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Research Article

Unsteady Flow Produced by Oscillations of Eccentric Rotating Disks

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While the disks are initially rotating eccentrically, the unsteady flow caused by their oscillations in their own planes and in the opposite directions is studied. The analytical solutions to the problem are obtained for both small and large times, and thus the velocity field is determined for every value of time. The variations of all the parameters on the flow are scrutinized by means of the graphical representations. In particular, the effect of the ratio of the frequency of oscillation to the angular velocity of the disks is analyzed. The dependence of the oscillations in both x- and y-directions on the flow is examined. The influence of the Reynolds number is also investigated.

1. Introduction

The instrument called the orthogonal rheometer consisting of two parallel disks rotating with the same angular velocity about noncoincident axes was originally developed by Maxwell and Chartoff [1]. Abbott and Walters [2] obtained an exact solution for the flow of a Newtonian fluid. They also found a solution by means of a perturbation analysis for a viscoelastic fluid. Later, Berker [3] proved that there is the existence of an infinite number of nontrivial solutions to the Navier-Stokes equations between eccentric rotating disks. Rajagopal [4] showed that this motion is one with constant stretch history. We refer the reader to the papers by Rajagopal [5] and Ersoy [6, 7] for a detailed list of references related to the flows between eccentric rotating disks.

Time-dependent flows between eccentric rotating disks have also attracted the attention of researchers [8–13]. In these studies, the unsteady flows are generated by the sudden motion of the disks rotating with the same angular velocity and the disks are not exposed to oscillation. Erdoğan [14, 15] was the first to study the unsteady motion induced by oscillations. He [14] studied the flow that the disks start to rotate eccentrically and the lower disk executes oscillations while the disks are initially rotating about a common axis. He [15] studied the flow that the disks start to rotate eccentrically and both the disks execute oscillations in the same direction while the disks are initially rotating about a common axis. It is clear that these flows are not symmetrical as indicated by Erdoğan [14, 15].

In addition, the reader may consult the references [16–24] for the analytical solutions about the unsteady flows induced by eccentric rotations of an oscillating disk and a fluid at infinity under various effects.

In this paper, the disks are initially rotating about noncoincident axes. For this reason, the initial condition is the solution obtained by Abbott and Walters [2]. The disks start to execute oscillations in their own planes and in the opposite directions, and thus the symmetrical condition is satisfied at all times. In order to obtain a more general solution, the oscillating disk velocity has two components. The problem is solved for both small and large times, and the two solutions are matched at a specific value of time. In other words, the velocity field is obtained for all times. The influences of the parameters acting on the flow are elucidated with the help of the figures.

2. Basic Equations

The flow field of the problem is bounded by two disks located at $z = h$ and $z = -h$. Initially, the top and bottom disks are rotating about the z'- and z''-axes with the same angular velocity Ω, respectively. The distance between the axes of rotation is shown in the y-direction by 2ℓ and the region between the disks is occupied by an incompressible Newtonian fluid. The motion of the fluid is examined after the disks start to execute oscillations in their own planes and in the opposite directions. The upper and lower disks oscillate in their own planes with the velocities $\mathbf{U}$ and $-\mathbf{U}$, respectively, where $\mathbf{U} = (U_x \sin nt, U_y \sin nt, 0)$ and n is the frequency of the oscillation. It should be emphasized that the distance between the axes of rotation is fixed during the motion. Furthermore, a physical reality of the oscillations of the disks requires that the sine oscillation is more reasonable than the cosine oscillation in this paper. The geometry of the problem is shown in Figure 1.

Therefore, the initial and boundary conditions can be written in the following form:

$$u = -\Omega y + \hat{f}(z), \quad v = \Omega x + \hat{g}(z) \quad \text{at } t = 0 \text{ for } -h \le z \le h, \tag{2.1a}$$

$$u = -\Omega(y - \ell) + U_x \sin nt, \quad v = \Omega x + U_y \sin nt \quad \text{at } z = h \text{ for } t \ge 0, \tag{2.1b}$$

$$u = -\Omega y, \quad v = \Omega x \quad \text{at } z = 0 \text{ for } t \ge 0, \tag{2.1c}$$

$$u = -\Omega(y + \ell) - U_x \sin nt, \quad v = \Omega x - U_y \sin nt \quad \text{at } z = -h \text{ for } t \ge 0, \tag{2.1d}$$

where u and v denote the velocity components in the x- and y-directions, respectively. The functions $\hat{f}(z)$ and $\hat{g}(z)$ obtained by Abbott and Walters [2] represent the eccentric symmetrical rotation for a Newtonian fluid and are given by

$$\hat{f}(z) + i\hat{g}(z) = \Omega\ell \frac{\sinh Kz}{\sinh Kh}, \tag{2.2}$$

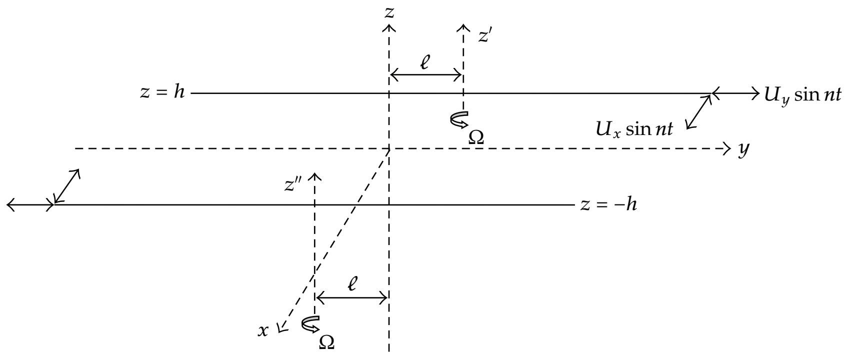

Figure 1: Flow geometry.

where $i = \sqrt{-1}$, $K = \sqrt{\Omega/(2\nu)}(1+i)$, and ν denotes the kinematic viscosity of the fluid. Equation (2.1c) reflects the symmetrical condition.

The velocity field for the flow under consideration is given by

$$u = -\Omega y + f(z,t), \qquad \nu = \Omega x + g(z,t). \tag{2.3}$$

We should note that this flow does not bring out a velocity component in the z-direction. Substituting (2.3) into the Navier-Stokes equations, one obtains

$$\begin{aligned} \nu\frac{\partial^2 f}{\partial z^2} - \frac{\partial f}{\partial t} + \Omega g = C_1(t), \\ \nu\frac{\partial^2 g}{\partial z^2} - \frac{\partial g}{\partial t} - \Omega f = C_2(t). \end{aligned} \tag{2.4}$$

Using (2.1a)–(2.1d) and (2.3), we get

$$f(z,0) = \hat{f}(z), \qquad g(z,0) = \hat{g}(z), \tag{2.5a}$$

$$f(\pm h,t) = \pm(\Omega\ell + U_x \sin nt), \qquad g(\pm h,t) = \pm U_y \sin nt, \tag{2.5b}$$

$$f(0,t) = 0, \qquad g(0,t) = 0. \tag{2.5c}$$

Introducing $F(z,t) = f(z,t) + ig(z,t)$ and using (2.4), we have

$$\nu\frac{\partial^2 F}{\partial z^2} - \frac{\partial F}{\partial t} - \Omega i F = C(t). \tag{2.6}$$

The symmetrical condition gives $C(t) = 0$, which implies the absence of the Poiseuille-type pressure gradient. The conditions for (2.6) become

$$\begin{gathered} F(z,0) = \hat{f}(z) + i\hat{g}(z), \\ F(\pm h,t) = \pm(\Omega\ell + U_x \sin nt) \pm iU_y \sin nt, \\ F(0,t) = 0. \end{gathered} \tag{2.7}$$

3. Solution for Small Times

Putting $F(z,t) = H(z,t)e^{-i\Omega t}$, (2.6) takes the form

$$\nu\frac{\partial^2 H}{\partial z^2} = \frac{\partial H}{\partial t}, \tag{3.1}$$

with the conditions

$$\begin{gathered} H(z,0) = \Omega\ell\frac{\sinh Kz}{\sinh Kh}, \\ H(\pm h,t) = \pm\left[\Omega\ell + \left(U_x + iU_y\right)\sin nt\right]e^{i\Omega t}, \\ H(0,t) = 0. \end{gathered} \tag{3.2}$$

The Laplace transform of $H(z,t)$ is defined by the equation

$$\overline{H}(z,s) = \int_0^\infty H(z,t)e^{-st}dt. \tag{3.3}$$

Taking the Laplace transform of (3.1) with the conditions (3.2), we have

$$\overline{H}'' - \frac{s}{\nu}\overline{H} = -\frac{\Omega\ell}{\nu\sinh Kh}\sinh Kz, \tag{3.4}$$

$$\overline{H}(\pm h) = \pm\frac{\Omega\ell}{s - i\Omega} \pm \frac{(U_x + iU_y)n}{n^2 + (s - i\Omega)^2}, \qquad \overline{H}(0) = 0, \tag{3.5}$$

where a prime denotes differentiation with respect to z. Applying the conditions (3.5), the solution of (3.4) is

$$\overline{H} = \frac{\Omega\ell}{s - i\Omega}\frac{\sinh Kz}{\sinh Kh} + \frac{(U_x + iU_y)n}{n^2 + (s - i\Omega)^2}\frac{\sinh\sqrt{s/\nu}z}{\sinh\sqrt{s/\nu}h}. \tag{3.6}$$

Letting $\varphi = \sqrt{s/\nu}$, (3.6) can be written as follows:

$$\overline{H} = \frac{1}{s}\frac{\Omega\ell}{1 - i\Omega/s}\frac{\sinh Kz}{\sinh Kh} - \frac{\left(U_y - iU_x\right)}{2s}\left[\frac{1}{1 + i(n-\Omega)/s} - \frac{1}{1 - i(n+\Omega)/s}\right]\frac{e^{-\varphi(h-z)} - e^{-\varphi(h+z)}}{1 - e^{-2\varphi h}}. \tag{3.7}$$

It is well known that the series $\sum_{q=0}^{\infty} X^q$ converges to $(1-X)^{-1}$ for $|X| < 1$. Using this binomial series, it is possible to obtain the solution for small times. Equation (3.7) can be written in the following form:

$$\begin{aligned}\overline{H} = {}& \Omega\ell\frac{\sinh Kz}{\sinh Kh}\sum_{q=0}^{\infty}\frac{(i\Omega)^q}{s^{q+1}} - \frac{\left(U_y - iU_x\right)}{2} \\ &\times \sum_{p=0}^{\infty}\sum_{m=0}^{\infty}\left\{\left([i(\Omega - n)]^p - [i(n+\Omega)]^p\right)\frac{e^{-\varphi(h-z+2hm)} - e^{-\varphi(h+z+2hm)}}{s^{p+1}}\right\}.\end{aligned} \tag{3.8}$$

The inverse Laplace transform of (3.8) gives

$$\begin{aligned}\frac{F}{\Omega\ell} = {}& \frac{\sinh\sqrt{R/2}(1+i)\zeta}{\sinh\sqrt{R/2}(1+i)} - \frac{\left(V_y - iV_x\right)}{2}(\cos\tau - i\sin\tau) \\ &\times \sum_{p=0}^{\infty} i^p(4\tau)^p\left[(1-k)^p - (1+k)^p\right]\sum_{m=0}^{\infty}\left[\mathrm{i}^{2p}\operatorname{erfc}\frac{1-\zeta+2m}{2\sqrt{\tau/R}} - \mathrm{i}^{2p}\operatorname{erfc}\frac{1+\zeta+2m}{2\sqrt{\tau/R}}\right],\end{aligned} \tag{3.9}$$

or

$$\begin{aligned}\overline{f} = {}& \frac{P(1)P(\zeta) + Q(1)Q(\zeta)}{\Delta} + \frac{V_x\sin\tau - V_y\cos\tau}{2}\left(-\alpha_2T_4 + \alpha_4T_8 - \alpha_6T_{12} + \alpha_8T_{16} - \cdots\right) \\ &- \frac{V_x\cos\tau + V_y\sin\tau}{2}\left(\alpha_1T_2 - \alpha_3T_6 + \alpha_5T_{10} - \alpha_7T_{14} + \cdots\right), \\ \overline{g} = {}& \frac{P(1)Q(\zeta) - Q(1)P(\zeta)}{\Delta} + \frac{V_x\sin\tau - V_y\cos\tau}{2}\left(\alpha_1T_2 - \alpha_3T_6 + \alpha_5T_{10} - \alpha_7T_{14} + \cdots\right) \\ &+ \frac{V_x\cos\tau + V_y\sin\tau}{2}\left(-\alpha_2T_4 + \alpha_4T_8 - \alpha_6T_{12} + \alpha_8T_{16} - \cdots\right),\end{aligned} \tag{3.10}$$

where

$$\bar{f} = \frac{f}{\Omega \ell}, \qquad \bar{g} = \frac{g}{\Omega \ell}, \qquad R = \frac{\Omega h^2}{\nu}, \qquad \zeta = \frac{z}{h}, \qquad \tau = \Omega t, \qquad V_x = \frac{U_x}{\Omega \ell},$$
$$V_y = \frac{U_y}{\Omega \ell}, \qquad k = \frac{n}{\Omega}, \qquad P(\zeta) = \sinh\sqrt{\frac{R}{2}}\zeta \cos\sqrt{\frac{R}{2}}\zeta,$$
$$Q(\zeta) = \cosh\sqrt{\frac{R}{2}}\zeta \sin\sqrt{\frac{R}{2}}\zeta, \qquad \Delta = [P(1)]^2 + [Q(1)]^2, \tag{3.11}$$
$$\alpha_r = (4\tau)^r\left[(1-k)^r - (1+k)^r\right],$$
$$T_r = \sum_{m=0}^{\infty}\left[\mathrm{i}^r \operatorname{erfc}\frac{1-\zeta+2m}{2\sqrt{\tau/R}} - \mathrm{i}^r \operatorname{erfc}\frac{1+\zeta+2m}{2\sqrt{\tau/R}}\right].$$

The properties of a function that characterizes the functions T_r defined here are given by Ersoy [25]. The series solutions shown by (3.10) converge rapidly for small times but converge slowly when τ increases. These solutions cannot be used for very large times. For this reason, it is necessary to introduce a different method for large times.

4. Solution for Large Times

For large times, we suggest a solution of the form

$$F(z,t) = F_0(z) + F_1(z)\cos nt + F_2(z)\sin nt, \tag{4.1}$$

where $F_0(z)$ corresponds to the case of $n = 0$. Substituting (4.1) into (2.6), we get

$$\nu F_0'' - \Omega i F_0 = 0, \tag{4.2a}$$
$$\nu F_1'' - nF_2 - \Omega i F_1 = 0, \tag{4.2b}$$
$$\nu F_2'' + nF_1 - \Omega i F_2 = 0. \tag{4.2c}$$

The boundary conditions for (4.2a)–(4.2c) are

$$F_0(\pm h) = \pm\Omega\ell, \qquad F_1(\pm h) = 0, \qquad F_2(\pm h) = \pm\left(U_x + iU_y\right),$$
$$F_0(0) = 0, \qquad F_1(0) = 0, \qquad F_2(0) = 0. \tag{4.3}$$

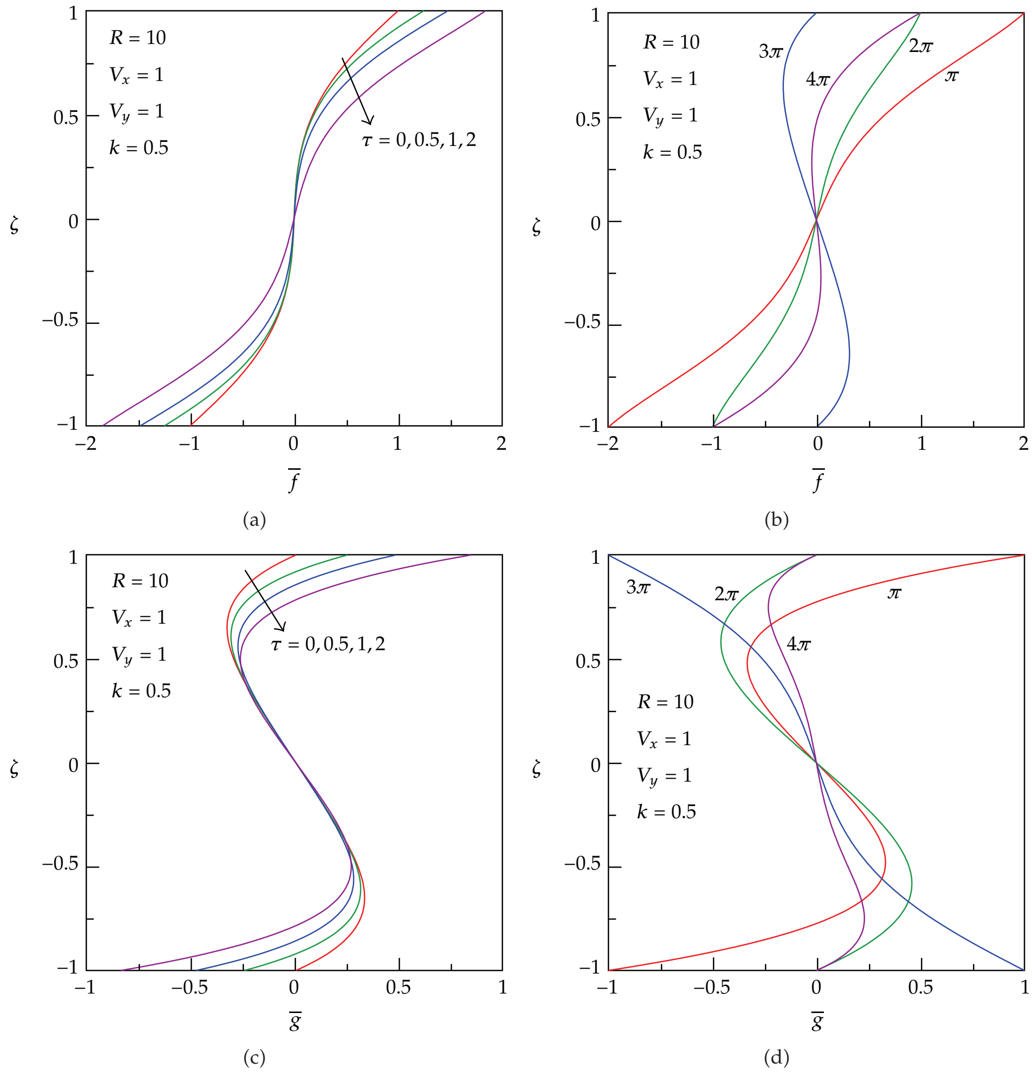

Figure 2: Variations of $\overline{f}$ and $\overline{g}$ with τ for $k = 0.5$ ($R = 10$, $V_x = 1$, $V_y = 1$).

From the solutions of (4.2a)–(4.2c) by the conditions (4.3), we have

$$\begin{aligned}\frac{F}{\Omega \ell} = {} & \frac{\sinh\sqrt{R/2}(1+i)\zeta}{\sinh\sqrt{R/2}(1+i)} \\ & - \frac{(V_y - iV_x)}{2}\left[\frac{\sin\sqrt{(k-1)R/2}(1+i)\zeta}{\sin\sqrt{(k-1)R/2}(1+i)} - \frac{\sinh\sqrt{(k+1)R/2}(1+i)\zeta}{\sinh\sqrt{(k+1)R/2}(1+i)}\right]\cos k\tau \\ & + \frac{(V_x + iV_y)}{2}\left[\frac{\sin\sqrt{(k-1)R/2}(1+i)\zeta}{\sin\sqrt{(k-1)R/2}(1+i)} + \frac{\sinh\sqrt{(k+1)R/2}(1+i)\zeta}{\sinh\sqrt{(k+1)R/2}(1+i)}\right]\sin k\tau,\end{aligned} \tag{4.4}$$

or

$$\begin{aligned}
\overline{f} = {} & \frac{P(1)P(\zeta) + Q(1)Q(\zeta)}{\Delta} \\
& + \left\{ V_x \left[\frac{J(1)K(\zeta) - K(1)J(\zeta)}{2L} - \frac{A(1)B(\zeta) - B(1)A(\zeta)}{2D} \right] \right. \\
& \qquad \left. + V_y \left[\frac{J(1)J(\zeta) + K(1)K(\zeta)}{2L} - \frac{A(1)A(\zeta) + B(1)B(\zeta)}{2D} \right] \right\} \cos k\tau \\
& + \left\{ V_x \left[\frac{A(1)A(\zeta) + B(1)B(\zeta)}{2D} + \frac{J(1)J(\zeta) + K(1)K(\zeta)}{2L} \right] \right. \\
& \qquad \left. - V_y \left[\frac{A(1)B(\zeta) - B(1)A(\zeta)}{2D} + \frac{J(1)K(\zeta) - K(1)J(\zeta)}{2L} \right] \right\} \sin k\tau, \\
\overline{g} = {} & \frac{P(1)Q(\zeta) - Q(1)P(\zeta)}{\Delta} \\
& + \left\{ V_x \left[\frac{A(1)A(\zeta) + B(1)B(\zeta)}{2D} - \frac{J(1)J(\zeta) + K(1)K(\zeta)}{2L} \right] \right. \\
& \qquad \left. + V_y \left[\frac{J(1)K(\zeta) - K(1)J(\zeta)}{2L} - \frac{A(1)B(\zeta) - B(1)A(\zeta)}{2D} \right] \right\} \cos k\tau \\
& + \left\{ V_x \left[\frac{A(1)B(\zeta) - B(1)A(\zeta)}{2D} + \frac{J(1)K(\zeta) - K(1)J(\zeta)}{2L} \right] \right. \\
& \qquad \left. + V_y \left[\frac{A(1)A(\zeta) + B(1)B(\zeta)}{2D} + \frac{J(1)J(\zeta) + K(1)K(\zeta)}{2L} \right] \right\} \sin k\tau,
\end{aligned} \tag{4.5}$$

where

$$\begin{aligned}
A(\zeta) &= \cosh \sqrt{\frac{(k-1)R}{2}}\zeta \sin \sqrt{\frac{(k-1)R}{2}}\zeta, \\
B(\zeta) &= \sinh \sqrt{\frac{(k-1)R}{2}}\zeta \cos \sqrt{\frac{(k-1)R}{2}}\zeta, \\
J(\zeta) &= \sinh \sqrt{\frac{(k+1)R}{2}}\zeta \cos \sqrt{\frac{(k+1)R}{2}}\zeta, \\
K(\zeta) &= \cosh \sqrt{\frac{(k+1)R}{2}}\zeta \sin \sqrt{\frac{(k+1)R}{2}}\zeta, \\
D &= [A(1)]^2 + [B(1)]^2, \qquad L = [J(1)]^2 + [K(1)]^2.
\end{aligned} \tag{4.6}$$

Table 1: Comparison of the two solutions of $\overline{f}$ when τ increases ($\zeta = 0.5$, $R = 10$, $V_x = 1$, $V_y = 1$, $k = 0.5$).

	Small-time solution ($\overline{f}$)	Large-time solution ($\overline{f}$)
$\tau = 0.1$	0.1799927555	0.0407381339
$\tau = 0.5$	0.1913381458	0.1385613370
$\tau = 0.8$	0.2231963485	0.2132298660
$\tau = 1$	0.2542444915	0.2626974023
$\tau = 1.5$	0.3528926478	0.3816911322
$\tau = 2$	0.4604354856	0.4881440727
$\tau = 3$	0.6281582038	0.6381439189
$\tau = 4$	0.6757990334	0.6759717470
$\tau = 5$	0.5936835522	0.5923659856
$\tau = 6$	0.4083508823	0.4077962408
$\tau = 7$	0.1674919804	0.1674516233

The solution for $k = 1$ is

$$\begin{aligned}
\overline{f} &= \frac{P(1)P(\zeta) + Q(1)Q(\zeta)}{\Delta} \\
&\quad + \left[-\frac{V_y\zeta}{2} + V_x\frac{M(1)N(\zeta) - N(1)M(\zeta)}{2E} + V_y\frac{M(1)M(\zeta) + N(1)N(\zeta)}{2E}\right]\cos\tau \\
&\quad + \left[\frac{V_x\zeta}{2} + V_x\frac{M(1)M(\zeta) + N(1)N(\zeta)}{2E} - V_y\frac{M(1)N(\zeta) - N(1)M(\zeta)}{2E}\right]\sin\tau, \\
\overline{g} &= \frac{P(1)Q(\zeta) - Q(1)P(\zeta)}{\Delta} \\
&\quad + \left[\frac{V_x\zeta}{2} - V_x\frac{M(1)M(\zeta) + N(1)N(\zeta)}{2E} + V_y\frac{M(1)N(\zeta) - N(1)M(\zeta)}{2E}\right]\cos\tau \\
&\quad + \left[\frac{V_y\zeta}{2} + V_x\frac{M(1)N(\zeta) - N(1)M(\zeta)}{2E} + V_y\frac{M(1)M(\zeta) + N(1)N(\zeta)}{2E}\right]\sin\tau,
\end{aligned} \tag{4.7}$$

where

$$\begin{aligned}
M(\zeta) &= \sinh\sqrt{R}\zeta\cos\sqrt{R}\zeta, \\
N(\zeta) &= \cosh\sqrt{R}\zeta\sin\sqrt{R}\zeta, \\
E &= [M(1)]^2 + [N(1)]^2.
\end{aligned} \tag{4.8}$$

The solutions shown by (4.5) and (4.7) are not valid for small times. Table 1 compares the two different solutions of $\overline{f}$ for the given specific values when the time varies.

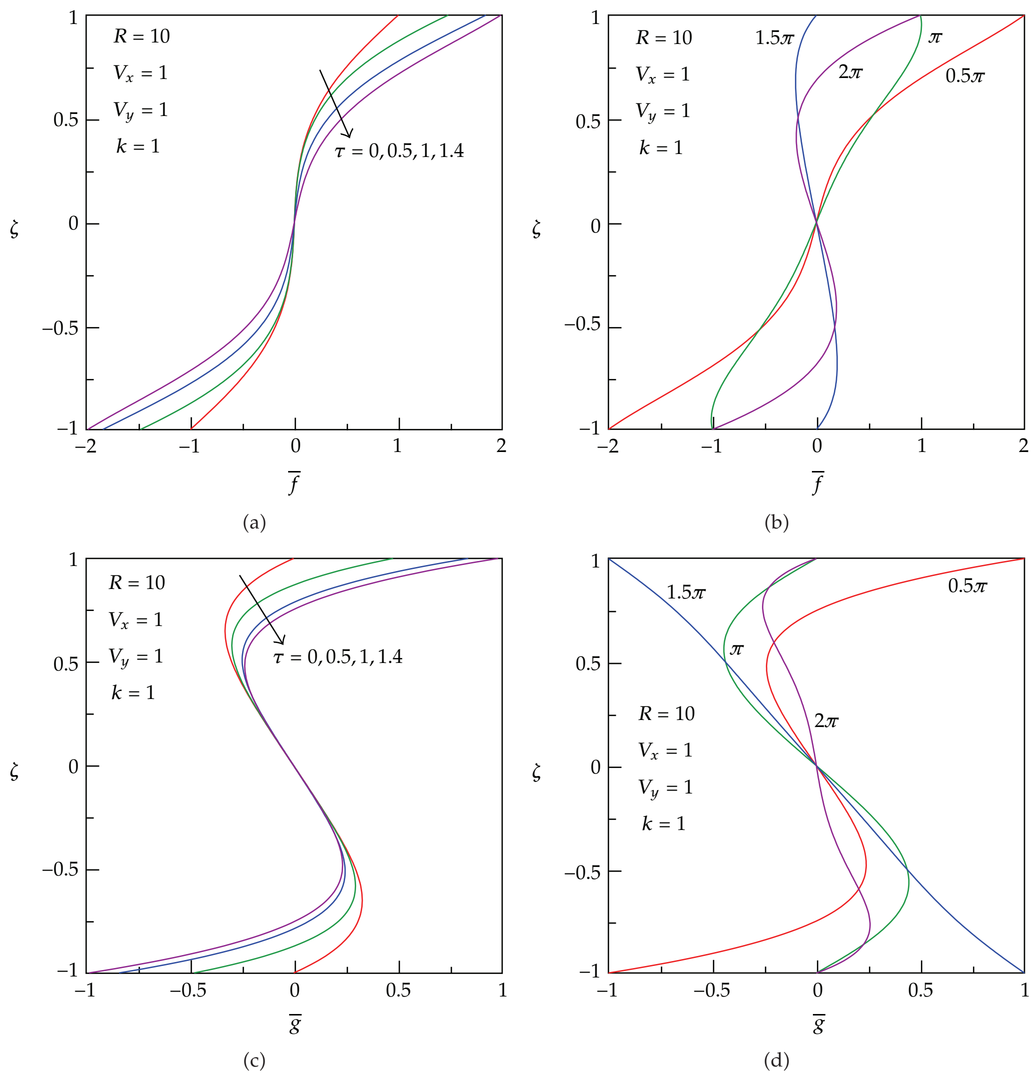

Figure 3: Variations of $\overline{f}$ and $\overline{g}$ with τ for $k = 1$ ($R = 10$, $V_x = 1$, $V_y = 1$).

5. Results and Discussion

In this paper, the motion of the fluid between the disks executing oscillations in their own planes and in the opposite directions is studied while they are initially rotating noncoaxially. There is no flow perpendicular to the disks due to the fact that they are rotating with the same angular velocity at all times. The fluid layer in the plane $z = 0$ rotates as if a rigid body about z-axis, which implies that the symmetrical condition is satisfied, since the disks oscillate in the opposite directions. The effects of all the parameters acting on the flow are revealed by means of Figures 2, 3, 4, 5, 6, and 7. Figures 2–4 display the effect of the frequency of oscillation. The influence of the oscillating direction of the disks on the flow is illustrated in Figures 5 and 6. Figure 7 shows how the flow depends on the Reynolds number. The main findings of the present analysis are pointed out below.

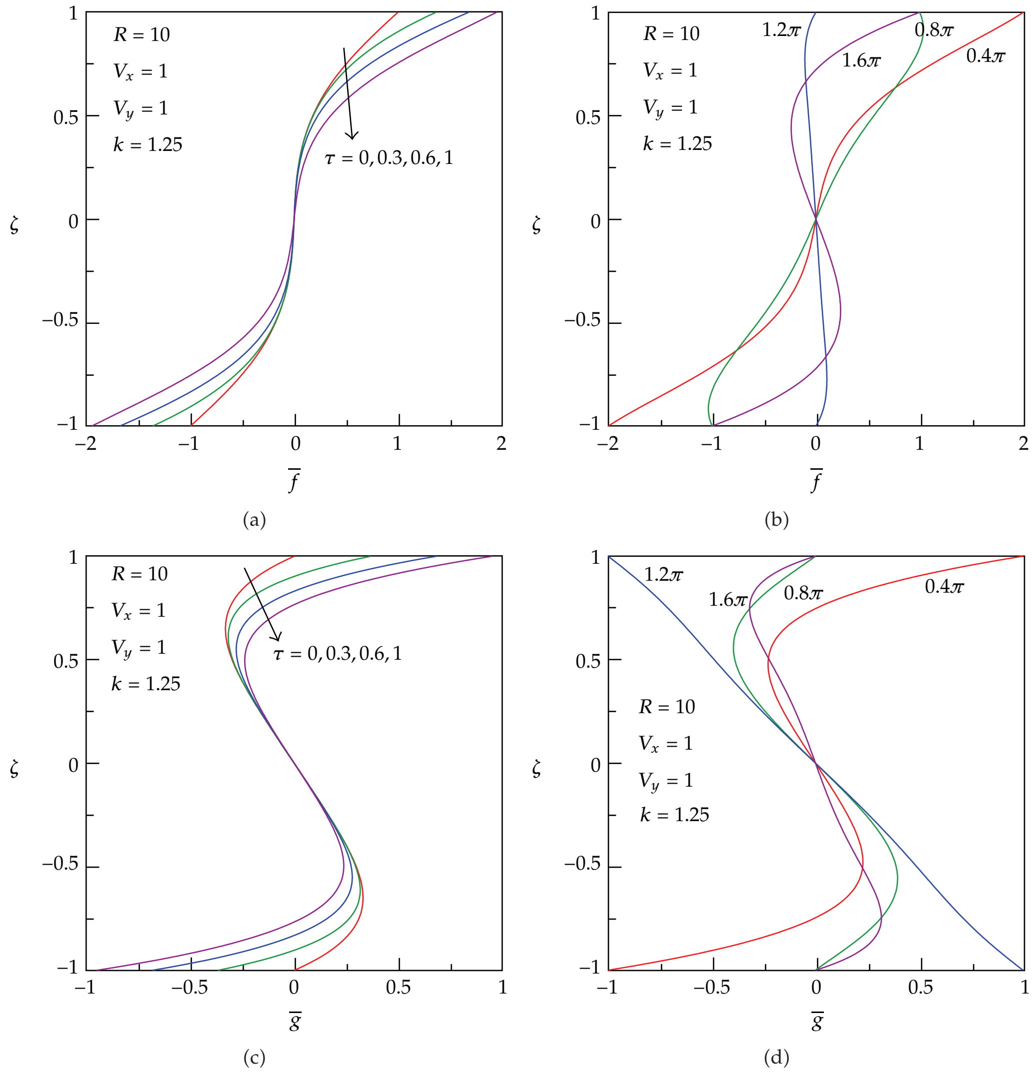

Figure 4: Variations of $\overline{f}$ and $\overline{g}$ with τ for $k = 1.25$ ($R = 10$, $V_x = 1$, $V_y = 1$).

(i) The solution of the problem is obtained for both small and large times. At some specific time, it is shown that the solutions for small times are in good agreement with those for large times. Thus, the velocity field is determined at all times.

(ii) It is shown that the velocity increases for the same small times when the frequency of oscillation increases. With the decrease of the frequency, the periodic motion takes place later.

(iii) When the oscillation takes place along the eccentricity direction, the y-component of the translational velocity is considerably affected, but the change in the x-component is almost imperceptible. On the other hand, a reverse effect is observed when the disks are forced to oscillate in the x-direction. In this case, the change in

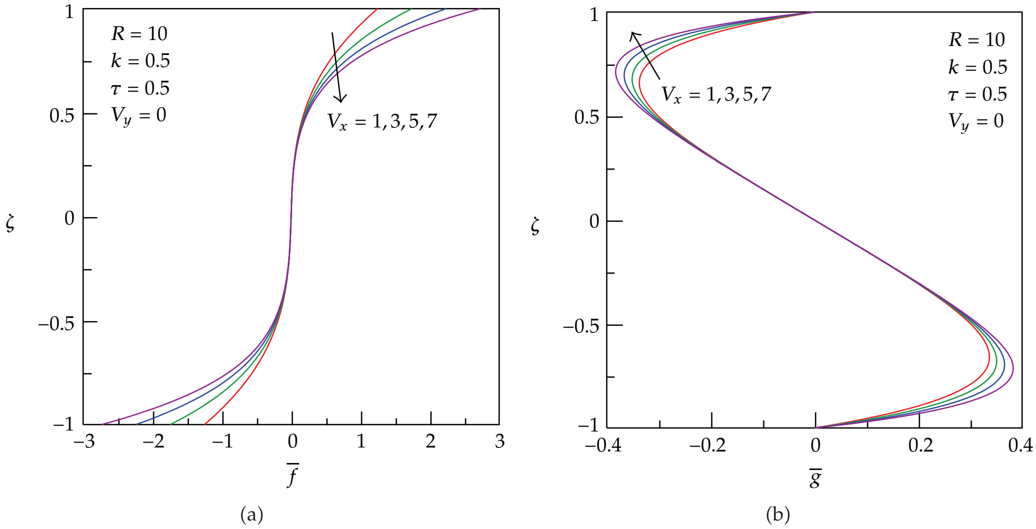

Figure 5: Variations of $\overline{f}$ and $\overline{g}$ with V_x for $V_y = 0$ ($R = 10$, $k = 0.5$, $\tau = 0.5$).

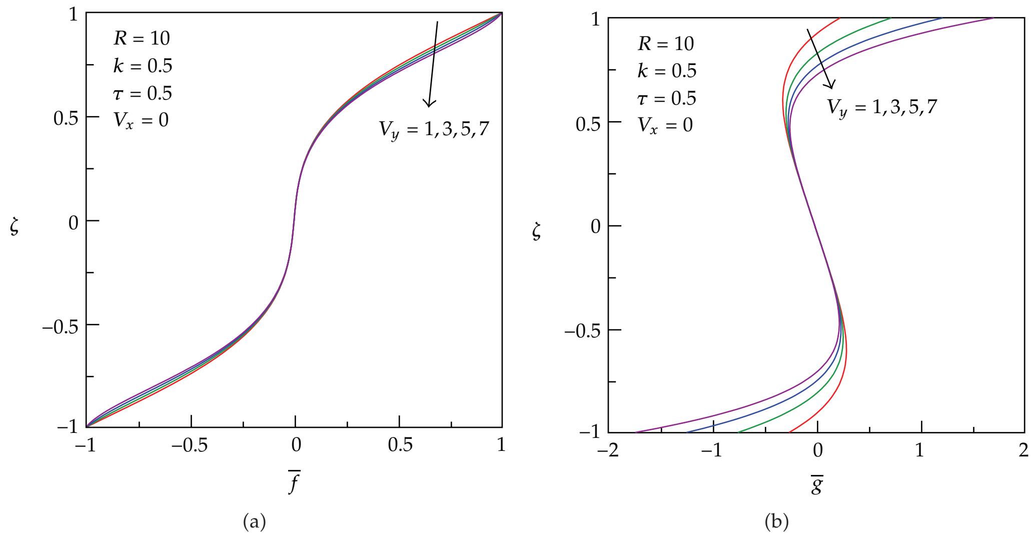

Figure 6: Variations of $\overline{f}$ and $\overline{g}$ with V_y for $V_x = 0$ ($R = 10$, $k = 0.5$, $\tau = 0.5$).

the x-component of the translational velocity is noticed clearly but the change in the y-component is insignificant.

(iv) Increasing the Reynolds number has the effect of decreasing the thickness of the boundary layer.

(v) It is observed that the periodic motion presupposed in the solution for large times occurs.

(vi) As it is expected, there exists a phase lag between the flow velocity and the disk oscillation when the periodic motion occurs.

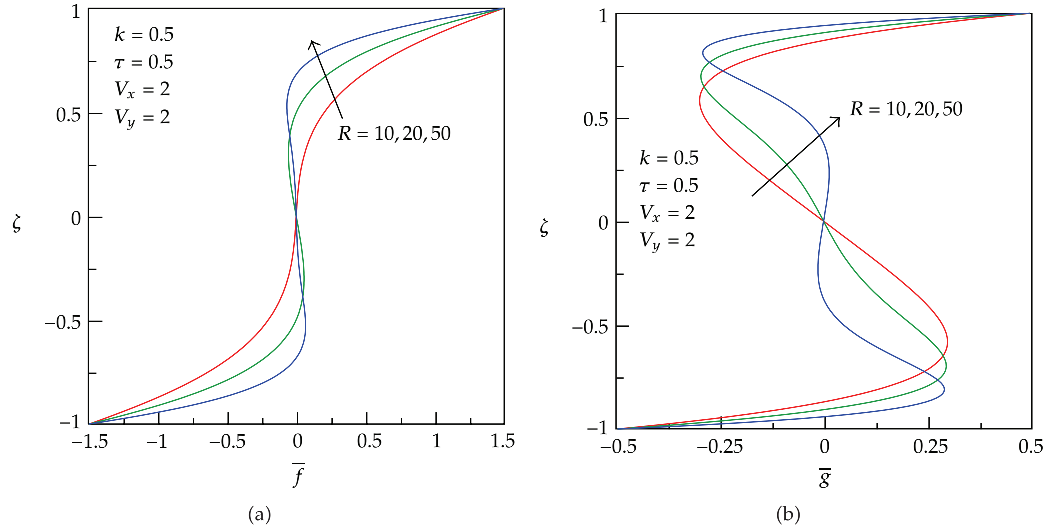

Figure 7: Variations of $\overline{f}$ and $\overline{g}$ with R ($k = 0.5$, $\tau = 0.5$, $V_x = 2$, $V_y = 2$).

(vii) It is shown that a solution can be obtained even when the angular velocity of the disk is equal to the frequency of oscillation.

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