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Research Article

Robust H_2/H_∞ Filter Design for a Class of Nonlinear Stochastic Systems with State-Dependent Noise

Weihai Zhang,[1] Bor-Sen Chen,[2] Li Sheng,[3] and Ming Gao[1]

[1] *College of Information and Electrical Engineering, Shandong University of Science and Technology, Qingdao 266590, China*
[2] *Department of Electrical Engineering, National Tsing Hua University, Hsinchu 30013, Taiwan*
[3] *College of Information and Control Engineering, China University of Petroleum (East China), Qingdao 266555, China*

Correspondence should be addressed to Weihai Zhang, w_hzhang@163.com

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This paper investigates the problem of robust filter design for a class of nonlinear stochastic systems with state-dependent noise. The state and measurement are corrupted by stochastic uncertain exogenous disturbance and the dynamic system is modeled by Itô-type stochastic differential equations. For this class of nonlinear stochastic systems, the robust H_∞ filter can be designed by solving linear matrix inequalities (LMIs). Moreover, a mixed H_2/H_∞ filtering problem is also solved by minimizing the total estimation error energy when the worst-case disturbance is considered in the design procedure. A numerical example is provided to illustrate the effectiveness of the proposed method.

1. Introduction

Over the past decades, the robust H_∞ filtering problem has been investigated extensively since it is very useful in signal processing and engineering applications [1–5]. The so-called H_∞ filtering problem is to design an estimator to estimate the unknown state combination via measurement output, which guarantees the $\mathcal{L}_2$ gain (from the external disturbance to the estimation error) to be less than a prescribed level $\gamma > 0$. In contrast to classical Kalman filter, it is not necessary to know the exact statistic information about the external disturbance in the H_∞ filter design. Obviously, there may be more than one solution to H_∞ filtering problem with a desired robustness. Since the H_2 performance is appealing for engineering, it naturally leads to the mixed H_2/H_∞ filtering problem [6–8]. Compared with the sole H_∞ filter, the mixed H_2/H_∞ filter is more attractive in engineering practice, since the former is a worst-case design which tends to be conservative whereas the latter minimizes the average performance with a guaranteed worst-case performance. The robust H_2/H_∞ filtering problem for linear perturbed systems with steady-state error variance constraints was investigated in [6], and the mixed H_2/H_∞ filter for polytopic discrete-time systems was discussed in [7].

On the other hand, stochastic H_∞ control and filtering problems for systems expressed by stochastic Itô-type differential equations have attracted a great deal of attention [9–13, 23]. A bounded real lemma was proposed for linear continuous-time stochastic systems [11], according to which full- and reduced-order robust H_∞ problems for linear stochastic systems were investigated by [12, 13], respectively. Most of the aforementioned works were limited to linear stochastic systems. Recently, the H_∞ filtering problem for nonlinear stochastic systems has become another popular research topic [14–20]. Wang et al. [14] studied the robust H_∞ filtering problem for a class of uncertain time-delay stochastic systems with sector-bounded nonlinearities. For general nonlinear stochastic systems, Zhang et al. [15] found that the H_∞ filter can be obtained by solving a second-order Hamilton-Jacobi inequality (HJI). Considering that it is difficult to solve the HJI, Tseng [17] designed the H_∞ fuzzy filter for nonlinear stochastic systems via solving LMIs instead of an HJI. However, there is little work dealing with the H_2/H_∞ filtering problem for nonlinear stochastic systems.

In this paper, we will deal with the robust filtering problem for a class of nonlinear stochastic systems. The state is corrupted not only by white noise but also by exogenous disturbance signal, and the measurement equation also includes noises. Our goal in this paper is to construct an asymptotically stable observer that leads to a mean square stable estimation error process whose $\mathcal{L}_2$ gain with respect to disturbance signal is less than a prescribed level. Moreover, a stochastic H_2/H_∞ filtering is designed for the nonlinear stochastic systems. Our main results are expressed in linear matrix inequalities (LMIs), which are more easily computed in practical application.

This paper is organized as follows: in Section 2, some definitions and notations are introduced; Section 3 treats with the H_∞ and mixed H_2/H_∞ filtering problems, and the main outcomes of this section are Theorems 3.2 and 3.6; a numerical example is presented to illustrate the effectiveness of the proposed filtering method in Section 4; Section 5 concludes this paper.

Notations. For convenience, we adopt the following notations. $\mathcal{S}_n$: the set of all $n\times n$ symmetric matrices; its components may be complex. A': the transpose of the corresponding matrix A. $A \geq 0$ $(A > 0)$: A is positive semidefinite (positive definite) symmetric matrix. $|x| := (\sum_{i=1}^{n} x_i^2)^{1/2}$, that is, $|x|$ denotes the Euclidean 2-norm of x, where $x = (x_1, x_2, \ldots, x_n)' \in \mathcal{R}^n$. $\mathcal{L}^2(\mathcal{R}_+, \mathcal{R}^l)$: the space of nonanticipative stochastic processes $y(t)$ with respect to filter $\mathcal{F}_t$ satisfying $\|y(t)\|_{L_2}^2 := E\int_0^\infty |y(t)|^2 dt < \infty$. $C_2^0(\{t > 0\} \times U)$: class of functions $V(t,x)$ twice continuously differential with respect to $x \in U$ and once continuously differential with respect to $t > 0$ except possibly at the point $x = 0$.

2. Problem Setting

Consider the following nonlinear stochastic system governed by Itô differential equation:

$$dx(t) = \big(f(x(t)) + B_0 w(t)\big)dt + \sigma(x(t))dw_0(t), \tag{2.1}$$

with the following measurement equation:

$$dy(t) = (A_1 x(t) + B_1 w(t))dt + C_1 x(t) dw_1(t), \tag{2.2}$$

and the controlled output

$$z(t) = Dx(t). \tag{2.3}$$

In the above, $x(t) \in \mathcal{R}^n$ is called the system state, $y(t) \in \mathcal{R}^r$ is the measurement output, $z(t)$ is the state combination to be estimated. $w_0(t), w_1(t)$ are the standard Wiener processes defined on the probability space $(\Omega, \mathcal{F}, \mathcal{P})$ related to an increasing family $(\mathcal{F}_t)_{t\in\mathcal{R}_+}$ of σ-algebras $\mathcal{F}_t \subset \mathcal{F}$. Without loss of generality, we can suppose $w_0(t), w_1(t)$ are one-dimensional, mutually uncorrelated. B_0, A_1, B_1, C_1, D are constant matrices of suitable dimensions, $w \in \mathcal{L}^2(\mathcal{R}_+, \mathcal{R}^q)$ represents the exogenous disturbance signal. Under very general conditions on f and σ, stochastic systems (2.1)-(2.2) have, respectively, a unique strong solution $x_{s,\xi}(t)$ for any $t \geq s \geq 0$ and initial state $x(s) = \xi \in \mathcal{R}^n$; see [21].

Now, we first introduce the following definitions.

Definition 2.1 (see [9]). We say that the equilibrium point $x \equiv 0$ of system

$$dx(t) = f(x(t))dt + \sigma(x(t))dw_0(t) \tag{2.4}$$

is exponentially mean square stable, if for some positive constants ρ, ϱ,

$$E|x(t)|^2 \leq \rho|x(0)|^2 \exp(-\varrho t), \quad t \geq 0. \tag{2.5}$$

Remark 2.2. It is well known that for stochastic linear time-invariant systems, the exponential mean square stability is equivalent to asymptotical mean square stability [9].

Definition 2.3. Nonlinear stochastic uncertain system (2.1) is said to be internally stable at the origin, if (2.1) with $w = 0$ is exponentially mean square stable.

Lemma 2.4 (see [9]). *The trivial solution of* (2.4) *is exponentially mean square stable for $t \geq 0$ if there exists $V(t,x) \in C_2^0(\{t > 0\} \times \mathcal{R}^n)$ such that*

$$k_1|x|^2 \leq V(t,x) \leq k_2|x|^2, \qquad \mathcal{L}V(t,x) \leq -k_3|x|^2 \tag{2.6}$$

for some positive constants k_1, k_2, k_3, where $\mathcal{L}$ is the so-called an infinitesimal generator of (2.4).

Now, suppose $f(x)$ and $\sigma(x)$ can be linearized, respectively, as

$$\begin{aligned} f(x) &= Ax + F_0(x), \qquad F_0(0) = 0, \\ \sigma(x) &= Cx + F_1(x), \qquad F_1(0) = 0, \end{aligned} \tag{2.7}$$

then the linearized stochastic system of (2.1) *becomes*

$$dx = (Ax + B_0 w + F_0(x))dt + (Cx + F_1(x))dw_0, \tag{2.8}$$

where A and C are constant matrices.

Consider the following filter for the estimation of $z(t)$*:*

$$d\hat{x} = A_f \hat{x} dt + B_f dy, \qquad \hat{x}(0) = \hat{x}_0, \qquad \hat{z} = D\hat{x}, \tag{2.9}$$

where $\hat{x} \in \mathcal{R}^n$*. Let* $\xi' = [x' \ \ x' - \hat{x}']$, $\tilde{z} = z - \hat{z}$*, then*

$$d\xi = \widetilde{A}\xi dt + \widetilde{D}_1 \xi dw_0 + \widetilde{D}_2 \xi dw_1 + \widetilde{F}_1 dt + \widetilde{F}_2 dw_0 + \widetilde{F}_3 w dt, \tag{2.10}$$

where

$$\begin{gathered} \widetilde{A} = \begin{bmatrix} A & 0 \\ A - B_f A_1 - A_f & -A_f \end{bmatrix}, \qquad \widetilde{D}_1 = \begin{bmatrix} C & 0 \\ C & 0 \end{bmatrix}, \qquad \widetilde{D}_2 = \begin{bmatrix} 0 & 0 \\ -B_f C_1 & 0 \end{bmatrix}, \\ \widetilde{F}_1 = \begin{bmatrix} F_0(x) \\ F_0(x) \end{bmatrix}, \qquad \widetilde{F}_2 = \begin{bmatrix} F_1(x) \\ F_1(x) \end{bmatrix}, \qquad \widetilde{F}_3 = \begin{bmatrix} B_0 \\ B_0 - B_f B_1 \end{bmatrix}. \end{gathered} \tag{2.11}$$

For any given disturbance attenuation level $\gamma > 0$*, one wants to find* A_f, B_f*, such that*

$$\|\tilde{z}(t)\|_{L_2}^2 < \gamma^2 \|w(t)\|_{L_2}^2 \tag{2.12}$$

holds for any $w \in \mathcal{L}^2(\mathcal{R}_+, \mathcal{R}^q)$*. Define the* H_∞ *performance index as*

$$J_s = \|\tilde{z}(t)\|_{L_2}^2 - \gamma^2 \|w(t)\|_{L_2}^2. \tag{2.13}$$

Obviously, (2.12) *holds iff* $J_s < 0$*. As in [12],* H_∞ *and mixed* H_2/H_∞*-based robust state estimation problems are formulated as follows.*

(i) *Stochastic* H_∞ *filtering problem: given* $\gamma > 0$*, find an estimator* $\hat{x}$ *of the form* (2.9) *leading* (2.10) *to being internally stable; Moreover,* $J_s < 0$ *for all nonzero* $w \in \mathcal{L}^2(\mathcal{R}_+, \mathcal{R}^n)$ *with* $\xi(0) = 0$.

(ii) *Stochastic* H_2/H_∞ *filtering problem: of all the* H_∞ *filter of (i), one finds the one that minimizes the steady error variance*

$$\lim_{t \to \infty} E\left[\tilde{z}'(t)\tilde{z}(t)\right], \tag{2.14}$$

where in this case, $w(t) = \dot{\eta}$, η *is taken as a standard Wiener process, independent of* $w_0(t)$ *and* $w_1(t)$*, so* $w(t)$ *is a white noise.* (2.2) *and* (2.8) *can be written as (see, e.g., [22])*

$$\begin{gathered} dy(t) = A_1 x(t) dt + B_1 d\eta(t) + C_1 x(t) dw_1(t), \\ dx(t) = (Ax(t) + F_0(x(t)))dt + (Cx(t) + F_1(x(t)))dw_0(t) + B_0 d\eta(t), \end{gathered} \tag{2.15}$$

respectively.

3. Stochastic H_∞ and Mixed H_2/H_∞ Filter Design

In this section, we will discuss, respectively, stochastic H_∞ and mixed H_2/H_∞ filtering problems.

3.1. Stochastic H_∞ Filter Design

In this section, some sufficient conditions are given for H_∞ filter design; our main results are as follows.

Theorem 3.1. *Suppose there exists a scalar $\lambda > 0$, such that*

$$|F_i(x)| \le \lambda |x|, \quad i = 0, 1, \ \forall x \in \mathcal{R}^n. \tag{3.1}$$

If the following matrix inequalities

$$P\widetilde{A} + \widetilde{A}'P + 2\widetilde{D}_1'P\widetilde{D}_1 + \widetilde{D}_2'P\widetilde{D}_2 + P + 6\lambda^2\alpha I + Q + \frac{1}{\gamma^2}P\widetilde{F}_3\widetilde{F}_3'P < 0, \tag{3.2}$$

$$0 < P \le \alpha I \tag{3.3}$$

have a solution $P > 0$, $\alpha > 0$, then (2.10) *is internally stable and H_∞ filtering performance $J_s < 0$, where $Q = (0\ D)'(0\ D)$.*

Proof. We first show (2.10) to be internally stable, that is, the following system

$$d\xi = \widetilde{A}\xi dt + \widetilde{D}_1\xi dw_0 + \widetilde{D}_2\xi dw_1 + \widetilde{F}_1 dt + \widetilde{F}_2 dw_0(t) \tag{3.4}$$

is asymptotically mean square stable. Let $\mathcal{L}_\xi$ be the infinitesimal operator of (3.4), $V(\xi) = \xi' P\xi$ with $\alpha I \ge P > 0$ to be determined. According to Lemma 2.4, in order to show (3.4) to be internally stable, we only need to show

$$\mathcal{L}_\xi V(\xi) \le -k_3|\xi|^2 \tag{3.5}$$

for some $k_3 > 0$. Note that

$$\begin{aligned}\mathcal{L}_\xi V(\xi) &= \frac{\partial V'(\xi)}{\partial \xi}\left(\widetilde{A}\xi + \widetilde{F}_1\right) + \frac{1}{2}\left(\widetilde{D}_1\xi + \widetilde{F}_2\right)'\frac{\partial^2 V(\xi)}{\partial \xi^2}\left(\widetilde{D}_1\xi + \widetilde{F}_2\right) + \frac{1}{2}\left(\widetilde{D}_2\xi\right)'\frac{\partial^2 V(\xi)}{\partial \xi^2}\left(\widetilde{D}_2\xi\right) \\ &= \xi'\left(P\widetilde{A} + \widetilde{A}'P + \widetilde{D}_1'P\widetilde{D}_1 + \widetilde{D}_2'P\widetilde{D}_2\right)\xi + 2\widetilde{F}_1'P\xi + \widetilde{F}_2'P\widetilde{F}_2 + 2\xi'\widetilde{D}_1'P\widetilde{F}_2.\end{aligned} \tag{3.6}$$

By condition (3.1), we have

$$\begin{aligned}2\widetilde{F}_1'P\xi &\le \xi'P\xi + \widetilde{F}_1'P\widetilde{F}_1 \le \xi'P\xi + \alpha\widetilde{F}_1'\widetilde{F}_1 = \xi'P\xi + 2\alpha F_0'F_0 \\ &\le \xi'P\xi + 2\alpha|F_0|^2 \le \xi'P\xi + 2\alpha\lambda^2|\xi|^2.\end{aligned} \tag{3.7}$$

Similarly,

$$\begin{aligned}2\xi'\widetilde{D}_1'P\widetilde{F}_2 &\le \xi'\widetilde{D}_1'P\widetilde{D}_1\xi + 2\alpha\lambda^2|\xi|^2, \\ \widetilde{F}_2'P\widetilde{F}_2 &\le 2\alpha\lambda^2|\xi|^2.\end{aligned} \tag{3.8}$$

Substituting (3.7), (3.8) into (3.6) and considering (3.2), it follows

$$\begin{aligned}\mathcal{L}_\xi V(\xi) &\le \xi'\left(P\widetilde{A} + \widetilde{A}'P + 2\widetilde{D}_1'P\widetilde{D}_1 + \widetilde{D}_2'P\widetilde{D}_2 + P + 6\alpha\lambda^2 I\right)\xi \\ &< -\xi'\left(Q + \frac{1}{\gamma^2}P\widetilde{F}_3\widetilde{F}_3'P\right)\xi \le 0.\end{aligned} \tag{3.9}$$

By Lemma 2.4, the internal stability of (2.10) is proved.

Secondly, we further show the H_∞ filtering performance $J_s < 0$. Let $\mathcal{L}_{\xi,w}$ be the infinitesimal generator of (2.10). For $V(\xi) = \xi'P\xi$, it is easy to show that

$$\mathcal{L}_{\xi,w}V(\xi) = \mathcal{L}_\xi V(\xi) + 2\xi'P\widetilde{F}_3 w. \tag{3.10}$$

For any $T > 0$ and $\xi(0) = 0$, we have

$$\begin{aligned}J_s(T) &:= E\int_0^T \left[|\widetilde{z}(t)|^2 - \gamma^2|w(t)|^2\right]dt \\ &= E\int_0^T \left\{\left[|\widetilde{z}(t)|^2 - \gamma^2|w(t)|^2\right]dt + d\left(\xi'P\xi\right)\right\} - E[\xi(T)P\xi(T)] \\ &\le E\int_0^T \left[|\widetilde{z}(t)|^2 - \gamma^2|w(t)|^2 + \mathcal{L}_{\xi,w}V(\xi)\right]dt.\end{aligned} \tag{3.11}$$

Note that

$$\begin{aligned}\mathcal{L}_{\xi,w}V(\xi) &\le \xi'\left(P\widetilde{A} + \widetilde{A}'P + 2\widetilde{D}_1'P\widetilde{D}_1 + \widetilde{D}_2'P\widetilde{D}_2 + P + 6\lambda^2\alpha I\right)\xi + 2\xi'P\widetilde{F}_3 w, \\ |\widetilde{z}(t)|^2 &= \xi'Q\xi.\end{aligned} \tag{3.12}$$

So

$$|\widetilde{z}(t)|^2 - \gamma^2|w(t)|^2 + \mathcal{L}_{\xi,w}V(\xi) \le \begin{bmatrix}\xi \\ w\end{bmatrix}'\begin{bmatrix}\Lambda_{11} & P\widetilde{F}_3 \\ \widetilde{F}_3'P & -\gamma^2 I\end{bmatrix}\begin{bmatrix}\xi \\ w\end{bmatrix} < 0, \tag{3.13}$$

where

$$\Lambda_{11} := P\widetilde{A} + \widetilde{A}'P + 2\widetilde{D}_1'P\widetilde{D}_1 + \widetilde{D}_2'P\widetilde{D}_2 + P + 6\lambda^2\alpha I + Q. \tag{3.14}$$

By the well-known Schur's complement and (3.2), there exists $\varepsilon > 0$, such that

$$\begin{bmatrix} \Lambda_{11} & P\widetilde{F}_3 \\ \widetilde{F}_3'P & -\gamma^2 I \end{bmatrix} < -\varepsilon I. \tag{3.15}$$

Summarizing the above analysis, (3.11) yields

$$J_s(T) \leq -\varepsilon E \int_0^T \left(|\xi(t)|^2 + w(\mathrm{t})|^2\right) dt \leq -\varepsilon E \int_0^T |w(t)|^2 dt. \tag{3.16}$$

So for any $T > 0$, $E\int_0^T |\widetilde{z}(t)|^2 dt \leq (\gamma^2 - \varepsilon)E\int_0^T |w(t)|^2 dt$.

Let $T \to \infty$, then

$$\|\widetilde{z}(t)\|_{L_2}^2 \leq (\gamma^2 - \varepsilon)\|w(t)\|_{L_2}^2 \tag{3.17}$$

which yields $J_s < 0$. This theorem is proved. □

Theorem 3.1 only has theoretical sense, because it is difficult to be used in designing H_∞ filter. The following result is of more important in practice.

Theorem 3.2. *Under the condition of Theorem 3.1, if the following LMIs*

$$\begin{bmatrix} P_{11} - \alpha I & 0 \\ 0 & P_{22} - \alpha I \end{bmatrix} < 0, \tag{3.18}$$

$$\begin{bmatrix} a_{11} & A'^{P_{22}} - A_1'Z_1' - Z' & \sqrt{2}C'^{P_{11}} & \sqrt{2}C'^{P_{22}} & -C_1'Z_1' & P_{11}B_0 \\ P_{22}A - Z_1A_1 - Z & a_{22} & 0 & 0 & 0 & P_{22}B_0 - Z_1B_1 \\ \sqrt{2}P_{11}C & 0 & -P_{11} & 0 & 0 & 0 \\ \sqrt{2}P_{22}C & 0 & 0 & -P_{22} & 0 & 0 \\ -Z_1C_1 & 0 & 0 & 0 & -P_{22} & 0 \\ B_0'P_{11} & B_0'P_{22} - B_1'Z_1' & 0 & 0 & 0 & -\gamma^2 I \end{bmatrix} < 0 \tag{3.19}$$

have solutions $P_{11} > 0, P_{22} > 0, \alpha > 0, Z_1 \in \mathcal{R}^{n\times r}$, $Z \in \mathcal{R}^{n\times n}$, *then* (2.10) *is internally stable and* $J_s < 0$.

Moreover,

$$d\widehat{x} = P_{22}^{-1}Z\widehat{x}dt + P_{22}^{-1}Z_1 dy \tag{3.20}$$

is the corresponding H_∞ *filter. In* (3.19), $a_{11} = P_{11}A + A'P_{11} + 6\lambda^2\alpha I + P_{11}$, $a_{22} = -Z - Z' + 6\lambda^2\alpha I + D'D + P_{22}$.

Proof. By Schur's complement, (3.2) is equivalent to

$$\begin{bmatrix} P\widetilde{A} + \widetilde{A}'P + P + 6\lambda^2\alpha I + Q & \sqrt{2}\widetilde{D}_1'P & \widetilde{D}_2'P & P\widetilde{F}_3 \\ \sqrt{2}P\widetilde{D}_1 & -P & 0 & 0 \\ P\widetilde{D}_2 & 0 & -P & 0 \\ \widetilde{F}_3'P & 0 & 0 & -\gamma^2 I \end{bmatrix} < 0. \tag{3.21}$$

Taking $P = \text{diag}(P_{11}, P_{22})$ and substituting (2.11) into (3.21), we have

$$\begin{bmatrix} \Psi_{11} & \Psi_{12}' & \Psi_{13}' & \phi_{14}' \\ \Psi_{12} & \Psi_{22} & 0 & 0 \\ \Psi_{13} & 0 & \Psi_{33} & 0 \\ \Psi_{14} & 0 & 0 & \Psi_{44} \end{bmatrix} < 0, \tag{3.22}$$

where

$$\Psi_{11} = \begin{bmatrix} P_{11}A + A'P_{11} + 6\lambda^2\alpha I + P_{11} & (A - B_f A_1 - A_f)'P_{22} \\ P_{22}(A - B_f A_1 - A_f) & -P_{22}A_f - A_f'P_{22} + 6\lambda^2\alpha I + P_{22} + D'D \end{bmatrix},$$

$$\Psi_{22} = \Psi_{33} = -P = \begin{bmatrix} -P_{11} & 0 \\ 0 & -P_{22} \end{bmatrix}, \qquad \Psi_{44} = -\gamma^2 I, \tag{3.23}$$

$$\Psi_{12}' = \begin{bmatrix} \sqrt{2}C'P_{11} & \sqrt{2}C'P_{22} \\ 0 & 0 \end{bmatrix}, \qquad \Psi_{13}' = \begin{bmatrix} 0 & -C_1'B_f'P_{22} \\ 0 & 0 \end{bmatrix}, \qquad \Psi_{14}' = \begin{bmatrix} P_{11}B_0 \\ P_{22}(B_0 - B_f B_1) \end{bmatrix}.$$

(3.22) is equivalent to

$$\begin{bmatrix} \overline{a}_{11} & (A - B_f A_1 - A_f)'P_{22} & \sqrt{2}C'P_{11} & \sqrt{2}C'P_{22} & -C_1'B_f'P_{22} & P_{11}B_0 \\ P_{22}(A - B_f A_1 - A_f) & \overline{a}_{22} & 0 & 0 & 0 & P_{22}(B_0 - B_f B_1) \\ \sqrt{2}P_{11}C & 0 & -P_{11} & 0 & 0 & 0 \\ \sqrt{2}P_{22}C & 0 & 0 & -P_{22} & 0 & 0 \\ -P_{22}B_f C_1 & 0 & 0 & 0 & -P_{22} & 0 \\ B_0'P_{11} & (B_0 - B_f B_1)'P_{22} & 0 & 0 & 0 & -\gamma^2 I \end{bmatrix} < 0, \tag{3.24}$$

where $\overline{a}_{11} = P_{11}A + A'P_{11} + 6\lambda^2\alpha I + P_{11}$, $\overline{a}_{22} = -P_{22}A_f - A'_f P_{22} + 6\lambda^2\alpha I + P_{11}$. Let $P_{22}A_f = Z, P_{22}B_f = Z_1$, then (3.22) becomes (3.19). From our assumption, $A_f = P_{22}^{-1}Z, B_f = P_{22}^{-1}Z_1$, so an H_∞ filtering equation is constructed as in the form of (3.20). Theorem 3.2 is proved. □

3.2. Mixed H_2/H_∞ *Filtering*

To design the mixed stochastic H_2/H_∞ filter, we need to choose the one from the set of all H_∞ filters, which also minimizes the estimation error variance, or concretely speaking, minimizes the H_2 performance

$$\begin{aligned} J_2 &:= \lim_{t\to\infty} E\{\tilde{z}'(t)\tilde{z}(t)\} = \lim_{t\to\infty} E\{\xi'(t)(0\ I)'D'D(0\ I)\xi(t)\} \\ &= \lim_{t\to\infty} \mathrm{Tr}\{D(0\ I)E\xi(t)\xi'(t)(0\ I)'D'\}. \end{aligned} \tag{3.25}$$

Two performances J_s in (2.13) and J_2 in (3.25) associated with H_∞ robustness and H_2 optimization have constructed, respectively. Now, we need to design the mixed H_2/H_∞ filter to maximize J_s and minimize J_2. Consider the following linear stochastic constant system

$$d\xi = A_{11}\xi dt + \sum_{i=1}^{l} B_{ii}\xi dw_i, \tag{3.26}$$

where $\{w_i,\ i = 1,\dots,l\}$ are independent, standard Wiener processes. The following lemma will be used in this section.

Lemma 3.3 (see [23]). *System* (3.26) *is exponentially mean square stable iff for any* $R > 0$, *the following Lyapunov-type equation*

$$PA_{11} + A'_{11}P + \sum_{i=1}^{l} B'_{ii}PB_{ii} = -R \tag{3.27}$$

has a unique positive definite solution $P > 0$.

In the next, for simplicity, when (3.26) *is exponentially stable, one also says* $(A_{11}, B_{11}, \dots, B_{ll})$ *is stable.*

As we have pointed out before, at this stage, we assume $w(t) = \dot{\eta}(t)$; (2.10) *accordingly becomes*

$$d\xi = \widetilde{A}\xi dt + \widetilde{D}_1\xi dw_0 + \widetilde{D}_2\xi dw_1 + \widetilde{F}_1 dt + \widetilde{F}_2 dw_0 + \widetilde{F}_3 d\eta. \tag{3.28}$$

Let $X(t) = E[\xi(t)\xi'(t)]$ *in* (3.28), *then by Itô's formula, we have*

$$\begin{aligned} \dot{X}(t) = {} & \widetilde{A}X(t) + X(t)\widetilde{A}' + E\left[\widetilde{F}_1\xi' + \xi\widetilde{F}_1'\right] + \widetilde{D}_1X\widetilde{D}_1' \\ & + E\left[\widetilde{D}_1\xi\widetilde{F}_2' + \widetilde{F}_2\xi'\widetilde{D}_1'\right] + E\left[\widetilde{F}_2\widetilde{F}_2'\right] + \widetilde{D}_2X(t)\widetilde{D}_2' + \widetilde{F}_3\widetilde{F}_3'. \end{aligned} \tag{3.29}$$

By means of

$$\begin{aligned} E\left[\widetilde{F}_1\xi' + \xi\widetilde{F}_1'\right] &\le E\left[\widetilde{F}_1\widetilde{F}_1'\right] + X(t), \\ E\left[\widetilde{D}_1\xi\widetilde{F}_2' + \widetilde{F}_2\xi'\widetilde{D}_1'\right] &\le \widetilde{D}_1X\widetilde{D}_1' + E\left[\widetilde{F}_2\widetilde{F}_2'\right], \end{aligned} \tag{3.30}$$

we have

$$\begin{aligned}\dot{X}(t) \leq\ & \widetilde{A}X(t) + X(t)\widetilde{A}' + 2\widetilde{D}_1X(t)\widetilde{D}_1' + \widetilde{D}_2X(t)\widetilde{D}_2' \\ & + X(t) + 2E\left[\widetilde{F}_2\widetilde{F}_2'\right] + E\left[\widetilde{F}_1\widetilde{F}_1'\right] + \widetilde{F}_3\widetilde{F}_3'.\end{aligned} \tag{3.31}$$

Now, we suppose $F_i(x)$ $(i = 0, 1)$ *satisfy*

$$F_i(x)F_i'(x) \leq G_ixx'G_i', \quad i = 0, 1,\ \forall x \in \mathcal{R}^n, \tag{3.32}$$

where G_1, G_2 *are constant matrices of suitable dimensions. At this stage,*

$$\begin{aligned}\widetilde{F}_i\widetilde{F}_i' &= \begin{bmatrix} I & 0 \\ I & I \end{bmatrix} \begin{bmatrix} F_iF_i' & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} I & I \\ 0 & I \end{bmatrix} \\ &\leq \begin{bmatrix} I & 0 \\ I & I \end{bmatrix} \begin{bmatrix} G_ixx'G_i' & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} I & I \\ 0 & I \end{bmatrix} \\ &= \begin{bmatrix} I & 0 \\ I & I \end{bmatrix} \begin{bmatrix} G_i & 0 \\ 0 & 0 \end{bmatrix} \xi\xi' \begin{bmatrix} G_i' & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} I & I \\ 0 & I \end{bmatrix} \\ &= \begin{bmatrix} G_i & 0 \\ G_i & 0 \end{bmatrix} \xi\xi' \begin{bmatrix} G_i' & G_i' \\ 0 & 0 \end{bmatrix} \\ &:= \widetilde{G}_i\xi\xi'\widetilde{G}_i', \quad i = 0, 1,\end{aligned} \tag{3.33}$$

where

$$\widetilde{G}_i = \begin{bmatrix} G_i & 0 \\ G_i & 0 \end{bmatrix}. \tag{3.34}$$

So (3.31) *becomes*

$$\dot{X}(t) \leq \widetilde{A}X(t) + X(t)\widetilde{A}' + 2\widetilde{D}_1X(t)\widetilde{D}_1' + \widetilde{D}_2X(t)\widetilde{D}_2' + X(t) + 2\widetilde{G}_2X(t)\widetilde{G}_2' + \widetilde{G}_1X(t)\widetilde{G}_1' + \widetilde{F}_3\widetilde{F}_3'. \tag{3.35}$$

In addition, if $X_1(t)$ *solves*

$$\begin{aligned}\dot{X}_1(t) &= \widetilde{A}X_1(t) + X_1(t)\widetilde{A}' + 2\widetilde{D}_1X_1(t)\widetilde{D}_1' + \widetilde{D}_2X_1(t)\widetilde{D}_2' + X_1(t) \\ &\quad + 2\widetilde{G}_2X_1(t)\widetilde{G}_2' + \widetilde{G}_1X_1(t)\widetilde{G}_1' + \widetilde{F}_3\widetilde{F}_3' \\ X_1(0) &= X(0)\end{aligned} \tag{3.36}$$

then it is easy to prove that $X(t) \leq X_1(t)$. *Denoting* $\overline{X}_1 := \lim_{t\to\infty}X_1(t)$, *where* $\overline{X}_1$ *satisfies*

$$\widetilde{A}\overline{X}_1 + \overline{X}_1\widetilde{A}' + 2\widetilde{D}_1\overline{X}_1\widetilde{D}_1' + \widetilde{D}_2\overline{X}_1\widetilde{D}_2' + 2\widetilde{G}_2\overline{X}_1\widetilde{G}_2' + \widetilde{G}_1\overline{X}_1\widetilde{G}_1' + \overline{X}_1 + \widetilde{F}_3\widetilde{F}_3' = 0. \tag{3.37}$$

Obviously, $\lim_{t\to\infty} X(t) \le \overline{X}_1$*, accordingly,*

$$J_2 \le \mathrm{Tr}\left\{D(0\ I)\overline{X}_1(0\ I)'D'\right\} = \mathrm{Tr}\left\{\overline{X}_1 Q\right\}. \tag{3.38}$$

As in [12, 24], it is easily seen the following fact.

Lemma 3.4. *If* $\widehat{P}$ *is a solution of*

$$\widetilde{A}'\widehat{P} + \widehat{P}\widetilde{A} + 2\widetilde{D}_1'\widehat{P}\widetilde{D}_1 + \widetilde{D}_2'\widehat{P}\widetilde{D}_2 + 2\widetilde{G}_2'\widehat{P}\widetilde{G}_2 + \widetilde{G}_1'\widehat{P}\widetilde{G}_1 + Q + \widehat{P} = 0 \tag{3.39}$$

then $\mathrm{Tr}(\overline{X}_1 Q) = \mathrm{Tr}(\widehat{P}(\widetilde{F}_3\widetilde{F}_3'))$.

Secondly, suppose $P > 0$ *satisfies*

$$\widetilde{A}'P + P\widetilde{A} + 2\widetilde{D}_1'P\widetilde{D}_1 + \widetilde{D}_2'P\widetilde{D}_2 + Q + P + 2\widetilde{G}_2'P\widetilde{G}_2 + \widetilde{G}_1'P\widetilde{G}_1 < 0. \tag{3.40}$$

By means of Lemma 3.3, one can show $P > \widehat{P}$*. So we have the following lemma.*

Lemma 3.5. $P > \widehat{P}$*, where* P *and* $\widehat{P}$ *stand for the positive definite solutions of* (3.40) *and* (3.39)*, respectively.*

From Lemmas 3.4–3.5, it gives

$$\begin{aligned} J_2 &= \lim_{t\to\infty} \mathrm{Tr}\{D(0\ I)X(t)(0\ I)'D'\} \\ &\le \lim_{t\to\infty} \mathrm{Tr}\{D(0\ I)X_1(t)(0\ I)'D'\} \\ &= \mathrm{Tr}\left\{D(0\ I)\overline{X}_1(0\ I)'D'\right\} \\ &= \mathrm{Tr}\left\{\overline{X}_1 Q\right\} = \mathrm{Tr}\left(\widehat{P}\widetilde{F}_3\widetilde{F}_3'\right) \\ &= \mathrm{Tr}\left(\widetilde{F}_3'\widehat{P}\widetilde{F}_3\right) \\ &\le \mathrm{Tr}\left(\widetilde{F}_3'P\widetilde{F}_3\right) := \widehat{J}_2. \end{aligned} \tag{3.41}$$

Hence, to solve the mixed stochastic H_2/H_∞ *filtering problem, we seek to minimize an upper-bound on* $\widehat{J}_2$ *subject to* (3.2), (3.3), *and*

$$P\widetilde{A} + \widetilde{A}'P + 2\widetilde{D}_1'P\widetilde{D}_1 + \widetilde{D}_2'P\widetilde{D}_2' + P + 2\widetilde{G}_2'P\widetilde{G}_2 + \widetilde{G}_1'P\widetilde{G}_1 + Q < 0. \tag{3.42}$$

(3.42) *having a positive definite solution* $P > 0$ *is equivalent to*

$$\begin{bmatrix} P\widetilde{A} + \widetilde{A}'P + P + Q & \sqrt{2}\widetilde{D}_1'P & \widetilde{D}_2'P & \widetilde{G}'_1P & \sqrt{2}\widetilde{G}_2'P \\ \sqrt{2}P\widetilde{D}_1 & -P & 0 & 0 & 0 \\ P\widetilde{D}_2 & 0 & -P & 0 & 0 \\ P\widetilde{G}_1 & 0 & 0 & -P & 0 \\ \sqrt{2}P\widetilde{G}_2 & 0 & 0 & 0 & -P \end{bmatrix} < 0. \tag{3.43}$$

A suboptimal H_2/H_∞ *filtering can be obtained by minimizing* $\mathrm{Tr}(H)$ *subject to* (3.2), (3.3), (3.43), *and*

$$H - \widetilde{F}_3'P\widetilde{F}_3 > 0. \tag{3.44}$$

(3.44) *is equivalent to*

$$\begin{bmatrix} H & \widetilde{F}_3'P \\ P\widetilde{F}_3 & P \end{bmatrix} > 0. \tag{3.45}$$

We still take $P = \mathrm{diag}(P_{11}, P_{22}) > 0$, $P_{22}B_f = Z_1$, $P_{22}A_f = Z$, *then* (3.3), (3.2), (3.43), *and* (3.45) *become, respectively, as* (3.18), (3.19),

$$\begin{bmatrix} \gamma_{11} & \gamma_{12} & \sqrt{2}C'P_{11} & \sqrt{2}C'P_{22} & -C_1'Z_1' & G_1'P_{11} & G_1'P_{22} & G_2'P_{11} & G_2'P_{22} \\ \gamma_{21} & \gamma_{22} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \sqrt{2}P_{11}C & 0 & -P_{11} & 0 & 0 & 0 & 0 & 0 & 0 \\ \sqrt{2}P_{22}C & 0 & 0 & -P_{22} & 0 & 0 & 0 & 0 & 0 \\ -Z_1C_1 & 0 & 0 & 0 & -P_{22} & 0 & 0 & 0 & 0 \\ P_{11}G_1 & 0 & 0 & 0 & 0 & -P_{11} & 0 & 0 & 0 \\ P_{22}G_1 & 0 & 0 & 0 & 0 & 0 & -P_{22} & 0 & 0 \\ P_{11}G_2 & 0 & 0 & 0 & 0 & 0 & 0 & -P_{11} & 0 \\ P_{22}G_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -P_{22} \end{bmatrix} < 0,$$
$$\begin{bmatrix} H & B_0'P_{11} & B_0'P_{22} - B_1'Z_1' \\ P_{11}B_0 & P_{11} & 0 \\ P_{22}B_0 - Z_1B_1 & 0 & P_{22} \end{bmatrix} > 0, \tag{3.46}$$

where $\gamma_{11} = P_{11}A + A'P_{11} + P_{11}$, $\gamma_{12} = A'P_{22} - A_1'Z_1' - Z'$, $\gamma_{21} = P_{22}A - Z_1A_1 - Z$, $\gamma_{22} = -Z - Z' + D'D + P_{22}$. *Therefore, we have the following theorem.*

Theorem 3.6. *Under the conditions of Theorem 3.2 and assumption* (3.32), *if there exists a solution* $(P_{11} > 0, P_{22} > 0, Z, Z_1, \alpha > 0)$ *to* (3.18), (3.19), (3.46), *then a suboptimal mixed stochastic* H_2/H_∞ *filtering is obtained by solving* P_{11} *and* P_{22} *from the following convex optimization problem:* $\min_{P_{11}, P_{22}, Z, Z_1, \alpha} \mathrm{Tr}(H)$ *subject to* (3.18),(3.19), (3.46), *and the corresponding filter is given by* (3.20).

Remark 3.7. In the proof of Theorems 3.2 and 3.6, the matrix P is chosen as $\mathrm{diag}(P_{11}, P_{22})$ for simplicity. In order to reduce the conservatism of the conditions, the matrix P can also be chosen as $\begin{bmatrix} P_{11} & P_{12} \\ P_{12}' & P_{22} \end{bmatrix}$. However, this case will increase the complexity of computation.

4. Numerical Example

Example 4.1. Consider the following nonlinear stochastic system governed by Itô differential equation

$$\begin{aligned} dx &= (Ax + B_0 w + F_0(x))dt + (Cx + F_1(x))dw_0, \\ dy &= (A_1 + B_1 w)dt + C_1 x dw_1, \qquad z = Dx, \end{aligned} \tag{4.1}$$

where

$$\begin{gathered} A = \begin{bmatrix} -3 & 1/2 \\ -1 & -3 \end{bmatrix}, \qquad B_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \qquad C = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \\ F_0(x) = 0.3\tanh(x), \qquad F_1(x) = 0.3\sin(x), \\ A_1 = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}, \qquad B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \qquad C_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \\ D = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad w = \frac{1}{1+2t}, \qquad t \geq 0. \end{gathered} \tag{4.2}$$

Consider the following filter for the estimation of $z(t)$:

$$d\hat{x} = A_f \hat{x} dt + B_f dy, \qquad \hat{z} = D\hat{x}. \tag{4.3}$$

Setting $\gamma = 0.9$, and using the LMI control toolbox of Matlab, the estimation gains of H_∞ filter are derived from Theorem 3.2:

$$A_f = \begin{bmatrix} 5.6231 & 3.7259 \\ -0.1617 & 8.2289 \end{bmatrix}, \qquad B_f = \begin{bmatrix} 0.1812 & -1.8190 \\ -0.2525 & 0.4635 \end{bmatrix}. \tag{4.4}$$

From Theorem 3.6, the estimation gains of H_2/H_∞ filter are obtained as follows:

$$A_f = \begin{bmatrix} 4.1449 & 3.4665 \\ -0.2469 & 6.3382 \end{bmatrix}, \qquad B_f = \begin{bmatrix} 0.5270 & -1.2388 \\ -0.3693 & 0.3445 \end{bmatrix}. \tag{4.5}$$

The initial condition in the simulation is assumed to be $\xi_0 = [0.3\ 0.2\ -0.02\ -0.05]'$. Figures 1 and 2 show the trajectories of $x_1(t)$, $\hat{x}_1(t)$, $x_2(t)$, $\hat{x}_2(t)$ by using the proposed H_∞ and H_2/H_∞ filters, respectively. The trajectories of the estimation error $\tilde{z}(t)$ for H_∞ and H_2/H_∞ filters are shown in Figures 3 and 4, respectively. From Figures 3 and 4, it is obvious that the performance of the proposed H_2/H_∞ filter is better than that of the H_∞ filter.

In [15], the H_∞ and H_2/H_∞ filters for general nonlinear stochastic systems were obtained by solving a second-order nonlinear HJI. Generally, it is difficult to solve the HJI. In fact, for the special nonlinear stochastic system (4.1), the H_∞ and H_2/H_∞ filtering problems can be solved via the LMI technique instead of the HJI according to Theorems 3.2 and 3.6 in this paper. Simulation results show the effectiveness of the proposed method.

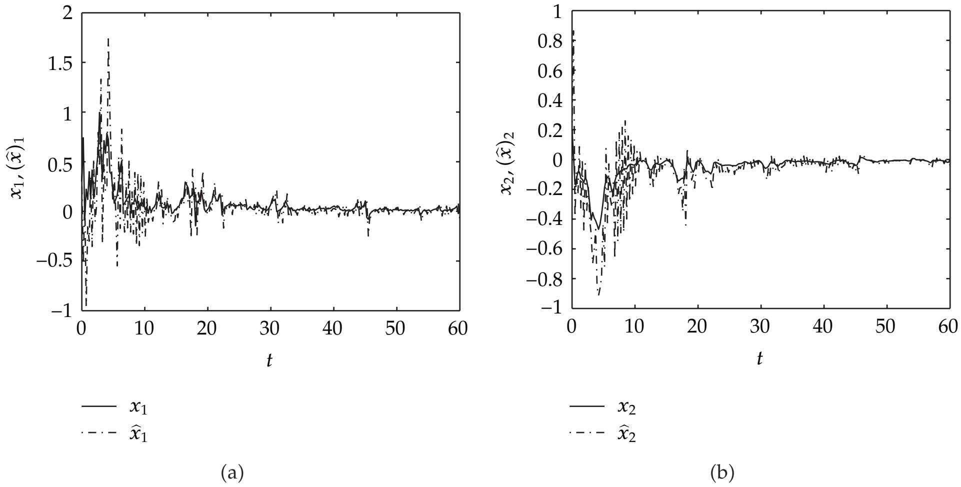

Figure 1: Trajectories of $x_1(t),\ \hat{x}_1(t)$ and $x_2(t),\ \hat{x}_2(t)$ for the proposed H_∞ filter.

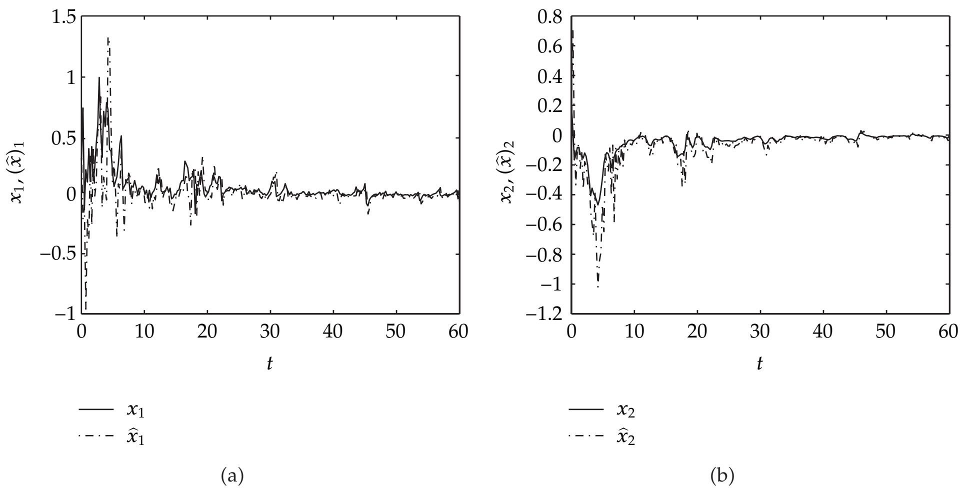

Figure 2: Trajectories of $x_1(t)$, $\hat{x}_1(t)$ and $x_2(t)$, $\hat{x}_2(t)$ for the proposed H_2/H_∞ filter.

5. Conclusions

In this paper, we have discussed the robust H_∞ filtering problem for a class of nonlinear stochastic systems. Meanwhile, the mixed H_2/H_∞ filtering analysis is also considered. Since the results can be solved by LMIs, the proposed method has much advantage in practical computation. Although we only demand the state equation to be nonlinear, one can tackle the case that when both the state and measurement equations are nonlinear.

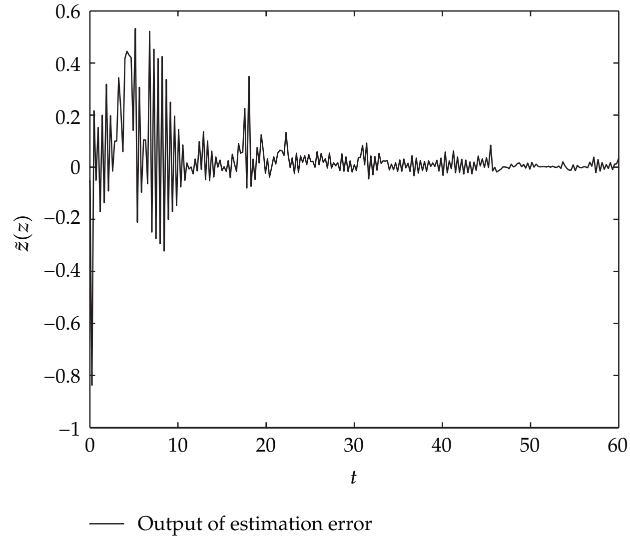

Figure 3: Trajectory of the estimation error $\tilde{z}(t)$ for the proposed H_∞ filter.

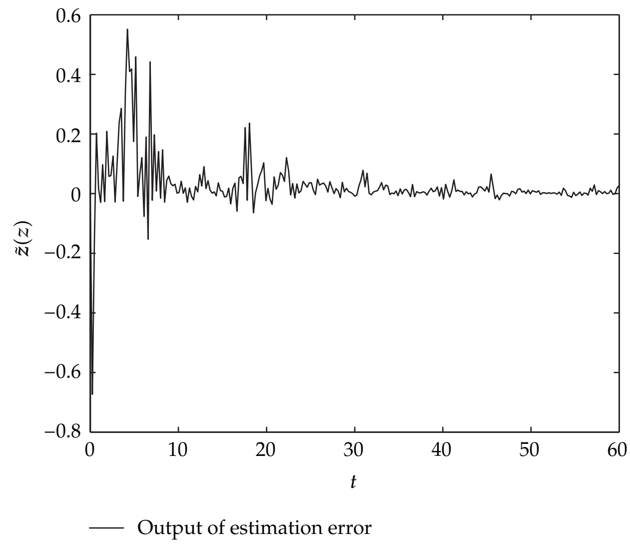

Figure 4: Trajectory of the estimation error $\tilde{z}(t)$ for the proposed H_2/H_∞ filter.

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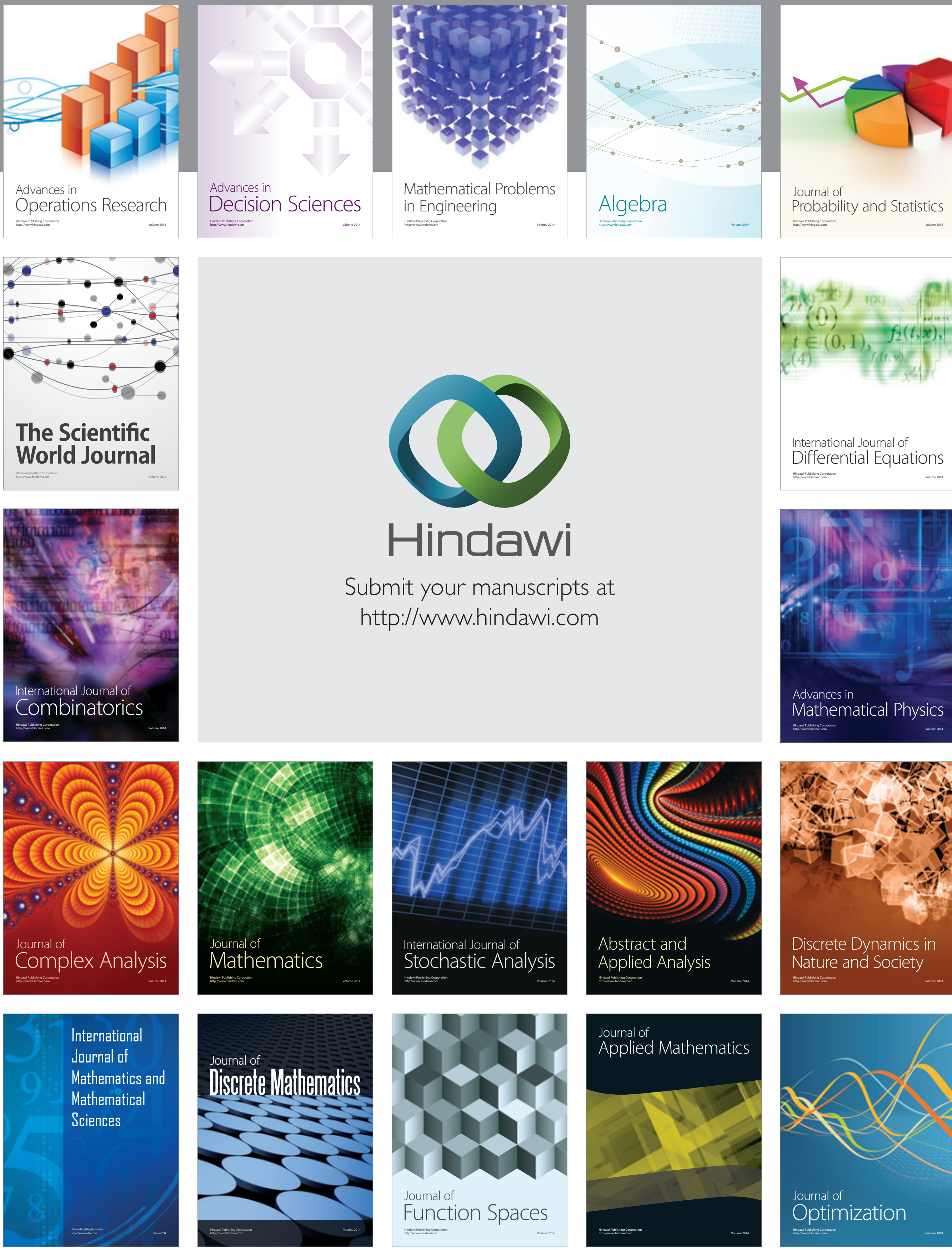
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