

# An Algorithm for Model Reduction in Large Electric Power Systems

A.I. ZEČEVIĆ\* and N. GAČIĆ

*Department of Electrical Engineering, Santa Clara University,  
Santa Clara, CA 95053, USA*

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Model reduction has become an important issue in the analysis of electric power systems, due to their constantly increasing size and complexity. In this paper we present a decomposition algorithm which is capable of reducing the number of equations in the model, while preserving the potential for parallel computation. A variety of experimental results are provided to illustrate the performance of the algorithm.

*Keywords:* Power systems; Model reduction; Differential-algebraic equations; Epsilon decomposition; Transient stability; Newton's method

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## 1. INTRODUCTION

The dynamic behavior of large electric power systems can be described by a system of differential-algebraic equations (DAE)

$$\dot{x} = f(x, y), \quad 0 = g(x, y), \quad (1.1)$$

where  $x$  is the set of state variables of all the dynamic devices (generators), and  $y$  represents voltage magnitudes and angles at the buses [2]. Given the physical interpretation of vectors  $x$  and  $y$ , in the

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\* Corresponding author. Fax: 408-554-5474. E-mail: azecevic@scuacc.scu.edu.

following we will refer to their components as *dynamic* and *static* variables, respectively.

The constant increase in the size and complexity of power systems has created a need for algorithms that can decrease the number of equations in (1.1), while preserving an acceptable level of accuracy. One possible way to accomplish this is to equivalently reduce groups of generators in the system, which results in an approximate model with fewer dynamic variables. A variety of methods exist for this type of dynamic reduction, utilizing concepts such as electromechanical distances [3,6], clustering of swing curves [7] and analysis of system eigenvectors [1]. For large systems, however, these methods are generally complex and result in a considerable computational effort. They also typically require either the construction of new matrices or substantial changes in the system model, both of which further complicate the procedure.

In addition to dynamic reduction, in the transient analysis of power systems it is common practice to eliminate the algebraic network equations in (1.1), thus transforming the problem into a substantially smaller system of ordinary differential equations. However, such a transformation carries a price – the once sparsely connected nodes now acquire additional links, and all related matrices become much denser. As a result, the initial sparsity and potential for parallel computing are largely lost.

The main objective of this paper is to develop a decomposition which is capable of reducing the number of equations that model the power system, while preserving the potential for parallel computation. In the following, we will present an efficient algorithm for dynamic reduction which requires only a modest computational effort, no new matrices and no changes to the model; these features make it very attractive in comparison with existing methods. We will then demonstrate how epsilon decomposition [8,9,11] can be utilized to solve the reduced system of differential equations in parallel.

Experimental results will be provided for two power systems – the IEEE 39 bus system with 10 generators and the NPCC 140 bus system with 48 generators. In both cases, it will be shown that the reduction and parallelization can indeed be performed efficiently without significant loss of accuracy in the simulation process.

## 2. EPSILON DECOMPOSITION

Our algorithm for the reduction of large power systems is based on the concept of epsilon decomposition. The basic idea of the method is remarkably simple – given a matrix  $A = [a_{ij}]$  and a value of parameter  $\varepsilon > 0$ , all elements satisfying  $|a_{ij}| < \varepsilon$  are set to zero. The resulting sparsified matrix is then permuted into a block diagonal form, and all the variables in the same block are considered to be strongly coupled. After such a permutation, matrix  $A$  can be represented as

$$A = A_D + \varepsilon A_C, \quad (2.1)$$

where  $A_D$  is block diagonal and all elements of  $A_C$  are less than or equal to one in magnitude.

Results obtained by applying epsilon decompositions will depend on the choice of parameter  $\varepsilon$ . Naturally, as  $\varepsilon$  is increased, more elements of  $A$  are discarded and the diagonal blocks will become smaller. It is therefore necessary to determine an appropriate value for  $\varepsilon$  that will identify strong coupling while preserving a desired block structure. In most applications such a value is difficult to obtain apriori, and some type of trial and error procedure needs to be performed in order to obtain satisfactory results. This is generally not a problem, due to the linear complexity of the decomposition algorithm.

We should also point out a potential difficulty that arises when epsilon decomposition is directly applied to a matrix with elements that widely vary in size. In such cases, it may not be possible to find a meaningful value for  $\varepsilon$ , as illustrated in (2.2):

$$A = \begin{bmatrix} 10 & 5 & 0.2 \\ 0.1 & 0.2 & 0.02 \\ 4.5 & 1 & 100 \end{bmatrix}. \quad (2.2)$$

In order to resolve this problem, each row should be scaled by the element with the maximal absolute value; this will always result in a more uniform distribution of elements. The application of scaling to the matrix in (2.2) is demonstrated in (2.3). The scaled matrix  $\bar{A}$

obviously has an epsilon decomposition  $\bar{A} = \bar{A}_D + \varepsilon \bar{A}_C$  for  $\varepsilon = 0.1$ ,

$$\bar{A}_D = \begin{bmatrix} 1 & 0.5 & 0 \\ 0.5 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}; \quad \bar{A}_C = \begin{bmatrix} 0 & 0 & 0.2 \\ 0 & 0 & 1 \\ 0.45 & 0.1 & 0 \end{bmatrix}. \quad (2.3)$$

Note that in this case  $A$  can be represented only as  $A = A_D + A_C$  (without  $\varepsilon$ ), but scaling does identify block diagonal dominance, since

$$\|A_D^{-1} A_C\|_\infty = \varepsilon \|\bar{A}_D^{-1} \bar{A}_C\|_\infty \ll 1. \quad (2.4)$$

### 3. IDENTIFICATION OF GENERATORS WITH SIMILAR DYNAMICS

The key step in any algorithm for dynamic reduction is to identify groups of generators that respond “similarly” to disturbances in the network; these groups then represent the basis for an equivalent transformation of the power system (e.g. [10]). When the network is large, standard identification techniques can be quite complex and time consuming, since they typically involve an analysis of the system eigenvectors or major modifications to the model. In contrast, the algorithm proposed in this section is linear in complexity, and involves only successive applications of epsilon decomposition.

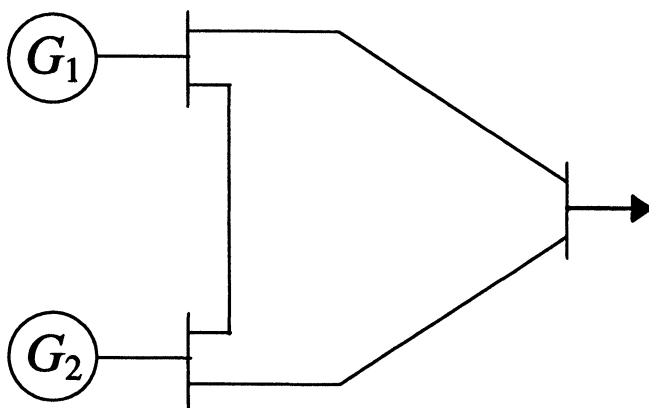


FIGURE 1 A power system with 2 generators and 3 buses.

We begin by considering the simple power system shown in Fig. 1, which consists of two generators and three buses [5].

The DAE that describe its dynamics have the form

$$\begin{aligned}
 f_1: \dot{x}_1 &= x_3 - \omega_0, \\
 f_2: \dot{x}_2 &= x_4 - \omega_0, \\
 f_3: \dot{x}_3 &= \frac{\omega_0}{2H_1} (P_{m1} - |E_1| |Y_{d1}| y_1 \sin(x_1 - y_3)), \\
 f_4: \dot{x}_4 &= \frac{\omega_0}{2H_2} (P_{m2} - |E_2| |Y_{d2}| y_2 \sin(x_2 - y_4)), \\
 g_1: 0 &= \sum_{k=1}^3 |Y_{1k}| y_k \cos(y_{k+3} + \phi_{1k}) - |Y_{d1}| |E_1| \sin x_1, \\
 g_2: 0 &= \sum_{k=1}^3 |Y_{2k}| y_k \cos(y_{k+3} + \phi_{2k}) - |Y_{d2}| |E_2| \sin x_2, \\
 g_3: 0 &= \sum_{k=1}^3 |Y_{1k}| y_k \sin(y_{k+3} + \phi_{1k}) + |Y_{d1}| |E_1| \cos x_1, \\
 g_4: 0 &= \sum_{k=1}^3 |Y_{2k}| y_k \sin(y_{k+3} + \phi_{2k}) + |Y_{d2}| |E_2| \cos x_2, \\
 g_5: 0 &= \sum_{k=1}^3 |Y_{3k}| y_k \cos(y_{k+3} + \phi_{3k}), \\
 g_6: 0 &= \sum_{k=1}^3 |Y_{3k}| y_k \sin(y_{k+3} + \phi_{3k}).
 \end{aligned} \tag{3.1}$$

When a short circuit fault occurs, the system is perturbed from its equilibrium state and it becomes necessary to solve Eqs. (3.1) in order to establish whether stability is preserved. This is typically done by discretizing the differential machine equations using the trapezoidal method, and combining them with the algebraic network equations [2]. In each time point  $t_k$ , this procedure results in a system of purely algebraic equations,

$$\begin{aligned}
 x_k &= x_{k-1} + \frac{h}{2} [f(x_{k-1}, y_{k-1}) + f(x_k, y_k)], \\
 0 &= g(x_k, y_k),
 \end{aligned} \tag{3.2}$$

where  $x_k \equiv x(t_k)$ ,  $y_k \equiv y(t_k)$  and  $h = t_k - t_{k-1}$  represents the discretization step.

For our purposes, it will be convenient to introduce a vector  $z_k \equiv [x_k^T y_k^T]^T$ , and order it so that all generator variables are numbered *first*. For the system in Fig. 1, these variables are  $\{x_1, x_3, y_1, y_3\}$  for generator 1 and  $\{x_2, x_4, y_2, y_4\}$  for generator 2; consequently,  $z_k$  would be ordered as

$$z_k = [x_1 \ x_3 \ y_1 \ y_3 \ | \ x_2 \ x_4 \ y_2 \ y_4 \ | \ y_5 \ y_6]. \quad (3.3)$$

Using this notation, we can now express the algebraic equations in (3.2) as

$$F(z_k) = 0 \quad (3.4)$$

and solve them for  $k = 1, 2, \dots$  by applying the modified Newton method [4],

$$z_k(i+1) = z_k(i) - [J_k(0)]^{-1} F(z_k(i)), \quad i = 0, 1, \dots \quad (3.5)$$

The iterative sequence generated by (3.5) starts from some initial approximation  $z_k(0)$ , and  $J_k(0)$  represents the Jacobian evaluated at this point.

Our algorithm for identifying strongly coupled generators is based on a decomposition of Jacobian  $J_1(0)$ , which corresponds to the *equilibrium* state of the power system. By choosing this particular Jacobian, we secure that the obtained groups of generators are independent of the fault location. In the following, it will be convenient to order the equations in (3.4) in accordance with vector  $z_k$ , so that  $J_1(0)$  can be partitioned as

$$J_1(0) \equiv \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix}, \quad (3.6)$$

where  $J_{11}$  corresponds exclusively to generator variables. For the system in Fig. 1, such an ordering would be

$$F = [f_1 \ f_3 \ g_1 \ g_3 \ | \ f_2 \ f_4 \ g_2 \ g_4 \ | \ g_5 \ g_6] \quad (3.7)$$

and the partitioned Jacobian has the form

$$J_1(0) = \begin{array}{cccc|cccc|cc} * & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 & 0 & 0 & 0 & 0 \\ * & 0 & * & * & 0 & 0 & * & * & * & * \\ * & 0 & * & * & 0 & 0 & * & * & * & * \\ \hline 0 & 0 & 0 & 0 & * & * & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & * & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 & * & * & * & * \\ 0 & 0 & * & * & * & 0 & * & * & * & * \\ \hline 0 & 0 & * & * & 0 & 0 & * & * & * & * \\ 0 & 0 & * & * & 0 & 0 & * & * & * & * \end{array} \quad (3.8)$$

In (3.8), \* denotes nonzero entries, and the two generator blocks in  $J_{11}$  are indicated by dashed lines.

### (i) Identification of Strong Direct Connections

The first step of the algorithm is to perform an epsilon decomposition *only* on  $J_{11}$ , which results in a block diagonal matrix  $J_{11}^\epsilon$ . Such a strategy is motivated by the fact that directly connected generators are usually the most likely to exhibit similar dynamics, and ought to be grouped first. There are two issues regarding the epsilon decomposition of  $J_{11}$  that need to be considered in more detail. It should first be observed that each generator actually corresponds to a  $4 \times 4$  submatrix in  $J_{11}$ , as indicated in (3.8). However, in order to simplify the application of epsilon decomposition, we will define the derivative with respect to the phase angle of the bus voltage as the “representative” of a generator; in our example, the representatives of generators 1 and 2 are  $\partial f_3/\partial y_3$  and  $\partial f_4/\partial y_4$ , respectively. Using this convention, two generators are assumed to belong to the same block if their “representatives” belong to that block. Experiments on a variety of power systems have shown that this simplification is justified, since the  $4 \times 4$  submatrices are regularly preserved in the process of epsilon decomposition.

The second issue that needs to be discussed is the choice of parameter  $\epsilon$ , since it is often difficult to determine a suitable value in advance. In order to resolve this problem, we propose to specify a *maximal allowable size* for a group of generators (denoted  $S_{\max}$ ), and

iteratively compute the smallest  $\varepsilon$  which will satisfy this constraint. A typical (and empirically justified) choice for  $S_{\max}$  is 15–20% of the total number of generators.

## (ii) Identification of Strong Indirect Connections

In order to obtain a meaningful grouping of the generators, it is usually not sufficient to consider only matrix  $J_{11}$ . Namely, two generators can also have similar dynamics if they are “indirectly” connected through the network, provided that these connections are sufficiently strong. We will distinguish between two types of indirect connections, as illustrated in Figs. 2 and 3.

It is natural to expect a greater impact from Type 1 connections, since they are the more direct ones; this fact will be reflected in all subsequent refinements to the original generator grouping.

In order to incorporate connections of Type 1 into our algorithm, it will be convenient to form an auxiliary matrix

$$J_A = \begin{bmatrix} J_{11}^\varepsilon & J_{12} \\ J_{21} & J_D \end{bmatrix}, \quad (3.9)$$

where  $J_D$  represents a matrix formed from the diagonal entries of  $J_{22}$ . It is not difficult to verify that all Type 1 connections are contained in

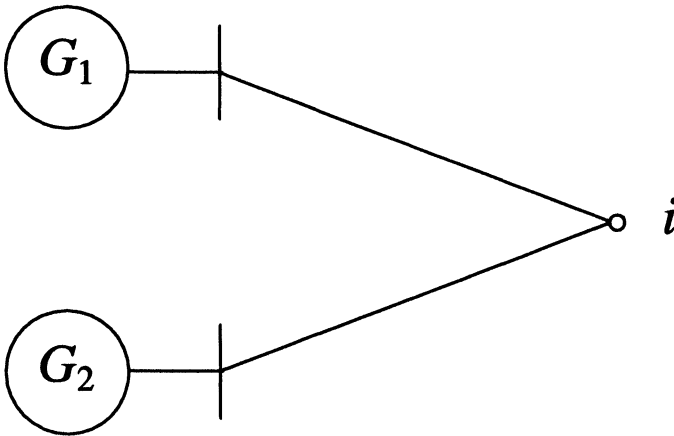


FIGURE 2 Indirect connections through a common bus (Type 1 connections).



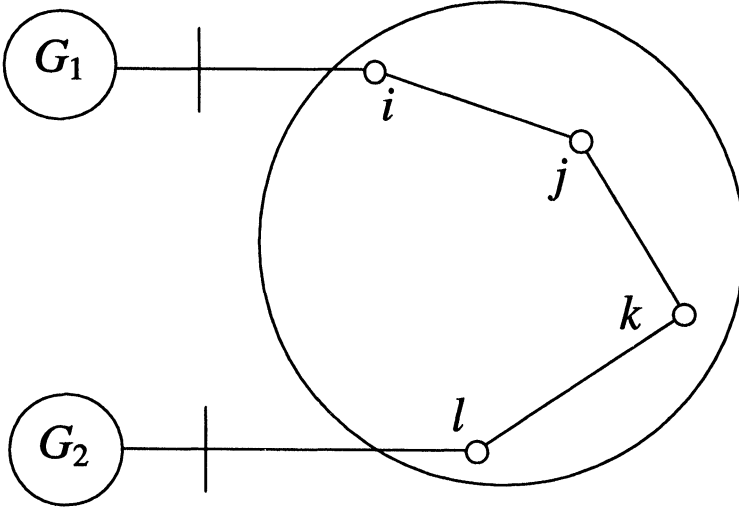


FIGURE 3 Indirect connections through the network (Type 2 connections).

blocks  $J_{12}$  and  $J_{21}$ , and the strongest among them can be identified by performing an epsilon decomposition of matrix  $J_A$ . Using this approach, we can now group certain previously unrelated generators, provided that this does not violate the original maximal block size  $S_{\max}$ . It is also important to point out that the value of  $\varepsilon$  used in the decomposition of  $J_A$  should be *larger* than the one applied to  $J_{11}$ , in order to reflect the hierarchy which exists among different types of generator connections. Experimental results indicate that  $\varepsilon_2 \approx 2\varepsilon_1$  is an appropriate choice.

As a final refinement in this algorithm, it is also possible to consider connections of Type 2. The strongest among these connections can once again be identified by an epsilon decomposition, this time of the entire matrix  $J_1(0)$ . Based on this decomposition, the groups of generators can be further enlarged within the constraints imposed by the choice of  $S_{\max}$ . Typically,  $\varepsilon_3 \approx 1.5\varepsilon_2$  is an adequate choice in this step (reflecting the fact that in practical power systems Type 1 connections are more dominant than Type 2 connections).

The performance of the proposed algorithm was tested on a number of power systems, including the IEEE 39 bus system with 10 generators

TABLE I Generator groups for the 39 bus system

<i>Group</i>	<i>Generators</i>
1	1, 8
2	2, 3
3	4, 5
4	6, 7
5	9
6	10

TABLE II Generator groups for the 140 bus system

<i>Group</i>	<i>Generator</i>
1	1, 2, 6, 7, 9
2	3, 4, 5
3	8
4	10
5	11, 12
6	13, 14, 24, 25, 26
7	15, 16, 17, 18, 19, 20, 23, 31
8	21, 22
9	27, 28, 29, 30
10	32
11	33, 34, 35
12	36
13	37, 38
14	39
15	40
16	41
17	42
18	43, 44, 45, 46, 47
19	48

and the NPCC 140 bus system with 48 generators. The resulting groups of generators for these two systems are shown in Tables I and II, respectively.

To demonstrate that generators in these groups indeed respond similarly to disturbances in the network, in Figs. 4(a) and (b) we show the transient behavior of groups 1 and 4 for the 39 bus systems, given a fault at bus #4.

Similar behavior can be observed in the 140 bus system, as indicated in Figs. 5(a) and (b) for generator groups 6 and 9; in this case, the fault was at generator #8.

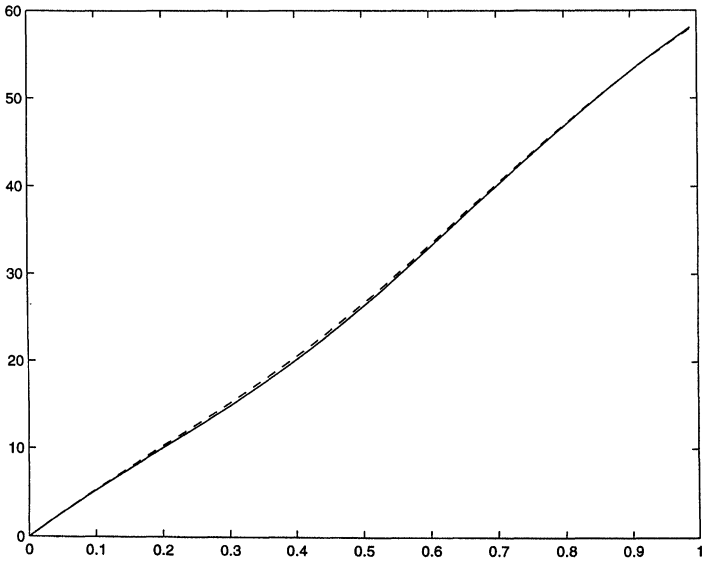


FIGURE 4(a) Rotor angle curves for generators in group 1 (39 bus system).

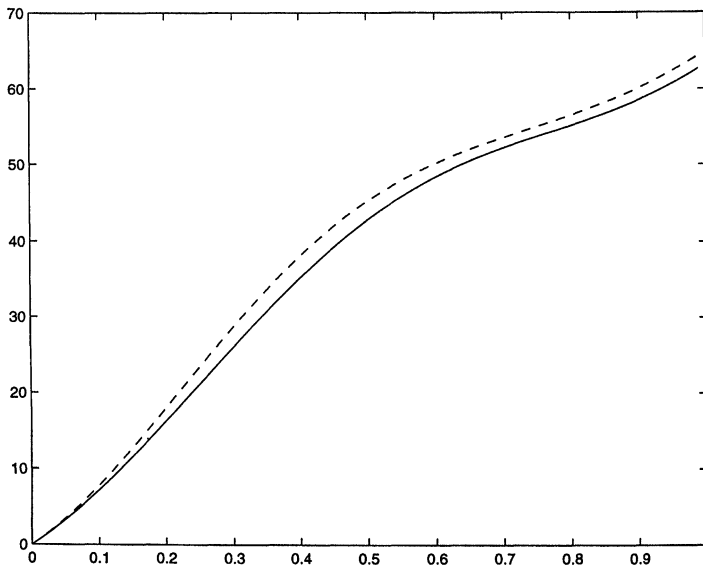


FIGURE 4(b) Rotor angle curves for generators in group 4 (39 bus system).

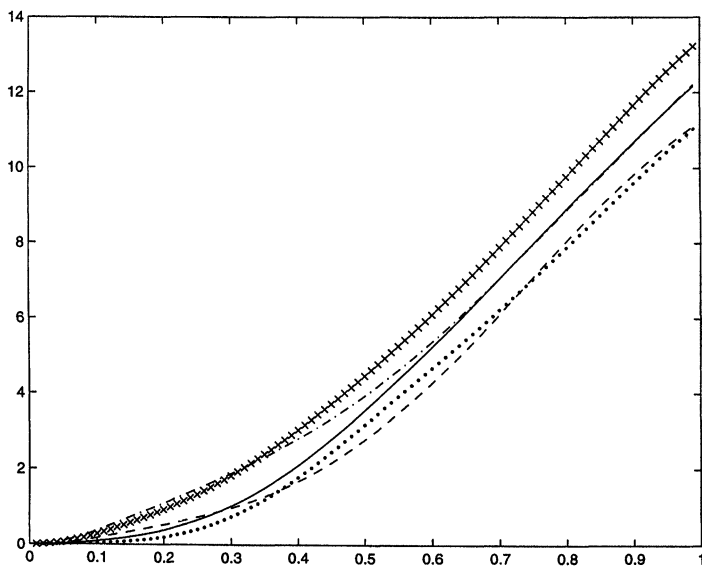


FIGURE 5(a) Rotor angle curves for generators in group 6 (140 bus system).

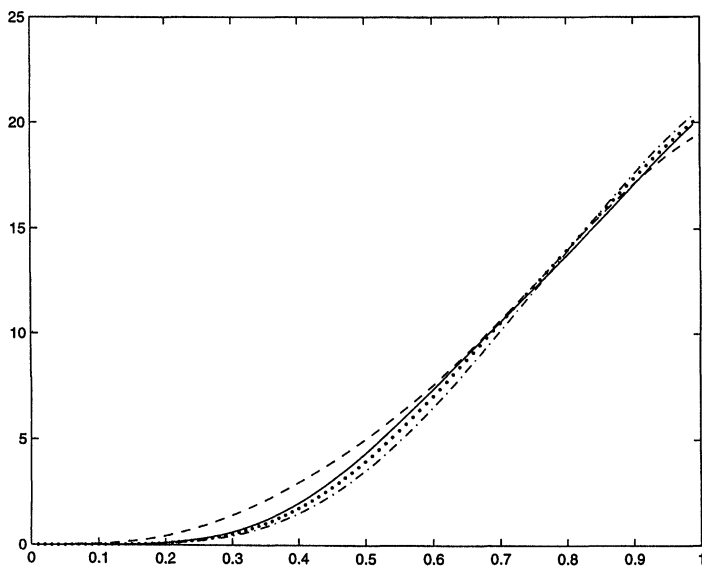


FIGURE 5(b) Rotor angle curves for generators in group 9 (140 bus system).

#### 4. REDUCTION OF LARGE POWER SYSTEM MODELS

Once groups of “similar” generators are identified, it is possible to replace each group with a single equivalent generator [1,7,10]. This transformation can substantially reduce the number of dynamic variables and accelerate the simulation process. It is important to note, however, that the resulting equivalent model is only an *approximation* of the original system, and that its accuracy depends heavily on an appropriate grouping of generators. To evaluate the performance of our method in this context, we applied Podmore’s aggregation algorithm [7] to reduce the number of dynamic variables in the 140 bus system from 96 to 50. The results shown in Figs. 6 and 7 indicate that the groups from Table II indeed produce accurate approximations of the exact solution; it follows, therefore, that epsilon

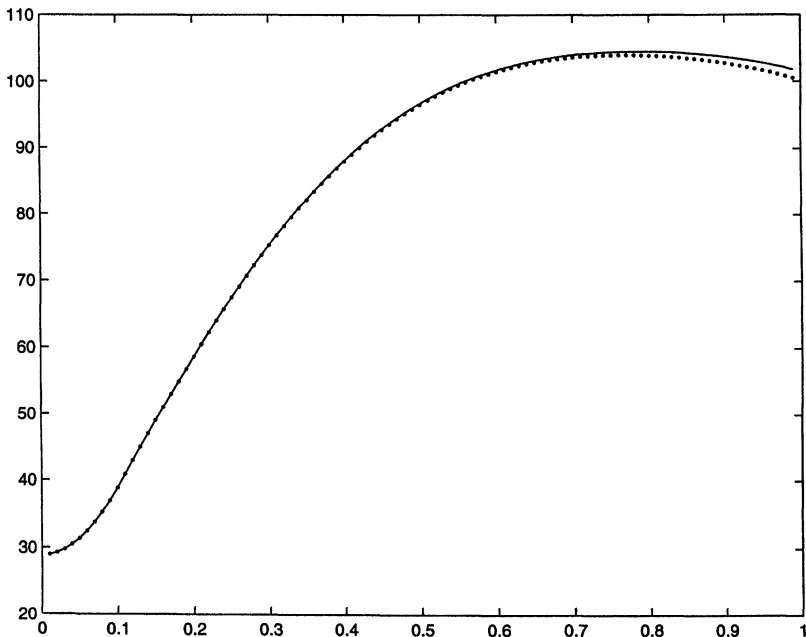


FIGURE 6 Rotor angle curves for generator #6, with a fault at bus #7 (after Podmore aggregation). Solid and dotted lines represent the original and reduced models, respectively.

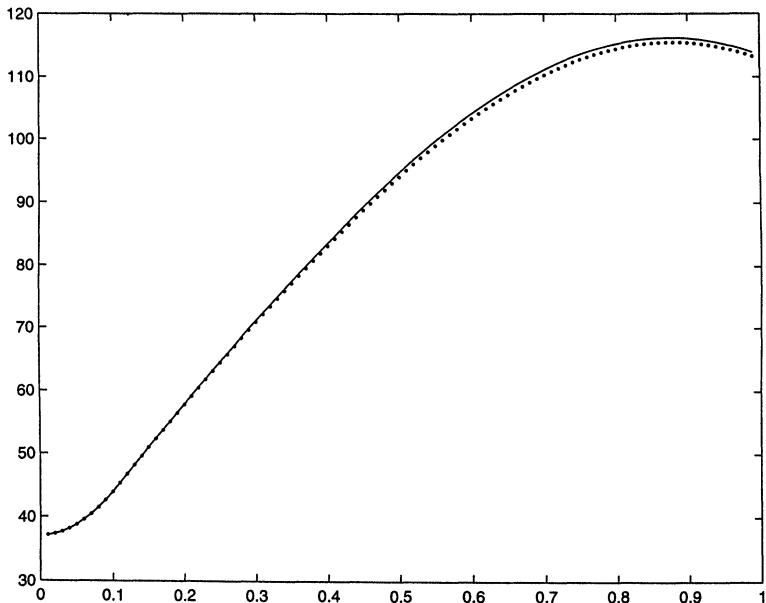


FIGURE 7 Rotor angle curves for generator #8, with a fault at bus #7 (after Podmore aggregation). Solid and dotted lines represent the original and reduced models, respectively.

decomposition can indeed be used effectively as a basis for dynamic reduction.

Epsilon decomposition can simplify the simulation process even further. Namely, in the transient analysis of large power systems it is common practice to transform the model into  $\tilde{n}$  purely differential equations

$$\dot{x} = \Phi(x), \quad (4.1)$$

where  $\tilde{n}$  is the number of dynamic variables that remain after the equivalent reduction of generators. However, in this process, the once sparsely connected nodes acquire additional links [10], and the Jacobian becomes much denser (offering very little in terms of parallel processing). In view of this fact, we now propose to eliminate the *weakest* among these newly established connections, by applying epsilon decomposition to the *Jacobian of  $\Phi(x)$* . The resulting decomposed Jacobian is now a block diagonal matrix with blocks  $J_{ii}$  ( $i = 1, 2, \dots, p$ ), and the corresponding

Newton method becomes completely decoupled

$$x_{k+1}^i = x_k^i - J_{ii}^{-1} \Phi_i(x), \quad i = 1, 2, \dots, p. \quad (4.2)$$

It is easy to see that such an iterative process is ideally suited for parallel computation.

The decoupled Newton process will typically result in a larger number of iterations, but the accuracy of the system model will not be affected. To evaluate this approach we applied epsilon decomposition to the Jacobian of the 140 bus system after its model was reduced to 50 ordinary differential equations. For  $\varepsilon = 0.05$  we obtained 3 decoupled subsystems, with dimensions  $20 \times 20$ ,  $16 \times 16$  and  $14 \times 14$ , respectively. In this case it was found that the total number of Newton iterations increased from 275 to 300; on the other hand, using three processors the computation time per iteration was reduced 2.4 times, resulting in an overall speedup of 2.2. As expected, the accuracy of the rotor angle curves was not affected by the decoupling.

## 5. CONCLUSIONS

In this paper we presented an algorithm for reducing the number of differential-algebraic equations that model large electric power systems. The algorithm is linear in complexity, and is substantially simpler than existing methods, which typically require the computation of eigenvectors and/or major modifications to the system.

The first step of the algorithm utilizes epsilon decompositions to efficiently identify groups of generators with similar transient behavior. Based on this grouping, the number of dynamic variables is reduced, and the model can be equivalently described by a system of ordinary differential equations. Epsilon decompositions are also used in the second step, this time to decouple and parallelize the Newton iterative method which arises in the simulation process. Experiments performed on a number of power systems confirm that the loss of accuracy due to reduction and decoupling is not significant.

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