Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2010, Article ID 898062, 40 pages
doi:10.1155/2010/898062

Research Article

The Wall Properties Effect on Peristaltic Transport of Micropolar Non-Newtonian Fluid with Heat and Mass Transfer

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Received 26 April 2010; Accepted 21 July 2010

Academic Editor: Mehrdad Massoudi

The problem of the unsteady peristaltic mechanism with heat and mass transfer of an incompressible micropolar non-Newtonian fluid in a two-dimensional channel. The flow includes the viscoelastic wall properties and micropolar fluid parameters using the equations of the fluid as well as of the deformable boundaries. A perturbation solution is obtained, which satisfies the momentum, angular momentum, energy, and concentration equations for case of free pumping (original stationary fluid). Numerical results for the stream function, temperature, and concentration distributions are obtained. Several graphs of physical interest are displayed and discussed.

1. Introduction

The peristaltic transport is traveling contraction wave along a tube-like structure, and it results physiologically from neuromuscular properties of any tubular smooth muscle. Peristalsis is now well known to physiologists to be one of the major mechanisms for fluid transport in many biological systems. In particular, a peristaltic mechanism may be involved in swallowing food through the esophagus, in urine transport from the kidney to the bladder through the ureter, movement of chyme in the gastrointestinal tract, in the transport of spermatozoa in the ducts efferentes of the male reproductive tracts and in the cervical canal, movement of ovum in the female fallopian tubes, the transport of lymph in the lymphatic vessels, and the vasomotion in small blood vessels such as arterioles, venues, and capillaries. In addition, peristaltic pumping occurs in many practical applications involving biomedical systems. It has now been accepted that most of the physiological fluids behave in general like suspensions of deformable or rigid particles in a Newtonian fluid. Blood, for example, is a suspension of red cells, white cells, and platelets in plasma. Another example is cervical mucus, which is a suspension of macromolecules in a water-like liquid [1]. However only a few studies have considered this aspect since the initial investigation by Raju and Devanathan [2, 3]. The biviscosity fluid can represent the behavior of blood in vessels of small diameter as the mean shear rate is about 20–150 s^{-1} [4].

Several studies have been made to analyze both theoretical and experimental aspects of the peristaltic motion of non-Newtonian fluids in different situations. Böhme and Friedrich [5] studied the mechanism of peristaltic transport of an incompressible viscoelastic fluid by means of an infinite train of sinusoidal waves traveling along the wall of the duct in the case of plane flow. The effects of an Oldroyd-B fluid on the peristaltic mechanism are examined by Hayat et al. [6], under the long wavelength assumption. They noticed that in the narrow part of the channel, the behavior of an Oldroyd-B fluid is much more different from that of a Newtonian fluid than in the wide part of the channel. El-Shehawey et al. [7] studied the peristaltic motion of an incompressible non-Newtonian fluid through a porous medium. They showed that the pressure rise increases as the permeability decreases and noted that both pressure rise and friction force do not depend on permeability parameter at a certain value of flow rate. Haroun [8] investigated the peristaltic flow of a third-order fluid in an asymmetric channel under the assumption of long wavelength approximation. He expanded the velocity components and pressure in a regular perturbation series in a small parameter Deborah number that contained the non-Newtonian coefficients appropriate to shear-thinning. The effects of slip boundary conditions on the dynamics of fluids in porous media is investigated by El-Shehawey et al. [9]. They studied the flow of non-Newtonian Maxwell fluids in an axisymmetric cylindrical tube (pore), in which the flow is induced by traveling transversal waves on the tube wall (peristaltic transport). Hakeem et al. [10] discussed the effect of an endoscope on the peristaltic mechanism of a generalized Newtonian fluid. Mekheimer [11] studied the effect of a uniform magnetic field on peristaltic transport of a blood in nonuniform two-dimensional channels, when blood is represented by a couple-stress fluid. Tsiklauri and Beresnev [12] analyzed the effect of viscoelasticity on the dynamics of fluids in porous media by studying the peristaltic flow of a Maxwell fluid in circular tube, in which the flow is induced by a wave traveling on the tube wall.

The micropolar fluid represents fluids, which consist of rigid, randomly oriented (or spherical) particles suspended in a viscous medium where the deformation of the particles is ignored. The theory of thermo-micropolar fluid was developed by Eringen [13]. Agrawal and Dhanapal [14] considered the micropolar fluid to be the model of blood flow in small arteries, and the calculation of theoretical velocity profiles is observed, in good agreement with experimental data [15]. They have many applications in physiological and chemical engineering [16]. Srinivasachrya et al. [17] studied the problem of peristaltic transport of a micropolar fluid in a circular tube. Girija Devi and Devanathan [18] studied the peristaltic flow of a micropolar fluid in a cylindrical tube with a sinusoidal deformation of small amplitude travelling down its flexible wall for the case of low Reynolds number flow devoid of wall properties like tension and damping. However, the wall properties are essential to be taken into consideration in various real situations. The peristaltic motion of a simple microfluid which accounts for microrotation and microstretching of the particles contained in a small volume element is studied by Philip and Chandra [19], using long wavelength approximation.

Combined heat and mass transfer problems are of importance in many processes and have, therefore, received a considerable amount of attention in recent years. In processes such as drying, evaporation at the surface of a water body, energy transfer in a wet cooling tower and the flow in a desert cooler, heat and mass transfer occurs simultaneously. Possible applications of this type of flow can be found in many industries. For example, in the power industry, among the methods of generating electric power is one in which electrical energy is extracted directly from a moving conducting fluid. Many practical diffusive operations involve the molecular diffusion of a species [20]. Radhakrishnamacharya and Radhakrishna [21] investigated the peristaltic flow with heat transfer in a nonuniform channel. The effect of elasticity of the flexible walls on peristaltic transport of an incompressible viscous fluid, with heat transfer, in a two-dimensional uniform channel under long wavelength approximation is explained by Radhakrishnamacharya and Srinivasulu [22]. El Dabe et al. [23] studied heat and mass transfer of a steady slow motion of a Rivlin-Ericksen fluid in tube of varying cross-section with suction. Agrawal et al. [24] obtained numerical solutions of flow and heat transfer of a micropolar fluid at a stagnation point on a porous stationary wall. They observed that the heat sources increase the velocity and temperature in the pipe while the heat sinks decrease them.

Muthu et al. [25] carried out a study of the peristaltic motion of an incompressible micropolar fluid in two-dimensional channel. They investigated the effects of viscoelastic wall properties and micropolar fluid parameters on the flow using the equations of the fluid as well as of the deformable boundaries. The non-Newtonian property and equations of heat and mass transfer were not taken into their consideration. Because of the wide range of practical importance of the heat and mass transfer, the present study considered the heat and mass transfer of an unsteady peristaltic motion of a micropolar non-Newtonian biviscosity fluid [4]. The following analysis includes the dynamic boundary condition. Analytical approximate solutions for the stream function, microrotation velocity, temperature, and concentration equations are obtained as a power series in terms of the small amplitude ratio. We have shown the relation between the different parameters of motion in order to investigate how to control the motion of the fluid by changing these parameters.

2. Formulation of the Problem

Consider a two-dimensional symmetric unsteady flow of an incompressible micropolar biviscosity fluid in an infinite channel of uniform thickness $2d$, with heat and mass transfer. The walls of the channel are flexible membranes on which they are imposed traveling sinusoidal waves of moderate amplitude (see Figure 1).

We choose a rectangular coordinate system such that the axes x and y are in the directions of wave propagation and normal to the mean position of the membranes, respectively. The origin is located at the center line of the channel. It was shown by Wilson and Taylor [26] that the limiting process by which the biviscosity model is approached with the limiting process implicit in lubrication theory. Models of this type are much easier to handle mathematically than many models such as Oldroyd's and represent the experimental facts just as well, at least in many cases.

The biviscosity model (which has been used in similar contexts by Nakayama and Sawada [4]) can be written as

$$\tau_{ij} = \begin{cases} 2\left(\mu_B + \dfrac{p_y}{\sqrt{2\pi}}\right)e_{ij}, & \pi \geq \pi_c, \\ 2\left(\mu_B + \dfrac{p_y}{\sqrt{2\pi_c}}\right)e_{ij}, & \pi < \pi_c. \end{cases} \tag{2.1}$$

The following quantity is introduced as a nondimensional parameter including π_c

$$\gamma = \mu_B \frac{\sqrt{2\pi_c}}{p_y}, \tag{2.2}$$

where μ_B is the plastic viscosity, p_y is the yielding stress, $\pi = e_{ij}e_{ij}$, where e_{ij} is the (i, j) component of the deformation rate and the value of γ denotes the upper limit of apparent viscosity coefficient. The biviscosity model is approached in the limit $\gamma \to \infty$ (see Nakayama and Sawada [4] for a few further details).

The governing continuity, momentum, angular momentum, temperature, and concentration equations for this problem can be written as [13]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2.3}$$

$$\rho\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] = -\frac{\partial P}{\partial x} + \left(\frac{2\mu_B(1+\gamma^{-1})+k}{2}\right)\nabla^2 u + k\frac{\partial N_\theta}{\partial y}, \tag{2.4}$$

$$\rho\left[\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right] = -\frac{\partial P}{\partial y} + \left(\frac{2\mu_B(1+\gamma^{-1})+k}{2}\right)\nabla^2 v - k\frac{\partial N_\theta}{\partial x}, \tag{2.5}$$

$$\rho J\left[\frac{\partial N_\theta}{\partial t} + u\frac{\partial N_\theta}{\partial x} + v\frac{\partial N_\theta}{\partial y}\right] = -2kN_\theta + \overline{\gamma}\ \nabla^2 N_\theta + k\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right], \tag{2.6}$$

$$\begin{aligned}\rho C_p\left[\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right] &= k_c\nabla^2 T + \left(\frac{2\mu_B(1+\gamma^{-1})+k}{2}\right)\\ &\quad\times\left[2\left(\frac{\partial u}{\partial x}\right)^2 + 2\left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2\right]\\ &\quad+2k\left[{N_\theta}^2 - N_\theta\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\right] + \overline{\gamma}\left[\left(\frac{\partial N_\theta}{\partial x}\right)^2 + \left(\frac{\partial N_\theta}{\partial y}\right)^2\right]\\ &\quad+\alpha_c\left[\frac{\partial T}{\partial x}\frac{\partial N_\theta}{\partial y} - \frac{\partial T}{\partial y}\frac{\partial N_\theta}{\partial x}\right],\end{aligned} \tag{2.7}$$

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right) + \frac{Dk_T}{T_m}\left(\frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial x^2}\right), \tag{2.8}$$

where $u(x, y, t)$ and $v(x, y, t)$ are the velocity components in the x and y directions, respectively, $N_\theta(x,y,t)$ is the microrotation velocity component in the direction normal to both the x and y axes. Here ρ is the density of the fluid, P is the pressure, J is the microinertia constant, k is the vortex viscosity coefficient (also known as the coefficient of gyroviscosity), and $\overline{\gamma}$ is the spin-gradient viscosity.

Further, the material constants μ_B and k satisfy the following inequalities [13]:

$$2\mu_B + k \geq 0, \quad k \geq 0, \ \mu_B \geq 0. \tag{2.9}$$

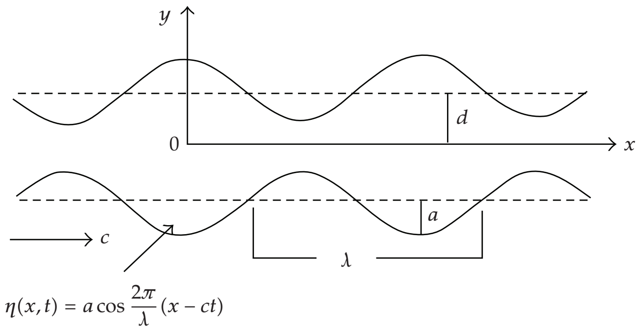

Figure 1: Geometry of two-dimensional peristaltic channel.

The temperature and concentration of fluid are $T(x,y,t)$ and $C(x,y,t)$, C_p is the specific heat at constant pressure, D is the coefficient of mass diffusivity, K_T is the thermal diffusion ratio, T_m is the mean fluid temperature, k_c is the thermal conductivity and α_c is the heat conduction parameter for a micropolar fluid. The last term in (2.8) signifies the thermal-diffusion effect.

We consider a symmetric motion of the flexible walls in the boundary conditions. Let the vertical displacements of the upper and lower walls be η and $-\eta$, respectively, where

$$\eta(x,t) = a\cos\frac{2\pi}{\lambda}(x-ct), \tag{2.10}$$

where a is the amplitude, λ is the wavelength, and c is the wave speed.

We assume that the walls are inextensible so that only lateral motion takes place and the horizontal displacement of the wall is zero [25].

Thus the no-slip boundary conditions for the velocity and microrotation are

$$u = 0, \quad N_\theta = 0 \quad \text{at } y = \pm(d+\eta(x,t)). \tag{2.11}$$

The dynamic boundary conditions are imposed on the fluid by the symmetric motion of the flexible walls, which can be written as [18]

$$\frac{\partial L(\eta)}{\partial x} = -\rho\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] + \left(\frac{2\mu_B(1+\gamma^{-1})+k}{2}\right)\nabla^2 u + k\frac{\partial N_\theta}{\partial y}, \quad \text{at } y = \pm(d+\eta(x,t)), \tag{2.12}$$

where

$$\frac{\partial L(\eta)}{\partial x} = -\ell\frac{\partial^3\eta}{\partial x^3} + m\frac{\partial^3\eta}{\partial t^2\partial x} + n\frac{\partial^3\eta}{\partial t\partial x}. \tag{2.13}$$

Here $L(\eta)$ is the pressure at the walls, ℓ is the tension in the membrane, m is the mass per unit area, and n is the coefficient of viscous damping force.

The plate temperature starts oscillating about a nonzero mean temperature. Under these physical conditions, the temperature and concentration at the upper wall give

$$T = T_w + \frac{a}{d}(T_w - T_m)\cos\left(\frac{2\pi(x-ct)}{\lambda}\right),$$
$$C = C_w + \frac{a}{d}(C_w - C_m)\cos\left(\frac{2\pi(x-ct)}{\lambda}\right), \quad \text{at } y = d + \eta(x,t), \tag{2.14}$$

where C_m is the mean fluid concentration, T_w and C_w are the uniform temperature and concentration at the lower wall, that is,

$$T = T_w, \quad C = C_w, \quad \text{at } y = -(d + \eta(x,t)). \tag{2.15}$$

Equation (2.3) allows the use of the stream function $\psi(x,y,t)$ such that

$$u = \frac{\partial \psi}{\partial y}, \qquad v = -\frac{\partial \psi}{\partial x}. \tag{2.16}$$

By introducing the following nondimensional quantities:

$$x' = \frac{x}{d}, \quad y' = \frac{y}{d}, \quad t' = \frac{tc}{d}, \quad u' = \frac{u}{c}, \quad v' = \frac{v}{c}, \quad N_\theta{}' = \frac{dN_\theta}{c}, \quad P' = \frac{P}{\rho c^2}, \quad \eta' = \frac{\eta}{d},$$
$$\psi' = \frac{\psi}{cd}, \quad J' = \frac{J}{d^2}, \quad T' = \frac{T - T_m}{T_w - T_m}, \quad C' = \frac{C - C_m}{C_w - C_m}, \quad \varepsilon = \frac{a}{d}, \quad \alpha = \frac{2\pi d}{\lambda}, \quad \mu_1 = \frac{k}{\mu_B},$$
$$\mathrm{Re} = \frac{\rho c d}{\mu_B}, \quad N = \left(\frac{\mu_1}{2 + \mu_1}\right)^{1/2}, \quad \overline{N} = \left(\frac{\mu_1}{2\,(1+\gamma^{-1}) + \mu_1}\right)^{1/2},$$
$$M = 2d\left(\frac{\mu_B}{\overline{\gamma}}\right)^{1/2}, \quad \overline{M} = \left(\frac{M^2\,(1-\overline{N}^2)}{(1-N^2)}\right)^{1/2}, \tag{2.17}$$
$$\Gamma = \frac{M}{\overline{\overline{M}}}, \quad R_\ell = \frac{4\mu\rho c d J}{\overline{\gamma}(2\mu_B + k)}, \quad K_2 = \frac{n\,d}{\mu_B}, \quad K_3 = \frac{\ell\rho d}{\mu_B{}^2}, \quad m_1 = \frac{m}{\rho d}, \quad \mathrm{Pr} = \frac{\mu_B C_\rho}{k_c},$$
$$\mathrm{Ec} = \frac{c^2}{C_\rho(T_w - T_m)}, \quad \mathrm{af} = \frac{\alpha_c c}{\mu_B C_\rho d}, \quad \mathrm{Sc} = \frac{\mu_B}{D\rho}, \quad \mathrm{Sr} = \frac{k_T(T_w - T_m)\rho D}{T_m(C_w - C_m)\mu_B},$$

substituting from (2.17) into (2.3)–(2.8), eliminating P between (2.4) and (2.5), and dropping the accent mark for simplicity, the governing equations may be written as

$$\frac{\partial}{\partial t}\nabla^2\psi + \psi_y\nabla^2\psi_x - \psi_x\nabla^2\psi_y = \frac{2\left(1+\gamma^{-1}\right)+\mu_1}{2\,\mathrm{Re}}\left[\nabla^2\nabla^2\psi\right] + \frac{\mu_1}{\mathrm{Re}}\nabla^2 N_\theta, \tag{2.18}$$

$$R_\ell\left(\frac{\partial N_\theta}{\partial t} + \psi_y\frac{\partial N_\theta}{\partial x} - \psi_x\frac{\partial N_\theta}{\partial y}\right) = 2\left(1-N^2\right)\nabla^2 N_\theta - N^2M^2\left(\nabla^2\psi + 2N_\theta\right), \tag{2.19}$$

$$\begin{aligned}\mathrm{Re}\cdot\mathrm{Pr}\left(\frac{\partial T}{\partial t} + \psi_y\frac{\partial T}{\partial x} - \psi_x\frac{\partial T}{\partial y}\right) &= \nabla^2 T + \frac{2\left(1+\gamma^{-1}\right)+\mu_1}{2}\mathrm{Pr}\cdot\mathrm{Ec}\times\left[4\psi_{xt}^2 + \left(\psi_{yy}-\psi_{xx}\right)^2\right]\\ &\quad + 2\mu_1\cdot\mathrm{Pr}\cdot\mathrm{Ec}\left(N_\theta^{\,2} + N_\theta\nabla^2\psi\right) + \frac{4}{M^2}\mathrm{Pr}\cdot\mathrm{Ec}\\ &\quad \times\left[\left(\frac{\partial N_\theta}{\partial x}\right)^2 + \left(\frac{\partial N_\theta}{\partial y}\right)^2\right] + \mathrm{af}\cdot\mathrm{Pr}\left[\frac{\partial T}{\partial x}\frac{\partial N_\theta}{\partial y} - \frac{\partial T}{\partial y}\frac{\partial N_\theta}{\partial x}\right],\end{aligned} \tag{2.20}$$

$$\mathrm{Sc}\cdot\mathrm{Re}\left(\frac{\partial C}{\partial t} + \psi_y\frac{\partial C}{\partial x} - \psi_x\frac{\partial C}{\partial y}\right) = \nabla^2 C + \mathrm{Sc}\cdot\mathrm{Sr}\cdot\nabla^2 T. \tag{2.21}$$

Also, the boundary conditions at $y = \pm(1+\eta(x,t))$, when $\eta(x,t) = \varepsilon\cos(\alpha(x-t))$, are

$$\begin{gathered}\psi_y = N_\theta = 0,\\ \frac{\partial L(\eta)}{\partial x} = -\left[\psi_{yt} + \psi_y\psi_{xy} - \psi_x\psi_{yy}\right] + \frac{2\left(1+\gamma^{-1}\right)+\mu_1}{2\,\mathrm{Re}}\left[\nabla^2\psi_y\right] + \frac{\mu_1}{\mathrm{Re}}\frac{\partial N_\theta}{\partial y},\end{gathered} \tag{2.22}$$

where

$$\frac{\partial L(\eta)}{\partial x} = -\frac{K_3}{\mathrm{Re}^2}\frac{\partial^3\eta}{\partial x^3} + m_1\frac{\partial^3\eta}{\partial t^2\partial x} + \frac{K_2}{\mathrm{Re}}\frac{\partial^2\eta}{\partial t\partial x}, \tag{2.23}$$

while

$$\begin{gathered}T = 1,\quad C = 1,\quad \text{at } y = -(1+\eta(x,\ t)),\\ T = 1+\varepsilon\cos(\alpha(x-t)),\quad C = 1+\varepsilon\cos(\alpha(x-t))\quad \text{at } y = 1+\eta(x,\ t).\end{gathered} \tag{2.24}$$

The parameter ε represents the amplitude ratio, α is wave number, Re is the Reynolds number, μ_1 is the ratio between the viscosity coefficient for micropolar fluids and the classical viscosity coefficient, and M is the micropolar fluid parameter characterizing spin-gradient viscosity. The parameter M can be thought of as a fluid property depending upon the size of the microstructure [18]. We note that R_ℓ is the modified Reynolds number and involves the quantity J. In this paper, we considered that the effect of microinertia is neglected and R_ℓ is taken to be zero [24]. The nondimensional quantities Pr are the Prandtl number, Ec is

Eckert number, af is the dimensionless heat conduction coefficient of the micropolar fluid, Sc is the Schmidt number, Sr is the Soret number and represents the thermal-diffusion effect, K_2 and K_3 represent the dissipative and rigiditive feature of walls, and m_1 indicates the stiffness property of walls.

3. Method of Solution

Assume the amplitude ratio ε of the wave to be small. As given by Muthu et al. [25], we seek for an approximate solution as a power series in terms of ε in the form

$$\begin{aligned}\psi(x,y,t) = \psi_0(y) &+ \frac{\varepsilon}{2}\left(\phi_1(y)e^{i\alpha(x-t)} + \phi_1^* e^{-i\alpha(x-t)}\right)\\ &+ \frac{\varepsilon^2}{2}\left(\phi_{20}(y) + \phi_{22}(y)e^{2i\alpha(x-t)} + \phi_{22}^*(y)e^{-2i\alpha(x-t)}\right) + o\left(\varepsilon^3\right),\end{aligned} \tag{3.1}$$

$$\begin{aligned}N_\theta(x,y,t) = N_0(y) &+ \frac{\varepsilon}{2}\left(\xi_1(y)e^{i\alpha(x-t)} + \xi_1^* e^{-i\alpha(x-t)}\right)\\ &+ \frac{\varepsilon^2}{2}\left(\xi_{20}(y) + \xi_{22}(y)e^{2i\alpha(x-t)} + \xi_{22}^*(y)e^{-2i\alpha(x-t)}\right) + o\left(\varepsilon^3\right),\end{aligned} \tag{3.2}$$

$$\begin{aligned}T(x,y,t) = T_0(y) &+ \frac{\varepsilon}{2}\left(T_1(y)e^{i\alpha(x-t)} + T_1^* e^{-i\alpha(x-t)}\right)\\ &+ \frac{\varepsilon^2}{2}\left(T_{20}(y) + T_{22}(y)e^{2i\alpha(x-t)} + T_{22}^*(y)e^{-2i\alpha(x-t)}\right) + o\left(\varepsilon^3\right),\end{aligned} \tag{3.3}$$

$$\begin{aligned}C(x,y,t) = C_0(y) &+ \frac{\varepsilon}{2}\left(C_1(y)e^{i\alpha(x-t)} + C_1^* e^{-i\alpha(x-t)}\right)\\ &+ \frac{\varepsilon^2}{2}\left(C_{20}(y) + C_{22}(y)e^{2i\alpha(x-t)} + C_{22}^*(y)e^{-2i\alpha(x-t)}\right) + o\left(\varepsilon^3\right).\end{aligned} \tag{3.4}$$

Here the asterisk denotes complex conjugate. The solutions for $\psi_0(y)$ and $N_0(y)$ are obtained as follows:

$$\begin{gathered}\frac{d\psi_0}{dy} = K'\left(y^2-1\right)\left(1+\frac{\overline{N}^2\Gamma^2}{(1-N^2)}\right) + \frac{2K'\overline{N}^2\Gamma^2}{(1-N^2)}\left(\frac{\cosh\left(N\overline{M}\right)-\cosh\left(N\overline{M}y\right)}{N\overline{M}\sinh\left(N\overline{M}\right)}\right),\\ N_0 = \frac{K'\Gamma^2}{(1-N^2)}\left(\frac{\sinh\left(N\overline{M}y\right)-y\sinh\left(N\overline{M}\right)}{\sinh\left(N\overline{M}\right)}\right),\end{gathered} \tag{3.5}$$

where $K' = (\mathrm{Re}/(2(1+\gamma^{-1})+\mu_1))(dP/dx)_0$ is the poiseuille flow parameter for the micropolar fluid. For pure peristalsis, which means that the flow is generated by wall motion only. The pressure gradient $(dP/dx)_0 = 0$; this implies that $K' = 0$, which gives $\psi_0(y) = 0$ and $N_0(y) = 0$. The expressions of ϕ_1 and ξ_1 for the case of free pumping can be written as

$$\phi_1(y) = A_3 \sinh \alpha y + A_4 \sinh \beta y + a_1 A_1 \sinh r_1 y + a_2 A_2 \sinh r_2 y, \tag{3.6}$$

$$\xi_1(y) = A_1 \sinh r_1 y + A_2 \sinh r_2 y, \tag{3.7}$$

where $\beta^2 = \alpha^2 - 2i\alpha \operatorname{Re}/(2\ (1+\gamma^{-1}) + \mu_1)$, $\lambda_1 = N^2 \overline{N}^2 M^2/(1-N^2)$, $\lambda_2 = N^2 M^2/(1-N^2)$,

$$\begin{aligned}
r_1 &= \left[\frac{1}{2}\left\{\lambda_2 - \lambda_1 + (\alpha^2+\beta^2) + \sqrt{((\lambda_1-\lambda_2)-(\alpha^2+\beta^2))^2 - 4(\alpha^2\beta^2 - \lambda_1\alpha^2 + \lambda_2\beta^2)}\right\}\right]^{1/2},\\
r_2 &= \left[\frac{1}{2}\left\{\lambda_2 - \lambda_1 + (\alpha^2+\beta^2) - \sqrt{((\lambda_1-\lambda_2)-(\alpha^2+\beta^2))^2 - 4(\alpha^2\beta^2 - \lambda_1\alpha^2 + \lambda_2\beta^2)}\right\}\right]^{1/2},\\
a_1 &= \frac{-2N^2}{r_1^2-\beta^2}, \quad a_2 = \frac{-2N^2}{r_2^2-\beta^2}, \quad A_1 = \frac{\operatorname{Re} N^2 M^2 \sinh r_2}{2d_1}, \quad A_2 = \frac{-\operatorname{Re} N^2 M^2 \sinh r_1}{2d_1},\\
d_1 &= \left(r_1^3 + \left(\lambda_1 - \lambda_2 - \alpha^2\right) r_1\right)\sinh r_2 \cosh r_1 - \left(r_2^3 + \left(\lambda_1 - \lambda_2 - \alpha^2\right) r_2\right)\sinh r_1 \cosh r_2,\\
A_3 &= \frac{1}{\alpha(\beta^2-\alpha^2)\cosh\alpha}\left(-\frac{2\operatorname{Re}\delta_1}{2(1+\gamma^{-1})+\mu_1} + 2N^2(r_1 A_1 \cosh r_1 + r_2 A_2 \cosh r_2)\right.\\
&\qquad \left. + a_1 A_1 r_1\left(r_1^2-\beta^2\right)\cosh r_1 + a_2 A_2 r_2 \left(r_2^2 - \beta^2\right)\cosh r_2\right),\\
A_4 &= \frac{1}{\beta(\beta^2-\alpha^2)\cosh\beta}\left(\frac{2\operatorname{Re}\delta_1}{2(1+\gamma^{-1})+\mu_1} - 2N^2(r_1 A_1 \cosh r_1 + r_2 A_2 \cosh r_2)\right.\\
&\qquad \left. - a_1 A_1 r_1\left(r_1^2-\alpha^2\right)\cosh r_1 + a_2 A_2 r_2 \left(r_2^2 - \alpha^2\right)\cosh r_2\right),\\
\delta_1 &= i\left(\frac{K_3\alpha^3}{\operatorname{Re}^2} - m_1\alpha^3\right) + K_2\frac{\alpha^2}{\operatorname{Re}}.
\end{aligned} \tag{3.8}$$

To obtain the mass and heat transfer, we shall use the perturbation scheme (3.3), (3.4) in (2.20) and (2.21), and equating the zero-order terms on both sides, we obtain the following set of equations:

$$\frac{d^2T_0}{dy^2} = -\operatorname{Ec}\cdot\operatorname{Pr}\left[\frac{d^2\psi_0}{dy^2}\left(2\mu_1 N_0 + \frac{2(1+\gamma^{-1})+\mu_1}{2}\frac{d^2\psi_0}{dy^2}\right) + 2\mu_1 N_0^2 + \frac{4}{M^2}\left(\frac{dN_0}{dy}\right)^2\right], \tag{3.9}$$

$$\frac{d^2C_0}{dy^2} = -\operatorname{Sc}\cdot\operatorname{Sr}\frac{d^2T_0}{dy^2}, \tag{3.10}$$

$$T_0(\pm 1) = 1, \qquad C_0(\pm 1) = 1. \tag{3.11}$$

In case of plane Poiseuille flow of micropolar fluid ($\varepsilon = 0$), the governing (3.9) and (3.10) may be solved in the light of the boundary conditions (3.11) with the constant pressure gradient $(\partial P/\partial x)_0$. At this stage, the temperature and concentration distributions T_0 and C_0, respectively, are given by

$$\begin{aligned}
T_0(y) = 1 - 2\mathrm{Pr}\cdot\mathrm{Ec}\cdot K'^2 \times \Bigg[&\frac{1}{6}\left(y^4-1\right) + \frac{1}{M^2\sinh^2 N\overline{M}} \\
&\times\Bigg\{\frac{1}{4}\left(y^2-1\right)\left(4\sinh^2 N\overline{M} + M^2\left(N^2\left(2-N^2\right) - \mu_1\left(1-N^2\right)^2\right)\right) \\
&- 4\sinh N\overline{M}\left(y\sinh N\overline{M}y - \sinh N\overline{M}\right) \\
&+ \frac{4\sinh N\overline{M}}{N\overline{M}}\left(\cosh N\overline{M}y - \cosh N\overline{M}\right) \\
&+ \frac{1}{8N^2}\left(N^2\left(2+N^2\right) + \mu_1\left(1-N^2\right)^2\right) \\
&\times\left(\cosh 2N\overline{M}y - \cosh 2N\overline{M}\right)\Bigg\}\Bigg], \\
C_0(y) = 1 + \mathrm{Sc}\,\mathrm{Sr}&\left(1 - T_0(y)\right).
\end{aligned} \tag{3.12}$$

Let us consider the case in which the pressure gradient $(\partial P/\partial x)_0$ vanishes. In this case there will be no flow if the wall motion stops. Hence $\psi_0 = N_0 = 0$ and $T_0 = C_0 = 1$, and equating the first-order terms on both sides of (2.20) and (2.21), we get the following set of equations:

$$\begin{aligned}
\frac{d^2T_1}{dy^2} - \gamma_1^2 T_1 = 0, \\
\frac{d^2C_1}{dy^2} - \gamma_2^2 C_1 = i\alpha\,\mathrm{Sc}\cdot\mathrm{Sr}\cdot\mathrm{Pr}\cdot\mathrm{Re}\cdot T_1,
\end{aligned} \tag{3.13}$$

where

$$\gamma_1^2 = \alpha^2 - i\alpha\cdot\mathrm{Pr}\cdot\mathrm{Re}, \qquad \gamma_2^2 = \alpha^2 - i\alpha\cdot\mathrm{Sc}\cdot\mathrm{Re}. \tag{3.14}$$

The boundary conditions are

$$T_1(-1) = C_1(-1) = 0, \qquad T_1(1) = C_1(1) = 1. \tag{3.15}$$

The solutions of (3.13) with boundary conditions (3.15) are

$$T_1(y) = \frac{\sinh(\gamma_1 + \gamma_1 y)}{\sinh 2\gamma_1}, \tag{3.16}$$

$$C_1(y) = \left(1 - \frac{\mathrm{Sc}\cdot\mathrm{Sr}\cdot\mathrm{Pr}}{(\mathrm{Sc}-\mathrm{Pr})}\right)\frac{\sinh(\gamma_2 + \gamma_2 y)}{\sinh 2\gamma_2} + \frac{\mathrm{Sc}\cdot\mathrm{Sr}\cdot\mathrm{Pr}}{(\mathrm{Sc}\cdot\mathrm{Pr})}\frac{\sinh(\gamma_1 + \gamma_1 y)}{\sinh 2\gamma_1}. \tag{3.17}$$

In this case, the governing equations for T_{20} and C_{20} reduce to

$$\begin{aligned}
\frac{d^2 T_{20}}{dy^2} = {} & \frac{i\alpha}{2}\mathrm{Pr}\cdot\mathrm{Re}\,\frac{d}{dy}\left(T_1\phi_1^* - \phi_1 T_1^*\right) \\
& - \mathrm{Pr}\cdot\mathrm{Ec}\Bigg[\frac{2(1+\gamma^{-1}) + \mu_1}{2} \\
& \qquad \times\left(4\alpha^2\frac{d\phi_1}{dy}\frac{d\phi_1^*}{dy} + \frac{d^2\phi_1}{dy^2}\frac{d^2\phi_1^*}{dy^2} + \alpha^4\phi_1\phi_1^* + \alpha^2\left(\phi_1\frac{d^2\phi_1^*}{dy^2} + \phi_1^*\frac{d^2\phi_1}{dy^2}\right)\right) \\
& \qquad + 2\mu_1\left(\xi_1\xi_1^* + \frac{1}{2}\left(\xi_1\frac{d^2\phi_1^*}{dy^2} + \xi_1^*\frac{d^2\phi_1}{dy^2} - \alpha^2\left(\xi_1\phi_1^* + \xi_1^*\phi_1\right)\right)\right) \\
& \qquad + \frac{4}{M^2}\left(\frac{d\xi_1}{dy}\frac{d\xi_1^*}{dy} + \alpha^2\xi_1\xi_1^*\right)\Bigg] - \frac{i\alpha}{2}\mathrm{af}\cdot\mathrm{Pr}\frac{d}{dy}\left(T_1\xi_1^* - \xi_1 T_1^*\right), \\
\frac{d^2 C_{20}}{dy^2} = {} & \frac{i\alpha}{2}\mathrm{Sc}\cdot\mathrm{Re}\,\frac{d}{dy}\left(C_1\phi_1^* - \phi_1 C_1^*\right) - \mathrm{Sc}\cdot\mathrm{Sr}\frac{d^2 T_{20}}{dy^2},
\end{aligned} \tag{3.18}$$

with the boundary conditions

$$T_{20}(\pm 1) = \mp\frac{1}{2}\left(T_{1y}(\pm 1) + T_{1y}^*(\pm 1)\right), \qquad C_{20}(\pm 1) = \mp\frac{1}{2}\left(C_{1y}(\pm 1) + C_{1y}^*(\pm 1)\right). \tag{3.19}$$

The solutions of (3.18) are

$$\begin{aligned}
T_{20}(y) = \frac{i\alpha}{4}\mathrm{Pr}\cdot\mathrm{Re}\Bigg[& \frac{1}{\sinh 2\gamma_1}\left(A_3^* g_1(\gamma_1,\alpha) + A_4^* g_1(\gamma_1,\beta^*) + A_1^* a_1^* g_1(\gamma_1, r_1^*) + A_2^* a_2^* g_1(\gamma_1, r_2^*)\right) \\
& - \frac{1}{\sinh 2\gamma_1^*}\left(A_3 g_1(\gamma_1^*,\alpha) + A_4 g_1(\gamma_1^*,\beta) + A_1 a_1 g_1(\gamma_1^*, r_1) + A_2 a_2 g_1(\gamma_1^*, r_2)\right)\Bigg]
\end{aligned}$$

$$
\begin{aligned}
&- \mathrm{Ec}\cdot\mathrm{Pr}\left[\frac{2\ \left(1+\gamma^{-1}\right)+\mu_1}{4}\right.\\
&\qquad\times\left(2\alpha^2 A_3 A_3^* \cosh 2\alpha y + 4\alpha^2\left(\alpha A_3 f_1 + r_1 A_1 a_1 f_2 + r_2 A_2 f_3 + \beta A_4 f_4\right)\right.\\
&\qquad\quad + 2\alpha^2 A_3 f_5 + \left(\alpha^2 + r_1^2\right) A_1 a_1 f_6\\
&\qquad\quad \left.+\left(\alpha^2 + r_2^2\right) A_2 a_2 f_7 + \left(\alpha^2+\beta^2\right) A_4 f_8\right) + \left(\frac{M^2\mu_1 + 2\alpha^2}{M^2}\right)\\
&\qquad\quad \times\left(A_1\left(A_1^* g_3\left(r_1, r_1^*\right) + A_2^* g_3\left(r_1, r_2^*\right)\right) + A_2\left(A_1^* g_3\left(r_2, r_1^*\right) + A_2^* g_3\left(r_2, r_2^*\right)\right)\right)\\
&\qquad + \frac{2}{M^2}\left(r_1 A_1\left(r_1^* A_1^* g_2\left(r_1, r_1^*\right) + r_2^* A_2^* g_2\left(r_1, r_2^*\right)\right)\right.\\
&\qquad\qquad \left.+ r_2 A_2\left(r_1^* A_1^* g_2\left(r_2, r_1^*\right) + r_2^* A_2^* g_2\left(r_2, r_2^*\right)\right)\right)\\
&\qquad \left.+\frac{\mu_1}{2}\left(A_1 f_9 + A_2 f_{10} + A_4\left(\beta^2-\alpha^2\right)\left(A_1^* g_3\left(\beta, r_1^*\right) + A_2^* g_3\left(\beta, r_2^*\right)\right)\right)\right]\\
&- \frac{i\alpha}{4}\mathrm{af}\cdot\mathrm{Pr}\left[\frac{1}{\sinh 2\gamma_1}\left(A_1^* g_1\left(\gamma_1, r_1^*\right) + A_2^* g_1\left(\gamma_1, r_2^*\right)\right)\right.\\
&\qquad \left.-\frac{1}{\sinh 2\gamma_1^*}\left(A_1 g_1\left(\gamma_1^*, r_1\right) + A_2 g_1\left(\gamma_1^*, r_2\right)\right)\right] + A_5 y + A_6,\\
C_{20}(y) &= A_8 + A_7 y - \mathrm{Sc}\cdot\mathrm{Sr}\cdot T_{20} + \frac{i\alpha}{4}\mathrm{Sc}\cdot\mathrm{Re}\\
&\quad\times\left\{\left(1-\frac{\mathrm{Sc}\cdot\mathrm{Sr}\cdot\mathrm{Pr}}{(\mathrm{Sc}-\mathrm{Pr})}\right)\frac{1}{\sinh 2\gamma_2}\right.\\
&\qquad\times\left(A_3^* g_1\left(\gamma_2, \alpha\right) + A_4^* g_1\left(\gamma_2, \beta^*\right) + A_1^* a_1^* g_1\left(\gamma_2, r_1^*\right) + A_2^* a_2^* g_1\left(\gamma_2, r_2^*\right)\right)\\
&\qquad + \frac{\mathrm{Sc}\cdot\mathrm{Sr}\cdot\mathrm{Pr}}{(\mathrm{Sc}-\mathrm{Pr})\sinh 2\gamma_1}\\
&\qquad\times\left(A_3^* g_1\left(\gamma_1, \alpha\right) + A_4^* g_1\left(\gamma_1, \beta^*\right) + A_1^* a_1^* g_1\left(\gamma_1, r_1^*\right) + A_2^* a_2^* g_1\left(\gamma_1, r_2^*\right)\right)\\
&\qquad + \left(1-\frac{\mathrm{Sc}\cdot\mathrm{Sr}\cdot\mathrm{Pr}}{(\mathrm{Sc}-\mathrm{Pr})}\right)\frac{1}{\sinh 2\gamma_2^*}\\
&\qquad\times\left(A_3 g_1\left(\gamma_2^*, \alpha\right) + A_4 g_1\left(\gamma_2^*, \beta\right) + A_1 a_1 g_1\left(\gamma_2^*, r_1\right) + A_2 a_2 g_1\left(\gamma_2^*, r_2\right)\right)\\
&\qquad \left.-\frac{\mathrm{Sc}\cdot\mathrm{Sr}\cdot\mathrm{Pr}}{(\mathrm{Sc}-\mathrm{Pr})}\frac{1}{\sinh 2\gamma_1^*}\left(A_3 g_1\left(\gamma_1^*, \alpha\right) + A_4 g_1\left(\gamma_1^*, \beta\right) + A_1 a_1 g_1\left(\gamma_1^*, r_1\right) + A_2 a_2 g_1\left(\gamma_1^*, r_2\right)\right)\right\},
\end{aligned}
\tag{3.20}
$$

where $g_1 - g_3$, $f_1 - f_{10}$ K_3 and $A_5 - A_8$ are given in the appendix.

Also, the equations governing for T_{22} and C_{22} are

$$\frac{d^2T_{22}}{dy^2}-\gamma_3^2T_{22}=\frac{i\alpha}{2}\mathrm{Pr}\cdot\mathrm{Re}\left(T_1\frac{d\phi_1}{dy}-\phi_1\frac{dT_1}{dy}\right)$$

$$-\mathrm{Pr}\cdot\mathrm{Ec}\left[\frac{2(1+\gamma^{-1})+\mu_1}{4}\left(\left(\frac{d^2\phi_1}{dy^2}\right)^2+\alpha^4\phi_1^2+2\alpha^2\left(\phi_1\frac{d^2\phi_1}{dy^2}-2\left(\frac{d\phi_1}{dy}\right)^2\right)\right)\right.$$

$$\left.+\mu_1\left(\xi_1^2+\xi_1\frac{d^2\phi_1}{dy^2}-\alpha^2\xi_1\phi_1\right)+\frac{2}{M^2}\left(\left(\frac{d\xi_1}{dy}\right)^2-\alpha^2\xi_1^2\right)\right]$$

$$-\frac{i\alpha}{2}\mathrm{af}\cdot\mathrm{Pr}\left(T_1\frac{d\xi_1}{dy}-\xi_1\frac{dT_1}{dy}\right),$$

$$\frac{d^2C_{22}}{dy^2}-\gamma_4^2C_{22}=\frac{i\alpha}{2}\mathrm{Sc}\cdot\mathrm{Re}\left(C_1\frac{d\phi_1}{dy}-\phi_1\frac{dC_1}{dy}\right)-\mathrm{Sc}\cdot\mathrm{Sr}\left(\frac{d^2T_{22}}{dy^2}-4\alpha^2T_{22}\right), \tag{3.21}$$

where

$$\gamma_3^2=4\alpha^2-2i\alpha\,\mathrm{Pr}\cdot\mathrm{Re},\qquad \gamma_4^3=4\alpha^2-2i\alpha\,\mathrm{Sc}\cdot\mathrm{Re}, \tag{3.22}$$

with boundary conditions

$$T_{22}(\pm1)=\mp\frac{1}{2}T_{1y}(\pm1),\qquad C_{22}(\pm1)=\mp\frac{1}{2}C_{1y}(\pm1). \tag{3.23}$$

Using boundary conditions (3.23), the expressions of T_{22} and C_{22} are

$$T_{22}(y)=A_9\cosh\gamma_3y+A_{10}\sinh\gamma_3y+\frac{i\alpha}{4}\mathrm{Pr}\frac{1}{\sinh2\gamma_1}$$

$$\times\left[\mathrm{Re}(\alpha A_3g_4(\gamma_1,\alpha)+\beta A_4g_4(\gamma_1,\beta)-\gamma_1A_3g_5(\gamma_1,\alpha)-\gamma_1A_4g_5(\gamma_1,\beta))\right.$$

$$+(a_1\,\mathrm{Re}-\mathrm{af})(r_1A_1g_4(\gamma_1,r_1)-\gamma_1A_1g_5(\gamma_1,r_1))$$

$$\left.+(a_2\,\mathrm{Re}-\mathrm{af})(r_2A_2g_4(\gamma_1,r_2)-\gamma_1A_2g_5(\gamma_1,r_2))\right]$$

$$
\begin{aligned}
&- \Pr\cdot \mathrm{Ec}\Bigg\{ \frac{2\left(1+\gamma^{-1}\right)+\mu_1}{8}\Big(4\alpha^4 A_3^2 g_8(\alpha)+\left(r_1^2+\alpha^2\right)^2 A_1^2 a_1^2 g_8(r_1)\\
&\qquad +\left(r_2^2+\alpha^2\right)^2 A_2^2 a_2^2 g_8(r_2)+\left(\beta^2+\alpha^2\right)^2 A_4^2 g_8(\beta)\\
&\qquad +4\alpha^2 A_3 f_{11}+2A_1 a_1\left(\alpha^2+r_1^2\right) f_{12}+2A_2 a_2\left(\alpha^2+r_2^2\right) f_{13}\\
&\qquad -4\alpha^2\Big(\alpha^2 A_3^2 g_9(\alpha)+r_1^2 A_1^2 \alpha_1^2 g_9(r_1)\\
&\qquad\quad +r_2^2 A_2^2 a_2^2 g_9(r_2)+\beta^2 A_4^2 g_9(\beta)+2\alpha A_3 f_{14}\\
&\qquad\quad +2r_1 A_1 a_1 f_{15}+2r_2 A_2 a_2 f_{16}\Big)\Big)\\
&\quad +\frac{\mu_1}{2}\Big(\Big(1+\left(r_1^2-\alpha^2\right)a_1\Big)A_1^2 g_8(r_1)+\Big(1+\left(r_2^2-\alpha^2\right)a_2\Big)A_2^2 g_8(r_2)\\
&\qquad +A_1 f_{17}+A_2 A_4\left(\beta^2-\alpha^2\right) g_6(r_2,\beta)\Big)\\
&\quad +\frac{1}{M^2}\Big(r_1^2 A_1^2 g_9(r_1)+r_2^2 A_2^2 g_9(r_2)+2r_1 r_2 A_1 A_2 g_7(r_1,r_2)\\
&\qquad -\alpha^2\Big(A_1^2 g_8(r_1)+A_2^2 g_8(r_2)+2A_1 A_2 g_6(r_1,r_2)\Big)\Big)\Bigg\},
\end{aligned}
\tag{3.24}
$$

$$
\begin{aligned}
C_{22}(y) ={}& A_{11}\cosh\gamma_4 y + A_{12}\sinh\gamma_4 y\\
&+\frac{i\alpha}{4}\mathrm{Sc}\cdot\mathrm{Re}\left\{\left(1-\frac{\mathrm{Sc}\cdot\mathrm{Sr}\cdot\Pr}{(\mathrm{Sc}-\Pr)}\right)\frac{f_{18}}{\sinh 2\gamma_2}+\frac{\mathrm{Sc}\cdot\mathrm{Sr}\cdot\Pr}{(\mathrm{Sc}-\Pr)}\frac{f_{19}}{\sinh 2\gamma_1}\right\}\\
&+\frac{\Pr\cdot\mathrm{Sc}\cdot\mathrm{Sr}}{(\mathrm{Sc}-\Pr)}\left(A_9\cosh\gamma_3 y + A_{10}\sinh\gamma_3 y\right)-\frac{i\alpha}{4}\mathrm{Sc}\cdot\mathrm{Sr}\cdot\Pr\cdot\mathrm{Re}\,\frac{f_{20}}{\sinh 2\gamma_1}+\Pr\cdot\mathrm{Ec}\cdot\mathrm{Sc}\cdot\mathrm{Sr}\\
&\times\Bigg[\frac{2\left(1+\gamma^{-1}\right)+\mu_1}{8}\Bigg(\frac{32\alpha^6 A_3^2}{(\gamma_3\gamma_4)^2}+\left(r_1^2+\alpha^2\right)^2 A_1^2 a_1^2 g_{14}(r_1)+\left(r_2^2+\alpha^2\right)^2 A_2^2 a_2^2 g_{14}(r_2)\\
&\qquad +\left(\beta^2+\alpha^2\right)^2 A_4^2 g_{14}(\beta)+4\alpha^2 A_3 f_{21}\\
&\qquad +2\left(\alpha^2+r_1^2\right)A_1 a_1 f_{22}+2\left(\alpha^2+r_2^2\right)A_2 a_2 f_{23}\\
&\qquad -4\alpha^2\Big(r^2 A_1^2 a_1^2 g_{15}(r_1)+r_2^2 A_2^2 a_2^2 g_{15}(r_2)+\beta^2 A_4^2 g_{15}(\beta)\\
&\qquad\quad +2\alpha A_3 f_{24}+2r_1 A_1 a_1 f_{25}+2r_2 A_2 a_2 f_{26}\Big)\Bigg)
\end{aligned}
$$

$$+\frac{\mu_1}{2}\Big(\Big(1+\Big(r_1^2-\alpha^2\Big)a_1\Big)A_1^2 g_{14}(r_1)+\Big(1+\Big(r_2^2-\alpha^2\Big)a_2\Big)A_2^2 g_{14}(r_2)$$

$$+A_1 f_{27}+A_2A_4\Big(\beta^2-\alpha^2\Big)g_{16}(r_2,\beta)\Big)$$

$$+\frac{1}{M^2}\Big(r_1^2A_1^2g_{15}(r_1)+r_2^2A_2^2g_{15}(r_2)+2r_1r_2A_1A_2g_{17}(r_1,r_2)$$

$$-\alpha^2\Big(A_1^2g_{14}(r_1)+A_2^2g_{14}(r_2)+2A_1A_2g_{16}(r_1,r_2)\Big)\Big)\Big]$$

$$+\frac{i\alpha}{4}\mathrm{af}\cdot\mathrm{Pr}\cdot\mathrm{Sc}\cdot\mathrm{Sr}\cdot\frac{f_{28}}{\sinh 2\gamma_1}. \tag{3.25}$$

Hence g_4-g_{17}, $f_{11}-f_{28}$ and A_9-A_{12} are given in the appendix.

Using the expressions for Ψ_1, T_1, C_1, T_{20}, C_{20}, T_{22}, and C_{22} in (3.1), (3.3), and (3.4), we can determine the stream function $\Psi(x,y,t)$, temperature $T(x,y,t)$, and concentration $C(x,y,t)$ of micropolar fluid for the case of free pumping as

$$\psi(x,y,t)=\frac{\varepsilon}{2}\Big(\phi_1(y)e^{i\alpha(x-t)}+\phi_1^*e^{-i\alpha(x-t)}\Big)+o\Big(\varepsilon^2\Big), \tag{3.26}$$

$$\begin{aligned}T(x,y,t)&=1+\frac{\varepsilon}{2}\Big(T_1(y)e^{i\alpha(x-t)}+T_1^*(y)e^{-i\alpha(x-t)}\Big)\\&\quad+\frac{\varepsilon^2}{2}\Big(T_{20}(y)+T_{22}e^{2i\alpha(x-t)}+T_{22}^*(y)e^{-2i\alpha(x-t)}\Big),\end{aligned} \tag{3.27}$$

$$\begin{aligned}C(x,y,t)&=1+\frac{\varepsilon}{2}\Big(C_1(y)e^{i\alpha(x-t)}+C_1^*(y)e^{-i\alpha(x-t)}\Big)\\&\quad+\frac{\varepsilon^2}{2}\Big(C_{20}(y)+C_{22}e^{2i\alpha(x-t)}+C_{22}^*(y)e^{-2i\alpha(x-t)}\Big),\end{aligned} \tag{3.28}$$

where ϕ_1^*, $T_1^*(y)$, $C_1^*(y)$, $T_{22}^*(y)$, and $C_{22}^*(y)$ are conjugate to (3.6), (3.16), (3.17), (3.24), and (3.25), respectively.

4. Results and Discussion

To discuss the effect of the parameters of micropolar, non-Newtonian fluid, and viscoelastic membrane properties on the solutions of the considered problem. A numerical results are calculated from formula (3.6), (3.27), and (3.28) for the temperature $T(x,y,t)$, concentration $C(x,y,t)$ and stream function $\psi(x,y,t)$ and shown by Figures 2–24. Take into account the parameters t and x have the numerical value 2.0, while parameter α and amplitude ratio ε take the value 0.5 and 0.01, respectively, but α takes the value 2 for the stream function.

Figures 2–4 illustrate the effects of $\mu_1=(k/\mu_B)$, $M=(2d\ (\mu_B/\overline{\gamma})^{0.5})$, and af, in order, on the temperature distribution with taking into account that the elastic wall ($K_2=0$). The

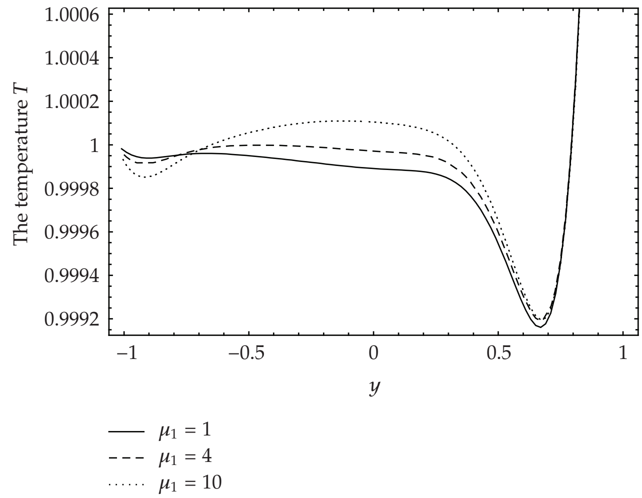

Figure 2: The variation of temperature distribution versus y, with fixed $\varepsilon = 0.01$, $\alpha = 2$, $\mathrm{Re} = 10$, $\gamma = 0.8$, $\mathrm{Pr} = 5$, $\mathrm{Ec} = 0.5$, $M = 10$, af $= 1$, $K_2 = m_1 = 0$, and $K_3 = 10$ for various values of μ_1.

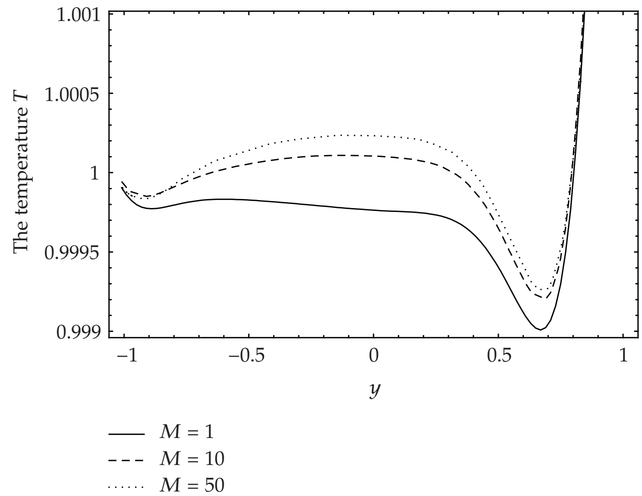

Figure 3: The variation of temperature distribution versus y, with fixed $\varepsilon = 0.01$, $\alpha = 2$, $\mathrm{Re} = 10$, $\gamma = 0.8$, $\mathrm{Pr} = 5$, $\mathrm{Ec} = 0.5$, $\mu_1 = 10$, af $= 1$, $K_2 = m_1 = 0$, and $K_3 = 10$ for various values of M.

ratios k/μ_B and $\overline{\gamma}/\mu_B$ measure the relative strengths of the vortex viscosity coefficient to the viscosity coefficient and the spin gradient viscosity coefficient to the viscosity coefficient, respectively. These coefficients μ_B, k, and $\overline{\gamma}$ may be greater or equal to zero. For example, the blood 50% hematocrit has the values $\mu_B = 0.0029$, $k = 0.000232$, and $\overline{\gamma} = 0.000001$ [27]. When the viscous effects are much larger than the spin gradient viscosity effects, $(\overline{\gamma}/\mu_B)$ may tend to be zero, and the parameter M will tend to infinity. Ahmadi [28] has stated that the parameter μ_1 depends on the shape and concentration of the microelements while the parameter M signifies the size of microstructure. In this context, small (large) value of μ_1 means low (high) concentration of the microelements. Similarly, a given small (large) value of M is related to small (large) size of the particles. It is noted from Figure 2 that the temperature distribution

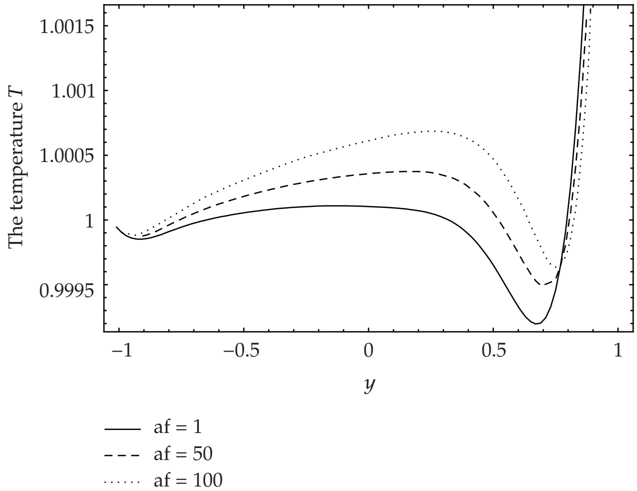

Figure 4: The variation of temperature distribution versus y, with fixed $\varepsilon = 0.01$, $\alpha = 2$, Re $= 10$, $\gamma = 0.8$, Pr $= 5$, Ec $= 0.5$, $\mu_1 = 10$, $M = 10$, $K_2 = m_1 = 0$, and $K_3 = 10$ for various values of af.

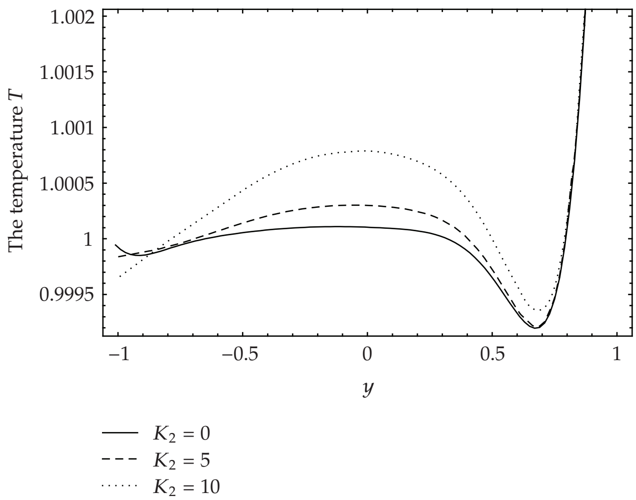

Figure 5: The variation of temperature distribution versus y, with fixed $\varepsilon = 0.01$, $\alpha = 2$, Re $= 10$, $\gamma = 0.8$, Pr $= 5$, Ec $= 0.5$, $\mu_1 = 10$, $M = 10$, af $= 1$, $m_1 = 0$, and $K_3 = 10$ for various values of K_2.

decreases with the increasing of μ_1 near the lower wall, while it increases as μ_1 increases when $y \geq -0.7$. In Figure 3, it can be seen that the temperature distribution increases as M increases. Also, Figure 4 shows that the temperature increases as af increases but at $y > 0.8$, an opposite effect is noticed near the upper wall. The results in our study are consistent with those which are obtained by Agarwal et al. [24] in case of isothermal wall.

The choice $K_2 = 0$ implies that the walls move up and down with no damping force on them and hence indicates that the membrane is treated as an elastic wall. The parameter K_2 depends upon the wall tension and represents the rigid nature of the walls. Figures 5, 6, and 7 illustrate the effect of the parameters of the viscoelastic wall properties K_2, K_3, and m_1 on the temperature distribution, respectively. It is found that the temperature distribution

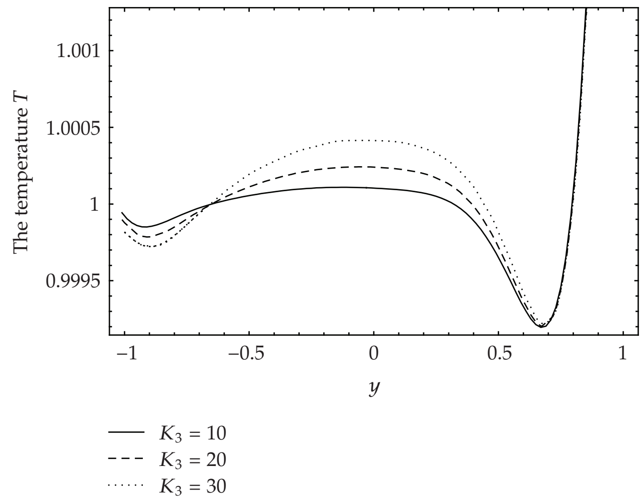

Figure 6: The variation of temperature distribution versus y, with fixed $\varepsilon = 0.01$, $\alpha = 2$, $\mathrm{Re} = 10$, $\gamma = 0.8$, $\mathrm{Pr} = 5$, $\mathrm{Ec} = 0.5$, $\mu_1 = 10$, $M = 10$, af = 1 and $K_2 = m_1 = 0$ for various values of K_3.

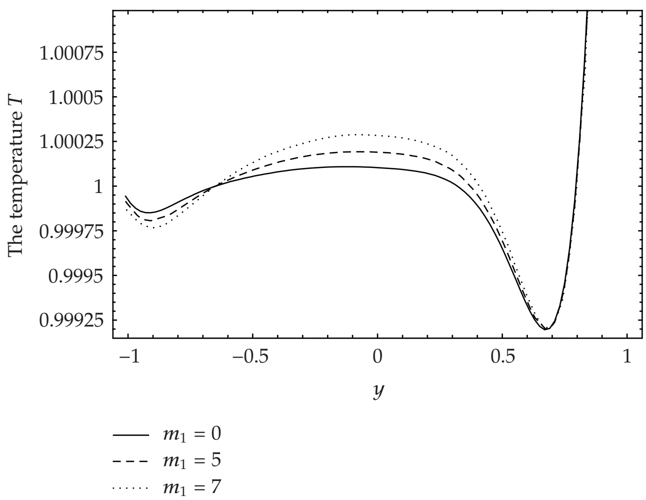

Figure 7: The variation of temperature distribution versus y, with fixed $\varepsilon = 0.01$, $\alpha = 2$, $\mathrm{Re} = 10$, $\gamma = 0.8$, $\mathrm{Pr} = 5$, $\mathrm{Ec} = 0.5$, $\mu_1 = 10$, $M = 10$, af = 1, $K_2 = 0$ and $K_3 = 10$ for various values of m_1.

increases as K_2 increases as shown in Figure 5, but near the lower wall, it decreases with the increase of K_2. The temperature distribution decreases with the increase of both K_3 and m_1, but when $y \geq -0.7$, it starts increasing as both K_3 and m_1 increase for the case of elastic walls ($K_2 = 0$), as shown in Figures 6 and 7. This phenomenon reflects the fact that the increasing of the values of tension or dissipative for the membrane leads to loss of heat from the walls and gives it to the fluid. Therefore, the temperature distribution of fluid will increase.

In Figure 8, we see the effect of Reynolds number Re on the temperature distribution. It is observed that the temperature distribution behaves a dual role with the variation of Re. The variations of temperature distribution for various values of Prandtl number Pr and

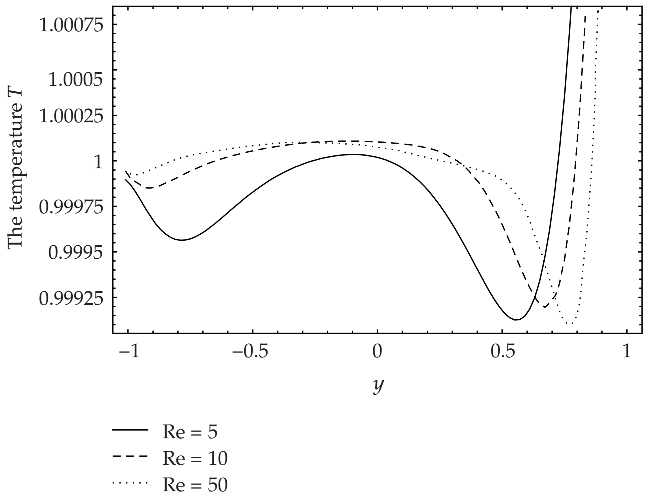

Figure 8: The variation of temperature distribution versus y, with fixed $\varepsilon = 0.01$, $\alpha = 2$, $\gamma = 0.8$, $\Pr = 5$, $\mathrm{Ec} = 0.5$, $\mu_1 = 10$, $M = 10$, $\mathrm{af} = 1$, $K_2 = m_1 = 0$ and $K_3 = 10$ for various values of Re.

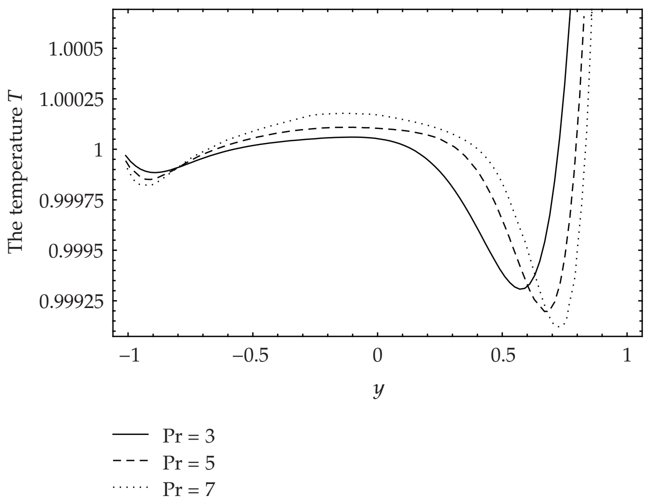

Figure 9: The variation of temperature distribution versus y, with fixed $\varepsilon = 0.01$, $\alpha = 2$, $\mathrm{Re} = 10$, $\gamma = 0.8$, $\mathrm{Ec} = 0.5$, $\mu_1 = 10$, $M = 10$, $\mathrm{af} = 1$, $K_2 = m_1 = 0$ and $K_3 = 10$ for various values of Pr.

Eckert number Ec are displayed in Figures 9 and 10, respectively. The graphical results of Figures 9 and 10 indicate that the temperature of fluid increases as Pr and Ec increase. While for $|y| > 0.7$, near the lower and upper wall, the temperature decreases with the increase of Pr and Ec. In the case when the temperature of the plate is periodic, Das et al. [29] showed that the temperature decreases with the increase of the Prandtle number of Newtonian fluid. In Figure 11, the temperature is plotted against y for various values of the upper limit apparent viscosity coefficient γ; it is observed that an increase of γ increases the temperature.

Figures 12–18 reveal the influence of physical parameters entering in the problem on the concentration distribution of fluid. The effect of the coupling parameter μ_1 on concentration is indicated in Figure 12; we find that the concentration distribution of fluid

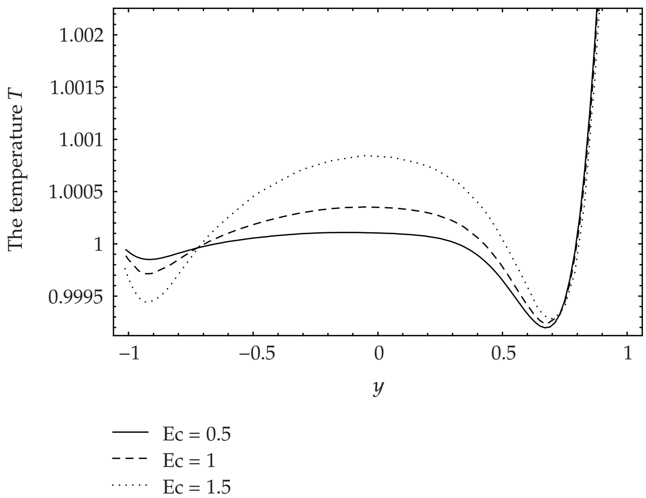

Figure 10: The variation of temperature distribution versus y, with fixed $\varepsilon = 0.01$, $\alpha = 2$, $\mathrm{Re} = 10$, $\gamma = 0.8$, $\mathrm{Pr} = 5$, $\mu_1 = 10$, $M = 10$, $\mathrm{af} = 1$, $K_2 = m_1 = 0$ and $K_3 = 10$ for various values of Ec.

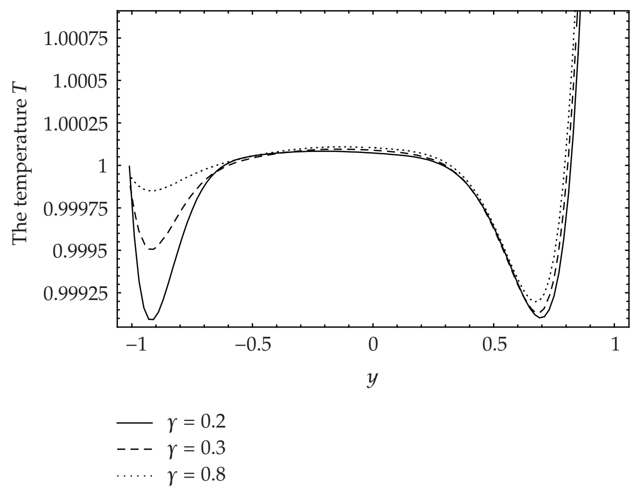

Figure 11: The variation of temperature distribution versus y, with fixed $\varepsilon = 0.01$, $\alpha = 2$, $\mathrm{Re} = 10$, $\mathrm{Pr} = 5$, $\mathrm{Ec} = 0.5$, $\mu_1 = 10$, $M = 10$, $\mathrm{af} = 1$, $K_2 = m_1 = 0$ and $K_3 = 10$ for various values of γ.

decreases with the increase of μ_1. It has been observed that the concentration distribution decreases with the increase of af, but an opposite effect is seen at $y > 0.8$. This is noticed in Figure 13. Also, in Figures 12 and 13, for small values of μ_1 and af, its effects are rather insignificant. In Figures 14, 15, and 16, it is found that the increase of the dissipative, rigid nature of the walls and stiffness in the walls causes a decrease of the concentration distribution. The behavior of the concentration distribution with the nondimensional distance y for various values of Soret number Sr is depicted in Figure 17. It is clear, from Figure 17, that the effect of Sr on the concentration can be neglected near the lower wall. When $-0.8 < y < 0.5$, the concentration decreases with the increase of Sr, but at $y > 0.5$, an increase in Sr leads to

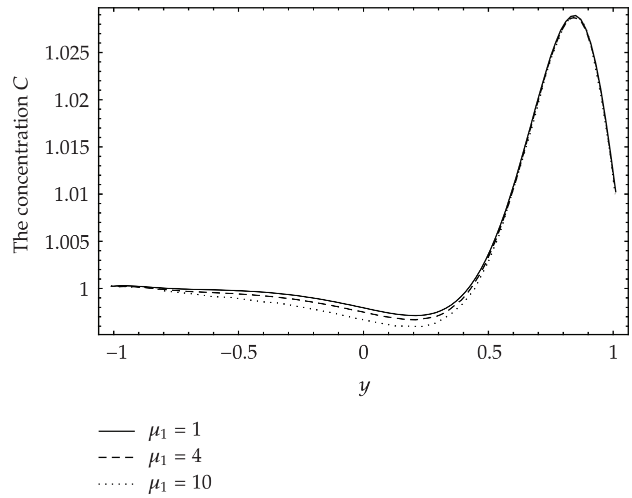

Figure 12: The variation of concentration distribution versus y, with fixed $\varepsilon = 0.01$, $\alpha = 2$, $\mathrm{Re} = 10$, $\gamma = 0.8$, $\mathrm{Pr} = 5$, $\mathrm{Ec} = 0.5$, $M = 10$, $\mathrm{af} = 1$, $K_2 = m_1 = 0$, $K_3 = 10$, $\mathrm{Sc} = 1$ and $\mathrm{Sr} = 1$ for various values of μ_1.

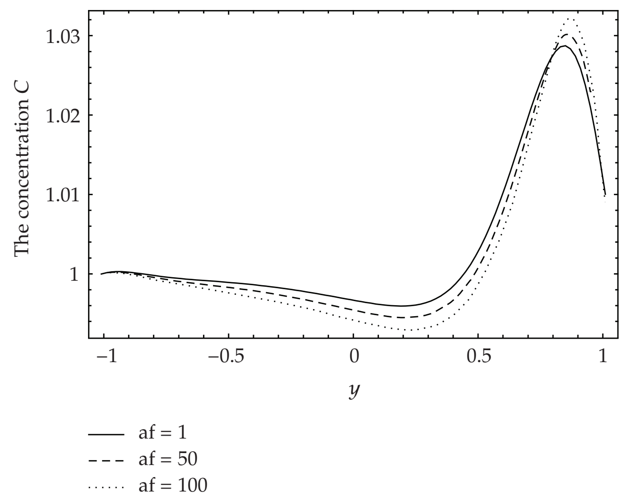

Figure 13: The variation of concentration distribution versus y, with fixed $\varepsilon = 0.01$, $\alpha = 2$, $\mathrm{Re} = 10$, $\gamma = 0.8$, $\mathrm{Pr} = 5$, $\mathrm{Ec} = 0.5$, $\mu_1 = 10$, $M = 10$, $K_2 = m_1 = 0$, $K_3 = 10$, $\mathrm{Sc} = 1$ and $\mathrm{Sr} = 1$ for various values of af.

increase the concentration distribution. The effect of the thermal-diffusion parameter Sr on the concentration distribution, in case of non-Newtonian fluid, has been studied by El Dabe et al. [23]. The result in Figure 17, at the upper wall, is in agreement with those obtained by El Dabe et al. [23]. Figure 9 illustrates the effects of the upper limit apparent viscosity coefficient γ on the concentration distribution. It is found that the concentration decreases near lower and upper walls, but it increases with the increase of γ at the middle of the channel.

The effects of various physical parameters on the stream function ψ are indicated in Figures 19–24. In these figures the stream function is plotted versus the dimensionless distance y, and it is clear that there is an inversely non-linear relation between y and ψ

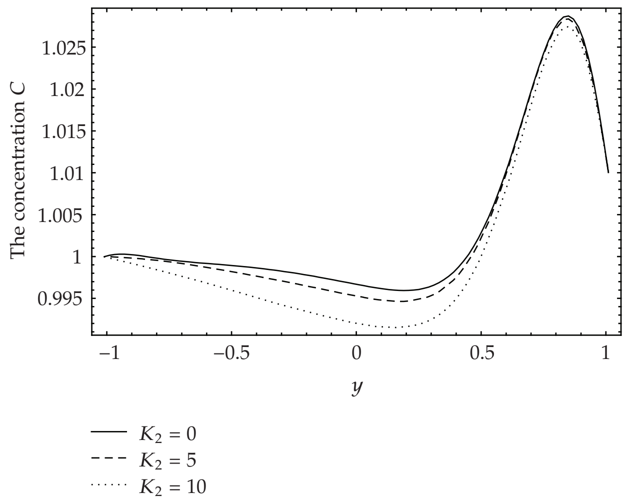

Figure 14: The variation of concentration distribution versus y, with fixed $\varepsilon = 0.01$, $\alpha = 2$, $\mathrm{Re} = 10$, $\gamma = 0.8$, $\mathrm{Pr} = 5$, $\mathrm{Ec} = 0.5$, $\mu_1 = 10$, $M = 10$, $\mathrm{af} = 1$, $m_1 = 0$, $K_3 = 10$, $\mathrm{Sc} = 1$ and $\mathrm{Sr} = 1$ for various values of K_2.

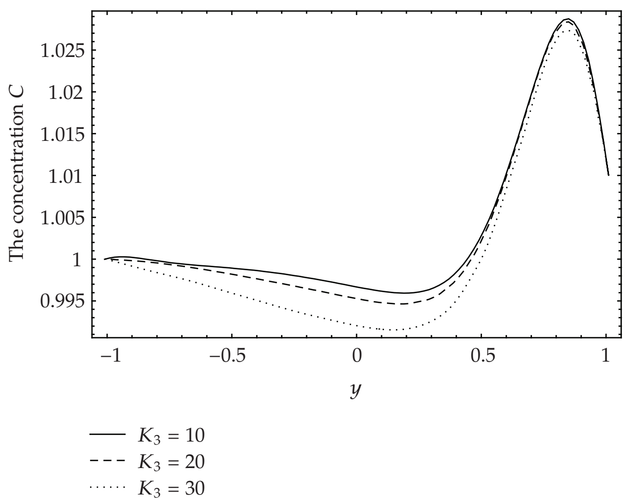

Figure 15: The variation of concentration distribution versus y, with fixed $\varepsilon = 0.01$, $\alpha = 2$, $\mathrm{Re} = 10$, $\gamma = 0.8$, $\mathrm{Pr} = 5$, $\mathrm{Ec} = 0.5$, $\mu_1 = 10$, $M = 10$, $\mathrm{af} = 1$, $K_2 = m_1 = 0$, $\mathrm{Sc} = 1$ and $\mathrm{Sr} = 1$ for various values of K_3.

and that the obtained curves of all figures will intersect at the origin where $y = 0$, $\psi = 0$. Figure 19 shows the variation of ψ with y for different values of the coupling parameter μ_1; it is observed that, in the region $-1 \leq y < 0$, the stream function decreases by increasing μ_1, while in the region $0 < y \leq 1$, ψ increases by increasing μ_1. Note that, when $\mu_1 \geq 1$, the peristaltic pumping region becomes slightly wider as μ_1 increases, and if $\mu_1 < 1$, it is slightly narrow as μ_1 decreases. Figure 20 illustrates the stream function ψ for different values of M. From this figure, it is observed that the effect of M on ψ is similar to the effect of μ_1 on ψ illustrated in Figure 19. In Figures 21 and 22, the effects of Reynolds number Re and the parameter of dissipates of walls K_2 on the stream function ψ, respectively, are reverse to the

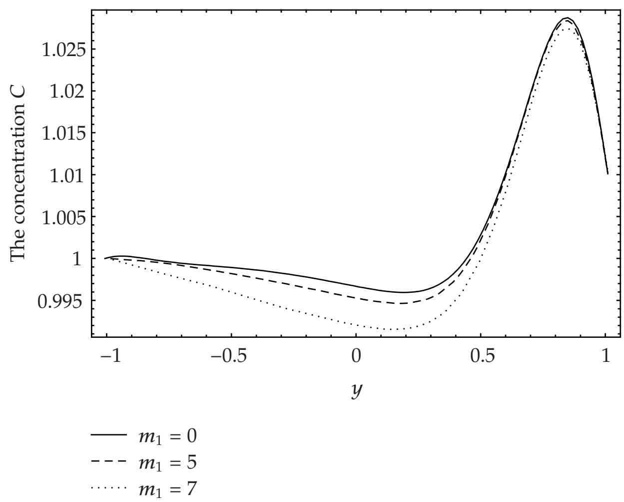

Figure 16: The variation of concentration distribution versus y, with fixed $\varepsilon = 0.01$, $\alpha = 2$, $\mathrm{Re} = 10$, $\gamma = 0.8$, $\mathrm{Pr} = 5$, $\mathrm{Ec} = 0.5$, $\mu_1 = 10$, $M = 10$, $\mathrm{af} = 1$, $K_2 = 0$, $K_3 = 10$, $\mathrm{Sc} = 1$ and $\mathrm{Sr} = 1$ for various values of m_1.

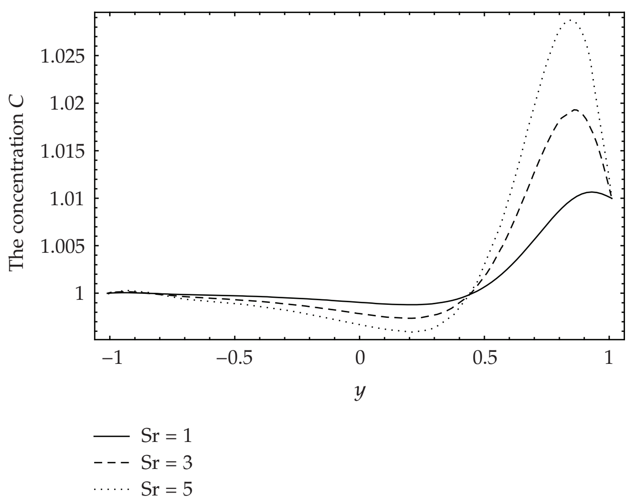

Figure 17: The variation of concentration distribution versus y, with fixed $\varepsilon = 0.01$, $\alpha = 2$, $\mathrm{Re} = 10$, $\gamma = 0.8$, $\mathrm{Pr} = 5$, $\mathrm{Ec} = 0.5$, $\mu_1 = 10$, $M = 10$, $\mathrm{af} = 1$, $K_2 = m_1 = 0$, $K_3 = 10$ and $\mathrm{Sc} = 1$ for various values of Sr.

effect of μ_1 and M on ψ, that is, in the region $-1 \leq y < 0$, ψ increases by increasing both of Re and K_2, while in the region $0 < y \leq 1$, ψ decreases by increasing both of Re and K_2. Note that the peristaltic pumping region becomes slightly wider as Re and K_2 increase. Figures 23 and 24 show that the behaviors of the stream function ψ with the dimensionless distance y for different values of the parameter of rigidities feature of walls K_3 and upper limit apparent viscosity coefficient γ, respectively, are similar to the behaviors of ψ for different values of μ_1 and M illustrated in Figures 19 and 20, with the only difference that the peristaltic pumping will disappear in these cases.

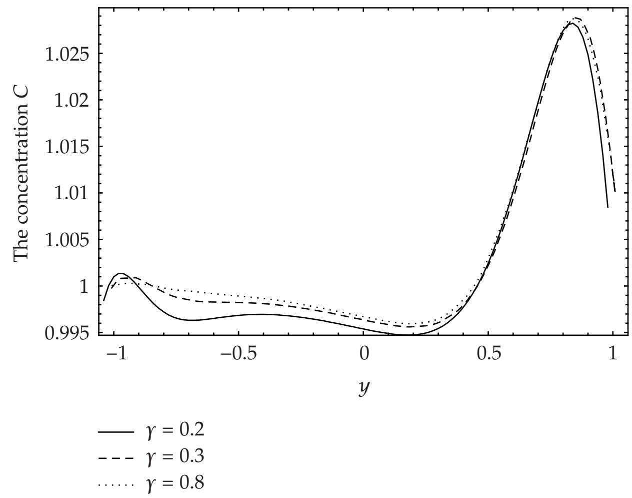

Figure 18: The variation of concentration distribution versus y, with fixed $\varepsilon = 0.01$, $\alpha = 2$, Re $= 10$, Pr $= 5$, Ec $= 0.5$, $\mu_1 = 10$, $M = 10$, af $= 1$, $K_2 = m_1 = 0$, $K_3 = 10$, Sc $= 1$ and Sr $= 1$ for various values of γ.

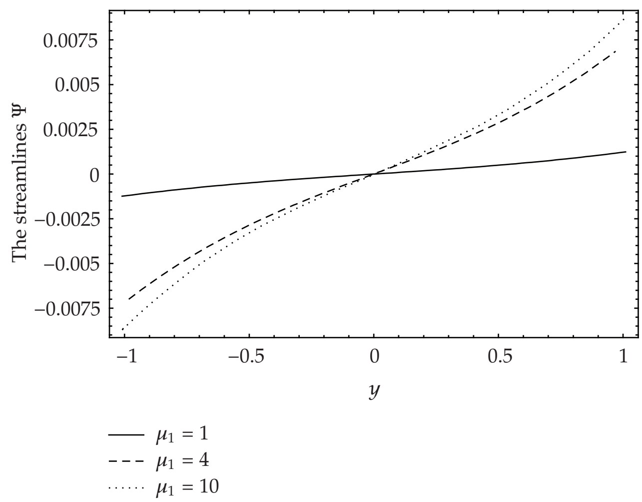

Figure 19: The variation of streamlines versus y, with fixed $\varepsilon = 0.01$, $\alpha = 0.5$, Re $= 10$, $\gamma = 0.8$, $M = 10$, $K_2 = m_1 = 0$ and $K_3 = 10$ for various values of μ_1.

5. Conclusion

In this paper, we study the peristaltic flow of a micropolar biviscosity fluid in channel with dynamic boundary condition. Also, we take into considerations that the temperature and concentration of lower wall are constant while they are periodic for the upper wall. This study is an extension of the work of Muthu et al. [25]. The analytical expressions are constructed for the stream function, microrotation velocity, temperature, and concentration distributions as a power series in terms of small amplitude ratio in case of free pumping. The effects of various physical parameters acting on the problem are discussed by a set of graphs. The micropolar

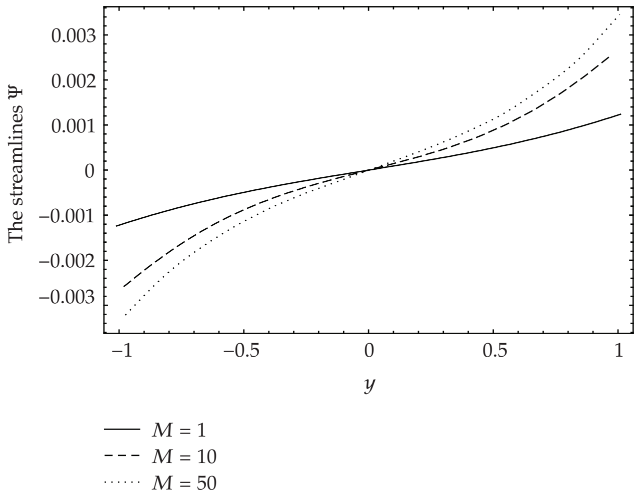

Figure 20: The variation of streamlines versus y, with fixed $\varepsilon = 0.01$, $\alpha = 0.5$, $\mathrm{Re} = 10$, $\gamma = 0.8$, $\mu_1 = 1$, $K_2 = m_1 = 0$ and $K_3 = 10$ for various values of M.

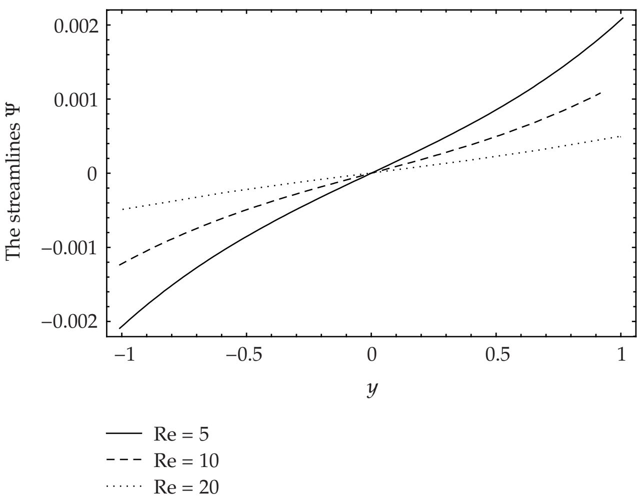

Figure 21: The variation of streamlines versus y, with fixed $\varepsilon = 0.01$, $\alpha = 0.5$, $\gamma = 0.8$, $\mu_1 = 1$, $M = 10$, $K_2 = m_1 = 0$ and $K_3 = 10$ for various values of Re.

parameters μ_1, M, and af and wall parameters K_2, K_3, and m_1 which describe the viscoelastic behaviors of the flexible wall are discussed through numerical computations. The obtained results can be outlined and summarized as follows.

(1) The temperature increases with the increase each of K_2, K_3, and m_1, but near the lower wall it decreases.

(2) The temperature for different values of K_2, K_3, and m_1 becomes greater with increasing the dimensionless distance y and reaches maximum at $y = 1$.

(3) The concentration decreases with the increase each of K_2, K_3, and m_1.

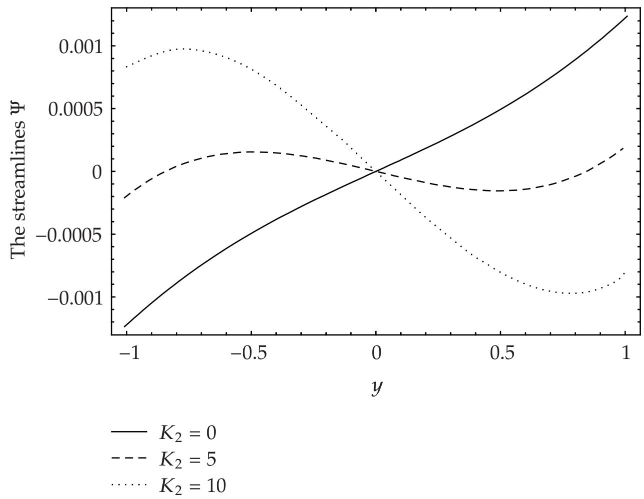

Figure 22: The variation of streamlines versus y, with fixed $\varepsilon = 0.01$, $\alpha = 0.5$, Re $= 10$, $\gamma = 0.8$, $\mu_1 = 1$, $M = 10$, $m_1 = 0$ and $K_3 = 10$ for various values of K_2.

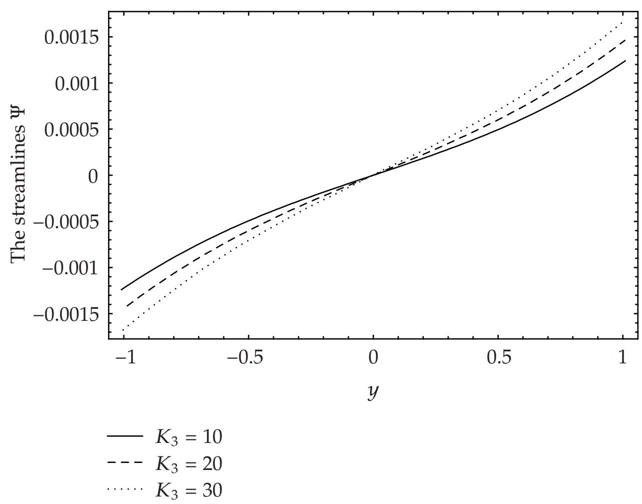

Figure 23: The variation of streamlines versus y, with fixed $\varepsilon = 0.01$, $\alpha = 0.5$, Re $= 10$, $\gamma = 0.8$, $\mu_1 = 1$, $M = 10$ and $K_2 = m_1 = 0$ for various values of K_3.

(4) The temperature and concentration increase as γ increases, but the concentration decreases near the lower and upper walls.

(5) The streamlines intersect at a critical value of y (=0) after which all the obtained curves will have an opposite behavior.

(6) For any value $y < 0.8$, the temperature increases with the increase of af, while for any value $y \geq 0.8$, all the obtained lines will behave in an opposite manner to these behavior when $y < 0.8$.

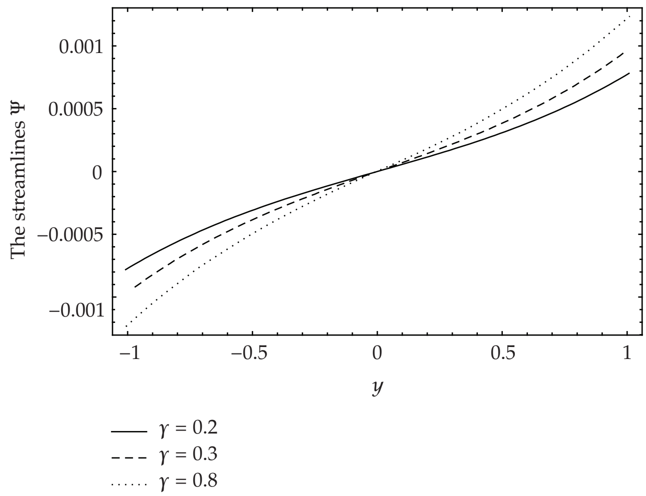

Figure 24: The variation of streamlines versus y, with fixed $\varepsilon = 0.01$, $\alpha = 0.5$, $\mathrm{Re} = 10$, $\mu_1 = 1$, $M = 10$, $K_2 = m_1 = 0$ and $K_3 = 10$ for various values of γ.

(7) The concentration has an opposite behavior compared to the temperature with af (i.e., for any value $y < 0.8$, the concentration decreases with the increase of af, while for any value $y \geq 0.8$, it increases).

Appendix

The functions $g_1 - g_{17}$, $f_1 - f_{28}$ and the constants $A_5 - A_{12}$ are given by

$$g_1(a,b) = \frac{\sinh(a + (a+b)y)}{a+b} - \frac{\sinh(a + (a-b)y)}{a-b},$$

$$g_2(a,b) = \frac{\cosh((a+b)y)}{(a+b)^2} + \frac{\cosh((a-b)y)}{(a-b)^2},$$

$$g_3(a,b) = \frac{\cosh((a+b)y)}{(a+b)^2} - \frac{\cosh((a-b)y)}{(a-b)^2},$$

$$f_1 = r_1^* A_1^* a_1^* g_2(\alpha, r_1^*) + r_2^* A_2^* a_2^* g_2(\alpha, r_2^*) + \beta^* A_4^* g_2(\alpha, \beta^*),$$

$$f_2 = \alpha A_3^* g_2(r_1, \alpha) + r_1^* A_1^* a_1^* g_2(r_1, r_1^*) + r_2^* A_2^* a_2^* g_2(r_1, r_2^*) + \beta^* A_4^* g_2(r_1, \beta^*),$$

$$f_3 = \alpha A_3^* g_2(r_2, \alpha) + r_1^* A_1^* a_1^* g_2(r_2, r_1^*) + r_2^* A_2^* a_2^* g_2(r_2, r_2^*) + \beta^* A_4^* g_2(r_2, \beta^*),$$

$$f_4 = \alpha A_3^* g_2(\beta, \alpha) + r_1^* A_1^* a_1^* g_2(\beta, r_1^*) + r_2^* A_2^* a_2^* g_2(\beta, r_2^*) + \beta^* A_4^* g_2(\beta, \beta^*),$$

$$f_5 = \left(\alpha^2 + r_1^{*2}\right) A_1^* a_1^* g_3(\alpha, r_1^*) + \left(\alpha^2 + r_1^{*^2}\right) A_1^* a_2^* g_3(\alpha, r_2^*) + \left(\alpha^2, \beta^{*^2}\right) + A_4^* g_2(\alpha, \beta^*),$$

$$f_6 = 2\alpha^2 A_3^* g_3(r_1,\alpha) + \left(\alpha^2 + r_1^{*^2}\right) A_1^* a_1^* g_3(r_1, r_1^*)$$

$$+ \left(\alpha^2, r_2^{*^{2^2}}\right) A_2^* a_2^* g_3(r_1, r_2^*) + \left(\alpha^2 + \beta^{*^2}\right) A_4^* g_3(r_1, \beta^*),$$

$$f_7 = 2\alpha^2 A_3^* g_3(r_2,\alpha) + \left(\alpha^2 + r_1^{*^2}\right) A_1^* a_1^* g_3(r_1, r_1^*)$$

$$+ \left(\alpha^2 + r_2^{*^2}\right) A_2^* a_2^* g_3(r_1, r_2^*) + \left(\alpha^2 + \beta^{*^2}\right) A_4^* g_3(r_2, \beta^*),$$

$$f_8 = 2\alpha^2 A_3^* g_3(\beta,\alpha) + \left(\alpha^2 + r_1^{*^2}\right) A_1^* a_1^* g_3(\beta, r_1^*)$$

$$+ \left(\alpha^2 + r_2^{*^2}\right) A_2^* a_2^* g_3(\beta, r_2^*) + \left(\alpha^2 + \beta^{*^2}\right) A_4^* g_3(\beta, \beta^*),$$

$$f_9 = A_1^*\left(a_1^*\left(r_1^{*^2} - \alpha^2\right) + a_1\left(r_1^2 - \alpha^2\right)\right) g_3(r_1, r_1^*)$$

$$+ A_2^*\left(a_2^*\left(r_2^{*^2} - \alpha^2\right) + a_1\left(r_1^2 - \alpha^2\right)\right) g_3(r_1, r_2^*) + A_4^*\left(\beta^{*^2} - \alpha^2\right) g_3(r_1, \beta^*),$$

$$f_{10} = A_1^*\left(a_1^*\left(r_1^{*^2} - \alpha^2\right) + a_2\left(r_2^2 - \alpha^2\right)\right) g_3(r_2, r_1^*)$$

$$+ A_2^*\left(a_2^*\left(r_2^{*^2} - \alpha^2\right) + a_2\left(r_2^2 - \alpha^2\right)\right) g_3(r_2, r_2^*) + A_4^*\left(\beta^{*^2} - \alpha^2\right) g_3(r_2, \beta^*),$$

$$\delta_2(a,b) = \frac{\sinh(2a+b)}{a+b} - \frac{\sinh(2a-b)}{a-b} - \frac{2a\sinh b}{a^2-b^2},$$

$$\delta_3(a,b) = \frac{\cosh(a+b)}{(a+b)^2} + \frac{\cosh(a-b)}{(a-b)^2},$$

$$\delta_4(a,b) = \frac{\cosh(a+b)}{(a+b)^2} - \frac{\cosh(a-b)}{(a-b)^2},$$

$$\delta_5(a,b) = \frac{\sinh(2a+b)}{a+b} - \frac{\sinh(2a-b)}{a-b} + \frac{2a\sinh b}{a^2-b^2},$$

$$\lambda_3 = r_1^* A_1^* \delta_3(\alpha, r_1^*) + r_2^* A_2^* a_2^* \delta_3(\alpha, r_2^*) + \beta^* A_4^* \delta_3(\alpha, \beta^*),$$

$$\lambda_4 = \alpha A_3^* \lambda_3(r_1,\alpha) + r_1^* A_1^* a_1^* \delta_3(r_1, r_1^*) + r_2^* A_2^* a_2^* \delta_3(r_1, r_2^*) + \beta^* A_4^* \delta_3(r_1, \beta^*),$$

$$\lambda_5 = \alpha A_3^* \delta_3(r_2,\alpha) + r_1^* A_1^* a_1^* \delta_3(r_2, r_1^*) + r_2^* A_2^* a_2^* \delta_3(r_2, r_2^*) + \beta^* A_4^* \delta_3(r_2, \beta^*),$$

$$\lambda_6 = \alpha A_3^* \delta_3(\beta,\alpha) + r_1^* A_1^* a_1^* \delta_3(\beta, r_1^*) + r_2^* A_2^* a_2^* \delta_3(\beta, r_2^*) + \beta^* A_4^* \delta_3(\beta, \beta^*),$$

$$\lambda_7 = \left(\alpha^2 + r_1^{*^2}\right) A_1^* a_1^* \delta_4(\alpha, r_1^*) + \left(\alpha^2 + r_2^{*^2}\right) A_2^* a_2^* \delta_4(\alpha, r_2^*) + \left(\alpha^2 + \beta^{*^2}\right) + A_4^* \delta_4(\alpha, \beta^*),$$

$$A_5 = -\frac{1}{4}\left(\frac{\gamma_1(1+\cosh 2\gamma_1)}{\sinh 2\gamma_1}1 + \frac{\gamma_1^*(1+\cosh 2\gamma^*)}{\sinh 2\gamma_1^*}\right)$$

$$-\frac{i\alpha}{8}\Pr\cdot\mathrm{Re}\left(\frac{1}{\sinh 2\gamma_1}\left(\left\{\frac{\sinh(2\gamma_1+\alpha)}{\gamma_1+\alpha} - \frac{\sinh(2\gamma_1-\alpha)}{\gamma_1-\alpha} + \frac{2\gamma_1\sinh\alpha}{\gamma_1^2-\alpha^2}\right\}A_3^*\right.\right.$$

$$\left.\left. + \left\{\frac{\sinh(2\gamma_1+\beta^*)}{\gamma_1+\beta^*} - \frac{\sinh(2\gamma_1-\beta^*)}{\gamma_1-\beta^*} + \frac{2\gamma_1\sinh\beta^*}{\gamma_1^2-\alpha^2}\right\}A_4^*\right)\right.$$

$$-\left\{\frac{1}{\sinh 2\gamma_1^*}\left(\left\{\frac{\sinh(2\gamma_1^*+\alpha)}{\gamma_1^*+\alpha} - \frac{\sinh(2\gamma_1^*-\alpha)}{\gamma_1^*-\alpha} + \frac{2\gamma_1^*\sinh\alpha}{\gamma_1^{*^2}-\alpha^2}\right\}A_3\right.\right.$$

$$\left.\left.\left. + \left\{\frac{\sinh(2\gamma_1^*+\beta)}{\gamma_1^*+\beta} - \frac{\sinh(2\gamma_1^*-\beta)}{\gamma_1^*-\beta} + \frac{2\gamma_1^*\sinh\beta}{\gamma_1^{*^2}-\beta^2}\right\}A_4\right)\right)\right.$$

$$+\frac{i\alpha}{8}\cdot\Pr\left[\frac{1}{\sinh 2\gamma_1}\right.$$

$$\times\left((\mathrm{af}-\mathrm{Re}\,a_1^*)A_1^*\left\{\frac{\sinh(2\gamma_1+r_1^*)}{\gamma_1+r_1^*} - \frac{\sinh(2\gamma_1-r_1^*)}{\gamma_1-r_1^*} + \frac{2\gamma_1\sinh r_1^*}{\gamma_1^2-r_1^{*^2}}\right\}\right.$$

$$\left. +(\mathrm{af}-\mathrm{Re}\,a_2^*)A_2^*\left\{\frac{\sinh(2\gamma_1+r_2^*)}{\gamma_1+r_2^*} - \frac{\sinh(2\gamma_1-r_2^*)}{\gamma_1-r_2^*} + \frac{2\gamma_1\sinh r_2^*}{\gamma_1^2-r_2^{*^2}}\right\}\right)$$

$$+\frac{1}{\sinh 2\gamma_1^*}\left((a_1\,\mathrm{Re}-\mathrm{af})A_1\right.$$

$$\times\left\{\frac{\sinh(2\gamma_1^*+r_1)}{\gamma_1^*+r_1} - \frac{\sinh(2\gamma_1^*-r_1)}{\gamma_1^*-r_1} + \frac{2\gamma_1^*\sinh r_1}{\gamma_1^{*^2}-r_1^2}\right\}$$

$$+(a_2\,\mathrm{Re}-\mathrm{af})A_2$$

$$\left.\left.\times\left\{\frac{\sinh(2\gamma_1^*+r_2)}{\gamma_1^*+r_2} - \frac{\sinh(2\gamma_1^*-r_2)}{\gamma_1^*-r_2} + \frac{2\gamma_1^*\sinh r_2}{\gamma_1^{*^2}-r_2^2}\right\}\right)\right],$$

$$\lambda_8 = 2\alpha^2 A_3^*\delta_4(r_1,\alpha) + \left(\alpha^2 + r_1^{*^2}\right)A_1^* a_1^*\delta_4(r_1,r_1^*)$$

$$+\left(\alpha^2 + r_2^{*^2}\right)A_2^* a_2^*\delta_4(r_1,r_2^*) + \left(\alpha^2+\beta^{*^2}\right)A_4^*\delta_4(r_1,\beta^*),$$

$$\lambda_9 = 2\alpha^2 A_3^*\delta_4(r_2,\alpha) + \left(\alpha^2 + r_1^{*^2}\right)A_1^* a_1^*\delta_4(r_2,r_1^*)$$

$$+\left(\alpha^2 + r_2^{*^2}\right)A_2^* a_2^*\delta_4(r_2,r_2^*) + \left(\alpha^2+\beta^{*^2}\right)A_4^*\delta_4(r_2,\beta^*),$$

$$\lambda_{10} = 2\alpha^2 A_3^* \delta_4(\beta,\alpha) + \left(\alpha^2 + r_1^{*^2}\right) A_1^* a_1^* \delta_4(\beta, r_1^*) + \left(\alpha^2 + r_2^{*^2}\right) A_2^* a_2^* \delta_4(\beta, r_2^*)$$

$$+ \left(\alpha^2 + \beta^{*^2}\right) A_4^* \delta_4(\beta, \beta^*),$$

$$\lambda_{11} = A_1^*\left(a_1^*\left(r_1^{*^2} - \alpha^2\right) + a_1\left(r_1^2 - \alpha^2\right)\right)\delta_4(r_1, r_1^*)$$

$$+ A_2^*\left(a_2^*\left(r_2^{*^2} - \alpha^2\right) + a_1\left(r_1^2 - \alpha^2\right)\right)\delta_4(r_1, r_2^*) + A_4^*\left(\beta^{*^2} - \alpha^2\right)\delta_4(r_1, \beta^*),$$

$$\lambda_{12} = A_1^*\left(a_1^*\left(r_1^{*^2} - \alpha^2\right) + a_2\left(r_2^2 - \alpha^2\right)\right)\delta_4(r_2, r_1^*)$$

$$+ A_2^*\left(a_2^*\left(r_2^{*^2} - \alpha^2\right) + a_2\left(r_2^2 - \alpha^2\right)\right)\delta_4(r_2, r_2^*) + A_4^*\left(\beta^{*^2} - \alpha^2\right)\delta_4(r_2, \beta^*),$$

$$f_{11} = A_1 a_1\left(r_1^2 + \alpha^2\right) g_6(\alpha, r_1) + A_2 a_2\left(r_2^2 + \alpha^2\right) g_6(\alpha, r_2) + A_4\left(\beta^2 + \alpha^2\right) g_6(\alpha, \beta),$$

$$f_{12} = A_2 a_2\left(r_2^2 + \alpha^2\right) g_6(r_1, r_2) + A_4\left(\beta^2 + \alpha^2\right) g_6(r_1, \beta),$$

$$f_{13} = A_4\left(\beta^2 + \alpha^2\right) g_6(r_2, \beta),$$

$$f_{14} = r_1 A_1 a_1 g_7(\alpha, r_1) + r_2 A_2 a_2 g_7(\alpha, r_2) + \beta A_4 g_7(\alpha, \beta),$$

$$f_{15} = r_2 A_2 a_2 g_7(r_1, r_2) + \beta A_4 g_7(r_1, \beta),$$

$$f_{16} = \beta A_4 g_7(r_2, \beta),$$

$$f_{17} = \left(2 + \left(r_1^2 - \alpha^2\right) a_1 + \left(r_2^2 - \alpha^2\right) a_2\right) A_2 g_6(r_1, r_2) + \left(\beta^2 - \alpha^2\right) A_4 g_6(r_1, \beta),$$

$$f_{18} = \alpha A_3 g_{10}(\gamma_2, \alpha) + \beta A_4 g_{10}(\gamma_2, \beta) + r_1 A_1 a_1 g_{10}(\gamma_2, r_1) + r_2 A_2 a_2 g_{10}(\gamma_2, r_2)$$

$$- \gamma_2(A_3 g_{11}(\gamma_2, \alpha) + A_4 g_{11}(\gamma_2, \beta) + A_1 a_1 g_{11}(\gamma_2, r_1) + A_2 a_2 g_{11}(\gamma_2, r_2)),$$

$$f_{19} = \alpha A_3 g_{10}(\gamma_1, \alpha) + \beta A_4 g_{10}(\gamma_1, \beta) + r_1 A_1 a_1 g_{10}(\gamma_1, r_1) + r_2 A_2 a_2 g_{10}(\gamma_1, r_2)$$

$$- \gamma_1(A_3 g_{11}(\gamma_1, \alpha) + A_4 g_{11}(\gamma_1, \beta) + A_1 a_1 g_{11}(\gamma_1, r_1) + A_2 a_2 g_{11}(\gamma_1, r_2)),$$

$$f_{20} = \alpha A_3 g_{12}(\gamma_1, \alpha) + \beta A_4 g_{12}(\gamma_1, \beta) + r_1 A_1 a_1 g_{12}(\gamma_1, r_1) + r_2 A_2 a_2 g_{12}(\gamma_1, r_2)$$

$$- \gamma_1(A_3 g_{12}(\gamma_1, \alpha) + A_4 g_{12}(\gamma_1, \beta) + A_1 a_1 g_{12}(\gamma_1, r_1) + A_2 a_2 g_{12}(\gamma_1, r_2)),$$

$$f_{21} = \left(r_1^2 + \alpha^2\right) A_1 a_1 g_{16}(\alpha, r_1) + \left(r_2^2 + \alpha^2\right) A_2 a_2 g_{16}(\alpha, r_2) + \left(\beta^2 + \alpha^2\right) A_4 g_{16}(\alpha, \beta),$$

$$f_{22} = \left(r_2^2 + \alpha^2\right) A_2 a_2 g_{16}(r_1, r_2) + \left(\beta^2 + \alpha^2\right) A_4 g_{16}(r_1, \beta),$$

$$f_{23} = \left(\beta^2 + \alpha^2\right) A_4 g_{16}(r_2, \beta),$$

$$f_{24} = r_1 A_1 a_1 g_{17}(\alpha, r_1) + r_2 A_2 a_2 g_{17}(\alpha, r_2) + \beta A_4 g_{17}(\alpha, \beta),$$

$$f_{25} = r_2 A_2 a_2 g_{17}(r_1, r_2) + \beta A_4 g_{17}(r_1, \beta),$$

$$A_6 = \frac{1}{4}\left(\frac{\gamma_1(1-\cosh 2\gamma_1)}{\sinh 2\gamma_1} + \frac{\gamma_1^*(1-\cosh 2\gamma_1^*)}{\sinh 2\gamma_1^*}\right)$$

$$+ \frac{i\alpha}{8}\frac{\Pr}{\sinh 2\gamma_1}\left[-\operatorname{Re}\left(A_3^*\delta_2(\gamma_1, \alpha) + A_4^*\delta_2(\gamma_1, \beta^*)\right) + A_1^*(\mathrm{af} - \operatorname{Re} a_1^*)\delta_2(\gamma_1, r_1^*)\right.$$

$$\left. + A_2^*(\mathrm{af} - \operatorname{Re} a_2^*)\delta_2(\gamma_1, r_2^*)\right] + \frac{i\alpha}{8}\frac{\Pr}{\sinh 2\gamma_1^*}$$

$$\times\left[\operatorname{Re}\left(A_3\delta_2(\gamma_1^*, \alpha) + A_4\delta_2(\gamma_1^*, \beta)\right) + A_1(a_1 \operatorname{Re} - \mathrm{af})\delta_2(\gamma_1^*, r_1) A_2^*(a_2 \operatorname{Re} - \mathrm{af})\delta_2(\gamma_1^*, r_2)\right]$$

$$+ \mathrm{Ec}\cdot\Pr\left[\frac{2(1+\gamma^{-1})+\mu_1}{4}\right.$$

$$\times\left(2\alpha^2 A_3 A_3^* \cosh 2\alpha + 4\alpha^2(\alpha A_3\lambda_3 + r_1 A_1 a_1 \lambda_4 + r_2 A_2 \lambda_5 + \beta A_4 \lambda_6) + 2\alpha^2 A_3 \lambda_7\right.$$

$$\left. + \left(\alpha^2 + r_1^2\right) A_1 a_1 \lambda_8 + \left(\alpha^2 + r_2^2\right) A_2 a_2 \lambda_9 + \left(\alpha^2 + \beta^2\right) A_4 \lambda_{10}\right) + \frac{2}{M^2}$$

$$\times\left(r_1 A_1 (r_1^* A_1^* \delta_3(r_1, r_1^*) + r_2^* A_2^* \delta_3(r_1, r_2^*)\right)$$

$$+ r_2 A_2\left(r_1^* A_1^* \delta_3(r_2, r_1^*) + r_2^* A_2^* \delta_3(r_2, r_2^*)\right)) + \left(\mu_1 + \frac{2\alpha^2}{M^2}\right)$$

$$\times\left(A_1\left(A_1^*\delta_4(r_1, r_1^*) + A_2^*\delta_4(r_1, r_2^*)\right) + A_2\left(A_1^*\delta_4(r_2, r_1^*) + A_2^*\delta_4(r_2, r_2^*)\right)\right)$$

$$\left. + \frac{\mu_1}{2}\left(A_1\lambda_{11} + A_2\lambda_{12} + A_4\left(\beta^2 - \alpha^2\right)\left(A_1^*\delta_4(\beta, r_1^*) + A_2^*\delta_4(\beta, r_2^*)\right)\right)\right],$$

$$A_7 = -\frac{1}{4}\left\{\left(1-\frac{\mathrm{Sc}\cdot\mathrm{Sr}\cdot\mathrm{Pr}}{(\mathrm{Sc}-\mathrm{Pr})}\right)\left[\gamma_2\left(\frac{1+\cosh 2\gamma_2}{\sinh 2\gamma_2}\right)+\gamma_2^*\left(\frac{1+\cosh 2\gamma_2^*}{\sinh 2\gamma_2^*}\right)\right]\right.$$
$$\left.+\left(\frac{\mathrm{Pr}}{(\mathrm{Sc}-\mathrm{Pr})}+1\right)\mathrm{Sc}\cdot\mathrm{Sr}\left[\gamma_1\left(\frac{1+\cosh 2\gamma_1}{\sinh 2\gamma_1}\right)+\gamma_1^*\left(\frac{1+\cosh 2\gamma_1^*}{\sinh 2\gamma_1^*}\right)\right]\right\}$$
$$-\frac{i\alpha}{8}\mathrm{Sc}\cdot\mathrm{Re}\left\{\left(1-\frac{\mathrm{Sc}\cdot\mathrm{Sr}\cdot\mathrm{Pr}}{(\mathrm{Sc}-\mathrm{Pr})}\right)\right.$$
$$\times\left[\frac{1}{\sinh 2\gamma_2}\left(A_3^*\delta_5(\gamma_2,\alpha)+A_4^*\delta_5(\gamma_2,\beta^*)\right.\right.$$
$$\left.+A_1^*a_1^*\delta_5(\gamma_2,r_1^*)+A_2^*a_2^*\delta_5(\gamma_2,r_2^*)\right)$$
$$-\frac{1}{\sinh 2\gamma_2^*}\left(A_3\delta_5(\gamma_2^*,\alpha)+A_4\delta_5(\gamma_2^*,\beta)\right.$$
$$\left.\left.+A_1a_1\delta_5(\gamma_2^*,r_1)+A_2a_2\delta_5(\gamma_2^*,r_2)\right)\right]$$
$$+\frac{\mathrm{Sc}\cdot\mathrm{Sr}\cdot\mathrm{Pr}}{(\mathrm{Sc}-\mathrm{Pr})}\left[\frac{1}{\sinh 2\gamma_1}\left(A_3^*\delta_5(\gamma_1,\alpha)+A_4^*\delta_5(\gamma_1,\beta^*)+A_1^*a_1^*\delta_5(\gamma_1,r_1^*)\right.\right.$$
$$\left.+A_2^*a_2^*\delta_5(\gamma_1,r_2^*)\right)-\frac{1}{\sinh 2\gamma_1^*}$$
$$\times\left(A_3\delta_5(\gamma_1^*,\alpha)+A_5\delta_5(\gamma_1^*,\beta)+A_1a_1\delta_5(\gamma_1^*,r_1)\right.$$
$$\left.\left.\left.+A_2a_2\delta_5(\gamma_1^*,r_2)\right)\right]\right\},$$

$$A_8 = \frac{1}{4}\left\{\left(1-\frac{\mathrm{Sc}\cdot\mathrm{Sr}\cdot\mathrm{Pr}}{(\mathrm{Sc}-\mathrm{Pr})}\right)\left[\gamma_2\left(\frac{1-\cosh 2\gamma_2}{\sinh 2\gamma_2}\right)+\gamma_2^*\left(\frac{1-\cosh 2\gamma_2^*}{\sinh 2\gamma_2^*}\right)\right]\right.$$
$$\left.+\left(\frac{\mathrm{Pr}}{(\mathrm{Sc}-\mathrm{Pr})}+1\right)\mathrm{Sc}\cdot\mathrm{Sr}\left[\gamma_1\left(\frac{1-\cosh 2\gamma_1}{\sinh 2\gamma_1}\right)+\gamma_1^*\left(\frac{1-\cosh 2\gamma_1^*}{\sinh 2\gamma_1^*}\right)\right]\right\}$$
$$-\frac{i\alpha}{8}\mathrm{Sc}\cdot\mathrm{Re}\left\{\left(1-\frac{\mathrm{Sc}\cdot\mathrm{Sr}\cdot\mathrm{Pr}}{(\mathrm{Sc}-\mathrm{Pr})}\right)\right.$$
$$\times\left[\frac{1}{\sinh 2\gamma_2}\left(A_3^*\delta_2(\gamma_2,\alpha)+A_4^*\delta_2(\gamma_2,\beta^*)+A_1^*a_1^*\delta_2(\gamma_2,r_1^*)\right.\right.$$
$$\left.+A_2^*a_2^*\delta_2(\gamma_2,r_2^*)\right)-\frac{1}{\sinh 2\gamma_2^*}$$
$$\left.\times\left(A_3\delta_2(\gamma_2^*,\alpha)+A_4\delta_2(\gamma_2^*,\beta)+A_1a_1\delta_2(\gamma_2^*,r_1)+A_2a_2\delta_2(\gamma_2^*,r_2)\right)\right]$$

$$+\frac{\mathrm{Sc}\cdot\mathrm{Sr}\cdot\mathrm{Pr}}{(\mathrm{Sc}-\mathrm{Pr})}\Bigg[\frac{1}{\sinh 2\gamma_1}\left(A_3^*\delta_2(\gamma_1,\alpha)+A_4^*\delta_2(\gamma_1,\beta^*)+A_1^*a_1^*\delta_2(\gamma_1,r_1^*)+A_2^*a_2^*\delta_2(\gamma_1,r_2^*)\right)$$

$$-\frac{1}{\sinh 2\gamma_1^*}\left(A_3\delta_2(\gamma_1^*,\alpha)+A_4\delta_2(\gamma_1^*,\beta)+A_1a_1\delta_2(\gamma_1^*,r_1)+A_2a_2\delta_2(\gamma_1^*,r_2)\right)\Bigg]\Bigg\},$$

$$g_4(a,b)=\frac{\sinh(a+(a+b)y)}{(a+b)^2-\gamma_3^2}+\frac{\sinh(a+(a-b)y)}{(a-b)^2-\gamma_3^2},$$

$$g_5(a,b)=\frac{\sinh(a+(a+b)y)}{(a+b)^2-\gamma_3^2}-\frac{\sinh(a+(a-b)y)}{(a-b)^2-\gamma_3^2},$$

$$g_6(a,b)=\frac{\cosh((a+b)y)}{(a+b)^2-\gamma_3^2}-\frac{\cosh((a-b)y)}{(a-b)^2-\gamma_3^2},$$

$$g_7(a,b)=\frac{\cosh((a+b)y)}{(a+b)^2-\gamma_3^2}+\frac{\cosh((a-b)y)}{(a-b)^2-\gamma_3^2},$$

$$g_8(a)=\frac{\cosh(2ay)}{4a^2-\gamma_3^2}+\frac{1}{\gamma_3^2},$$

$$g_9(a)=\frac{\cosh(2ay)}{4a^2-\gamma_3^2}-\frac{1}{\gamma_3^2},$$

$$g_{10}(a,b)=\frac{\sinh(a+(a+b)y)}{(a+b)^2-\gamma_4^2}+\frac{\sinh(a+(a-b)y)}{(a+b)^2-\gamma_4^2},$$

$$g_{11}(a,b)=\frac{\sinh(a+(a+b)y)}{(a+b)^2-\gamma_4^2}-\frac{\sinh(a+(a-b)y)}{(a+b)^2-\gamma_4^2},$$

$$g_{12}(a,b)=\frac{(a+b)^2-4\alpha^2}{(a+b)^2-\gamma_4^2}\frac{\sinh(a+(a+b)y)}{(a+b)^2-\gamma_3^2}+\frac{(a-b)^2-4\alpha^2}{(a-b)^2-\gamma_4^2}\frac{\sinh(a+(a-b)y)}{(a-b)^2-\gamma_3^2},$$

$$g_{13}(a,b)=\frac{(a+b)^2-4\alpha^2}{(a+b)^2-\gamma_4^2}\frac{\sinh(a+(a+b)y)}{(a+b)^2-\gamma_3^2}-\frac{(a-b)^2-4\alpha^2}{(a-b)^2-\gamma_4^2}\frac{\sinh(a+(a-b)y)}{(a-b)^2-\gamma_3^2},$$

$$g_{14}(a)=\frac{4(a^2-\alpha^2)}{(4a^2-\gamma_4^2)}\frac{\cosh(2ay)}{(4a^2-\gamma_3^2)}+\frac{4\alpha^2}{(\gamma_3\gamma_4)^2},$$

$$g_{15}(a)=\frac{4(a^2-\alpha^2)}{(4a^2-\gamma_4^2)}\frac{\cosh(2ay)}{(4a^2-\gamma_3^2)}-\frac{4\alpha^2}{(\gamma_3\gamma_4)^2},$$

$$g_{16}(a,b)=\frac{(a+b)^2-4\alpha^2}{(a+b)^2-\gamma_4^2}\frac{\cosh(a+(a+b)y)}{(a+b)^2-\gamma_3^2}-\frac{(a-b)^2-4\alpha^2}{(a-b)^2-\gamma_4^2}\frac{\cosh((a-b)y)}{(a-b)^2-\gamma_3^2},$$

$$g_{17}(a,b)=\frac{(a+b)^2-4\alpha^2}{(a+b)^2-\gamma_4^2}\frac{\cosh\left(a+(a+b)y\right)}{(a+b)^2-\gamma_3^2}+\frac{(a-b)^2-4\alpha^2}{(a-b)^2-\gamma_4^2}\frac{\cosh\left((a-b)y\right)}{(a-b)^2-\gamma_3^2},$$

$$f_{26}=\beta A_4 g_{17}(r_2,\beta),$$

$$f_{27}=\left(2+\left(r_1^2-\alpha^2\right)a_1+\left(r_2^2-\alpha^2\right)a_2\right)A_2 g_{16}(r_1,r_2)+A_4\left(\beta^2-\alpha^2\right)g_{16}(r_1,\beta),$$

$$f_{28}=r_1A_1g_{12}(\gamma_1,r_1)+r_2A_2g_{12}(\gamma_1,r_2)-\gamma_1A_1g_{13}(\gamma_1,r_1)-\gamma_1A_2g_{13}(\gamma_1,r_2),$$

$$\delta_6(a,b)=\frac{\sinh(2a+b)-\sinh b}{(a+b)^2-\gamma_3^2}+\frac{\sinh(2a-b)-\sinh b}{(a-b)^2-\gamma_3^2},$$

$$\delta_7(a,b)=\frac{\sinh(2a+b)-\sinh b}{(a+b)^2-\gamma_3^2}-\frac{\sinh(2a-b)+\sinh b}{(a-b)^2-\gamma_3^2},$$

$$\delta_8(a,b)=\frac{\sinh(2a+b)+\sinh b}{(a+b)^2-\gamma_3^2}+\frac{\sinh(2a-b)-\sinh b}{(a-b)^2-\gamma_3^2},$$

$$\delta_9(a,b)=\frac{\sinh(2a+b)+\sinh b}{(a+b)^2-\gamma_3^2}-\frac{\sinh(2a-b)-\sinh b}{(a-b)^2-\gamma_3^2},$$

$$\delta_{10}(a)=\frac{\cosh(2a)}{4a^2-\gamma_3^2}+\frac{1}{\gamma_3^2},$$

$$\delta_{11}(a)=\frac{\cosh(2a)}{4a^2-\gamma_3^2}-\frac{1}{\gamma_3^2},$$

$$\delta_{12}(a,b)=\frac{\cosh(a+b)}{(a+b)^2-\gamma_3^2}+\frac{\cosh(a-b)}{(a-b)^2-\gamma_3^2},$$

$$\delta_{13}(a,b)=\frac{\cosh(a+b)}{(a+b)^2-\gamma_3^2}-\frac{\cosh(a-b)}{(a-b)^2-\gamma_3^2},$$

$$\delta_{14}(a,b)=\frac{\sinh(2a+b)-\sinh b}{(a+b)^2-\gamma_4^2}+\frac{\sinh(2a-b)+\sinh b}{(a-b)^2-\gamma_4^2},$$

$$\delta_{15}(a,b)=\frac{\sinh(2a+b)-\sinh b}{(a+b)^2-\gamma_4^2}-\frac{\sinh(2a-b)+\sinh b}{(a-b)^2-\gamma_4^2},$$

$$\delta_{16}(a,b)=\frac{\sinh(2a+b)+\sinh b}{(a+b)^2-\gamma_4^2}+\frac{\sinh(2a-b)-\sinh b}{(a-b)^2-\gamma_4^2},$$

$$\delta_{17}(a,b)=\frac{\sinh(2a+b)+\sinh b}{(a+b)^2-\gamma_4^2}-\frac{\sinh(2a-b)-\sinh b}{(a-b)^2-\gamma_4^2},$$

$$\delta_{18}(a,b) = \frac{(a+b)^2-4\alpha^2}{(a+b)^2-\gamma_4^2}\frac{\sinh(2a+b)-\sinh b}{(a+b)^2-\gamma_3^2} + \frac{(a-b)^2-4\alpha^2}{(a-b)^2-\gamma_4^2}\frac{\sinh(2a-b)+\sinh b}{(a-b)^2-\gamma_3^2},$$

$$\delta_{19}(a,b) = \frac{(a+b)^2-4\alpha^2}{(a+b)^2-\gamma_4^2}\frac{\sinh(2a+b)-\sinh b}{(a+b)^2-\gamma_3^2} - \frac{(a-b)^2-4\alpha^2}{(a-b)^2-\gamma_4^2}\frac{\sinh(2a-b)+\sinh b}{(a-b)^2-\gamma_3^2},$$

$$\delta_{20}(a,b) = \frac{(a+b)^2-4\alpha^2}{(a+b)^2-\gamma_4^2}\frac{\sinh(2a+b)+\sinh b}{(a+b)^2-\gamma_3^2} + \frac{(a-b)^2-4\alpha^2}{(a-b)^2-\gamma_4^2}\frac{\sinh(2a-b)-\sinh b}{(a-b)^2-\gamma_3^2},$$

$$\delta_{21}(a,b) = \frac{(a+b)^2-4\alpha^2}{(a+b)^2-\gamma_4^2}\frac{\sinh(2a+b)-\sinh b}{(a+b)^2-\gamma_3^2} - \frac{(a-b)^2-4\alpha^2}{(a-b)^2-\gamma_4^2}\frac{\sinh(2a-b)+\sinh b}{(a-b)^2-\gamma_3^2},$$

$$\delta_{22}(a) = \frac{4\left(a^2-\alpha^2\right)}{4a^2-\gamma_4^2}\frac{\cosh(2a)}{4a^2-\gamma_3^2} + \frac{4\alpha^2}{(\gamma_3\gamma_4)^2},$$

$$\delta_{23}(a) = \frac{4\left(a^2-\alpha^2\right)}{4a^2-\gamma_4^2}\frac{\cosh(2a)}{4a^2-\gamma_3^2} - \frac{4\alpha^2}{(\gamma_3\gamma_4)^2},$$

$$\delta_{24}(a,b) = \frac{(a+b)^2-4\alpha^2}{(a+b)^2-\gamma_4^2}\frac{\cosh(a+b)}{(a+b)^2-\gamma_3^2} + \frac{(a+b)^2-4\alpha^2}{(a-b)^2-\gamma_4^2}\frac{\cosh(a-b)}{(a-b)^2-\gamma_3^2},$$

$$\delta_{25}(a,b) = \frac{(a+b)^2-4\alpha^2}{(a+b)^2-\gamma_4^2}\frac{\cosh(a+b)}{(a+b)^2-\gamma_3^2} - \frac{(a+b)^2-4\alpha^2}{(a-b)^2-\gamma_4^2}\frac{\cosh(a-b)}{(a-b)^2-\gamma_3^2},$$

$$\lambda_{13} = A_1a_1\left(r_1^2+\alpha^2\right)\delta_{13}(\alpha,r_1) + A_2a_2\left(r_2^2+\alpha^2\right)\delta_{13}(\alpha,r_2) + A_4\left(\beta^2+\alpha^2\right)\delta_{13}(\alpha,\beta),$$

$$\lambda_{14} = A_2a_2\left(r_2^2+\alpha^2\right)\delta_{13}(r_1,r_2) + A_4\left(\beta^2+\alpha^2\right)\delta_{13}(r_1,\beta),$$

$$\lambda_{15} = A_4\left(\beta^2+\alpha^2\right)\delta_{13}(r_2,\beta),$$

$$\lambda_{16} = r_1A_1a_1\delta_{12}(\alpha,r_1) + r_2A_2a_2\delta_{12}(\alpha,r_2) + \beta A_4\delta_{12}(\alpha,\beta),$$

$$\lambda_{17} = r_2A_2a_2\delta_{12}(r_1,r_2) + \beta A_4\delta_{12}(r_1,\beta),$$

$$\lambda_{18} = \beta A_4\delta_{12}(r_2,\beta),$$

$$\lambda_{19} = \left(2+\left(r_1^2-\alpha^2\right)a_1+\left(r_2^2-\alpha^2\right)a_2\right)A_2\delta_{13}(r_1,r_2) + \left(\beta^2-\alpha^2\right)A_4\delta_{13}(r_1,\beta),$$

$$\begin{aligned}\lambda_{20} = {} & \alpha A_3\delta_{14}(\gamma_2,\alpha) + \beta A_4\delta_{14}(\gamma_2,\beta) + r_1A_1a_1\delta_{14}(\gamma_2,r_1) + r_2A_2a_2\delta_{14}(\gamma_2,r_2) \\ & - \gamma_2(A_3\delta_{15}(\gamma_2,\alpha) + A_4\delta_{14}(\gamma_2,\beta) + A_1a_1\delta_{15}(\gamma_2,r_1) + A_2a_2\delta_{15}(\gamma_2,r_2)),\end{aligned}$$

$$\lambda_{21} = \alpha A_3\delta_{14}(\gamma_1,\alpha) + \beta A_4\delta_{14}(\gamma_1,\beta) + r_1A_1a_1\delta_{14}(\gamma_1,r_1) + r_2A_2a_2\delta_{14}(\gamma_1,r_2)$$
$$- \gamma_1(A_3\delta_{15}(\gamma_1,\alpha) + A_4\delta_{14}(\gamma_1,\beta) + A_1a_1\delta_{15}(\gamma_1,r_1) + A_2a_2\delta_{15}(\gamma_1,r_2)),$$

$$\lambda_{22} = \alpha A_3\delta_{18}(\gamma_1,\alpha) + \beta A_4\delta_{18}(\gamma_1,\beta) + r_1A_1a_1\delta_{18}(\gamma_1,r_1) + r_2A_2a_2\delta_{18}(\gamma_1,r_2)$$
$$- \gamma_1(A_3\delta_{19}(\gamma_1,\alpha) + A_4\delta_{19}(\gamma_1,\beta) + A_1a_1\delta_{19}(\gamma_1,r_1) + A_2a_2\delta_{19}(\gamma_1,r_2)),$$

$$\lambda_{23} = \left(r_1^2+\alpha^2\right)A_1a_1\delta_{25}(\alpha,r_1) + \left(r_2^2+\alpha^2\right)A_2a_2\delta_{25}(\alpha,r_2) + \left(\beta^2+\alpha^2\right)A_4\delta_{25}(\alpha,\beta),$$

$$\lambda_{24} = \left(r_2^2+\alpha^2\right)A_2a_2\delta_{25}(r_1,r_2) + \left(\beta^2+\alpha^2\right)A_4\delta_{25}(r_1,\beta),$$

$$\lambda_{25} = \left(\beta^2+\alpha^2\right)A_4\delta_{25}(r_2,\beta),$$

$$\lambda_{26} = r_1A_1a_1\delta_{24}(\alpha,r_1) + r_2A_2a_2\delta_{24}(\alpha,r_2) + \beta A_4\delta_{24}(\alpha,\beta),$$

$$\lambda_{27} = r_2A_2a_2\delta_{24}(r_1,r_2) + \beta A_4\delta_{24}(r_1,\beta),$$

$$\lambda_{28} = \beta A_4\delta_{24}(r_2,\beta),$$

$$\lambda_{29} = \left(2+\left(r_1^2-\alpha^2\right)a_1+\left(r_2^2-\alpha^2\right)a_2\right)A_2\delta_{25}(r_1,r_2) + A_4\left(\beta^2-\alpha^2\right)\delta_{25}(r_1,\beta),$$

$$\lambda_{30} = r_1A_1\delta_{18}(\gamma_1,r_1) + r_2A_2\delta_{18}(\gamma_1,r_2) - \gamma_1A_1\delta_{19}(\gamma_1,r_1) - \gamma_1A_2\delta_{19}(\gamma_1,r_2),$$

$$\lambda_{31} = \alpha A_3\delta_{16}(\gamma_2,\alpha) + \beta A_4\delta_{16}(\gamma_2,\beta) + r_1A_1a_1\delta_{16}(\gamma_2,r_1) + r_2A_2a_2\delta_{16}(\gamma_2,r_2)$$
$$- \gamma_2(A_3\delta_{17}(\gamma_2,\alpha) + A_4\delta_{17}(\gamma_2,\beta) + A_1a_1\delta_{17}(\gamma_2,r_1) + A_2a_2\delta_{17}(\gamma_2,r_2)),$$

$$\lambda_{32} = \alpha A_3\delta_{16}(\gamma_1,\alpha) + \beta A_4\delta_{16}(\gamma_1,\beta) + r_1A_1a_1\delta_{16}(\gamma_1,r_1) + r_2A_2a_2\delta_{16}(\gamma_1,r_2)$$
$$- \gamma_1(A_3\delta_{17}(\gamma_1,\alpha) + A_4\delta_{17}(\gamma_1,\beta) + A_1a_1\delta_{17}(\gamma_1,r_1) + A_2a_2\delta_{17}(\gamma_1,r_2)),$$

$$\lambda_{33} = \alpha A_3\delta_{20}(\gamma_1,\alpha) + \beta A_4\delta_{20}(\gamma_1,\beta) + r_1A_1a_1\delta_{20}(\gamma_1,r_1) + r_2A_2a_2\delta_{20}(\gamma_1,r_2)$$
$$- \gamma_1(A_3\delta_{21}(\gamma_1,\alpha) + A_4\delta_{21}(\gamma_1,\beta) + A_1a_1\delta_{21}(\gamma_1,r_1) + A_2a_2\delta_{21}(\gamma_1,r_2)),$$

$$\lambda_{34} = r_1A_1\delta_{20}(\gamma_1,r_1) + r_2A_2\delta_{21}(\gamma_1,r_2) - \gamma_1A_1\delta_{21}(\gamma_1,r_1) - \gamma_1A_2\delta_{21}(\gamma_1,r_2),$$

$$A_9 = \frac{1}{2\cosh\gamma_3}$$

$$\times\left\{\frac{1}{2}\gamma_1\left(\frac{1-\cosh 2\gamma_1}{\sinh 2\gamma_1}\right) - \frac{i\alpha}{4}\frac{\Pr}{\sinh 2\gamma_1}\right.$$

$$\times\left[\operatorname{Re}\left(\alpha A_3\delta_6(\gamma_1,\alpha) + \beta A_4\delta_6(\gamma_1,\beta) - \gamma_1 A_3\delta_7(\gamma_1,\alpha) - \gamma_1 A_4\delta_7(\gamma_1,\beta)\right) + (a_1\operatorname{Re} - \mathrm{af})\right.$$

$$\left.\times\left(r_1A_1\delta_6(\gamma_1,r_1) - \gamma_1A_1\delta_7(\gamma_1,r_1)\right) + (a_2\operatorname{Re} - \mathrm{af})\left(r_2A_2\delta_6(\gamma_1,r_2) - \gamma_1A_2\delta_7(\gamma_1,r_2)\right)\right]$$

$$+ \Pr\cdot\mathrm{Ec}\left[\frac{2(1+\gamma^{-1}) + \mu_1}{4}\right.$$

$$\times\left(4\alpha^4A_3^2\delta_{10}(\alpha) + (r_1^2+\alpha^2)^2A_1^2a_1^2\delta_{10}(r_1) + (r_2^2+\alpha^2)^2A_2^2a_2^2\delta_{10}(r_2)\right.$$

$$+\left(\beta^2+\alpha^2\right)^2A_4^2\delta_{10}(\beta) + 4\alpha^2A_3\lambda_{13} + 2A_1a_1\left(\alpha^2+r_1^2\right)\lambda_{14}$$

$$+2A_2a_2\left(\alpha^2+r_2^2\right)\lambda_{15}$$

$$-4\alpha^2\left(\alpha^2A_3^2\delta_{11}(\alpha) + r_1^2A_1^2a_1^2\delta_{11}(r_1) + r_2^2A_2^2a_2^2\delta_{11}(r_2) + \beta^2A_4^2\delta_{11}(\beta)\right.$$

$$\left.\left.+2\alpha A_3\lambda_{16} + 2r_1A_1a_1\lambda_{17} + 2r_2A_2a_2\lambda_{18}\right)\right)$$

$$+\mu_1\left(\left(1+\left(r_1^2-\alpha^2\right)a_1\right)A_1^2\delta_{10}(r_1) + \left(1+\left(r_2^2-\alpha^2\right)a_2\right)A_2^2\delta_{10}(r_2)\right.$$

$$\left.+A_1\lambda_{19} + A_2A_4\left(\beta^2-\alpha^2\right)\delta_{13}(r_2,\beta)\right)$$

$$+\frac{2}{M^2}\left(r_1^2A_1^2\delta_{11}(r_1) + r_2^2A_2^2\delta_{11}(r_2) + 2r_1r_2A_1A_2\delta_{12}(r_1,r_2)\right.$$

$$\left.\left.\left.-\alpha^2\left(A_1^2\delta_{10}(r_1) + A_2^2\delta_{10}(r_2) + 2A_1A_2\delta_{13}(r_1,r_2)\right)\right)\right]\right\},$$

$$A_{10} = -\frac{1}{2\cosh\gamma_3}\left\{\frac{1}{2}\gamma_1\left(\frac{1+\cosh 2\gamma_1}{\sinh 2\gamma_1}\right) + \frac{i\alpha}{4}\frac{\Pr}{\sinh 2\gamma_1}\right.$$

$$\times\left[\operatorname{Re}\left(\alpha A_3\delta_8(\gamma_1,\alpha) + \beta A_4\delta_8(\gamma_1,\beta) - \gamma_1A_3\delta_9(\gamma_1,\alpha) - \gamma_1A_4\delta_9(\gamma_1,\beta)\right)\right.$$

$$+ (a_1\operatorname{Re} - \mathrm{af})\left(r_1A_1\delta_8(\gamma_1,r_1) - \gamma_1A_1\delta_9(\gamma_1,r_1)\right)$$

$$\left.\left.+(a_2Re - af)\left(r_2A_2\delta_8(\gamma_1,r_2) - \gamma_1A_2\delta_9(\gamma_1,r_2)\right)\right]\right\},$$

$$A_{11} = \frac{1}{2\cosh\gamma_4}\left\{\frac{1}{2}\left[\left(1-\frac{\mathrm{Sc}\cdot\mathrm{Sr}\cdot\mathrm{Pr}}{(\mathrm{Sc}-\mathrm{Pr})}\right)\gamma_2\left(\frac{1-\cosh 2\gamma_2}{\sinh 2\gamma_2}\right)+\frac{\gamma_1\cdot\mathrm{Sc}\cdot\mathrm{Sr}\cdot\mathrm{Pr}}{\mathrm{Sc}-\mathrm{Pr}}\left(\frac{1-\cosh 2\gamma_1}{\sinh 2\gamma_1}\right)\right]\right.$$

$$-\frac{i\alpha}{4}\mathrm{Sc}\cdot\mathrm{Re}\left\{\left(1-\frac{\mathrm{Sc}\cdot\mathrm{Sr}\cdot\mathrm{Pr}}{(\mathrm{Sc}-\mathrm{Pr})}\right)\frac{\lambda_{20}}{\sinh 2\gamma_2}+\frac{\mathrm{Sc}\cdot\mathrm{Sr}\cdot\mathrm{Pr}}{(\mathrm{Sc}-\mathrm{Pr})}\frac{\lambda_{21}}{\sinh 2\gamma_1}\right\}$$

$$-\frac{\mathrm{Pr}\cdot\mathrm{Sc}\cdot\mathrm{Sr}}{\mathrm{Sc}-\mathrm{Pr}}2A_9\cosh\gamma_3+\frac{i\alpha}{4}\mathrm{Sc}\cdot\mathrm{Sr}\cdot\mathrm{Pr}\cdot\mathrm{Re}\,\frac{\lambda_{22}}{\sinh 2\gamma_1}$$

$$-\mathrm{Pr}\cdot\mathrm{Ec}\cdot\mathrm{Sc}\cdot\mathrm{Sr}\left[\frac{\left(1+\gamma^{-1}\right)+\mu_1}{2}\right.$$

$$\times\left(\frac{32\alpha^6A_3^2}{\left(\gamma_3\gamma_4\right)^2}+\left(r_1^2+\alpha^2\right)^2A_1^2a_1^2\delta_{22}(r_1)\right.$$

$$+\left(r_2^2+\alpha^2\right)^2A_2^2a_2^2\delta_{22}(r_2)$$

$$+\left(\beta^2+\alpha^2\right)^2A_4^2\delta_{22}(\beta)+4\alpha^2A_3\lambda_{23}$$

$$+2\left(\alpha^2+r_1^2\right)A_1a_1\lambda_{24}$$

$$+2\left(\alpha^2+r_2^2\right)A_2a_2\lambda_{25}$$

$$-4\alpha^2\left(r_1^2A_1^2a_1^2\delta_{23}(r_1)+r_2^2A_2^2a_2^2\delta_{23}(r_2)+\beta^2A_4^2\delta_{23}(\beta)\right.$$

$$\left.\left.+2\alpha A_3\lambda_{26}+2r_1A_1a_1\lambda_{27}+2r_2A_2a_2\lambda_{28}\right)\right)$$

$$+\mu_1\left(\left(1+\left(r_1^2-\alpha^2\right)a_1\right)A_1^2\delta_{22}(r_1)+\left(1+\left(r_2^2-\alpha^2\right)a_2\right)\right.$$

$$\times A_2^2\delta_{22}(r_2)$$

$$\left.+A_1\lambda_{29}+A_2A_4\left(\beta^2-\alpha^2\right)\delta_{25}(r_2,\beta)\right)$$

$$+\frac{2}{M^2}\left(r_1^2A_1^2\delta_{23}(r_1)+r_2^2A_2^2\delta_{23}(r_2)+2r_1r_2A_1A_2\delta_{24}(r_1,r_2)\right.$$

$$\left.-\alpha^2\left(A_1^2\delta_{22}(r_1)+A_2^2\delta_{22}(r_2)+2A_1A_2\delta_{25}(r_1,r_2)\right)\right)$$

$$\left.\left.-\frac{i\alpha}{4}\mathrm{af}\cdot\mathrm{Pr}\cdot\mathrm{Sc}\cdot\mathrm{Sr}\cdot\frac{\lambda_{30}}{\sinh 2\gamma_1}\right]\right\},$$

$$A_{12} = \frac{-1}{2\cosh\gamma_4}\left\{\frac{1}{2}\left[\left(1-\frac{\mathrm{Sc}\cdot\mathrm{Sr}\cdot\mathrm{Pr}}{(\mathrm{Sc}-\mathrm{Pr})}\right)\gamma_2\left(\frac{1+\cosh 2\gamma_2}{\sinh 2\gamma_2}\right)+\frac{\gamma_1\cdot\mathrm{Sc}\cdot\mathrm{Sr}\cdot\mathrm{Pr}}{\mathrm{Sc}-\mathrm{Pr}}\left(\frac{1+\cosh 2\gamma_1}{\sinh 2\gamma_1}\right)\right]\right.$$
$$+\frac{i\alpha}{4}\mathrm{Sc}\cdot\mathrm{Re}\left[\left(1-\frac{\mathrm{Sc}\cdot\mathrm{Sr}\cdot\mathrm{Pr}}{(\mathrm{Sc}-\mathrm{Pr})}\right)\frac{\lambda_{31}}{\sinh 2\gamma_2}+\frac{\mathrm{Sc}\cdot\mathrm{Sr}\cdot\mathrm{Pr}}{(\mathrm{Sc}-\mathrm{Pr})}\frac{\lambda_{32}}{\sinh 2\gamma_1}\right]$$
$$+\frac{\mathrm{Pr}\cdot\mathrm{Sc}\cdot\mathrm{Sr}}{\mathrm{Sc}-\mathrm{Pr}}2A_{10}\sinh\gamma_3-\frac{i\alpha}{4}\mathrm{Sc}\cdot\mathrm{Sr}\cdot\mathrm{Pr}\cdot\mathrm{Re}\,\frac{\lambda_{33}}{\sinh 2\gamma_1}$$
$$\left.+\frac{i\alpha}{4}\mathrm{af}\cdot\mathrm{Pr}\cdot\mathrm{Sc}\cdot\mathrm{Sr}\cdot\frac{\lambda_{34}}{\sinh 2\gamma_1}\right\}. \tag{A.1}$$

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