

Research Article

Nonlinear Observer Design of the Generalized Rössler Hyperchaotic Systems via DIL Methodology

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Received 29 February 2012; Accepted 13 April 2012

Academic Editor: Jun-Juh Yan

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The generalized Rössler hyperchaotic systems are presented, and the state observation problem of such systems is investigated. Based on the differential inequality with Lyapunov methodology (DIL methodology), a nonlinear observer design for the generalized Rössler hyperchaotic systems is developed to guarantee the global exponential stability of the resulting error system. Meanwhile, the guaranteed exponential decay rate can be accurately estimated. Finally, numerical simulations are provided to illustrate the feasibility and effectiveness of proposed approach.

1. Introduction

In recent decades, several kinds of chaotic systems have been widely explored; see, for instance, [1–11] and the references therein. This is due to theoretical interests as well as to an efficient tool for chaos synchronization and chaos control design. As a rule, chaos in many systems is a source of the generation of oscillation and a source of instability. Chaotic systems frequently exist in various fields of application, such as system identification, master-slave chaotic systems, secure communication, and ecological systems.

From practical considerations, it is either impossible or inappropriate to measure all the elements of the state vector. The state observer has come to take its pride of place in system identification, filter theory, and control design. As we know, the tasks of observer-based control systems (with or without chaos) can be divided into two categories: tracking (or synchronization) and observer-based stabilization (or regulation). The state observer can be skillfully applied in observer-based stabilization, synchronization of master-slave chaotic systems, and secure communication. For more detailed knowledge, one can refer to [1, 2, 7–9, 11–14]. However, the state observer design of dynamic systems with chaos is in general not as easy as that without chaos. Motivated by the above reasons, the observer design of chaotic

systems is actually crucial and meaningful. On the other hand, a variety of methods have been proposed for the observer design of systems, such as Chebyshev neural network (CNN), sliding-mode approach, passivation of error dynamics, separation principle, and frequency domain analysis; see, for instance, [15–20] and the references therein.

In this paper, the nonlinear state reconstructor of the generalized Rössler hyperchaotic systems is investigated. Using the DIL methodology, a nonlinear observer for such systems is provided to guarantee the global exponential stability of the resulting error system. Furthermore, the guaranteed exponential decay rate can be correctly estimated. Finally, numerical simulations are given to verify the effectiveness of proposed approach.

2. Problem Formulation and Main Result

In this paper, we consider the generalized Rössler hyperchaotic systems as follows:

$$\begin{aligned}
 \dot{x}_1(t) &= \alpha_1 x_1 + \alpha_2 x_2 + g_1(x_3, x_4), \\
 \dot{x}_2(t) &= \alpha_3 x_1 + \alpha_4 x_2 + g_2(x_4), \\
 \dot{x}_3(t) &= r_1 x_1 x_3 + g_3(x_4), \\
 \dot{x}_4(t) &= r_2 x_3 + g_4(x_4), \\
 y(t) &= x_4,
 \end{aligned} \tag{2.1}$$

where $x(t) := [x_1(t) \ x_2(t) \ x_3(t) \ x_4(t)]^T \in \mathfrak{R}^4$ is the state vector, $y(t) \in \mathfrak{R}$ is the system output, r_1, r_2 , and α_i , for all $i \in \{1, 2, 3, 4\}$ are the system parameters with $r_1 r_2 \neq 0$. For the existence and uniqueness of system (2.1), we assume that all the functions $g_i(\cdot)$, for all $i \in \{1, 2, 3, 4\}$, are sufficiently smooth.

The following assumption is made on system (2.1) throughout this paper.

(A1) There exists a constant h_1 such that

$$h_1 > \alpha_4, \quad h_1 \alpha_4 < -\alpha_2 \alpha_3. \tag{2.2}$$

Remark 2.1. It is noted that the Rössler hyperchaotic system [21] is the special cases of system (2.1) with

$$\begin{aligned}
 \alpha_1 &= 0, & \alpha_2 &= -1, & \alpha_3 &= 1, & \alpha_4 &= 0.25, & r_1 &= 1, & r_2 &= -0.5, \\
 g_1(x_3, x_4) &= -x_3, & g_2(x_4) &= x_4, & g_3(x_4) &= 3, & g_4(x_4) &= 0.05x_4.
 \end{aligned} \tag{2.3}$$

The objective of this paper is to search a nonlinear observer for system (2.1) such that the global exponential stability of the resulting error systems can be guaranteed. Before presenting the main result, let us introduce a definition which will be used in the main theorem.

Definition 2.2. System (2.1) is exponentially state reconstructible if there exist an observer $E\hat{\hat{x}}(t) = g(\hat{x}(t), y(t))$ and positive numbers k and α such that

$$\|e(t)\| := \|x(t) - \hat{x}(t)\| \leq k \exp(-\alpha t), \quad \forall t \geq 0, \quad (2.4)$$

where $\hat{x}(t)$ expresses the reconstructed state of system (2.1). In this case, the positive number α is called the exponential decay rate.

Now we present the main result for the state observer of system (2.1).

Theorem 2.3. *System (2.1) with (A1) is exponentially state reconstructible. Besides, a suitable nonlinear observer is given by*

$$\begin{aligned} r_1 \hat{x}_3 \dot{\hat{x}}_1 &= (\alpha_1 + h_1) [\dot{\hat{x}}_3 - g_3(y)] - h_1 r_1 \hat{x}_1 \hat{x}_3 + r_1 \hat{x}_3 [\alpha_2 \hat{x}_2 + g_1(\hat{x}_3, y)], \\ \dot{\hat{x}}_2 &= \alpha_3 \hat{x}_1 + \alpha_4 \hat{x}_2 + g_2(y), \\ \dot{\hat{x}}_3 &= \frac{1}{r_2} [\dot{y} - g_4(y)], \\ \dot{\hat{x}}_4 &= y. \end{aligned} \quad (2.5)$$

In this case, the guaranteed exponential decay rate is given by $\alpha := 1/\lambda_{\max}(P)$, where $P > 0$ is the unique solution to the following Lyapunov equation:

$$\begin{bmatrix} -h_1 & \alpha_2 \\ \alpha_3 & \alpha_4 \end{bmatrix}^T P + P \begin{bmatrix} -h_1 & \alpha_2 \\ \alpha_3 & \alpha_4 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}. \quad (2.6)$$

Proof. From (2.1), (2.5) with

$$e_i(t) := x_i(t) - \hat{x}_i(t), \quad \forall i \in \{1, 2, 3, 4\}, \quad (2.7)$$

it can be readily obtained that

$$\begin{aligned} e_4(t) &= x_4(t) - \hat{x}_4(t) = 0, \quad \forall t \geq 0, \\ e_3(t) &= x_3(t) - \hat{x}_3(t) \\ &= x_3(t) - \frac{1}{r_2} [\dot{y} - g_4(y)] \\ &= x_3(t) - \frac{1}{r_2} [\dot{x}_4 - g_4(x_4)] \\ &= x_3(t) - \frac{1}{r_2} [r_2 x_3(t) + g_4(x_4) - g_4(x_4)] \\ &= 0, \quad \forall t \geq 0, \\ \dot{e}_2(t) &= \dot{x}_2(t) - \dot{\hat{x}}_2(t) \\ &= \alpha_3 x_1 + \alpha_4 x_2 + g_2(x_4) - \alpha_3 \hat{x}_1 - \alpha_4 \hat{x}_2 - g_2(\hat{x}_4) \\ &= \alpha_3 (x_1 - \hat{x}_1) + \alpha_4 (x_2 - \hat{x}_2) + [g_2(x_4) - g_2(\hat{x}_4)] \end{aligned}$$

$$\begin{aligned}
&= \alpha_3 e_1 + \alpha_4 e_2 + [g_2(y) - g_2(y)] \\
&= \alpha_3 e_1(t) + \alpha_4 e_2(t), \quad \forall t \geq 0, \\
\dot{e}_1(t) &= \dot{x}_1(t) - \dot{\hat{x}}_1(t) \\
&= \alpha_1 x_1 + \alpha_2 x_2 + g_1(x_3, x_4) - \frac{(\alpha_1 + h_1)[\dot{\hat{x}}_3 - g_3(y)]}{r_1 \hat{x}_3} + h_1 \hat{x}_1 - \alpha_2 \hat{x}_2 - g_1(\hat{x}_3, y) \\
&= \alpha_1 x_1 + \alpha_2 e_2 + g_1(x_3, x_4) - \frac{(\alpha_1 + h_1)[\dot{x}_3 - g_3(x_4)]}{r_1 x_3} + h_1 \hat{x}_1 - g_1(x_3, x_4) \\
&= \alpha_1 x_1 + \alpha_2 e_2 - \frac{(\alpha_1 + h_1)[g_3(x_4) + r_1 x_1 x_3 - g_3(x_4)]}{r_1 x_3} + h_1 \hat{x}_1 \\
&= \alpha_1 x_1 + \alpha_2 e_2 - (\alpha_1 + h_1)x_1 + h_1 \hat{x}_1 \\
&= h_1 e_1(t) + \alpha_2 e_2(t), \quad \forall t \geq 0.
\end{aligned} \tag{2.8}$$

This implies that

$$\begin{bmatrix} \dot{e}_1(t) \\ \dot{e}_2(t) \end{bmatrix} = \begin{bmatrix} -h_1 & \alpha_2 \\ \alpha_3 & \alpha_4 \end{bmatrix} \begin{bmatrix} e_1(t) \\ e_2(t) \end{bmatrix}, \quad \begin{bmatrix} e_3(t) \\ e_4(t) \end{bmatrix} = 0, \quad \forall t \geq 0, \tag{2.9}$$

with $\sigma\left(\begin{bmatrix} -h_1 & \alpha_2 \\ \alpha_3 & \alpha_4 \end{bmatrix}\right) \subseteq C^-$, in view of (A1):

Let

$$W(t) := \begin{bmatrix} e_1(t) & e_2(t) \end{bmatrix} P \begin{bmatrix} e_1(t) \\ e_2(t) \end{bmatrix}. \tag{2.10}$$

The time derivative of $W(t)$ along the trajectories of dynamical error system, with (2.9), (2.10), and (2.6), is given by

$$\begin{aligned}
\dot{W}(t) &= \begin{bmatrix} \dot{e}_1 & \dot{e}_2 \end{bmatrix} P \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} + \begin{bmatrix} e_1 & e_2 \end{bmatrix} P \begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \end{bmatrix} \\
&= \begin{bmatrix} e_1 & e_2 \end{bmatrix} \begin{bmatrix} -h_1 & \alpha_2 \\ \alpha_3 & \alpha_4 \end{bmatrix}^T P \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} + \begin{bmatrix} e_1 & e_2 \end{bmatrix} P \begin{bmatrix} -h_1 & \alpha_2 \\ \alpha_3 & \alpha_4 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} \\
&= \begin{bmatrix} e_1 & e_2 \end{bmatrix} \left\{ \begin{bmatrix} -h_1 & \alpha_2 \\ \alpha_3 & \alpha_4 \end{bmatrix}^T P + P \begin{bmatrix} -h_1 & \alpha_2 \\ \alpha_3 & \alpha_4 \end{bmatrix} \right\} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} \\
&= -2 \begin{bmatrix} e_1 & e_2 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} \\
&\leq -\frac{2}{\lambda_{\max}(P)} W(t) \\
&= -2\alpha W(t), \quad \forall t \geq 0.
\end{aligned} \tag{2.11}$$

Thus, one has

$$e^{2\alpha t} \cdot W + e^{2\alpha t} \cdot 2\alpha W = \frac{d}{dt} [e^{2\alpha t} \cdot W] \leq 0, \quad \forall t \geq 0. \quad (2.12)$$

It follows that

$$\int_0^t \frac{d}{d\tau} [e^{2\alpha\tau} \cdot W(\tau)] d\tau = e^{2\alpha t} \cdot W(t) - W(0) \leq \int_0^t 0 d\tau = 0. \quad (2.13)$$

Consequently, we conclude that

$$\begin{aligned} \|e(t)\| &= \sqrt{e_1^2(t) + e_2^2(t) + e_3^2(t) + e_4^2(t)} \\ &= \sqrt{e_1^2(t) + e_2^2(t)} \\ &\leq \sqrt{\frac{W(t)}{\lambda_{\min}(P)}} \\ &\leq \sqrt{\frac{e^{-2\alpha t} W(0)}{\lambda_{\min}(P)}} \\ &= \sqrt{\frac{W(0)}{\lambda_{\min}(P)}} \cdot e^{-\alpha t}, \quad \forall t \geq 0, \end{aligned} \quad (2.14)$$

in view of (2.8), (2.10), and (2.13). This completes the proof. \square

3. Numerical Simulations

Consider the generalized hyperchaotic system:

$$\begin{aligned} \dot{x}_1(t) &= -x_2 - x_3, \\ \dot{x}_2(t) &= ax_1 + bx_2 + x_4, \\ \dot{x}_3(t) &= x_1x_3 + 3, \\ \dot{x}_4(t) &= -0.5x_3 + 0.05x_4, \\ y(t) &= x_4. \end{aligned} \quad (3.1)$$

Case 1 ($a = 1$, $b = 0.25$) or, equivalently, the Rössler hyperchaotic system). It can be verified that condition (A1) is satisfied with $h_1 = 1.2$. By Theorem 2.3, we conclude that system (3.1) with $a = 1$ and $b = 0.25$ is exponentially state reconstructible by the nonlinear observer:

$$\begin{aligned}\hat{x}_3\hat{x}_1 &= 1.2(\hat{x}_3 - 3) - 1.2\hat{x}_1\hat{x}_3 - \hat{x}_2\hat{x}_3 - \hat{x}_3^2, \\ \dot{\hat{x}}_2 &= \hat{x}_1 + 0.25\hat{x}_2 + y, \\ \hat{x}_3 &= 0.1y - 2\dot{y}, \\ \hat{x}_4 &= y,\end{aligned}\tag{3.2}$$

with the guaranteed exponential decay rate $\alpha = 0.164$.

Case 2 ($a = -20$, $b = -50$). It can be verified that condition (A1) is satisfied with $h_1 = 10$. By Theorem 2.3, we conclude that system (3.1) with $a = -20$ and $b = -50$ is exponentially state reconstructible by the nonlinear observer:

$$\begin{aligned}\hat{x}_3\hat{x}_1 &= 10(\hat{x}_3 - 3) - 10\hat{x}_1\hat{x}_3 - \hat{x}_2\hat{x}_3 - \hat{x}_3^2, \\ \dot{\hat{x}}_2 &= -20\hat{x}_1 - 50\hat{x}_2 + y, \\ \hat{x}_3 &= 0.1y - 2\dot{y}, \\ \hat{x}_4 &= y,\end{aligned}\tag{3.3}$$

with the guaranteed exponential decay rate $\alpha = 8.47$.

Case 3 ($a = 30$, $b = -40$). It can be verified that condition (A1) is satisfied with $h_1 = 5$. By Theorem 2.3, we conclude that system (3.1) with $a = 30$ and $b = -40$ is exponentially state reconstructible by the nonlinear observer:

$$\begin{aligned}\hat{x}_3\hat{x}_1 &= 5(\hat{x}_3 - 3) - 5\hat{x}_1\hat{x}_3 - \hat{x}_2\hat{x}_3 - \hat{x}_3^2, \\ \dot{\hat{x}}_2 &= 30\hat{x}_1 - 40\hat{x}_2 + y, \\ \hat{x}_3 &= 0.1y - 2\dot{y}, \\ \hat{x}_4 &= y,\end{aligned}\tag{3.4}$$

with the guaranteed exponential decay rate $\alpha = 3.79$.

The time response of error states for system (3.1) with Case 1–Case 3 is depicted in Figures 1, 2, and 3, respectively. From the foregoing simulations results, it is seen that system (3.1) with Case 1–Case 3, regardless of chaotic system or nonchaotic system, is exponentially state reconstructible by the nonlinear observers (3.2)–(3.4), respectively.

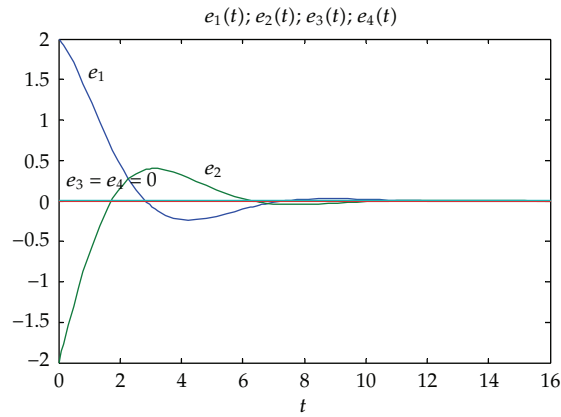


Figure 1: The time response of error states, with $a = 1$ and $b = 0.25$.

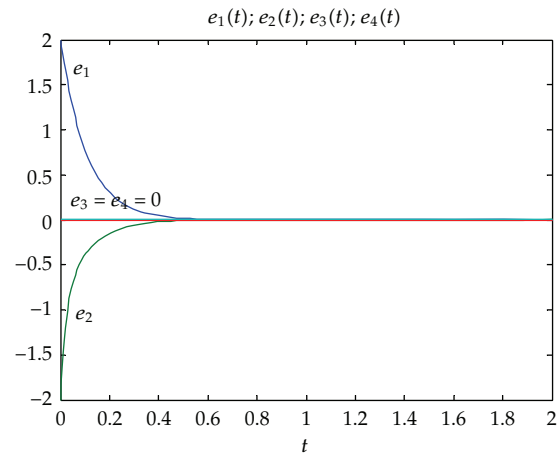


Figure 2: The time response of error states, with $a = -20$ and $b = -50$.

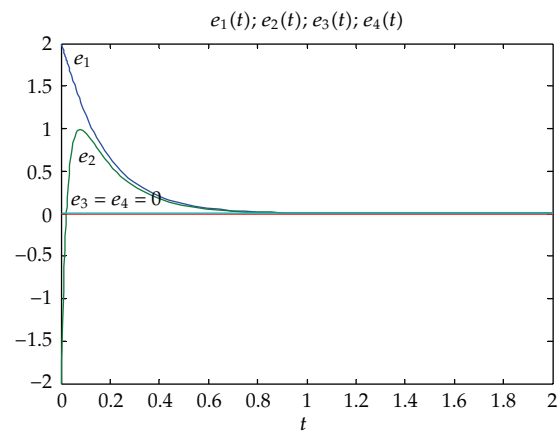


Figure 3: The time response of error states, with $a = 30$ and $b = -40$.

4. Conclusion

In this paper, the generalized Rössler hyperchaotic systems have been presented, and the state observation problem of such systems has been investigated. Based on the DIL methodology, a nonlinear state reconstructor of the generalized Rössler hyperchaotic systems has been developed to guarantee the global exponential stability of the resulting error system. Besides, the guaranteed exponential decay rate can be accurately estimated. However, the state observation design for more general uncertain hyperchaotic system still remains unanswered. This constitutes an interesting future research problem.

Nomenclature

\mathcal{R}^n :	The n -dimensional real space
C^- :	The set of $\{a + bj \mid a < 0, b \in \mathcal{R}\}$
$ a $:	The modulus of a real number a
$\ x\ $:	The Euclidean norm of the vector $x \in \mathcal{R}^n$
$\ A\ $:	The induced Euclidean norm of the matrix A
A^T :	The transpose of the matrix A
$\sigma(A)$:	The set of all eigenvalues of the matrix A
$P > 0$:	The symmetric matrix P is positive definite
$\lambda_{\max}(P)$:	The maximum eigenvalue of the symmetric matrix P with real eigenvalues
$\lambda_{\min}(P)$:	The minimum eigenvalue of the symmetric matrix P with real eigenvalues.

Acknowledgments

The author thanks the National Science Council of Republic of China for supporting this work under Grant NSC-100-2221-E-214-015. The author also wishes to thank the anonymous reviewers for providing constructive suggestions.

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