Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2010, Article ID 797084, 10 pages
doi:10.1155/2010/797084

Research Article

Exact Solutions to KdV6 Equation by Using a New Approach of the Projective Riccati Equation Method

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Received 21 January 2010; Revised 23 May 2010; Accepted 8 July 2010

Academic Editor: David Chelidze

We study a new integrable KdV6 equation from the point of view of its exact solutions by using an improved computational method. A new approach to the projective Riccati equations method is implemented and used to construct traveling wave solutions for a new integrable system, which is equivalent to KdV6 equation. Periodic and soliton solutions are formally derived. Finally, some conclusions are given.

1. Introduction

The sixth-order nonlinear wave equation

$$\left(\partial_x^3 + 8u_x\partial_x + 4u_{xx}\right)\left(u_t + u_{xxx} + 6u_x^2\right) = 0 \tag{1.1}$$

has been recently derived by Karasu-Kalkanl1 et al. [1] as a new integrable particular case of the general sixth-order wave equation

$$u_{xxxxxx} + \alpha u_x u_{xxxx} + \beta u_{xx}u_{xxx} + \gamma u_x^2 u_{xx} + \delta u_{tt} + \rho u_{xxxt} + \omega u_x u_{xt} + \sigma u_t u_{xx} = 0, \tag{1.2}$$

where, $\alpha, \beta, \gamma, \delta, \rho, \omega, \sigma$ are arbitrary parameters, and $u = u(x,t)$, is a differentiable function. By means of the change of variable

$$\begin{aligned} v &= u_x, \\ \widetilde{w} &= u_t + u_{xxx} + 6u_x^2, \end{aligned} \tag{1.3}$$

equation (1.1) converts to the Korteweg-de Vries equation with a source satisfying a third-order ordinary differential equation (KdV6)

$$\begin{aligned} v_t + v_{xxx} + 12vv_x - \widetilde{w}_x &= 0, \\ \widetilde{w}_{xxx} + 8v\widetilde{w}_x + 4\widetilde{w}v_x &= 0, \end{aligned} \tag{1.4}$$

which is regarded as a nonholonomic deformation of the KdV equation [2]. Setting

$$\begin{aligned} v(x,t) &= \frac{1}{2}u(x,-t), \\ \widetilde{w}(x,t) &= \frac{1}{2}w(x,t), \end{aligned} \tag{1.5}$$

the system (1.4) reduces to [2, 3]

$$\begin{aligned} u_t - 6uu_x - u_{xxx} + w_x &= 0, \\ w_{xxx} + 4uw_x + 2u_x w &= 0. \end{aligned} \tag{1.6}$$

A first study on the integrability of (1.6) has been done by Kupershmidt [2]. However, only at the end of the last year, Yao and Zeng [4] have derived the integrability of (1.6). More exactly, they showed that (1.6) is equivalent to the Rosochatius deformations of the KdV equation with self-consistent sources (RD-KdVESCS). This is a remarkable fact because the soliton equations with self-consistent sources (SESCS) have important physical applications. For instance, the KdV equation with self-consistent sources (KdVESCS) describes the interaction of long and short capillary-gravity waves [5]. On the other hand, when $w = 0$ the system (1.6) reduces to potential KdV equation, so that solutions of the potential KdV equation are solutions to (1.1). Furthermore, solving (1.6) we can obtain new solutions to (1.1). In the soliton theory, several computational methods have been implemented to handle nonlinear evolution equations. Among them are the tanh method [6], generalized tanh method [7, 8], the extended tanh method [9–11], the improved tanh-coth method [12, 13], the Exp-function method [14–16], the projective Riccati equations method [17], the generalized projective Riccati equations method [18–23], the extended hyperbolic function method [24], variational iteration method [25–27], He's polynomials [28], homotopy perturbation method [29–31], and many other methods [32–35], which have been used in a satisfactory way to obtain exact solutions to NLPDEs. Exact solutions to system (1.6) and (1.1) have been obtained using several methods [3, 4, 36–38]. In this paper, we obtain exact solutions to system (1.6). However, our idea is based on a new version of the projective Riccati method which can be considered as a generalized method, from which all other methods can be derived. This paper is organized as follows. In Section 2 we briefly review the new improved projective Riccati equations method. In Section 3 we give the mathematical framework to search exact for solutions to the system (1.6). In Section 4, we mention a new sixth-order KdV system from which novel solutions to (1.6) can be derived. Finally, some conclusions are given.

2. The Method

In the search of the traveling wave solutions to nonlinear partial differential equation of the form

$$P(u, u_x, u_t, u_{xx}, u_{xt}, u_{tt}, \dots) = 0, \tag{2.1}$$

the first step consists in use the wave transformation

$$u(x,t) = v(\xi), \quad \xi = x + \lambda t, \tag{2.2}$$

where λ is a constant. With (2.2), equation (2.1) converts to an ordinary differential equation (ODE) for the function $v(\xi)$

$$P\left(v, v', v'', \dots\right) = 0. \tag{2.3}$$

To find solutions to (2.3), we suppose that $v(\xi)$ can be expressed as

$$v(\xi) = H\left(f(\xi), g(\xi)\right), \tag{2.4}$$

where $H(f(\xi), g(\xi))$ is a *rational function* in the new variables $f(\xi)$, $g(\xi)$ which are solutions to the system

$$\begin{aligned} f'(\xi) &= \rho f(\xi) g(\xi), \\ g^2(\xi) &= R\left(f(\xi)\right), \end{aligned} \tag{2.5}$$

being $\rho \neq 0$ an arbitrary constant to be determinate and $R(f(\xi))$ a rational function in the variable $f(\xi)$. Taking

$$f(\xi) = \phi^N(\xi), \tag{2.6}$$

where $\phi(\xi) \neq 0$, and $N \neq 0$, then (2.5) reduces to

$$\begin{aligned} \phi'(\xi) &= \frac{\rho}{N} \phi(\xi) g(\xi), \\ g^2(\xi) &= R\left(\phi^N(\xi)\right). \end{aligned} \tag{2.7}$$

From (2.7) we obtain

$$\left(\phi'(\xi)\right)^2 = \frac{\rho^2}{N^2}\,\phi^2(\xi)R\left(\phi^N\right). \tag{2.8}$$

Let $N = -1$ and $R(f(\xi)) = \alpha + \beta f(\xi) + \gamma f(\xi)^2$, with $\alpha \neq 0$. In this case, (2.8) reduces to

$$\left(\phi'(\xi)\right)^2 = \rho^2\phi(\xi)^2 R\left(\phi(\xi)^{-1}\right) = \rho^2\left(\alpha\phi^2(\xi) + \beta\phi(\xi) + \gamma\right), \tag{2.9}$$

and (2.5) are transformed into

$$\begin{aligned} f'(\xi) &= \rho f(\xi) g(\xi), \\ g^2(\xi) &= \alpha + \beta f(\xi) + \gamma f(\xi)^2. \end{aligned} \tag{2.10}$$

The following are solutions to (2.9):

$$\begin{aligned} \phi_1(\xi) &= \frac{1}{4\alpha}\left(-2\beta + (1-\Delta)\sinh\left(\rho\sqrt{\alpha}\xi\right) + (1+\Delta)\cosh\left(\rho\sqrt{\alpha}\xi\right)\right), \\ \phi_2(\xi) &= \frac{1}{4\alpha}\left(-2\beta - (1-\Delta)\sinh\left(\rho\sqrt{\alpha}\xi\right) + (1+\Delta)\cosh\left(\rho\sqrt{\alpha}\xi\right)\right). \end{aligned} \tag{2.11}$$

Therefore, solutions to (2.10) are given by

$$\begin{aligned} f(\xi) &= \frac{-4\alpha}{2\beta \pm (1-\Delta)\sinh\left(\rho\sqrt{\alpha}\xi\right) - (1+\Delta)\cosh\left(\rho\sqrt{\alpha}\xi\right)}, \\ g(\xi) &= \frac{\sqrt{\alpha}\left((1+\Delta)\sinh\left(\rho\sqrt{\alpha}\xi\right) \mp (1-\Delta)\cosh\left(\rho\sqrt{\alpha}\xi\right)\right)}{2\beta \pm (1-\Delta)\sinh\left(\rho\sqrt{\alpha}\xi\right) - (1+\Delta)\cosh\left(\rho\sqrt{\alpha}\xi\right)}. \end{aligned} \tag{2.12}$$

In all cases $\Delta = \beta^2 - 4\alpha\gamma$.

3. Exact Solutions to the Integrable KdV6 System

Using the traveling wave transformation

$$\begin{aligned} u(x,t) &= v(\xi), \\ w(x,t) &= w(\xi), \\ \xi &= x + \lambda t + \xi_0, \end{aligned} \tag{3.1}$$

the system (1.6) reduces to

$$\left(\lambda v(\xi) - 3v^2(\xi) - v''(\xi) + w(\xi)\right)' = 0, \tag{3.2}$$

$$w'''(\xi) + 4v(\xi)w'(\xi) + 2v'(\xi)w(\xi) = 0. \tag{3.3}$$

Integrating (3.2) with respect to ξ and setting the constant of integration to zero we obtain

$$\begin{gathered} \lambda v(\xi) - 3v^2(\xi) - v''(\xi) + w(\xi) = 0, \\ w'''(\xi) + 4v(\xi)w'(\xi) + 2v'(\xi)w(\xi) = 0. \end{gathered} \tag{3.4}$$

Using the idea of the projective Riccati equations method [19–22], we seek solutions to (3.4) as follows:

$$\begin{gathered} v(\xi) = H_1\left(f(\xi), g(\xi)\right) = \sum_{0}^{M} a_i f^i(\xi) + \sum_{M+1}^{2M} a_i g(\xi) f^{i-(M+1)}(\xi), \\ w(\xi) = H_2\left(f(\xi), g(\xi)\right) = \sum_{0}^{N} b_i f^i(\xi) + \sum_{N+1}^{2N} b_i g(\xi) f^{i-(N+1)}(\xi), \end{gathered} \tag{3.5}$$

where $f(\xi)$ and $g(\xi)$ satisfy the system given by (2.10) (with $\rho = 1$). Substituting (3.5) into (3.4), after balancing we have that

$$M = 2, \tag{3.6}$$

and $N$is an arbitrary positive constant. By simplicity we take $N = M$. Therefore, (3.5) reduce to

$$\begin{gathered} v(\xi) = H_1\left(f(\xi), g(\xi)\right) = \sum_{0}^{2} a_i f^i(\xi) + \sum_{3}^{4} a_i g(\xi) f^{i-(3)}(\xi), \\ w(\xi) = H_2\left(f(\xi), g(\xi)\right) = \sum_{0}^{2} b_i f^i(\xi) + \sum_{3}^{4} b_i g(\xi) f^{i-(3)}(\xi). \end{gathered} \tag{3.7}$$

Substituting this last two equations into (3.4), using (2.10) we obtain an algebraic system in the unknowns a_0, a_1, a_2, a_3, a_4, b_0, b_1, b_2, b_3, b_4, λ, α, β, and γ. Solving it and using (3.7), (2.12), and (3.1) we have the following set of new nontrivial solutions to KdV6 system (1.6). In all cases, $a_1 = a_3 = b_1 = b_3 = \beta = 0$

$$\lambda = \frac{2b_4 \mp \sqrt{\alpha^2 a_4^2 + 9b_4^2}}{a_4}, \quad b_0 = \frac{1}{6}\left(-\frac{\alpha b_4}{a_4} - \frac{6b_4^2}{a_4^2} \pm \frac{2b_4\sqrt{\alpha^2 a_4^2 + 9b_4^2}}{a_4^2} \right),$$
$$b_2 = -a_4 b_4, \quad a_0 = \frac{-\alpha a_4 + 3b_4 \mp \sqrt{\alpha^2 a_4^2 + 9b_4^2}}{6a_4}, \quad a_2 = -a_4^2, \quad \gamma = a_4^2. \tag{3.8}$$

A combined formal soliton solution is:

$$\begin{aligned} u_1(x,t) &= \frac{-\alpha a_4 + 3b_4 \mp \sqrt{\alpha^2 a_4^2 + 9b_4^2}}{6a_4} \\ &\quad + \left(-a_4^2\right)\left(\frac{-4\alpha}{\pm(1+4\alpha a_4^2)\sinh[\sqrt{\alpha}\xi] - (1-4\alpha a_4^2)\cosh[\sqrt{\alpha}\xi]} \right)^2 \\ &\quad + a_4\left(\frac{-4\alpha}{\pm(1+4\alpha a_4^2)\sinh[\sqrt{\alpha}\xi] - (1-4\alpha a_4^2)\cosh[\sqrt{\alpha}\xi]} \right) \\ &\qquad \times \left(\frac{\sqrt{\alpha}\left((1-4\alpha a_4^2)\sinh[\sqrt{\alpha}\xi] \mp (1+4\alpha a_4^2)\cosh[\sqrt{\alpha}\xi]\right)}{\pm(1+4\alpha a_4^2)\sinh[\sqrt{\alpha}\xi] - (1-4\alpha a_4^2)\cosh[\sqrt{\alpha}\xi]} \right), \\ w_1(x,t) &= \frac{1}{6}\left(-\frac{\alpha b_4}{a_4} - \frac{6b_4^2}{a_4^2} \pm \frac{2b_4\sqrt{\alpha^2 a_4^2 + 9b_4^2}}{a_4^2} \right) \\ &\quad + (-a_4 b_4)\left(\frac{-4\alpha}{\pm(1+4\alpha a_4^2)\sinh[\sqrt{\alpha}\xi] - (1-4\alpha a_4^2)\cosh[\sqrt{\alpha}\xi]} \right)^2 \\ &\quad \times \left(\frac{\sqrt{\alpha}\left((1-4\alpha a_4^2)\sinh[\sqrt{\alpha}\xi] \mp (1+4\alpha a_4^2)\cosh[\sqrt{\alpha}\xi]\right)}{\pm(1+4\alpha a_4^2)\sinh[\sqrt{\alpha}\xi] - (1-4\alpha a_4^2)\cosh[\sqrt{\alpha}\xi]} \right), \end{aligned} \tag{3.9}$$

where a_4, b_4, α are arbitrary constants, and $\xi = x + \lambda t + \xi_0$.

Furthermore,

$$\lambda = \frac{8\alpha^2 + 20\alpha a_0 + 15a_0^2}{2\alpha + 3a_0}, \quad b_0 = -\frac{2(4\alpha^2 a_0 + 7\alpha a_0^2 + 3a_0^3)}{2\alpha + 3a_0},$$
$$b_4 = 0, \quad a_4 = 0, \quad b_2 = -\frac{2(4\alpha\gamma a_0 + 3\gamma a_0^2)}{2\alpha + 3a_0}, \quad a_2 = -2\gamma. \tag{3.10}$$

A soliton solution is given by

$$
\begin{aligned}
u_2(x,t) &= a_0 + (-2\gamma)\left(\frac{-4\alpha}{\pm(1+4\alpha\gamma)\sinh\left[\sqrt{\alpha}\xi\right] - (1-4\alpha\gamma)\cosh\left[\sqrt{\alpha}\xi\right]}\right)^2, \\
w_2(x,t) &= -\frac{2\left(4\alpha^2 a_0 + 7\alpha a_0^2 + 3a_0^3\right)}{2\alpha + 3a_0} \\
&\quad + \left(-\frac{2\left(4\alpha\gamma a_0 + 3\gamma a_0^2\right)}{2\alpha + 3a_0}\right) \\
&\quad \times \left(\frac{-4\alpha}{\pm(1+4\alpha\gamma)\sinh\left[\sqrt{\alpha}\xi\right] - (1-4\alpha\gamma)\cosh\left[\sqrt{\alpha}\xi\right]}\right)^2,
\end{aligned}
\tag{3.11}
$$

where a_0, α, γ are arbitrary constants and $\xi = x + \lambda t + \xi_0$.

3.1. A New System

A direct calculation shows that (1.1) reduces to

$$
u_{xxxxxxx} + 20u_x u_{xxxx} + 40u_{xx}u_{xxx} + 120u_x^2 u_{xx} + u_{xxxt} + 4u_{xx}u_t + 8u_x u_{xt} = 0. \tag{3.12}
$$

On the other hand, it is easy to see that (3.12) can be written as

$$
\left(\partial_x^2 + 4u_{xx}\partial_x^{-1} + 8u_x\right)\left(u_{xt} + u_{xxxx} + 12u_x u_{xx}\right) = 0. \tag{3.13}
$$

Using the analogy between KdV equation and MKdV equation and motivated by the structure of (3.13), the authors in [38] have introduced the so-called MKdV6 equation

$$
\left(\partial_x^3 + 8v_x^2\partial_x + 8v_{xx}\partial_x^{-1}v_x\partial_x\right)\left(v_t + v_{xxx} + 4v_x^3\right) = 0, \tag{3.14}
$$

and they showed that

$$
\begin{aligned}
\left(\partial_x^3 + 8u_x\partial_x + 4u_{xx}\right)\left(u_t + u_{xxx} + 6u_x^2\right) &= \left(2v_x + \frac{\sqrt{2}}{2i\partial_x}\right), \\
\left(\partial_x^3 + 8v_x^2\partial_x + 8v_{xx}\partial_x^{-1}v_x\partial_x\right)\left(v_t + v_{xxx} + 4v_x^3\right) &= 0,
\end{aligned}
\tag{3.15}
$$

where $v_x^2 + \sqrt{2}/2iv_{xx}$ is the Miura transformation between KdV6 equation (1.1) and MKdV6 equation (3.14). Therefore, solving (3.14), according to (3.15), solutions to (1.1) are obtained. Setting $w_x = v_x^2$, then the new MKdV6 equation is equivalent to new system

$$\begin{gathered} v_{xxxxxx} + 20v_x^2 v_{xxxx} + 80v_x v_{xx} v_{xxx} + 20v_{xx}^3 + 120v_x^4 v_{xx} + v_{xxxt} + 8v_x^2 v_{xt} + 4v_{xx} w_t = 0, \\ w_{xx} - 2v_x v_{xx} = 0. \end{gathered} \tag{3.16}$$

In equivalent form, with $s = v_x$, $w = v_t + v_{xxx} + 4v_x^3$, from (3.14) the following system is derived:

$$\begin{gathered} s_t + s_{xxx} + 12s^2 s_x - w_x = 0, \\ w_{xxx} + 8s^2 w_x + 8s_x z = 0, \\ z_x - s w_x = 0. \end{gathered} \tag{3.17}$$

We believe that traveling wave solutions to these systems can be obtained using the method used here. By reasons of space, we omit them.

4. Conclusions

In this paper we have derived two new soliton solutions to KdV6 system (1.2) by using a new approach of the improved projective Riccati equations method. The results show that the method is reliable and can be used to handle other NLPDE's. Other methods such as tanh, tanh-coth, and exp-function methods can be derived from the new version of the projective Riccati equation method. Moreover, new methods can be obtained using the exposed ideas in the present paper. Other methods related to the problem of solving nonlinear PDEs exactly may be found in [39, 40].

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