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Research Article

Peristaltic Flow of Carreau Fluid in a Rectangular Duct through a Porous Medium

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We have examined the peristaltic flow of Carreau fluid in a rectangular channel through a porous medium. The governing equations of motion are simplified by applying the long wavelength and low Reynolds number approximations. The reduced highly nonlinear partial differential equations are solved jointly by homotopy perturbation and Eigen function expansion methods. The expression for pressure rise is computed numerically by evaluating the numerical integration. The physical features of pertinent parameters have been discussed by plotting graphs of velocity, pressure rise, pressure gradient, and stream functions.

1. Introduction

Investigation of flow through a porous medium has many applications in various branches of science and technology. The applications in which flow through a porous medium is mostly prominent are filtration of fluids, seepage of water in river beds, movement of underground water and oils, limestone, rye bread, wood, the human lung, bile duct, gallbladder with stones, and small blood vessels which are few examples of flow through porous medium [1]. Peristaltic mechanism is another important phenomenon which has exploited the attention of many researchers due to its physiological and industrial applications. A large amount of literature is available on the peristalsis involving Newtonian and non-Newtonian fluids with different flow geometries [2–12]. The peristaltic flow through a porous medium has been also discussed by number of researchers. Pandey and Chaube [13] have examined the peristaltic flow of micropolar fluid through a porous medium in the presence of external magnetic field. They pointed out that the maximum pressure is strongly dependent on permeability of porous medium. Interaction of peristaltic flow with pulsatile fluid through a porous medium has been investigated by Afifi and Gad [14]. Rao and Mishra [15] have examined the peristaltic transport of a power law fluid in a porous tube. In another paper Mishra and Rao [16] have studied the peristaltic transport in a channel with a porous peripheral layer. Some other papers on this topic are given in the references [17–20]. The mathematical model of peristaltic flow in a two-dimensional symmetric and asymmetric channel was discussed by Eytan and Elad [17] because of its application in interuterine fluid flow in a nonpregnant uterus. A number of researchers have examined the peristaltic flow of two-dimensional flow in an asymmetric channel [18–25]. However Reddy et al. [26] have recently given the idea that the sagittal cross-section of the uterus may be better approximated by a tube of rectangular cross-section than a two-dimensional channel and presented the influence of lateral walls on peristaltic flow in a rectangular duct. Only few papers (two or three) have been given in literature which discuss the peristaltic flows in a rectangular channel [26, 27].

Motivated by the previous studies, in the present investigation, we have examined the peristaltic flow of Carreau fluid in a rectangular symmetric channel through a porous medium. In the laboratory frame under the assumptions of long wavelength and low Reynolds number, the solutions of the governing equations of Carreau fluid in a rectangular duct have been found by using homotopy perturbation method. The physical features of the pertinent parameters are discussed by plotting pressure rise, velocity, pressure gradient, and stream functions.

2. Mathematical Formulation

Consider the peristaltic flow of an incompressible Carreau fluid in a duct of rectangular cross-section through a porous medium having the channel width $2d$ and height $2a$. We are considering the Cartesian coordinate system in such a way that X-axis is taken along the axial direction, Y-axis is taken along the lateral direction, and Z-axis is along the vertical direction of rectangular channel.

The peristaltic waves on the walls are represented as

$$Z = H(X,t) = \pm a \pm b\cos\left[\frac{2\pi}{\lambda}(X-ct)\right], \tag{2.1}$$

where a and b are the amplitudes of the waves, λ is the wavelength, c is the velocity of the propagation, t is the time, and X is the direction of wave propagation. The walls parallel to XZ-plane remain undisturbed and are not subject to any peristaltic wave motion. We assume that the lateral velocity is zero as there is no change in lateral direction of the duct cross-section. Let $(U,0,W)$ be the velocity for a rectangular duct. The governing equations for the flow problem are stated as follows.

Continuity equation:

$$\frac{\partial U}{\partial X} + \frac{\partial W}{\partial Z} = 0, \tag{2.2}$$

X-momentum equation:

$$\rho\left(\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial X} + W\frac{\partial U}{\partial Z}\right) = -\frac{\partial P}{\partial Z} + \frac{\partial}{\partial X}S_{XX} + \frac{\partial}{\partial Y}S_{XY} + \frac{\partial}{\partial Z}S_{XZ} - \frac{\mu}{k_1}U, \tag{2.3}$$

Y-momentum equation:

$$0 = -\frac{\partial P}{\partial Y} + \frac{\partial}{\partial X} S_{YX} + \frac{\partial}{\partial Y} S_{YY} + \frac{\partial}{\partial Z} S_{YZ}, \tag{2.4}$$

Z-momentum equation:

$$\rho\left(\frac{\partial W}{\partial t} + U\frac{\partial W}{\partial X} + W\frac{\partial W}{\partial Z}\right) = -\frac{\partial P}{\partial Z} + \frac{\partial}{\partial X} S_{ZX} + \frac{\partial}{\partial Y} S_{ZY} + \frac{\partial}{\partial Z} S_{ZZ}, \tag{2.5}$$

in which ρ is the density, P is the pressure, and $\mathbf{S}$ is the stress tensor for Carreau fluid, which is defined as

$$\mathbf{S} = \mu\left(1 + \left(\Gamma \dot{\gamma}\right)^2\right)^{(n-1)/2} \dot{\gamma}\,. \tag{2.6}$$

Let us define a wave frame (x, y) moving with the velocity c away from the fixed frame (X, Y) by the transformation

$$x = X - ct, \qquad y = Y, \qquad z = Z, \qquad u = U - c, \qquad w = W, \qquad p(x, z) = P(X, Z, t). \tag{2.7}$$

Define the following nondimensional quantities:

$$\bar{x} = \frac{x}{\lambda}, \quad \bar{y} = \frac{y}{d}, \quad \bar{z} = \frac{z}{a}, \quad \bar{u} = \frac{u}{c}, \quad \bar{w} = \frac{w}{c\delta}, \quad \bar{t} = \frac{ct}{\lambda}, \quad \bar{h} = \frac{H}{a}, \quad \bar{p} = \frac{a^2 p}{\mu c \lambda},$$
$$\mathrm{Re} = \frac{\rho a c}{\mu}, \quad \delta = \frac{a}{\lambda}, \quad \phi = \frac{b}{a}, \quad \overline{S}_{\bar{x}\bar{x}} = \frac{a}{\mu c} S_{xx}, \quad \overline{S}_{\bar{x}\bar{y}} = \frac{d}{\mu c} S_{xy}, \quad k = \frac{k_1}{a^2},$$
$$\overline{S}_{\bar{x}\bar{z}} = \frac{a}{\mu c} S_{xz}, \quad \overline{S}_{\bar{y}\bar{z}} = \frac{d}{\mu c} S_{yz}, \quad \overline{S}_{\bar{z}\bar{z}} = \frac{\lambda}{\mu c} S_{zz}, \quad \overline{S}_{\bar{y}\bar{y}} = \frac{\lambda}{\mu c} S_{yy}, \quad \beta = \frac{a}{d},$$
$$\bar{\dot{\gamma}} = \frac{\dot{\gamma}\, d_1}{c}, \quad We = \frac{\Gamma c}{d_1}. \tag{2.8}$$

Using the previous nondimensional quantities in (2.2) to (2.5), the resulting equations (after dropping the bars) can be written as

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0, \tag{2.9}$$

$$\mathrm{Re}\,\delta\left(u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial x} + \delta\frac{\partial}{\partial x} S_{xx} + \beta^2\frac{\partial}{\partial y} S_{xy} + \frac{\partial}{\partial z} S_{xz} - \frac{1}{k}(u+1),$$
$$0 = -\frac{\partial p}{\partial y} + \delta^2\frac{\partial}{\partial x} S_{yx} + \delta^2\frac{\partial}{\partial y} S_{yy} + \delta\frac{\partial}{\partial z} S_{yz}, \tag{2.10}$$
$$\mathrm{Re}\,\delta^2\left(u\frac{\partial w}{\partial x} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial p}{\partial z} + \delta^2\frac{\partial}{\partial x} S_{zx} + \delta\beta^2\frac{\partial}{\partial y} S_{zy} + \delta^2\frac{\partial}{\partial z} S_{zz},$$

where

$$
\begin{aligned}
S_{xx} &= 2\delta\left(1+\frac{n-1}{2}We^2\dot{\gamma}^2\right)\frac{\partial u}{\partial x},\\
S_{xy} &= \left(1+\frac{n-1}{2}We^2\dot{\gamma}^2\right)\frac{\partial u}{\partial y},\\
S_{xz} &= \left(1+\frac{n-1}{2}We^2\dot{\gamma}^2\right)\left(\frac{\partial u}{\partial z}+\delta^2\frac{\partial w}{\partial x}\right),\\
S_{yy} &= 0,\\
S_{yz} &= \delta\left(1+\frac{n-1}{2}We^2\dot{\gamma}^2\right)\frac{\partial w}{\partial y},\\
S_{zz} &= 2\left(1+\frac{n-1}{2}We^2\dot{\gamma}^2\right)\frac{\partial w}{\partial z},\\
\dot{\gamma}^2 &= 2\delta^2\left(\frac{\partial u}{\partial x}\right)^2+\beta^2\left(\frac{\partial u}{\partial y}\right)^2+\delta^2\beta^2\left(\frac{\partial w}{\partial y}\right)^2+\delta^2\left(\frac{\partial w}{\partial z}\right)^2+\left(\delta^2\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right)^2.
\end{aligned}
\tag{2.11}
$$

Under the assumption of long wavelength $\delta \leq 1$ and low Reynolds number $\mathrm{Re} \rightarrow 0$, (2.10), take the form

$$
\begin{aligned}
\frac{dp}{dx} = {}&\beta^2\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}+\frac{n-1}{2}We^2\beta^4\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)^3+\frac{n-1}{2}We^2\frac{\partial}{\partial z}\left(\frac{\partial u}{\partial z}\right)^3+\frac{n-1}{2}We^2\beta^2\\
&\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\left(\frac{\partial u}{\partial z}\right)^2\right)+\frac{n-1}{2}We^2\beta^2\frac{\partial}{\partial z}\left(\frac{\partial u}{\partial z}\left(\frac{\partial u}{\partial y}\right)^2\right)-\frac{1}{k}(u+1).
\end{aligned}
\tag{2.12}
$$

The corresponding boundary conditions are

$$
\begin{aligned}
u &= -1 \quad \text{at } y = \pm 1,\\
u &= -1 \quad \text{at } z = \pm h(x) = \pm 1 \pm \phi\cos 2\pi x,
\end{aligned}
\tag{2.13}
$$

where $0 \leq \phi \leq 1$.

3. Solution of the Problem

The solution of the previously mentioned nonlinear partial differential equation has been calculated by homotopy perturbation method (HPM), which is defined as [28–44]

$$
\begin{aligned}
H(v,q) = L(v)-L(u_0)+qL(u_0)+q\Bigg(&\frac{n-1}{2}We^2\beta^4\frac{\partial}{\partial y}\left(\frac{\partial v}{\partial y}\right)^3+\frac{n-1}{2}We^2\frac{\partial}{\partial z}\left(\frac{\partial v}{\partial z}\right)^3\\
&+\frac{n-1}{2}We^2\beta^2\frac{\partial}{\partial y}\left(\frac{\partial v}{\partial y}\left(\frac{\partial v}{\partial z}\right)^2\right)\\
&+\frac{n-1}{2}We^2\beta^2\frac{\partial}{\partial z}\left(\frac{\partial v}{\partial z}\left(\frac{\partial v}{\partial y}\right)^2\right)-\frac{dp}{dx}-\frac{1}{k}\Bigg),
\end{aligned}
\tag{3.1}
$$

Table 1: Velocity for various values of z for fixed $\phi = 0.6$, $Q = 0.5$.

z	For Newtonian fluid, when $n = 0$, $We = 0$ $u(x, y, z)$ for $\beta = 0.5$ Exact solution [26]	Analytical solution	For Carreau fluid when $n = 0.9$, $We = 0$ $u(x, y, z)$ for $\beta = 0.5$ Analytical solution
−1.6	−1.0000	−1.0000	−1.0000
−1.2	0.2108	0.2115	−0.0938
−0.8	1.1161	1.1173	0.1677
−0.4	1.6746	1.6758	0.2445
0.0	1.8633	1.8645	0.2613
0.4	1.6746	1.6758	0.2445
0.8	1.1161	1.1173	0.1677
1.2	0.2108	0.2115	−0.0938
1.6	−1.0000	−1.0000	−1.0000

Table 2: Pressure rise for various values of Q.

Q	For Newtonian fluid, when $n = 0$, $We = 0$ Δp for $\beta = 2$, $\phi = 0.6$ Exact solution [26]	Analytical solution	For Carreau fluid when $n = 0.9$, $We = 0.9$ Δp for $\beta = 2$, $\phi = 0.6$ Analytical solution
−0.1	12.514	12.7087	12.8287
0.0	9.4799	9.4399	9.8599
0.1	6.4457	6.4712	6.8912
0.2	3.4115	3.4025	3.9225
0.3	0.3773	0.3338	0.9538
0.4	−2.6569	−2.6350	−2.0149
0.5	−5.6911	−5.6037	−4.9837
0.6	−8.7254	−8.7724	−7.9524
0.7	−11.7596	−11.7412	−10.9211
0.8	−14.7938	−14.7099	−13.8899

in which q is embedding parameter which has the range $0 \le q \le 1$. For our convenience we have taken $L = \beta^2(\partial^2/\partial y^2) + (\partial^2/\partial z^2) - 1/k$ as the linear operator. We choose the following initial guess:

$$u_0(y, z) = -\cosh \frac{y}{\beta\sqrt{2k}} \operatorname{sech} \frac{1}{\beta\sqrt{2k}} \cosh \frac{z}{\sqrt{2k}} \operatorname{sech} \frac{h}{\sqrt{2k}}. \tag{3.2}$$

Define

$$v(x, y, z, q) = v_0 + qv_1 + q^2 v_2 + \cdots. \tag{3.3}$$

Substituting (3.3) into (3.1) and then comparing the like powers of q, one obtains the following problems with the corresponding boundary conditions:

$$L(v_0) - L(u_0) = 0, \tag{3.4}$$

$$\begin{aligned} v_0 &= -1 \quad \text{at } y = \pm 1, \\ v_0 &= -1 \quad \text{at } z = \pm h(x), \end{aligned} \tag{3.5}$$

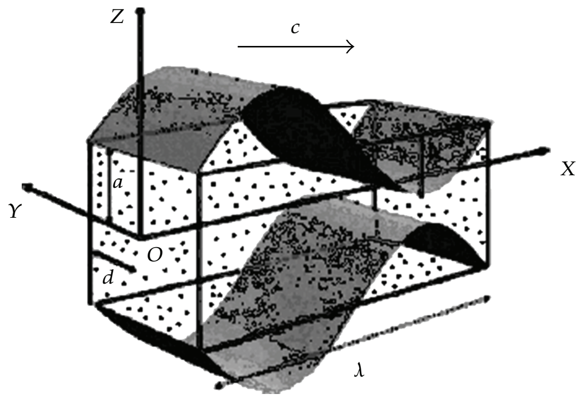


Figure 1: Schematic diagram for peristaltic flow in a rectangular duct.

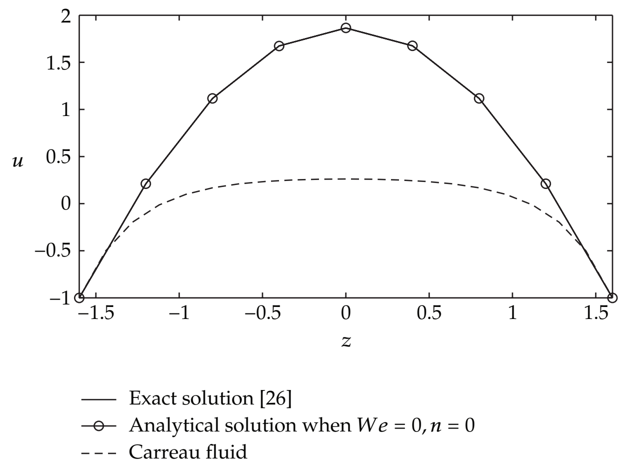


Figure 2: Velocity for various values of z for fixed $\phi = 0.6$, $Q = 0.5$, $x = 0$, $y = 0.5$, $\beta = 0.5$.

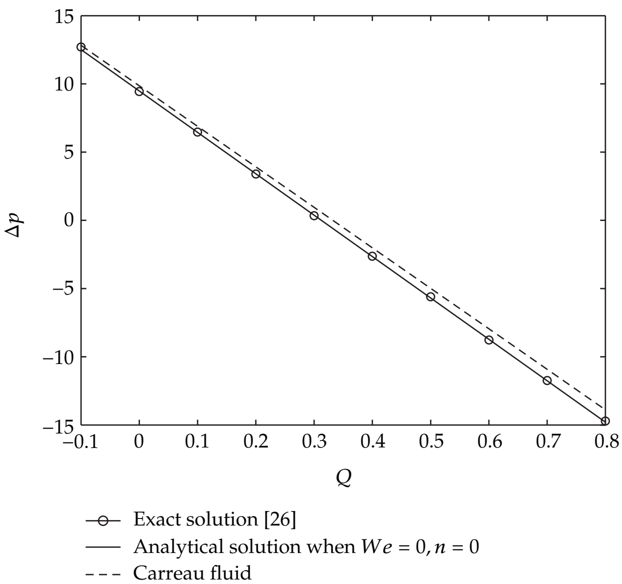


Figure 3: Pressure rise for various values of Q, when $\phi = 0.6$, $\beta = 2$.

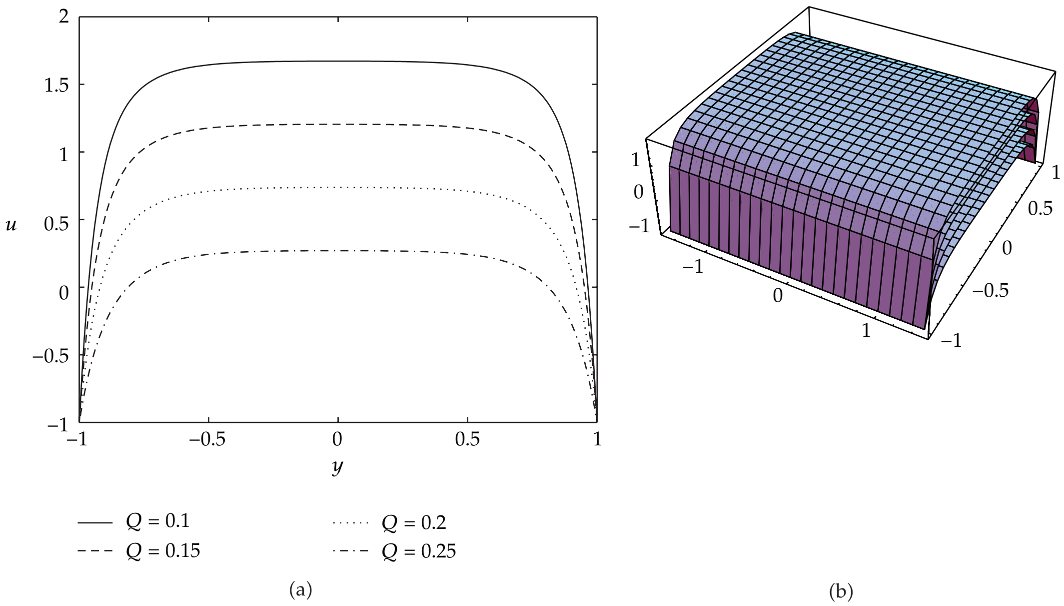


Figure 4: Velocity profile for different values of Q for fixed $k = 1,\ n = 0.9,\ We = 0.03,\ \beta = 0.1,\ \phi = 0.5,\ x = 0.5$. (a) For 2-dimensional and (b) for 3-dimensional.

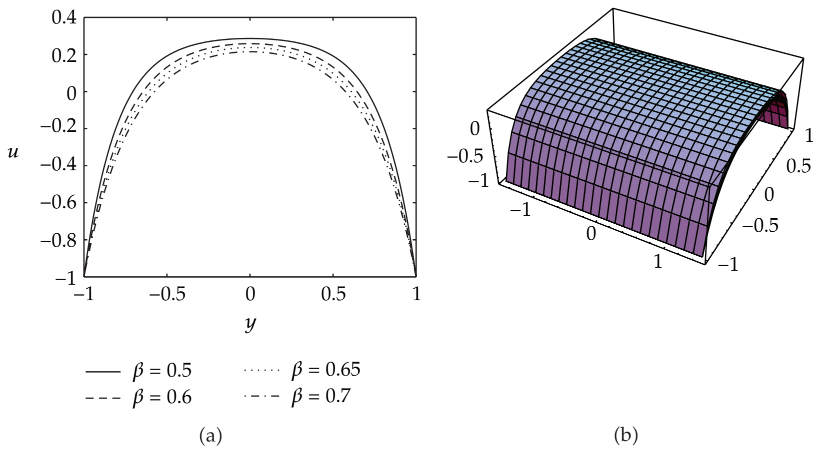
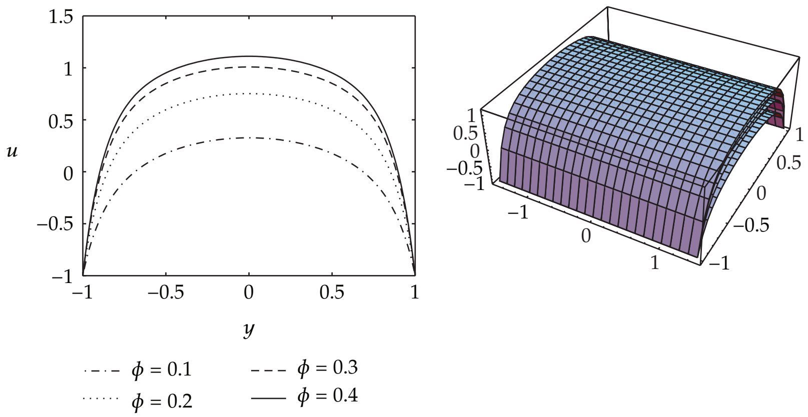


Figure 5: Velocity profile for different values of β for fixed $k = 0.1,\ n = 0.9,\ We = 0.03,\ \phi = 0.5,\ Q = 2,\ x = 0.5$. (a) For 2-dimensional and (b) For 3-dimensional.

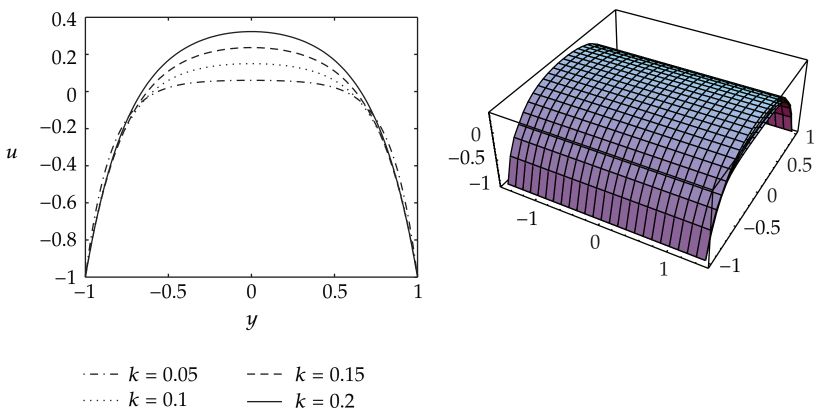
$$L(v_1) + L(u_0) + \left(\frac{n-1}{2} We^2 \beta^4 \frac{\partial}{\partial y}\left(\frac{\partial v_0}{\partial y}\right)^3 + \frac{n-1}{2} We^2 \frac{\partial}{\partial z}\left(\frac{\partial v_0}{\partial z}\right)^3 + \frac{n-1}{2} We^2 \beta^2 \times \frac{\partial}{\partial y}\left(\frac{\partial v_0}{\partial y}\left(\frac{\partial v_0}{\partial z}\right)^2\right) + \frac{n-1}{2} We^2 \beta^2 \frac{\partial}{\partial z}\left(\frac{\partial v_0}{\partial z}\left(\frac{\partial v_0}{\partial y}\right)^2\right) - \frac{dp}{dx} - \frac{1}{k} \right) = 0, \tag{3.6}$$

$$\begin{aligned} v_1 &= 0 \quad \text{at } y = \pm 1, \\ v_1 &= 0, \quad \text{at } z = \pm h(x). \end{aligned} \tag{3.7}$$



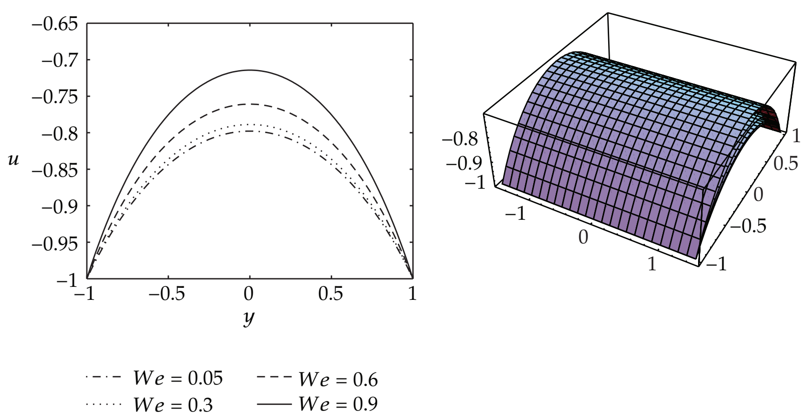
(a) (b)

Figure 6: Velocity profile for different values of ϕ for fixed $k = 0.5$, $n = 0.9$, $We = 0.03$, $\beta = 0.5$, $Q = 2$, $x = 0.5$. (a) For 2-dimensional and (b) for 3-dimensional.



(a) (b)

Figure 7: Velocity profile for different values of k for fixed $Q = 2$, $n = 0.9$, $We = 0.03$, $\beta = 0.5$, $\phi = 0.5$, $x = 0.5$. (a) For 2-dimensional and (b) for 3-dimensional.



(a) (b)

Figure 8: Velocity profile for different values of We for fixed $k = 0.4$, $Q = 2$, $n = 0.9$, $\beta = 2$, $\phi = 0.95$, $x = 0$. (a) For 2-dimensional and (b) for 3-dimensional.

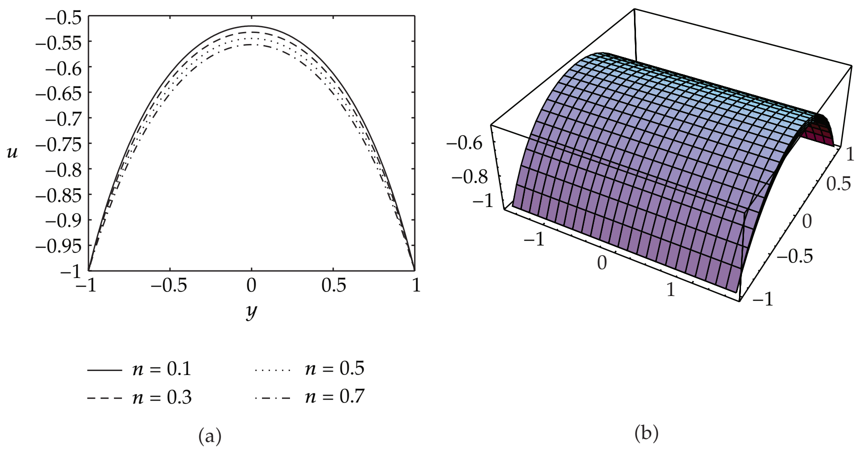


Figure 9: Velocity profile for different values of n for fixed $k = 0.1,\ Q = 2,\ We = 0.5,\ \beta = 2,\ \phi = 0.95,\ x = 0.5$. (a) For 2-dimensional and (b) for 3-dimensional.

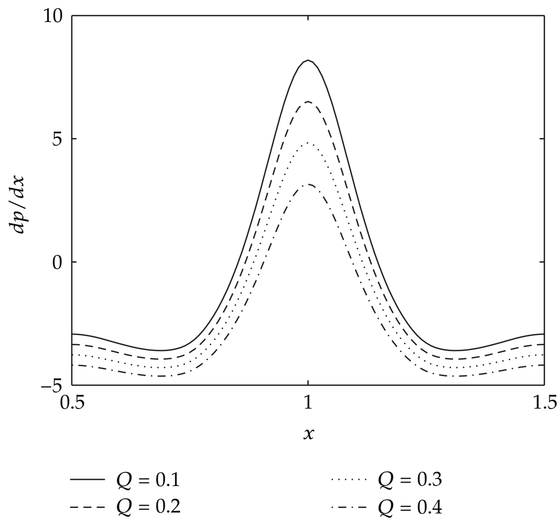


Figure 10: Variation of dp/dx with x for different values of Q at $k = 0.15,\ n = 0.9,\ We = 0.03,\ \beta = 0.05,\ \phi = 0.6$.

From (3.4) we have

$$v_0 = u_0 = -\cosh\frac{y}{\beta\sqrt{2k}}\operatorname{sech}\frac{1}{\beta\sqrt{2k}}\cosh\frac{z}{\sqrt{2k}}\operatorname{sech}\frac{h}{\sqrt{2k}}. \tag{3.8}$$

With the help of (3.8), (3) can be written as

$$\beta^2\frac{\partial^2 v_1}{\partial y^2} + \frac{\partial^2 v_1}{\partial z^2} - \frac{v_1}{k} = \frac{n-1}{2}We^2\frac{A^3}{k^2}\sinh^2\frac{y}{\beta\sqrt{2k}}\cosh^3\frac{z}{\sqrt{2k}}\cosh\frac{y}{\beta\sqrt{2k}} + \sinh^2\frac{y}{\beta\sqrt{2k}}$$
$$\sinh^2\frac{z}{\sqrt{2k}}\cosh\frac{z}{\sqrt{2k}}\cosh\frac{y}{\beta\sqrt{2k}} + \cosh^3\frac{z}{\sqrt{2k}}\cosh^3\frac{y}{\beta\sqrt{2k}} + \frac{dp}{dx} + \frac{1}{k}, \tag{3.9}$$

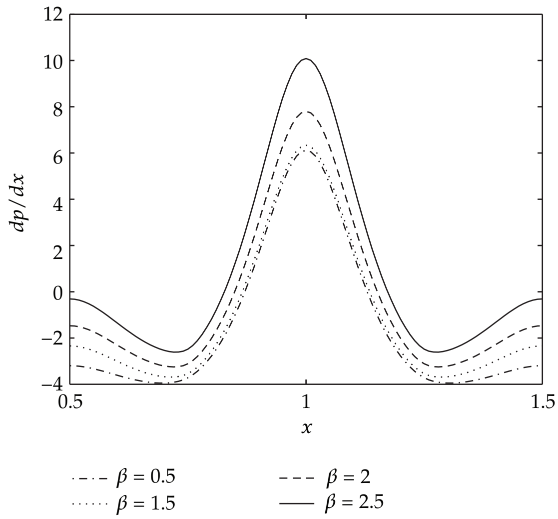


Figure 11: Variation of dp/dx with x for different values of β at $k = 0.15$, $n = 0.9$, $We = 0.03$, $Q = 0.2$, $\phi = 0.6$.

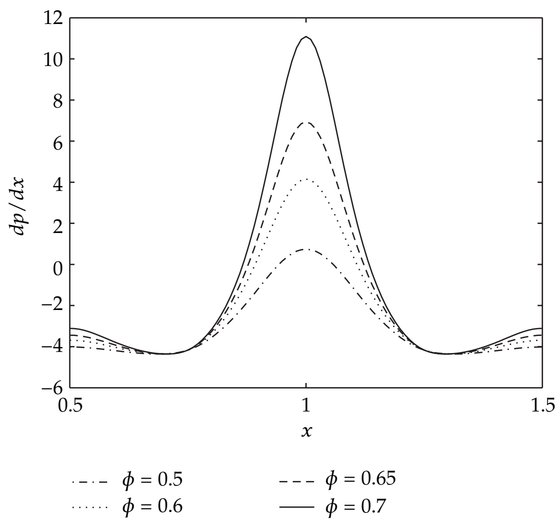


Figure 12: Variation of dp/dx with x for different values of ϕ at $k = 0.15$, $n = 0.9$, $We = 0.03$, $\beta = 0.05$, $Q = 0.3$.

in which

$$A = \operatorname{sech}\frac{1}{\beta\sqrt{2k}}\operatorname{sech}\frac{h}{\sqrt{2k}}. \tag{3.10}$$

The solution of (3.9) is calculated by eigenfunction expansion method and is defined as

$$v_1 = \sum_{m=1}^{\infty} b_m \cos(2m-1)\frac{\pi}{2}z, \tag{3.11}$$

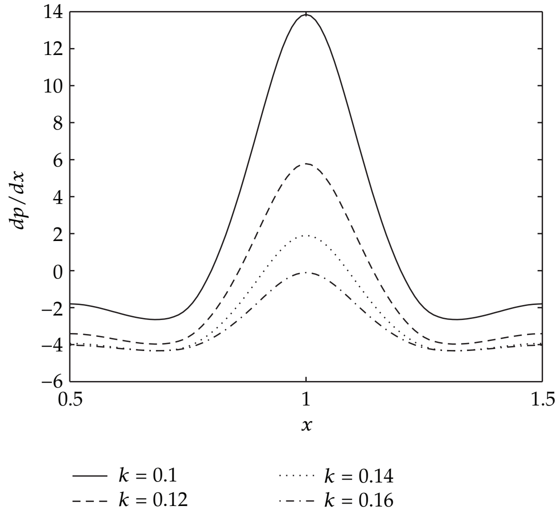


Figure 13: Variation of dp/dx with x for different values of k at $Q = 0.3,\ n = 0.9,\ We = 0.03,\ \beta = 0.05,\ \phi = 0.6$.

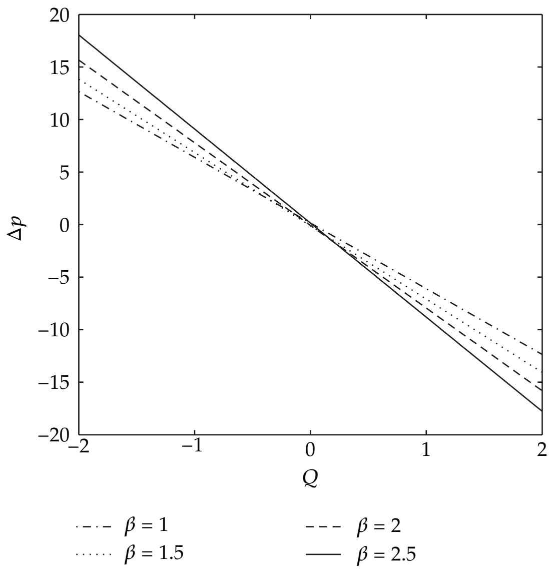


Figure 14: Variation of Δp with Q for different values of β at $k = 0.3,\ n = 0.9,\ We = 0.03,\ \phi = 0.5$.

where b_m and other constants are defined in the appendix section. The HPM solution up to first iteration is finally defined as (when $q \to 1$)

$$u(x, y, z) = v_0 + v_1, \tag{3.12}$$

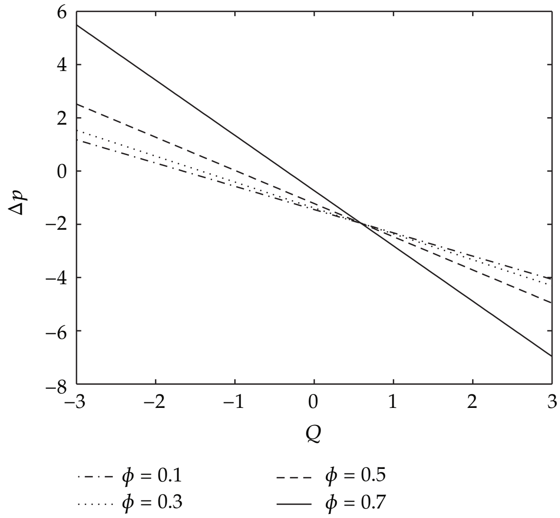


Figure 15: Variation of Δp with Q for different values of ϕ at $k = 0.5,\ n = 0.9,\ We = 0.03,\ \beta = 0.5$.

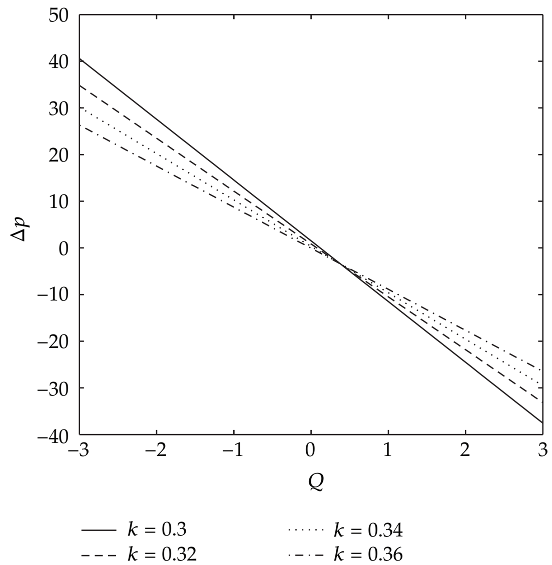


Figure 16: Variation of Δp with Q for different values of k at $\phi = 0.9,\ n = 0.9,\ We = 0.03,\ \beta = 0.5$.

where v_0 and v_1 are defined in (3.8) and (3.11). The volumetric flow rate f is calculated as

$$f = \int_0^1 \int_0^{h(x)} u(x, y, z)\, dy\, dz. \tag{3.13}$$

The instantaneous flux is given by

$$\overline{Q} = \int_0^1 \int_0^{h(x)} (u + 1)\, dy\, dz = f + h(x). \tag{3.14}$$

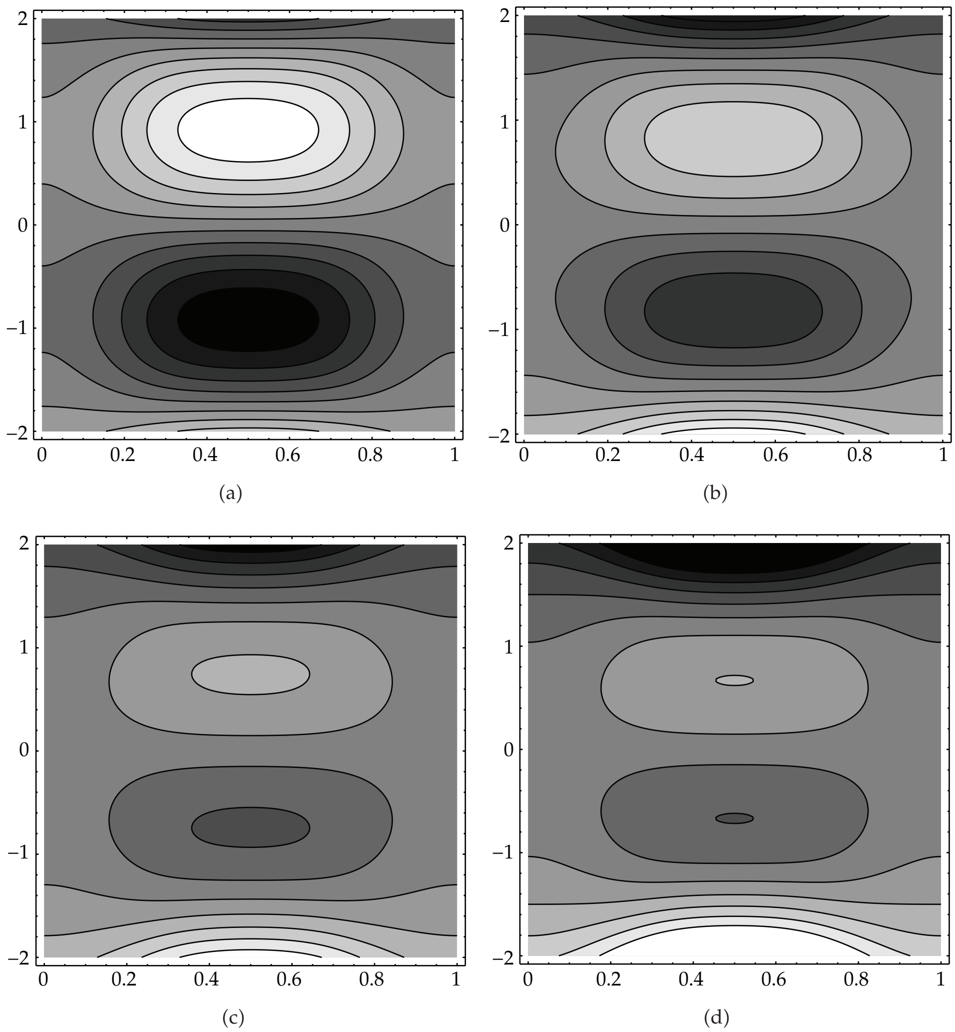


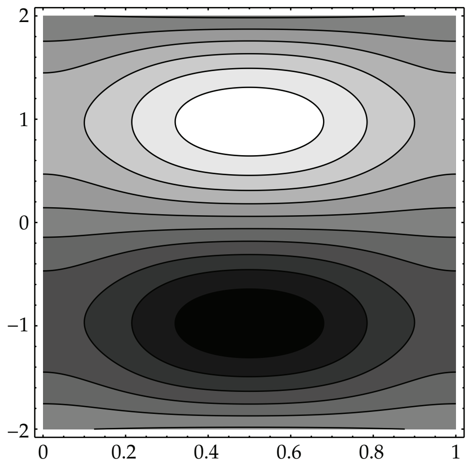
Figure 17: Stream lines for different values of β. (a) For $\beta = 0.2$, (b) for $\beta = 0.3$, (c) for $\beta = 0.4$, and (d) for $\beta = 0.5$. The other parameters are $k = 0.55$, $\phi = 0.7$, $Q = 1$, $n = 0.9$, $We = 0.03$.

The average volume flow rate over one period ($T = \lambda/c$) of the peristaltic wave is defined as

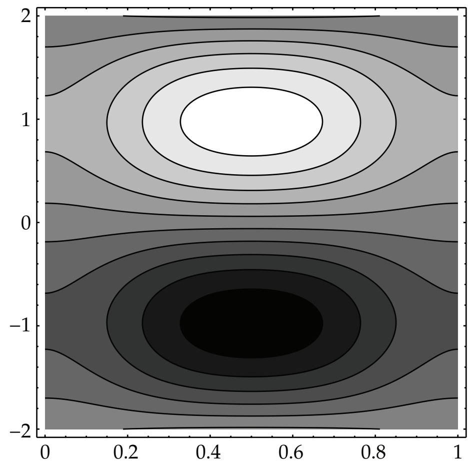
$$Q = \frac{1}{T}\int_0^T \overline{Q}dt = f + 1. \tag{3.15}$$

The pressure gradient dp/dx is obtained after solving (3.13) and (3.15). The pressure rise Δp is evaluated by using the following expression:

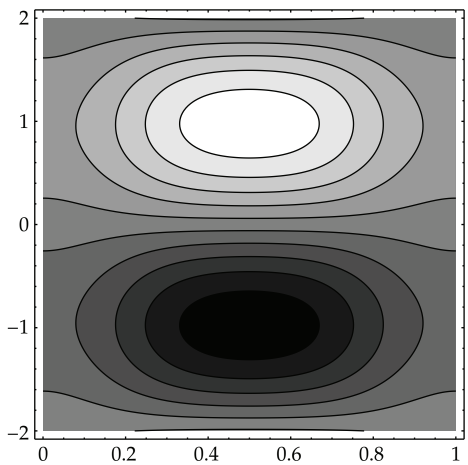
$$\Delta p = \int_0^1 \frac{dp}{dx}dx. \tag{3.16}$$



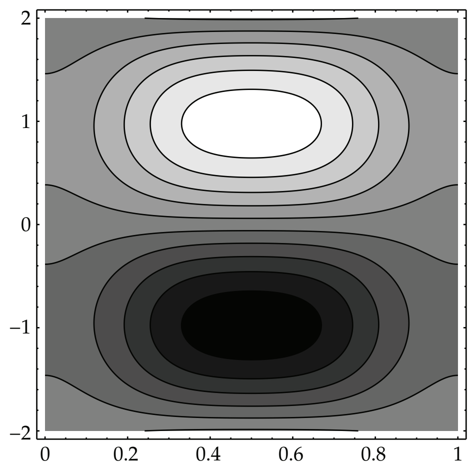
(a)



(b)



(c)



(d)

Figure 18: Stream lines for different values of ϕ. (a) For $\phi = 0.4$, (b) for $\phi = 0.5$, (c) for $\phi = 0.6$, and (d) for $\phi = 0.7$. The other parameters are $k = 1,\ \beta = 0.5,\ Q = 1,\ n = 0.9,\ We = 0.03$.

4. Results and Discussions

In this section, we have discussed the homotopy perturbation solution given in (3.12) through plotting the graphs for velocity, pressure gradient, pressure rise, and stream lines. In Figures 4(a) and 4(b), the velocity field is plotted for different values of flow rate Q, both for two dimensional and three-dimensional flows, respectively. It is observed that with the increase of flow rate Q, the velocity field decreases. The variation of velocity field for different values of β is displayed in Figures 5(a) and 5(b). It is seen that with the increase in β, the velocity field decreases, and the maximum velocity is at the centre of the channel for small values of β. In Figures 6(a) and 6(b), the velocity field increases with the increase in ϕ. However, with the increase in k, the velocity field decreases near the channel wall and increases in the middle (see Figures 7(a) and 7(b)). The velocity field distributions for different

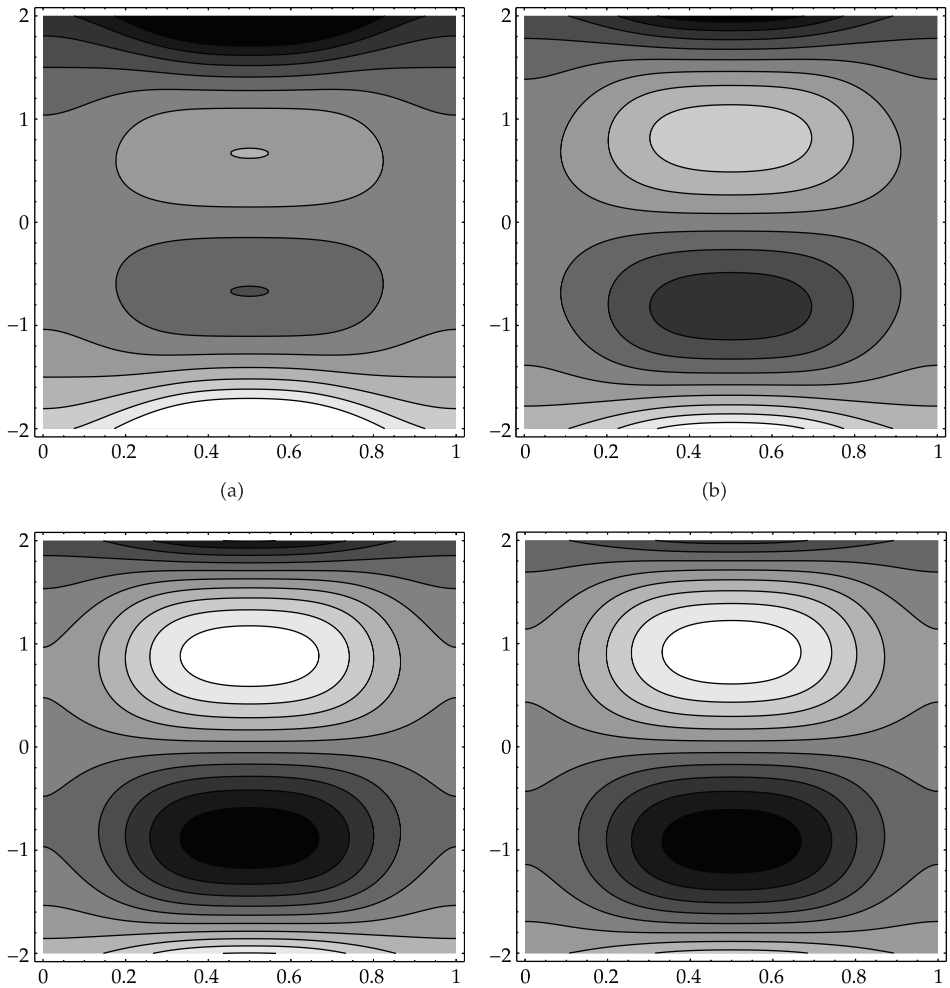


Figure 19: Stream lines for different values of k. (a) for $k = 0.55$, (b) for $k = 0.6$, (c) for $k = 0.65$, and (d) for $k = 0.7$. The other parameters are $\beta = 0.5,\ \phi = 0.7,\ Q = 1,\ n = 0.9,\ We = 0.03$.

values of We and n are shown in Figures 8 and 9. It is seen that the magnitude of velocity increases for different values of We, while decreasing for n. The pressure gradient graphs for various values of Q, β, ϕ, and k are plotted in Figures 10, 11, 12, and 13. It is observed that with the increase in Q and k, the pressure gradient decreases but increases with β and ϕ, and the maximum pressure gradient occurs in the middle of the channel for small values of the parameters. It means that flow can easily pass in the centre of the channel. The plots for pressure rise Δp for different values of β, ϕ, and k are portrayed in Figures 14, 15, and 16. It is seen that pressure rise increases with the increase in β in the peristaltic pumping region; that is, when $\Delta p > 0$, $Q < 0$ ($Q \in [-2, 0]$) and in the augmented pumping region $Q \in [0, 2]$, the pressure rise decreases (see Figure 14). The pressure rise increases with the increase in ϕ in the region $Q \in [-3, 0.6]$ and gives opposite behavior in the region [0.6,3] (see Figure 15). In Figure 16, it is depicted that with the increase in k, pressure rise decreases for small Q and increases for large Q. The trapping phenomenon is presented by plotting stream lines and is shown in Figures 17, 18, and 19. The stream lines for various values of β are shown in Figure 17. It is seen that the trapping bolus decreases with the increase in β, while size of the bolus is reduced with β. The stream lines for different values of ϕ and k are sketched in Figures 18 and 19. It is depicted that with the increase in both parameters, the numbers of trapping bolus increase. It is also noted that bolus becomes small in size for ϕ but enlarges for the parameter k.

Tables 1 and 2 are placed to compare the present work with [26]. Figures 2 and 3 are drawn according to the data given in Tables 1 and 2. From these figures it is clear to see that the analytical solution is in good agreement with the exact solution. Also, it is observed that velocity is reduced in the case of non-Newtonian (Carreau) fluid (see Figure 1). From Figure 2, it is seen that pressure rise is increased for the present fluid model.

Appendix

Consider

$$
\begin{aligned}
b_m = \Big(& e^{-3hl-6cy-y\gamma_n/\beta} \\
& \times \Big(e^{-3hl+6cy} h(-1+2m)\pi \left(144l^4 + 40l^2(1-2m)^2\pi^2 + (1-2m)^4\pi^4\gamma_n^2\right) \\
& \quad \times \left(9c^4\beta^4 - 10c^2\beta^2\gamma_n^2 + \gamma_n^4\right)\left(e^{2y\gamma_n/\beta}C_1 + C_2\right) + 8A^3 l(-1+2m)(-1+n)\pi W e^2 \gamma_n^2 \\
& \quad \times \Big(c^2 \Big(\left(\left(4l^2 + (1-2m)^2\pi^2\right) - \Big(9e^{y(3c\beta+\gamma_n)/\beta} + 9e^{6hl+3cy+y\gamma_n/\beta} - 27e^{5cy+y\gamma_n/\beta} \right. \\
& \qquad\qquad + 27e^{6hl+5cy+y\gamma_n/\beta} - 27e^{7cy+y\gamma_n/\beta} + 27e^{6hl+7cy+y\gamma_n/\beta} \\
& \qquad\qquad - 9e^{9cy+y\gamma_n/\beta} + 9e^{6hl+9cy+y\gamma_n/\beta}\Big) + \left(36l^2 + (1-2m)^2\pi^2\right) \\
& \qquad \times \Big(-5e^{2hl+3cy+y\gamma_n/\beta} + 5e^{4hl+3cy+y\gamma_n/\beta} - 63e^{2hl+5cy+y\gamma_n/\beta} + 63e^{4hl+5cy+y\gamma_n/\beta} \\
& \qquad\qquad - 63e^{2hl+7cy+y\gamma_n/\beta} + 63e^{4hl+7cy+y\gamma_n/\beta} - 5e^{2hl+9cy+y\gamma_n/\beta} + 5e^{4hl+9cy+y\gamma_n/\beta}\Big)\Big)\beta^2 \\
& \quad + \left(4l^2 + (1-2m)^2\pi^2\right)\Big(9e^{y(3c\beta+\gamma_n)/\beta} - 9e^{6hl+3cy+y\gamma_n/\beta} + 3e^{5cy+y\gamma_n/\beta} \\
& \qquad\qquad - 3e^{6hl+5cy+y\gamma_n/\beta} + 3e^{7cy+y\gamma_n/\beta} - 3e^{6hl+7cy+y\gamma_n/\beta} \\
& \qquad\qquad + 9e^{9cy+y\gamma_n/\beta} - 9e^{6hl+9cy+y\gamma_n/\beta}\Big) + \left(36l^2 + (1-2m)^2\pi^2\right) \\
& \quad \times \Big(5e^{2hl+3cy+y\gamma_n/\beta} - 5e^{4hl+3cy+y\gamma_n/\beta} + 7e^{2hl+5cy+y\gamma_n/\beta} - 7e^{4hl+5cy+y\gamma_n/\beta} \\
& \qquad + 7e^{2hl+7cy+y\gamma_n/\beta} - 7e^{4hl+7cy+y\gamma_n/\beta} + 5e^{2hl+9cy+y\gamma_n/\beta} - 5e^{4hl+9cy+y\gamma_n/\beta}\Big)\gamma_n^2\Big) \\
& \quad \times \cos\left(\frac{1}{2}h(-1+2m)\pi\right) + 4
\end{aligned}
$$

$$
\begin{aligned}
&-\Big(1152c^4e^{3hl+6cy+y\gamma_n/\beta}k(1+kp)\Big(144l^4+40l^2(1-2m)^2\pi^2+(1-2m)^4\pi^4\Big)\beta^4+c^2\\
&\quad\times\Big(1280e^{3hl+6cy+y\gamma_n/\beta}k(1+kp)\Big(144l^4+40l^2(1-2m)^2\pi^2+(1-2m)^4\pi^4\Big)\\
&\qquad+(1-2m)^2(-1+n)\pi^2\Big(4l^2+(1-2m)^2\pi^2\Big)We^2\\
&\qquad\times\Big(3A^3e^{(y(3c\beta+\gamma_n))/\beta}+3A^3e^{6hl+3cy+y\gamma_n/\beta}+9A^3e^{5cy+y\gamma_n/\beta}+9A^3e^{6hl+5cy+y\gamma_n/\beta}\\
&\qquad\quad+9A^3e^{7cy+y\gamma_n/\beta}+9A^3e^{6hl+7cy+y\gamma_n/\beta}+3A^3e^{9cy+y\gamma_n/\beta}\\
&\qquad\quad+3A^3e^{6hl+9cy+y\gamma_n/\beta}+5A^3e^{2hl+3cy+y\gamma_n/\beta}(1-2m)^2(-1+n)\pi^2\\
&\qquad\quad\times\Big(36l^2+(1-2m)^2\pi^2\Big)We^2+5A^3e^{4hl+3cy+y\gamma_n/\beta}(1-2m)^2(-1+n)\pi^2\\
&\qquad\quad\times\Big(36l^2+(1-2m)^2\pi^2\Big)We^2+63A^3e^{2hl+5cy+y\gamma_n/\beta}(1-2m)^2(-1+n)\pi^2\\
&\qquad\quad\times\Big(36l^2+(1-2m)^2\pi^2\Big)We^2+63A^3e^{4hl+5cy+y\gamma_n/\beta}(1-2m)^2(-1+n)\pi^2\\
&\qquad\quad\times\Big(36l^2+(1-2m)^2\pi^2\Big)We^2+63A^3e^{2hl+7cy+y\gamma_n/\beta}(1-2m)^2(-1+n)\pi^2\\
&\qquad\quad\times\Big(36l^2+(1-2m)^2\pi^2\Big)We^2+63A^3e^{4hl+7cy+y\gamma_n/\beta}(1-2m)^2(-1+n)\pi^2\\
&\qquad\quad\times\Big(36l^2+(1-2m)^2\pi^2\Big)We^2+5A^3e^{2hl+9cy+y\gamma_n/\beta}(1-2m)^2(-1+n)\pi^2\\
&\qquad\quad\times\Big(36l^2\Big)+(1-2m)^2\pi^2\Big)We^2+5A^3e^{4hl+9cy+y\gamma_n/\beta}(1-2m)^2(-1+n)\\
&\qquad\times\pi^2 36l^2+(1-2m)^2\pi^2\Big)We^2\Big)\beta^2\gamma_n^2-128e^{3hl+6cy+y\gamma_n/\beta}k(1+kp)\\
&\times\Big(144l^4+40l^2(1-2m)^2\pi^2+(1-2m)^4\pi^4\Big)+3A^3e^{y(3c\beta+\gamma_n)/\beta}(1-2m)^2(-1+n)\pi^2\\
&\times\Big(4l^2+(1-2m)^2\pi^2\Big)We^2+3A^3e^{6hl+3cy+y\gamma_n/\beta}(1-2m)^2(-1+n)\pi^2\\
&\times\Big(4l^2+(1-2m)^2\pi^2\Big)We^2+A^3e^{5cy+y\gamma_n/\beta}(1-2m)^2(-1+n)\pi^2\Big(4l^2+(1-2m)^2\pi^2\Big)\\
&\times We^2+A^3e^{6hl+5cy+y\gamma_n/\beta}(1-2m)^2(-1+n)\pi^2\Big(4l^2+(1-2m)^2\pi^2\Big)We^2+A^3e^{7cy+y\gamma_n/\beta}\\
&\times(1-2m)^2(-1+n)\pi^2\Big(4l^2+(1-2m)^2\pi^2\Big)We^2+A^3e^{6hl+7cy+y\gamma_n/\beta}(1-2m)^2(-1+n)\pi^2\\
&\times\Big(4l^2+(1-2m)^2\pi^2\Big)We^2+3A^3e^{9cy+y\gamma_n/\beta}(1-2m)^2(-1+n)\pi^2\Big(4l^2+(1-2m)^2\pi^2\Big)\\
&\times We^2+3A^3e^{6hl+9cy+y\gamma_n/\beta}(1-2m)^2(-1+n)\pi^2\Big(4l^2+(1-2m)^2\pi^2\Big)We^2\\
&+5A^3e^{2hl+3cy+y\gamma_n/\beta}(1-2m)^2(-1+n)\pi^2\Big(4l^2+(1-2m)^2\pi^2\Big)We^2+5A^3e^{4hl+3cy+y\gamma_n/\beta}\\
&\times(1-2m)^2(-1+n)\pi^2\Big(4l^2+(1-2m)^2\pi^2\Big)We^2+7A^3e^{2hl+5cy}e^{y\gamma_n/\beta}(1-2m)^2\\
&\times(-1+n)\pi^2\Big(4l^2+(1-2m)^2\pi^2\Big)We^2+7A^3e^{4hl+5cy+y\gamma_n/\beta}(1-2m)^2(-1+n)\pi^2
\end{aligned}
$$

$$\times \left(4l^2+(1-2m)^2\pi^2\right)We^2+7A^3e^{4hl+7cy+y\gamma_n/\beta}(1-2m)^2(-1+n)\pi^2\left(4l^2+(1-2m)^2\pi^2\right)$$
$$\times We^2 + 7A^3e^{2hl+5cy+y\gamma_n/\beta}(1-2m)^2(-1+n)\pi^2\left(4l^2+(1-2m)^2\pi^2\right)We^2$$
$$+ 5A^3e^{2hl+9cy+y\gamma_n/\beta}(1-2m)^2(-1+n)\pi^2\left(4l^2+(1-2m)^2\pi^2\right)We^2$$
$$+ 5A^3e^{4hl+5cy+y\gamma_n/\beta}(1-2m)^2(-1+n)\pi^2\left(4l^2+(1-2m)^2\pi^2\right)We^2\Big)\gamma_n^4\Big)$$
$$\times \sin\left(\frac{1}{2}h(-1+2m)\pi\right)$$
$$/\left(h(-1+2m)\pi\left(144l^4+40l^2(1-2m)^2\pi^2+(1-2m)^4\pi^4\right)\gamma_n^2\left(9c^4\beta^4-10c^2\beta^2\gamma_n^2+\gamma_n^4\right)\right), \tag{A.1}$$

while the other constants are defined as

$$C_1 = \Big(4e^{-3(c+hl)+\gamma_m/\beta}$$
$$-\Big(2A^3\left(1+e^{2c}\right)\left(-1+e^{2hl}\right)l(-1+2m)(-1+n)\pi We^2\gamma_m^2$$
$$\times \Big(c^2\Big(36\left(1+2e^{2c}+e^{4c}-6e^{2hl}+e^{4hl}+60e^{2(c+hl)}+e^{4(c+hl)}+6e^{4c+2hl}+2e^{2c+4hl}\right)l^2$$
$$-\left(9+18e^{2c}+9e^{4c}+14e^{2hl}+9e^{4hl}+76e^{2(c+hl)}+9e^{4(c+hl)}+14e^{4c+2hl}+18e^{2c+4hl}\right)$$
$$\times(\pi-2m\pi)^2\beta^2-12\Big(3-2e^{2c}+3e^{4c}+18e^{2hl}+3e^{4hl}+4e^{2(c+hl)}-3e^{4(c+hl)}$$
$$+18e^{4c+2hl}-2e^{2c+4hl}\Big)l^2$$
$$+\left(9-6e^{2c}+9e^{4c}+14e^{2hl}+9e^{4hl}-4e^{2(c+hl)}+9e^{4(c+hl)}+14e^{4c+2hl}-6e^{2c+4hl}\right)$$
$$\times(\pi-2m\pi)^2\Big)\gamma_m^2\Big)\cos\left(\frac{1}{2}h(-1+2m)\pi\right)$$
$$+\Big(1152c^4e^{3(c+hl)}k(1+kp)\left(144l^4+40l^2(1-2m)^2\pi^2+(1-2m)^4\pi^4\right)\beta^4-c^2$$
$$\times\Big(1280e^{3hl+3cy}k(1+kp)\left(144l^4+40l^2(1-2m)^2\pi^2+(1-2m)^4\pi^4\right)-3A^3(1-2m)^2$$
$$\times(-1+n)\pi^2\left(4l^2+(1-2m)^2\pi^2\right)We^2+9A^3e^{2c}(1-2m)^2(-1+n)\pi^2$$
$$\times\left(4l^2+(1-2m)^2\pi^2\right)We^2+9A^3e^{4c}(1-2m)^2(-1+n)\pi^2\left(4l^2+(1-2m)^2\pi^2\right)$$
$$\times We^2+3A^3e^{6c}(1-2m)^2(-1+n)\pi^2\left(4l^2+(1-2m)^2\pi^2\right)We^2+3A^3e^{6hl}$$
$$\times(1-2m)^2(-1+n)\pi^2\left(4l^2+(1-2m)^2\pi^2\right)We^2+9A^3e^{6(c+hl)}(1-2m)^2$$
$$\times(-1+n)\pi^2\left(4l^2+(1-2m)^2\pi^2\right)We^2+9A^3e^{2c+6hl}(1-2m)^2(-1+n)\pi^2$$
$$\times\left(4l^2+(1-2m)^2\pi^2\right)We^2+9A^3e^{4c+6hl}(1-2m)^2(-1+n)\pi^2$$

$$\times \left(4l^2+(1-2m)^2\pi^2\right)We^2+5A^3e^{2hl}(1-2m)^2(-1+n)\pi^2\left(36l^2+(1-2m)^2\pi^2\right)$$

$$\times We^2+5A^3e^{4hl}(1-2m)^2(-1+n)\pi^2\left(36l^2+(1-2m)^2\pi^2\right)We^2+63A^3e^{2(c+hl)}$$

$$\times (1-2m)^2(-1+n)\pi^2\left(36l^2+(1-2m)^2\pi^2\right)We^2+63A^3e^{4(c+hl)}(1-2m)^2$$

$$\times (-1+n)\pi^2\left(36l^2+(1-2m)^2\pi^2\right)We^2+63A^3e^{4c+2hl}(1-2m)^2(-1+n)\pi^2$$

$$\times\left(36l^2+(1-2m)^2\pi^2\right)We^2+5A^3e^{6c+2hl}(1-2m)^2(-1+n)\pi^2\left(36l^2+(1-2m)^2\pi^2\right)$$

$$\times We^2+63A^3e^{2c+4hl}(1-2m)^2(-1+n)\pi^2\left(36l^2+(1-2m)^2\pi^2\right)We^2+5A^3e^{6c+4hl}$$

$$\times(1-2m)^2(-1+n)\pi^2\left(36l^2+(1-2m)^2\pi^2\right)\beta^2\gamma_m^2+128e^{3(c+hl)}k(1+kp)$$

$$\times \left(144l^4+40l^2(1-2m)^2\pi^2+(1-2m)^4\pi^4\right)+3A^3(1-2m)^2(-1+n)\pi^2$$

$$\times \left(4l^2+(1-2m)^2\pi^2\right)We^2+A^3e^{2c}(1-2m)^2(-1+n)\pi^2\left(4l^2+(1-2m)^2\pi^2\right)We^2$$

$$+A^3e^{4c}(1-2m)^2(-1+n)\pi^2+3A^3e^{6c}(1-2m)^2(-1+n)\pi^2\left(4l^2+(1-2m)^2\pi^2\right)We^2$$

$$+3A^3e^{6c}(1-2m)^2(-1+n)\pi^2\left(4l^2+(1-2m)^2\pi^2\right)We^2+3A^3e^{6hl}(1-2m)^2$$

$$\times (-1+n)\pi^2\left(4l^2+(1-2m)^2\pi^2\right)We^2+3A^3e^{6(c+hl)}(1-2m)^2(-1+n)\pi^2$$

$$\times \left(4l^2+(1-2m)^2\pi^2\right)We^2+A^3e^{2c+6hl}(1-2m)^2\pi^2\left(4l^2+(1-2m)^2\pi^2\right)We^2$$

$$+A^3e^{4c+6hl}(1-2m)^2(-1+n)\pi^2\left(4l^2+(1-2m)^2\pi^2\right)We^2+5A^3e^{2hl}$$

$$\times (1-2m)^2(-1+n)\pi^2\left(36l^2+(1-2m)^2\pi^2\right)We^2+5A^3e^{4hl}(1-2m)^2(-1+n)\pi^2$$

$$\times\left(36l^2+(1-2m)^2\pi^2\right)We^2+7A^3e^{2(c+hl)}(1-2m)^2(-1+n)\pi^2\left(36l^2+(1-2m)^2\pi^2\right)$$

$$\times We^2+7A^3e^{4(c+hl)}(1-2m)^2(-1+n)\pi^2\left(36l^2+(1-2m)^2\pi^2\right)We^2+7A^3e^{4c+2hl}$$

$$\times (1-2m)^2(-1+n)\pi^2\left(36l^2+(1-2m)^2\pi^2\right)We^2+5A^3e^{6c+2hl}(1-2m)^2(-1+n)$$

$$\times\pi^2\left(36l^2+(1-2m)^2\pi^2\right)We^2+7A^3e^{2c+4hl}(1-2m)^2(-1+n)\pi^2\left(36l^2(1-2m)^2\pi^2\right)$$

$$\times We^2 5A^3e^{6c+4hl}(1-2m)^2(-1+n)\pi^2\left(36l^2+(1-2m)^2\pi^2\right)We^2\Big)\gamma_m^4\Big)$$

$$\times \sin\left(\frac{1}{2}h(-1+2m)\pi\right)$$

$$\Big/\bigg(\left(1+e^{2(\gamma_m/\beta)}\right)\left(h(-1+2m)\pi\left(144l^4+40l^2(1-2m)^2\pi^2+(1-2m)^4\pi^4\right)\gamma_n^2\right.$$

$$\left.\times\left(9c^4\beta^4-10c^2\beta^2\gamma_n^2+\gamma_n^4\right)\right)\bigg)+A^3e^{4c+6hl}(1-2m)^2(-1+n)\pi^2$$

$$\times \left(4l^2+(1-2m)^2\pi^2\right)We^2+5A^3e^{2hl}(1-2m)^2(-1+n)\pi^2$$

$$
\begin{aligned}
&\times \left(36l^2 + (1-2m)^2\pi^2\right)We^2 + 5A^3e^{4hl}(1-2m)^2(-1+n)\pi^2\left(36l^2 + (1-2m)^2\pi^2\right)\\
&\times We^2 + 7A^3e^{2(c+hl)}(1-2m)^2(-1+n)\pi^2\left(36l^2 + (1-2m)^2\pi^2\right)We^2 + 7A^3e^{4(c+hl)}\\
&\times (1-2m)^2(-1+n)\pi^2\left(36l^2 + (1-2m)^2\pi^2\right)We^2 + 7A^3e^{4c+2hl}(1-2m)^2(-1+n)\pi^2\\
&\times \left(36l^2 + (1-2m)^2\pi^2\right)We^2 + 5A^3e^{6c+2hl}(1-2m)^2(-1+n)\pi^2\left(36l^2 + (1-2m)^2\pi^2\right)\\
&\times We^2 + 7A^3e^{2c+4hl}(1-2m)^2(-1+n)\pi^2\left(36l^2 + (1-2m)^2\pi^2\right)We^2 + 5A^3e^{6c+4hl}\\
&\times (1-2m)^2(-1+n)\pi^2\left(36l^2 + (1-2m)^2\pi^2\right)We^2\Big)\gamma_m^4\Big)\sin\left(\frac{1}{2}h(-1+2m)\pi\right)\\
&/\left(1+e^{2(\gamma_m/\beta)}\right)h(-1+2m)\pi\left(144l^4 + 40l^2(1-2m)^2\pi^2 + (1-2m)^4\pi^4\right)\gamma_n^2\\
&\times\left(9c^4\beta^4 - 10c^2\beta^2\gamma_n^2 + \gamma_n^4\right),
\end{aligned}
\tag{A.2}
$$

$$
\begin{aligned}
C_2 = \Big(&4e^{-3(c+hl)+\gamma_m/\beta}\\
&-\Big(2A^3\left(1+e^{2c}\right)\left(-1+e^{2hl}\right)l(-1+2m)(-1+n)\pi We^2\gamma_m^2\\
&\times\Big(c^2\Big(36\left(1+2e^{2c}+e^{4c}-6e^{2hl}+e^{4hl}+60e^{2(c+hl)}+e^{4(c+hl)}+6e^{4c+2hl}+2e^{2c+4hl}\right)l^2\\
&\qquad -\left(9+18e^{2c}+9e^{4c}+14e^{2hl}+9e^{4hl}+76e^{2(c+hl)}+9e^{4(c+hl)}+14e^{4c+2hl}+18e^{2c+4hl}\right)\\
&\qquad\times(\pi-2m\pi)^2\beta^2 - 12\Big(3-2e^{2c}+3e^{4c}+18e^{2hl}+3e^{4hl}+4e^{2(c+hl)}-3e^{4(c+hl)}\\
&\qquad\qquad +18e^{4c+2hl}-2e^{2c+4hl}\Big)l^2\\
&\qquad +\Big(9-6e^{2c}+9e^{4c}+14e^{2hl}+9e^{4hl}-4e^{2(c+hl)}+9e^{4(c+hl)}+14e^{4c+2hl}-6e^{2c+4hl}\Big)\\
&\qquad\times(\pi-2m\pi)^2\Big)\gamma_m^2\Big)\cos\left(\frac{1}{2}h(-1+2m)\pi\right)\\
&+\Big(1152c^4e^{3(c+hl)}k(1+kp)\left(144l^4+40l^2(1-2m)^2\pi^2+(1-2m)^4\pi^4\right)\beta^4 - c^2\\
&\quad\times\Big(1280e^{3hl+3cy}k(1+kp)\left(144l^4+40l^2(1-2m)^2\pi^2+(1-2m)^4\pi^4\right) - 3A^3\\
&\qquad\times(1-2m)^2(-1+n)\pi^2\left(4l^2+(1-2m)^2\pi^2\right)We^2+9A^3e^{2c}(1-2m)^2(-1+n)\pi^2\\
&\qquad\times\left(4l^2+(1-2m)^2\pi^2\right)We^2+9A^3e^{4c}(1-2m)^2(-1+n)\pi^2\left(4l^2+(1-2m)^2\pi^2\right)We^2\\
&\qquad+3A^3e^{6c}(1-2m)^2(-1+n)\pi^2\left(4l^2+(1-2m)^2\pi^2\right)We^2+3A^3e^{6hl}(1-2m)^2\\
&\qquad\times(-1+n)\pi^2\left(4l^2+(1-2m)^2\pi^2\right)We^2+9A^3e^{6(c+hl)}(1-2m)^2(-1+n)\pi^2\\
&\qquad\times\left(4l^2+(1-2m)^2\pi^2\right)We^2+9A^3e^{2c+6hl}(1-2m)^2(-1+n)\pi^2\left(4l^2+(1-2m)^2\pi^2\right)\\
&\qquad\times We^2+9A^3e^{4c+6hl}(1-2m)^2(-1+n)\pi^2\left(4l^2+(1-2m)^2\pi^2\right)We^2+5A^3e^{2hl}\\
&\qquad\times(1-2m)^2(-1+n)\pi^2\left(36l^2+(1-2m)^2\pi^2\right)We^2+5A^3e^{4hl}(1-2m)^2
\end{aligned}
$$

$$\times(-1+n)\pi^2\left(36l^2+(1-2m)^2\pi^2\right)We^2+63A^3e^{2(c+hl)}(1-2m)^2(-1+n)\pi^2$$
$$\times\left(36l^2+(1-2m)^2\pi^2\right)We^2+63A^3e^{4(c+hl)}(1-2m)^2(-1+n)\pi^2$$
$$\times\left(36l^2+(1-2m)^2\pi^2\right)We^2+63A^3e^{4c+2hl}(1-2m)^2(-1+n)\pi^2\left(36l^2+(1-2m)^2\pi^2\right)$$
$$\times We^2+5A^3e^{6c+2hl}(1-2m)^2(-1+n)\pi^2\left(36l^2+(1-2m)^2\pi^2\right)We^2+63A^3e^{2c+4hl}$$
$$\times(1-2m)^2(-1+n)\pi^2\left(36l^2+(1-2m)^2\pi^2\right)We^2+5A^3e^{6c+4hl}(1-2m)^2$$
$$\times(-1+n)\pi^2\left(36l^2+(1-2m)^2\pi^2\right)\beta^2\gamma_m^2+128e^{3(c+hl)}k(1+kp)$$
$$\times\left(144l^4+40l^2(1-2m)^2\pi^2+(1-2m)^4\pi^4\right)+3A^3(1-2m)^2(-1+n)\pi^2$$
$$\times\left(4l^2+(1-2m)^2\pi^2\right)We^2+A^3e^{2c}(1-2m)^2(-1+n)\pi^2\left(4l^2+(1-2m)^2\pi^2\right)We^2$$
$$+A^3e^{4c}(1-2m)^2(-1+n)\pi^2+3A^3e^{6c}(1-2m)^2(-1+n)\pi^2\left(4l^2+(1-2m)^2\pi^2\right)We^2$$
$$+3A^3e^{6c}(1-2m)^2(-1+n)\pi^2\left(4l^2+(1-2m)^2\pi^2\right)We^2+3A^3e^{6hl}(1-2m)^2$$
$$\times(-1+n)\pi^2\left(4l^2+(1-2m)^2\pi^2\right)We^2+3A^3e^{6(c+hl)}(1-2m)^2(-1+n)\pi^2$$
$$\times\left(4l^2+(1-2m)^2\pi^2\right)We^2+A^3e^{2c+6hl}(1-2m)^2\pi^2\left(4l^2+(1-2m)^2\pi^2\right)$$
$$\times We^2+A^3e^{4c+6hl}(1-2m)^2(-1+n)\pi^2\left(4l^2+(1-2m)^2\pi^2\right)We^2+5A^3e^{2hl}$$
$$\times(1-2m)^2(-1+n)\pi^2\left(36l^2+(1-2m)^2\pi^2\right)We^2+5A^3e^{4hl}(1-2m)^2(-1+n)\pi^2$$
$$\times\left(36l^2+(1-2m)^2\pi^2\right)We^2+7A^3e^{2(c+hl)}(1-2m)^2(-1+n)\pi^2$$
$$\times\left(36l^2+(1-2m)^2\pi^2\right)We^2+7A^3e^{4(c+hl)}(1-2m)^2(-1+n)\pi^2\left(36l^2+(1-2m)^2\pi^2\right)$$
$$\times We^2+7A^3e^{4c+2hl}(1-2m)^2(-1+n)\pi^2\left(36l^2+(1-2m)^2\pi^2\right)We^2+5A^3e^{6c+2hl}$$
$$\times(1-2m)^2(-1+n)\pi^2\left(36l^2+(1-2m)^2\pi^2\right)We^2+7A^3e^{2c+4hl}(1-2m)^2$$
$$\times(-1+n)\pi^2\left(36l^2+(1-2m)^2\pi^2\right)We^2+5A^3e^{6c+4hl}(1-2m)^2(-1+n)\pi^2$$
$$\times\left(36l^2+(1-2m)^2\pi^2\right)We^2\Big)\gamma_m^4\Big)\sin\left(\frac{1}{2}h(-1+2m)\pi\right)$$
$$\Big/\bigg(\left(1+e^{2(\gamma_m/\beta)}\right)\Big(h(-1+2m)\pi\left(144l^4+40l^2(1-2m)^2\pi^2+(1-2m)^4\pi^4\right)\gamma_n^2$$
$$\times\left(9c^4\beta^4-10c^2\beta^2\gamma_n^2+\gamma_n^4\right)\Big)\bigg)+A^3e^{4c+6hl}(1-2m)^2$$
$$\times(-1+n)\pi^2\left(4l^2+(1-2m)^2\pi^2\right)We^2+5A^3e^{2hl}(1-2m)^2(-1+n)\pi^2$$
$$\times\left(36l^2+(1-2m)^2\pi^2\right)We^2+5A^3e^{4hl}(1-2m)^2(-1+n)\pi^2\left(36l^2+(1-2m)^2\pi^2\right)We^2$$
$$+7A^3e^{2(c+hl)}(1-2m)^2(-1+n)\pi^2\left(36l^2+(1-2m)^2\pi^2\right)We^2+7A^3e^{4(c+hl)}(1-2m)^2$$
$$\times(-1+n)\pi^2\left(36l^2+(1-2m)^2\pi^2\right)We^2+7A^3e^{4c+2hl}(1-2m)^2(-1+n)\pi^2$$

$$\times\left(36l^2+(1-2m)^2\pi^2\right)We^2+5A^3e^{6c+2hl}(1-2m)^2(-1+n)\pi^2\left(36l^2+(1-2m)^2\pi^2\right)We^2$$
$$+7A^3e^{2c+4hl}(1-2m)^2(-1+n)\pi^2\left(36l^2+(1-2m)^2\pi^2\right)We^2+5A^3e^{6c+4hl}(1-2m)^2$$
$$\times(-1+n)\pi^2\left(36l^2+(1-2m)^2\pi^2\right)We^2\Big)\gamma_m^4\Big)\sin\left(\frac{1}{2}h(-1+2m)\pi\right)$$
$$/\left(1+e^{2(\gamma_m/\beta)}\right)h(-1+2m)\pi\left(144l^4+40l^2(1-2m)^2\pi^2+(1-2m)^4\pi^4\right)\gamma_n^2$$
$$\times\left(9c^4\beta^4-10c^2\beta^2\gamma_n^2+\gamma_n^4\right). \tag{A.3}$$

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