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Research Article

Advanced Signal Processing and Command Synthesis for Memory-Limited Complex Systems

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This paper presents advanced signal processing methods and command synthesis for memory-limited complex systems. For accurate measurements performed on limited time interval, some specific methods should be added. For signal processing, a robust filtering and sampling procedure performed on a specific working interval is required, so as the influence of low-amplitude and high-frequency fluctuations to be diminished. This study shows that such a signal processing method for the case of memory-limited complex systems requires the use of certain differentiation/integration procedures performed by oscillating systems, so as robust results suitable for efficient command synthesis to be available. A brief comparison with uncertainty aspects in modern physics (where quantum aspects can be considered as features of complex systems) is also presented.

1. Introduction

As it is known, an important aspect in observing and modeling dynamic environmental phenomena consists in measuring with higher accuracy some physical quantities corresponding to changes in the environment. Yet for accurate measurements performed on limited time interval, for memory-limited complex systems, some specific methods should be used. Sudden (sharp) changes in the environment require a pair of consecutive values for the measured quantity so as any difference to be detected as soon as possible. Moreover, any value taken into consideration by the complex system should be established using a robust filtering and sampling procedure performed on a specific working interval, so as the influence of low-amplitude and high-frequency fluctuations to be decreased in a significant manner. Being quite possible for sharp (sudden) changes in the environment to appear during such a working interval (on which filtering and sampling procedures are performed), it results that specific signal processing methods based on the values achieved on a set of succesive time intervals are necessary.

Filtering and sampling devices usually consist of asymptotically stable systems, sometimes an integration of the output over a certain time interval being added. Yet such structures are very sensitive at random variations of the integration period, being recommended for the signal which is integrated to be approximately equal to zero at the end of the integration period. For this reason, oscillating systems for filtering the received signal should be used, so as the filtered signal and its slope to be approximately zero at the end of a certain time interval (at the end of an oscillation). For avoiding instability of such oscillating systems on extended time intervals, certain electronic devices (gates) controlled by computer commands should be added, so as to restore the initial null conditions for the oscillating system before a new working cycle to start [1].

The filtering performances of asymptotically stable systems are determined by their transfer function. a Filtering and sampling devices consisting of low-pass filters of first or second order having the transfer function

$$H(s) = \frac{1}{T_0 s + 1} \tag{1.1}$$

(for a first-order system) and

$$H(s) = \frac{1}{T_0^2 s^2 + 2bT_0 s + 1} \tag{1.2}$$

(for a second-order system) attenuate an alternating signal of angular frequency $\omega \gg \omega_0 = 1/T_0$ about ω/ω_0 times (for a first-order system) or about $(\omega/\omega_0)^2$ times (for a second-order system). The response time of such systems at a continuous useful signal is about $4-6T_0$ ($5T_0$ for the first-order system and $4T_0/b$ for the second-order system). If the signal given by the first- or second-order system is integrated over such a period, a supplementary attenuation for the alternating signal of about $4-6\omega/\omega_0$ can be obtained.

But such structures are very sensitive at the random variations of the integration period (for unity-step input, the signal which is integrated is equal to unity at the sampling moment of time), and the use of oscillators with a very high accuracy cannot solve the problem due to switching phenomena appearing at the end of the integration period (when an electric current charging a capacitor is interrupted).

These random variations cannot be avoided if we use asymptotically stable filters. For robustness, the signal processing structure based on an integration procedure should provide a null value for the integrating signal at the end of a certain working interval. This property is similar to wavelets aspects presented in [2, 3].

Mathematically, an ideal solution could consist in using an extended Dirac function for multiplying the received signal before the integration (see [1]) but is very hard to generate such extended Dirac functions (a kind of acausal pulses) using nonlinear differential equations for (i) symmetrical pulses (see [4]) or (ii) asymmetrical pulses (see [5] for more details).

A heuristic algorithm for generating practical test functions using MATLAB procedures was presented in [4]. First, it has been shown that ideal test functions cannot be generated by differential equations, being emphasized the fact that differential equations can only generate functions similar to test functions (defined as practical test functions). Then a step-by-step algorithm for designing the most simple differential equation able to generate a practical test function was presented, based on the invariance properties of the differential equation and on standard MATLAB procedures. The result of the algorithm consists in an oscillating second-order system working at the stability limit from initial null conditions, on a limited working interval corresponding to the period of the generated oscillations.

It was shown that the simplest structure possessing such properties is represented by an oscillating second-order system having the transfer function

$$H_{\text{osc}} = \frac{1}{T_0^2 s^2 + 1}, \tag{1.3}$$

receiving a step input and working on the time interval $[0, 2\pi T_0]$. For initial conditions equal to zero, the response of the oscillating system at a step input with amplitude A will have the form

$$y(t) = A\left(1 - \cos\left(\frac{t}{T_0}\right)\right). \tag{1.4}$$

By integrating this result on the time interval $[0, 2\pi T_0]$, we obtain the result $2\pi A T_0$, and we can also notice that the quantity which is integrated and its slope are equal to zero at the end of the integration period. Thus, the influence of the random variations of the integration period (generated by the switching phenomena) is practically rejected.

This oscillating system attenuates about $(\omega/\omega_0)^2$ times such an input, and the influence of the integrator consists in a supplementary attenuation of about

$$\left[\frac{1}{(2\pi)}\left(\frac{\omega}{\omega_0}\right)\right] \tag{1.5}$$

times. The oscillations having the form

$$y_{\text{osc}} = a\sin(\omega_0 t) + b\cos(\omega_0 t) \tag{1.6}$$

generated by the input alternating component have a lower amplitude and give a null result after an integration over the time interval $[0, 2\pi T_0]$.

These results have shown that such a structure provides practically the same performances as a structure consisting of an asymptotically stable second-order system and an integrator (response time of about $6T_0$, an attenuation of about $(1/6)(\omega/\omega_0)^3$ times for an alternating component having frequency ω) moreover being less sensitive at the random variations of the integration period. For restoring the initial null conditions after the sampling procedure (at the end of the working period), some electronic devices must be added. Yet the previous analysis is valid for step inputs which are active on the whole working interval (the integration period).

In [6] has been performed the analysis of this structure by considering that the input is represented by a unity short-step pulse (instead of a unity step pulse) which differs to zero on the time interval $[0, \tau]$. It was shown that certain free oscillations of the second-order oscillating system are generated for $t > \tau$ (when the action of the external short-step command u has ceased). These free oscillations have the angular velocity ω_0, the amplitude

$$A = 2\sin\left(\frac{\omega_0 \tau}{2}\right), \tag{1.7}$$

and the initial phase

$$\phi = -\frac{\omega_0 \tau}{2}. \tag{1.8}$$

Thus, the output $y(t)$ corresponding to the free oscillations of the system for $t > \tau$ (when the action of the external short-step command u has ceased) can be written as

$$y(t) = 2\sin\left(\frac{\omega_0 \tau}{2}\right)\sin\left(\omega_0 t - \frac{\omega_0 \tau}{2}\right). \tag{1.9}$$

However, we must notice that, usually, such a filtering and sampling structure receives an electronic signal presenting possible step changes from an already measured value to a final unknown value. Since the previously measured value can be substracted from the received signal during subsequent working intervals, the analysis of sudden (sharp) changes in the environment could start by considering that the input of the second-order oscillating system is represented by a null signal for $t \leq \tau$ (the first part of the working interval) and by a signal with amplitude A for $t > \tau$ (the second part of the working interval).

2. The Oscillating Signal Processing System for the Case of Short-Step Inputs

We will continue the analysis of this structure by considering that the input is represented by a short-step pulse which differs to zero on the time interval $[\tau, 2\pi T_0]$. This means that the input u can be represented under the form

$$\begin{aligned} u(t) &= 0, \quad \text{for } t < \tau, \\ u(t) &= A, \quad \text{for } t \in [\tau, 2\pi T_0], \end{aligned} \tag{2.1}$$

or using the Heaviside function

$$u(t) = Ah(t-\tau) \quad \text{for } t \in [0, \infty), \tag{2.2}$$

where $h(t)$ corresponds to the function $1/s$ if we apply the Laplace transformation.

The transfer function of the second-order oscillating system is

$$H(s) = \frac{1}{T_0^2 s^2 + 1}. \tag{2.3}$$

On the time interval $[0, \tau]$, the output of the second-order oscillating system equals zero

$$y(t) = 0. \tag{2.4}$$

On the time interval $[\tau, 2\pi T_0]$, the output of the second-order oscillating system is represented (using the Laplace transformation) as

$$y(s) = H(s)u(s) = \frac{1}{T_0^2 s^2 + 1}\frac{A}{s}\exp(-\tau s), \tag{2.5}$$

which corresponds to the output

$$y(t) = A\left(1 - \cos\left(\frac{t-\tau}{T_0}\right)\right), \tag{2.6}$$

which can be written as

$$y(t) = A[1 - \cos(\omega_0(t-\tau))], \tag{2.7}$$

where $\omega_0 = 1/T_0$.

By denoting with $z(t)$ the integral of $y(t)$ (considering as initial moment the zero moment of time), it results at the time moment $t_f = 2\pi T_0$ the set of values:

$$\begin{aligned} y(t_f) &= A[1-\cos(\omega_0(2\pi T_0 - \tau))] = A[1-\cos(\omega_0\tau)], \\ y'(t_f) &= A\omega_0 \sin(\omega_0(2\pi T_0 - \tau)) = -A\omega_0\sin(\omega_0\tau), \\ z(t_f) &= \int_\tau^{2\pi T_0} A[1-\cos(\omega_0(t-\tau))]dt = A\left(\frac{2\pi}{\omega_0} - \tau\right) + \frac{A}{\omega_0}\sin(\omega_0\tau). \end{aligned} \tag{2.8}$$

It can be easily noticed that

$$\omega_0 z(t_f) + \frac{1}{\omega_0}y'(t_f) = A(2\pi - \omega_0\tau), \tag{2.9}$$

which can be written also as

$$z(t_f) + \frac{1}{\omega_0^2}y'(t_f) = A(2\pi T_0 - \tau). \tag{2.10}$$

This result shows that the sampled values for $z(t)$ (the integral of $y(t)$) and for $y'(t)$ at the time moment $t_f = 2\pi T_0$ (the end of the working interval) can be used in a simple manner for obtaining the quantity

$$S(t_f) = A(2\pi T_0 - \tau) = At_A. \tag{2.11}$$

For this purpose, we can divide the sampled value for $y'(t)$ by $1/\omega_0^2$ and add this result to the sampled value for $z(t)$ (the integral of $y(t)$). All these operations can be performed electronically (using analog devices) in an accurate manner. It can be easily noticed that the quantity

$$t_A = 2\pi T_0 - \tau \tag{2.12}$$

represents the active time (on which the step input A acts upon the second-order oscillating system).

On the subsequent working interval, we can consider that the input of the second-order oscillating system equals A on the whole time interval $[0, 2\pi T_0]$. As a consequence, the integral of the generated output equals

$$z_{\text{next}} = 2\pi T_0 A, \tag{2.13}$$

which allows a robust estimation of the amplitude A of the step change for the input as

$$A = \frac{z_{\text{next}}}{2\pi T_0}, \tag{2.14}$$

where z_{next} is the sampled quantity for the integral over a period for the oscillating system output (starting to work from initial null conditions on the next working interval), and $2\pi T_0$ is a constant value. This operation can be also performed electronically in an easy manner.

However, this result is far of being useful for practical applications. Since the differential equation of the second-order oscillating system is

$$y(t) + \left(\frac{1}{\omega_0^2}\right) y''(t) = u(t), \tag{2.15}$$

it results that

$$S(t_f) = \int_0^{t_f} \left[y(t) + \left(\frac{1}{\omega_0^2}\right) y''(t) \right] dt = \int_0^{t_f} u(t) dt, \tag{2.16}$$

where $u(t)$ represents the amplitude of the received signal. So the algorithm previously presented performs the integral of the received signal on a period, without any filtering procedure. The influence of low-amplitude high-frequency alternating components of the received signal is not diminished in a significant manner, the advantages of a filtering procedure based on second-order systems being lost.

The previously presented algorithm could be accepted if the integral can be performed on an extended time interval. In this case, we can simply estimate the quantity $S(t_f)$ on a time period T_F several times greater that $2\pi T_0$ (the period of the oscillating system) so as requirements regarding filtering performances (for rejecting the influence of low-frequency high-amplitude components) to be fulfilled. Thus, the influence of an alternating component with time constant T and angular frequency $\omega = 2\pi/T$ is decreased about $\omega T_F = T_F/T$ times.

The sampling procedure is not robust any more (the signal which is integrated differs to zero at the end of the integration interval, in case of step changes) but for extended intervals, the relative error generated by switching phenomena can be neglected (the switching interval is very narrow as related to the integration interval, so the integral of stochastic switching phenomena represents a low value as related to the integral performed on the whole working interval).

3. Synthesis of Quick Command for Compensating Dynamic Environmental Changes

In case of electric drives, an extended integration interval for quantity $u(t)$ (as presented in the second section of this paper) could be allowed if it corresponds not to the controlled quantity (the angular frequency Ω of a shaft, for example) but to its derivative (supposing that the resistive torque can vary). As a consequence, $u(t)$ can be written as

$$u(t) = c\frac{d\Omega}{dt} \tag{3.1}$$

(c being a constant). It results that

$$S(t_f) = \int_0^{t_f} u(t)dt = \int_0^{t_f} c\frac{d\Omega}{dt}dt = c(\Omega(t_f) - \Omega(0)), \tag{3.2}$$

so it corresponds to the variation of the angular frequency Ω on the time interval T_F which should be adjusted by a supplementary active torque M_s represented by

$$M_s = M_a - M_r, \tag{3.3}$$

where M_a corresponds to the active torque and M_r corresponds to the resistive torque. The active and resistive torque should be equal in the stationary regime, so the supplementary torque should act on a limited time interval, so as to compensate the difference $\Omega(t_f) - \Omega(0)$ previously detected.

For this purpose, we could notice that quantity $S(t_f)$ is proportional to this difference, so a supplementary active torque can be transmitted in a limited time interval $(t_f, t_f + T_s)$ as

$$M_s(t) = b\left(1 - \cos\left(2\pi\frac{t - t_f}{T_s}\right)\right)S(t_f) \tag{3.4}$$

(b being a constant). When $t = t_f + T_s$, both function $M_s(t)$ and its derivative $C'(t)$ have null values, so the action of the supplementary torque can be stopped avoiding the influence of any switching phenomena (which can generate errors).

Since

$$M_s = M_a - M_r = c\frac{d\Omega}{dt}, \tag{3.5}$$

it results that the action of this supplementary torque on this time interval T_s can be represented as

$$\Omega(t_f + T_s) - \Omega(t_f) = \frac{1}{c}\int_{t_f}^{t_f+T_s} M_s(t)dt = \frac{b}{c}\int_{t_f}^{t_f+T_s}\left(1 - \cos\left(2\pi\frac{t-t_f}{T_s}\right)\right)S(t_f)dt. \quad (3.6)$$

While $S(t_f)$ is proportional to the difference $\Omega(t_f)-\Omega(0)$ (as has been shown), it finally results that

$$\Omega(t_f + T_s) - \Omega(t_f) = bT_s(\Omega(t_f) - \Omega(0)). \quad (3.7)$$

By adjusting the relation between b and T_s, we can set quantity bT_s to unity, so as the action of the step change in the environment upon the angular velocity to be compensated in a subsequent finite time interval.

This intuitive model is also valid for any complex (biological) system which should maintain its position or the velocity of certain components at a specific value.

4. Efficient Signal Processing Methods Based on Two State Variables

The signal processing method presented in previous paragraph is based on sampled values for three successive working intervals of the second-order oscillating system. Considering that a step change for the input is detected on a certain working interval, the previous value for the input is determined on an initial working interval (on which the input equals a certain value A_{in}), and the final value for the input is determined on a final working interval (when the input equals a final value A_{next}). During the middle working interval, the quantity A_{in} is substracted from the received signal, and the quantity

$$At_A = (A_{\text{next}} - A_{\text{in}})(2\pi T_0 - \tau) = z(t_f) + \frac{1}{\omega_0^2}y'(t_f) \quad (4.1)$$

is available at the end of the interval (using amplifying and sampling procedures). Finally, the step change $A = A_{\text{next}} - A_{\text{in}}$ and the time moment τ are determined (using substracting and dividing procedures).

However, the linearity of this second-order oscillating system allows a more efficient and robust procedure to be used. The algorithm presented in the previous paragraph requires two identical oscillating second-order systems working at the same time: one for processing the input (so as to determine the estimated values for an input considered to be constant on that interval) and another for processing the difference between the received signal and the previously sampled value (so as to detect a possible step change during this interval by determining the quantity At_A, where A stands for the step change and t_A stands for the active time). Moreover, filtering aspects could require extended time intervals for processing the input signal (as was shown), and the absence of any control action during such an interval could allow significant changes for the output of the complex system from the desired value.

In previous paragraph, it was shown that filtering properties and robustness require an extended time interval for processing the input signal (received from transducers). For this purpose, we can use either a second-order oscillating system with a period equal to the working interval (which means $2\pi T_0 = T_F$), or a second-order oscillating system with a period corresponding to a submultiple of the working interval (which means $2\pi T_0 = T_F/N$). The last choice is far more convenient, as it will be shown in this paragraph. Let us suppose that on the whole working interval the input $u(t)$ is represented just by the constant value A_{in}. In this case, the output $y(t)$ is represented by $A_{\text{in}}(1 - \cos(\omega_0 t))$. Supposing that quantity A_{in} was determined on previous working interval, we can determine in a very simple manner the expected values $y_{\text{ex}(k)}$ which would be sampled for $y(t)$ when phase ϕ equals $k\pi$ (this means $0, 2A_{\text{in}}, 0, -2A_{\text{in}}, \ldots$, the effect of high-frequency low-amplitude fluctuations being neglected). At these time moments quantity $y'(t)$ would be zero, so the sampling procedure would be robust as related to the constant input A_{in}.

This suggests the possibility of sampling $y(t)$ at these time moments (this means when $\omega_0 t$ equals $k\pi$) and subtracting the expected values previously presented. Due to the linearity of the second-order oscillating system, if a step change with amplitude A is detected during the working interval, the sampled values $y_{(k)}$ at the moments of time $t_k = k\pi/\omega_0$ will be represented by a sum of values determined by step input A_{in} and by a step change A starting to act at moment τ. Thus, the result is

$$y_{A(k)} = y_{(k)} - y_{\text{ex}(k)}. \tag{4.2}$$

Thus, the successive values $y_{A(k)}$ of the subtracting procedure will correspond just to the influence of the step change A.

A quick analysis for first pair of values for $y_{A(1)}$ and $y_{A(2)}$ after the step change starts to act (when significant differences from expected values are detected) can be performed by comparing quantities

$$\begin{aligned} y_{A(1)} &= A[1 - \cos(\omega_0(t_1 - \tau))] = A[1 - \cos\phi_1] = 2A\sin^2\frac{\phi_1}{2}, \\ y_{A(2)} &= A[1 - \cos(\omega_0(t_2 - \tau))] = A[1 - \cos\phi_2] = 2A\sin^2\frac{\phi_2}{2}, \end{aligned} \tag{4.3}$$

where

$$\begin{aligned} \phi_1 &= \omega_0(t_1 - \tau), \\ \phi_2 &= \omega_0(t_2 - \tau). \end{aligned} \tag{4.4}$$

The phase difference between ϕ_1 and ϕ_2 equals π (as it was shown). It results that

$$y_{A(1)} = 2A\sin^2\frac{\phi_1}{2}, \qquad y_{A(2)} = 2A\sin^2\frac{\phi_1 + \pi}{2} = 2A\cos^2\frac{\phi_1}{2}. \tag{4.5}$$

It can be easily noticed that the ratio $y_{A(1)}/y_{A(2)}$ can be written as

$$\frac{y_{A(1)}}{y_{A(2)}} = \tan^2\frac{\phi_1}{2}. \tag{4.6}$$

A quick comparison of $y_{A(1)}$ and $y_{A(2)}$ is useful for an approximation of ϕ_1. For high values for $y_{A(1)}$ and low values for $y_{A(2)}$, $\phi_1/2$ can be approximated as $\pi/2$, and $2A$ can be approximated as $y_{A(1)}$. For low values for $y_{A(1)}$ and high values for $y_{A(2)}$, $\phi_1/2$ can be approximated as 0 and $2A$ can be approximated as $y_{A(2)}$. For similar values for $y_{A(1)}$ and $y_{A(2)}$, $\phi_1/2$ can be approximated as $\pi/4$, and $2A$ can be approximated as $y_{A(1)}/2 = y_{A(2)}/2$. Using ϕ_1, τ can be determined as

$$\tau = \omega_0\left(t_1 - \phi_1\right), \tag{4.7}$$

where t_1 stands for the first time moment in the set determined by $\omega_0 t_k = k\pi, k \in Z$, for which significant differences from expected values $y_{\mathrm{ex}(k)}$ are detected for sampled values y_k. This algorithm is efficient because it allows a preliminary command to be transmitted to the controlled system before the working interval (on which the received signal is integrated and sampled) to come to an end. It is more effective if the working interval includes several periods of the second-order oscillating system (thus, more moments for estimative sampling are established inside the working interval, and the preliminary command is transmitted faster).

5. Aspects Connected with the Uncertainty Principle

In the third section of this paper has been presented an algorithm for a preliminary estimation of a step change in the environment based on two state variables (two successive values for a $\sin^2\phi$ function sampled at two successive time moments) with the phase difference corresponding to these time moments determined as

$$\Delta\phi = \phi_2 - \phi_1 = \frac{\pi}{2}. \tag{5.1}$$

It can be noticed that measurements performed for sinusoidal functions at time moments when the phase difference equals $\pi/2$ imply some aspects regarding opposite requirements for sampling moments of time. Let us suppose that we are sampling just a sine function for phase

$$\begin{aligned} \phi_1 &= \psi, \\ \phi_2 &= \psi + \frac{\pi}{2}. \end{aligned} \tag{5.2}$$

It results that sampled values correspond to the pair

$$\sin\psi, \quad \cos\psi. \tag{5.3}$$

This pair can be considered also as a value for a sine function and a value for its derivative for the same phase ψ. By taking into account the fact that any sampling procedure requires a nonzero time interval, it results that it is desirable for these functions $\sin\psi, \cos\psi$ to be almost constant on a very short time interval necessary for this sampling procedure.

Mathematically, this would imply the necessity of both functions to have null values at sampling moments of time. However, it is well known that a maximum/minimum value for sine function (when the differential of sine function equals zero) corresponds to a null value for its derivative, the cosine function (at these time moments, the differential of the cosine function has the greatest value for its slope, considered as modulus). Quite similar, a maximum/minimum value for cosine function (when the differential of cosine function equals zero) corresponds to a null value for its derivative, the − sin function (at these time moments, the differential of the sine function has the greatest value for its slope, considered as modulus). So it is impossible to select a suitable phase ψ so as the measurement accuracy for both sine and cosine functions to be the best possible if nonzero sampling intervals are taken into account.

This aspect is similar to the uncertainty principle in physics, when measurements corresponding to a certain physical variable and to its conjugated variable (usually corresponding to a derivative of a function in respect to the previous variable) are involved. Moreover, it should be noticed that for a cosine function

$$f = A\cos\psi = \operatorname{Re} A\exp(i\psi), \tag{5.4}$$

the action of the operator

$$\mathrm{Cr} = 1 - i\frac{d}{d\psi} \tag{5.5}$$

would correspond to

$$\mathrm{Cr}(f) = \left(1 - i\frac{d}{d\psi}\right)f = \operatorname{Re}\left[A\exp(i\psi) - (i)^2 A\exp(i\psi)\right] = \operatorname{Re}\left[2A\exp(i\psi)\right]. \tag{5.6}$$

It can be noticed that another cosine function $\cos\psi$ has been generated.

Quite similar, the action of the operator

$$\mathrm{An} = 1 + i\frac{d}{d\psi} \tag{5.7}$$

would correspond to

$$\mathrm{An}(f) = \left(1 + i\frac{d}{d\psi}\right)f = \operatorname{Re}\left[A\exp(i\psi) + (i)^2 A\exp(i\psi)\right] = 0. \tag{5.8}$$

It can be noticed that the cosine function $\cos\psi$ has been annihilated. This aspect is similar to creation/annihilation of particles in advanced quantum mechanics, where such operators are derived using the decomposition of certain fields in plane waves. However, aspects connected with momentum and position operators are hard to be noticed at this stage of research.

As a conclusion for memory-limited complex systems, we can notice that the use of second-order oscillating systems allows ot just robust sampling procedures on extended time intervals for certain quantities corresponding to step changes in the environment, but also the use of just two state variables (corresponding to sampled values for sine and cosine function at certain moments of time) for a preliminary estimation of such step changes during the working interval. For an extended working interval which includes several alternances of the oscillations generated by the received signal, this implies the possibility of transmitting quick preliminary commands towards the actuators, a final adjustment being determined at the end of the whole working interval, based on the difference between required action and the already-performed action. Similar to aspects presented in previous section, these preliminary commands $\mathrm{Com}(t)$ should be better transmitted as a pulse defined by

$$\mathrm{Com}(t) = C[1 - \cos(\omega_0(t - t_2))], \tag{5.9}$$

for $t \in (t_2, t_2 + 2\pi T_0)$ (a period of the oscillating signal considered from the second moment of time when the preliminary sampling procedure for $y(t)$ was performed, and the pair of values sampled at t_1 and t_2 has been analyzed). Thus, subsequent expected values for $y(t)$ and $y'(t)$ at time moments determined by $\omega_0 t = k\pi$ could be computed in an easy manner by the signal processing device, and any differences could be analyzed (for a preliminary conclusion) in an easy manner on next periods of the oscillating system until the working interval comes to an end.

For this reason, the algorithm previously presented is suitable for memory-limited complex systems since it performs both a preliminary analysis of signal received from environment for detecting step changes (with preliminary commands transmitted towards actuators) and a final accurate estimation for the required action on next extended time intervals (computed as a difference between the whole action required by the step change and the action already performed by preliminary commands).

As in case of biological systems, this algorithm is based on values sampled at some successive moments of time. It generates a sequence of certain commands towards the environment as a sequence of pulses, analyzes the difference between expected values and real values for the signal received from the environment, and adjusts the command with higher accuracy after an extended time interval (a kind of multilevel control and command). Another important similarity between this algorithm and behaviour of biological systems should be noticed; the sampling moments of time (when the processed signal is recorded) differ to the time moments when the filtered (processed) signal has a great slope (considered as modulus) so as to allow a robust estimation using just two sampled (recorded) values. The fact that less memory is involved is essential for complex systems which have to survey a great number of parameters (motion parameters, for instance) in the environment and to check the effect of commands transmitted towards a great number of actuators, see the case of vision processing studied in [7, 8].

6. Conclusions

This study has presented advanced signal processing methods and command synthesis for memory-limited complex systems. It was shown that for observing, modeling, and controlling dynamic environmental phenomena in case of memory-limited complex systems, some specific methods based on accurate measurements performed on limited time intervals are required. Starting from the necessity of a set of consecutive measurements performed in a robust manner for detecting step changes in the environment, it was shown that an extended time interval for processing the input signal is necessary. For this reason, the use of second-order oscillating systems was improved by adding a supplementary algorithm so as preliminary values for step changes in the environment to be available for control and command during the signal processing interval. A method for generating a robust command towards the control equipment on a limited time interval in order to compensate the step changes detected on previous working interval was also presented. Finally, similarities between measurements of a certain quantity and of its derivative for a sine function, by one side, and the uncertainty principle in physics (by the other side) were briefly mentioned.

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