqrt{\mu(t)}\right)\xi\right) - k^2\sigma(t)\exp\left(-\left(1/\sqrt{\mu(t)}\right)\xi\right)}, \\ v_8 &= \pm\frac{24\lambda k}{k^2\sigma(t)\exp\left(\left(1/\sqrt{\mu(t)}\right)\xi\right) - 24\lambda\exp\left(-\left(1/\sqrt{\mu(t)}\right)\xi\right)}. \end{aligned} \tag{4.2}$$

From (3.3)–(3.7) with $p = 2$:

$$\begin{aligned} v_9 &= \pm\frac{k}{2}\operatorname{csch}\left(\frac{1}{\sqrt{\mu(t)}}\xi\right), \qquad \lambda = \frac{k^2}{24}\sigma(t), \\ v_{10} &= \pm\frac{k}{2}\operatorname{sech}\left(\frac{1}{\sqrt{\mu(t)}}\xi\right), \qquad \lambda = -\frac{k^2}{24}\sigma(t), \\ v_{11} &= \pm\frac{k}{2}\csc\left(\frac{1}{\sqrt{-\mu(t)}}\xi\right), \qquad \lambda = -\frac{k^2}{24}\sigma(t), \\ v_{12} &= \pm\frac{k}{2}\sec\left(\frac{1}{\sqrt{\mu(t)}}\xi\right), \qquad \lambda = -\frac{k^2}{24}\sigma(t). \end{aligned} \tag{4.3}$$

Other exact solutions are:

$$v_{13} = \pm\frac{\left(3a^2\exp\left(2\sqrt{-2/\mu(t)}\xi\right) + 2\sqrt{55}a\exp\left(\sqrt{-2/\mu(t)}\xi\right) - 22\right)k}{3a^2\exp\left(2\sqrt{-2/\mu(t)}\xi\right) + 22a\exp\left(\sqrt{-2/\mu(t)}\xi\right) + 22}, \qquad \lambda = -\frac{1}{3}k^2\sigma(t),$$

$$v_{14} = \pm\frac{\left(3a^2 \pm 2\sqrt{55}a\exp\left(\sqrt{-2/\mu(t)}\xi\right) - 22\exp\left(2\sqrt{-2/\mu(t)}\xi\right)\right)k}{3a^2 + 22a\exp\left(\sqrt{-2/\mu(t)}\xi\right) + 22\exp\left(2\sqrt{-2/\mu(t)}\xi\right)}, \qquad \lambda = -\frac{1}{3}k^2\sigma(t),$$

$$v_{15} = \pm k\left(1 - \frac{44\left(8+\sqrt{55}\right)}{3a\left(11+\sqrt{55}\right)\exp\left(\sqrt{-2/\mu(t)}\xi\right) + 22\left(8+\sqrt{55}\right)}\right), \qquad \lambda = -\frac{1}{3}k^2\sigma(t), \tag{4.4}$$

$$v_{16} = \pm\frac{k\left(a \pm \sinh\left(\sqrt{-2/\mu(t)}\xi\right)\right)}{\sqrt{a^2+1} \pm \cosh\left(\sqrt{-2/\mu(t)}\xi\right)}, \qquad \lambda = -\frac{1}{3}k^2\sigma(t),$$

$$v_{17} = \pm\frac{k\left(a \pm \cosh\left(\sqrt{-2/\mu(t)}\xi\right)\right)}{\sqrt{a^2+1} \pm \sinh\left(\sqrt{-2/\mu(t)}\xi\right)}, \qquad \lambda = -\frac{1}{3}k^2\sigma(t),$$

$$v_{18} = \pm\frac{k\cos\left(\sqrt{2/\mu(t)}\xi\right)}{1 \pm \sin\left(\sqrt{2/\mu(t)}\xi\right)}, \qquad \lambda = \frac{k^2}{3}\sigma(t).$$

4.2. Case 2: Solutions to (2.14) When $p = 4$

Equation (2.14) takes the form

$$\lambda v^2(\xi) + \mu(t)\lambda(v')^2 - \frac{\lambda}{2}\mu(t)v(\xi)v''(\xi) + \frac{1}{5}\sigma(t)v(\xi)^4 = 0. \tag{4.5}$$

From (3.2) with $p = 4$:

$$v_{19} = \pm\frac{60\lambda k}{60\lambda\exp\left(\left(2/\sqrt{\mu(t)}\right)\xi\right) - k^2\sigma(t)\exp\left(-\left(2/\sqrt{\mu(t)}\right)\xi\right)}, \qquad \lambda = \lambda(t),$$

$$v_{20} = \pm\frac{60\lambda k}{\sigma(t)k^2\exp\left(\left(2/\sqrt{\mu(t)}\right)\xi\right) - 60\lambda\exp\left(-\left(2/\sqrt{\mu(t)}\right)\xi\right)}, \qquad \lambda = \lambda(t). \tag{4.6}$$

From (3.3)–(3.7) with $p = 4$:

$$\begin{aligned} v_{21} &= \pm\frac{k}{2}\operatorname{csch}\left(\frac{2}{\sqrt{\mu(t)}}\xi\right), \qquad \lambda = \frac{k^2}{60}\sigma(t), \\ v_{22} &= \pm\frac{k}{2}\operatorname{sech}\left(\frac{2}{\sqrt{\mu(t)}}\xi\right), \qquad \lambda = -\frac{k^2}{60}\sigma(t), \\ v_{23} &= \pm\frac{k}{2}\csc\left(\frac{2}{\sqrt{-\mu(t)}}\xi\right), \qquad \lambda = -\frac{k^2}{60}\sigma(t), \\ v_{24} &= \pm\frac{k}{2}\sec\left(\frac{2}{\sqrt{-\mu(t)}}\xi\right), \qquad \lambda = -\frac{k^2}{60}\sigma(t). \end{aligned} \tag{4.7}$$

Other exact solutions are:

$$\begin{aligned} v_{25} &= \pm\frac{k\left(a\exp\left(\left(2/\sqrt{-\mu(t)}\right)\xi\right)-4\right)^2}{a^2\exp\left(\left(4/\sqrt{-\mu(t)}\right)\xi\right)+16a\exp\left(\left(2/\sqrt{-\mu(t)}\right)\xi\right)+16}, \qquad \lambda = -\frac{1}{5}k^2\sigma(t), \\ v_{26} &= \pm\frac{k\left(4\exp\left(2/\sqrt{-\mu(t)}\xi\right)-a\right)^2}{a^2+16a\exp\left(\left(2/\sqrt{-\mu(t)}\right)\xi\right)+16\exp\left(\left(4/\sqrt{-\mu(t)}\right)\xi\right)}, \qquad \lambda = -\frac{1}{5}k^2\sigma(t), \\ v_{27} &= \pm 2k\left(1-\frac{3}{2\pm\cos\left(\left(2/\sqrt{\mu(t)}\right)\xi\right)}\right), \qquad \lambda = -\frac{4}{5}k^2\sigma(t), \\ v_{28} &= \pm 2k\left(1-\frac{3}{2\pm\sin\left(\left(2/\sqrt{\mu(t)}\right)\xi\right)}\right), \qquad \lambda = -\frac{4}{5}k^2\sigma(t). \end{aligned} \tag{4.8}$$

It is clear that using (2.13) we obtain solutions to (1.5). Finally, observe that if $u_0(x,t)$ is a solution of (1.5), then the solutions $u(x,t)$ to the generalized BBM equation (1.6) are obtained as follows:

$$u(x,t) = u_0(x,t) - 1. \tag{4.9}$$

5. Conclusions

We have considered the generalized EW-equation with variable coefficients and the generalized BBM-equation with variable coefficients. We obtained analytic solutions by using the variational iteration method combined with the exp-function method. With the aid of *Mathematica* we have derived a lot of different types of solutions for these two models. Combined formal soliton-like solutions as well as kink solutions have been formally derived.

The results obtained show that the technique used here can be considered as a powerful method to analyze other types of nonlinear wave equations.

According to [22], there are alternative iteration alorithms, which might be useful for future work. Furthermore, various modifications of the exp-function method have been appeared in open literature, for example, the double exp-function method [23, 24].

Other methods for solving nonlinear differential equations may be found in [25–35].

We think that the results presented in this paper are new in the literature.

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