#A55 INTEGERS 9 (2009), 725-734

PALINDROMES IN DIFFERENT BASES: A CONJECTURE OF J. ERNEST WILKINS

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Received: 4/26/09, Accepted: 10/13/09, Published: 12/23/09

Abstract

We show that there exist exactly 203 positive integers N such that for some integer $d \geq 2$ this number is a d-digit palindrome base 10 as well as a d-digit palindrome for some base b different from 10. To be more precise, such N range from 22 to 9986831781362631871386899.

–Dedicated to J. Ernest Wilkins

1. Introduction

During the summer of 2004, while meeting at the Conference for African-American Researchers in the Mathematical Sciences (CAARMS), the author had a short conversation with J. Ernest Wilkins. He was interested in palindromes which remain palindromes when expressed in a different base system. For example, 207702 is a 6-digit palindrome expressed in base 10 (written as $[2, 0, 7, 7, 0, 2]_{10}$) as well as 6-digit palindrome expressed in base 8 (written as $[6, 2, 5, 5, 2, 6]_8$). He posed the following question:

> *Does there exist a positive integer N which is an 8-digit palindrome base 10 as well as an 8-digit palindrome for some base b different from 10?*

He suspected that the answer would be no, but, without being well-versed in the art of computer programming, could not find a definitive proof using pen and paper.

It is natural to generalize this question to any number of digits. The main result of this exposition is as follows:

Theorem 1 *There exist exactly 203 positive integers N such that for some integer $d \geq 2$ this number is a d-digit palindrome base 10 as well as a d-digit palindrome for some base b different from 10. To be more precise, such N range from 22 to 9986831781362631871386899, and*

$$d = 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 9,\ 11,\ 13,\ 15,\ 17,\ 19,\ 21,\ 23,\ 25.$$

(We assume that $d \geq 2$ because any positive integer $N < 10$ is trivially a 1-digit palindrome base b for any $b > N$.) A complete list of these palindromes can be found in the appendix.

The author found this result using a few theoretically trivial yet computationally frustrating inequalities, then parallelized the search using several high-powered machines at Purdue University. Almost all of the computations were done using `gridMathematica`. Wilkins conjectured that the only *even* d are $d = 2, 4, 6$; this result is positive verification of his conjecture. This article is a bit different from [4]: Those authors consider positive integers $N = 10^n \pm 1$ which are palindromes base 10 as well as palindromes base 2 – but the number of digits is not fixed among the bases.

2. Computational Set-Up

Fix an integer $b \geq 2$. For each positive integer N, the expression $[a_d, \dots, a_2, a_1]_b$ will denote the unique base b expansion

$$N = a_d \, b^{d-1} + \cdots + a_2 \, b + a_1 \qquad \text{where} \qquad 0 \leq a_k < b.$$

We will call this the *base b representation of N*. Moreover, we will say that N is a *d-digit number base b* if a_d is nonzero; and that such a d-digit number is a *d-digit palindrome base b* if $a_{d-k+1} = a_k$ for $k = 1, 2, \dots, d$. For example, $N = 207702$ may be expressed as $[2, 0, 7, 7, 0, 2]_{10}$ for $b = 10$, or $[6, 2, 5, 5, 2, 6]_8$ for $b = 8$. In particular, N is a 6-digit palindrome base 10 as well as a 6-digit palindrome base 8.

There are only finitely many d-digit numbers base 10 which are also d-digit numbers for some other base b:

Lemma 2 *If N is a positive d-digit number base 10 which is also d-digit number for some base $b \neq 10$ then $d \leq 26$.*

Proof. (Wilkins himself suggested this proof.) Upon fixing such an integer N, the base b must satisfy

$$\left\lfloor \frac{\log N}{\log 10} \right\rfloor + 1 = d = \left\lfloor \frac{\log N}{\log b} \right\rfloor + 1$$

in terms of the greatest integer function $\lfloor \cdot \rfloor$. In fact, given any real number x we have the inequality $x \leq \lfloor x \rfloor + 1 \leq x + 1$, so it is easy to see that

$$-1 \leq \frac{\log N}{\log 10} - \frac{\log N}{\log b} \leq 1 \qquad \Longrightarrow \qquad 10^{1/(1+\log 10/\log N)} \leq b \leq 10^{1/(1-\log 10/\log N)}.$$

When $N \geq 10^{26}$, i.e., $d > 26$, this forces $b = 10$. □

The following result gives computable ranges for N:

Lemma 3 *Say that N is a positive integer N which is both a d-digit palindrome base 10 as well as a d-digit palindrome for some base $b \neq 10$. Then either $d \leq 14$, or else d, b, and N are related as in the following table:*

Digits d	Base b	Range for N
15	9	$10^{14} < N < 9^{15}$
	11	$11^{14} < N < 10^{15}$
16	9	$10^{15} < N < 9^{16}$
17	9	$10^{16} < N < 9^{17}$
	11	$11^{16} < N < 10^{17}$
18	9	$10^{17} < N < 9^{18}$
19	9	$10^{18} < N < 9^{19}$
	11	$11^{18} < N \leq 10^{19}$
20	9	$10^{19} < N < 9^{20}$
21	9	$10^{20} < N < 9^{21}$
	11	$11^{20} < N \leq 10^{21}$
23	11	$11^{22} < N < 10^{23}$
25	11	$11^{24} < N < 10^{25}$

Proof. According to Lemma 2, it suffices to consider those integers N satisfying the double inequality $10^1 < N < 10^{26}$. Recall that

$$d = \left\lfloor \frac{\log N}{\log b} \right\rfloor + 1.$$

When $15 \leq d \leq 21$, this forces $9 \leq b \leq 11$; and when $22 \leq d \leq 26$, this forces $10 \leq b \leq 11$. It suffices then to show that $d \neq 22, 24, 26$. Following an observation of Wilkins, we see that no d-digit palindrome base 10 can also be a d-digit palindrome base 11 when d is even: Indeed, write $[c_d, \ldots, c_2, c_1]_{10}$ and $[a_d, \ldots, a_2, a_1]_{11}$ as the base 10 and base 11 representations of N, respectively, where the leading coefficient satisfies $0 < a_d < 11$. Then we find

$$\begin{aligned} a_d = a_1 &\equiv a_d \cdot 11^{d-1} + a_{d-1} \cdot 11^{d-2} + \cdots + a_2 \cdot 11 + a_1 \pmod{11} \\ &= N = c_d \cdot 10^{d-1} + c_{d-1} \cdot 10^{d-2} + \cdots + c_2 \cdot 10 + c_1 \\ &\equiv 0 + \cdots + (c_{d-1} - c_2) - (c_d - c_1) \pmod{11} \\ &\equiv 0 \pmod{11} \end{aligned} \tag{1}$$

which is a contradiction. □

3. Implementation

Here is the actual Mathematica code. Given a pair of d-digit integers $\{N_1, N_2\}$ and a base b, the output is a list of d-digit palindromes base b in the range $N_1 \le N < N_2$ which are palindromes for some base different from b. In practice, we set $N_1 = 10$, $N_2 = 10^{26}$, and $b = 10$ – although for large enough N it seems computationally more efficient to set $b = 11$. The built-in Mathematica command RealDigits[N,b] returns $\{\{a_d, \ldots, a_1\}, d\}$, as related to the base b expansion $N = a_d\, b^{d-1} + \cdots + a_2\, b + a_1$; while FromDigits[list, b] undoes this command and returns N.

```
PalindromeSearch[{N1_Integer, N2_Integer, b_Integer}] :=
  Module[{d, FoundList, TestPalindrome, BaseList},

     d = RealDigits[N1,b][[2]]; (* number of digits *)
     FoundList = {}; (* list of found palindromes *)

     For[
        n = Floor[ N1/b^Floor[d/2] ],
        n < Floor[ N2/b^Floor[d/2] ],
        n++,
        (* n denotes the first d/2 digits of the palindrome *)


        TestPalindrome = FromDigits[Join[
           RealDigits[n,b][[1]],
           Take[ Reverse[RealDigits[n,b][[1]]], -Floor[d/2] ]
        ],b];
        (* reconstructs the d-digit palindrome from n *)


        BaseList = Select[
           Range[
              Ceiling[ b^(1/(1+Log[TestPalindrome,b])) ],
              Floor[ b^(1/(1-Log[TestPalindrome,b])) ]
           ],
           RealDigits[TestPalindrome,#][[1]] ==
           Reverse[ RealDigits[TestPalindrome,#][[1]] ]
           &&
           RealDigits[TestPalindrome,#][[2]] == d &
        ];
        (* a list of bases for which TestPalindrome is also a
        d-digit palindrome *)
```

```
      If[
         Length[BaseList] > 1,
         AppendTo[ FoundList, {TestPalindrome, BaseList} ];
         (* if base is not b, add to list *)
      ];

   ];

   Return[FoundList]; (* return complete list *)
]
```

4. Enumeration of Palindromes

Lemma 3 gives effective computing ranges. The first $d \leq 14$ digits took about a day. These last $15 \leq d < 26$ took about fifteen months running on twenty processors each – for a total of about twelve years computing time! The results form the basis of Theorem 1 and the table below. Recall that $[a_d, \ldots, a_2, a_1]_b$ denotes the expansion $N = a_d\, b^{d-1} + \cdots + a_2\, b + a_1$.

Digits d	Integer N	Base b Representations
2	22	$[2,2]_{10}$, $[1,1]_{21}$
	33	$[3,3]_{10}$, $[1,1]_{32}$
	44	$[4,4]_{10}$, $[2,2]_{21}$, $[1,1]_{43}$
	55	$[5,5]_{10}$, $[1,1]_{54}$
	66	$[6,6]_{10}$, $[3,3]_{21}$, $[2,2]_{32}$, $[1,1]_{65}$
	77	$[7,7]_{10}$, $[1,1]_{76}$
	88	$[8,8]_{10}$, $[4,4]_{21}$, $[2,2]_{43}$, $[1,1]_{87}$
	99	$[9,9]_{10}$, $[3,3]_{32}$, $[1,1]_{98}$
3	111	$[3,0,3]_6$, $[1,1,1]_{10}$
	121	$[2,3,2]_7$, $[1,7,1]_8$, $[1,2,1]_{10}$
	141	$[3,5,3]_6$, $[1,4,1]_{10}$
	171	$[3,3,3]_7$, $[1,7,1]_{10}$
	181	$[1,8,1]_{10}$, $[1,3,1]_{12}$
	191	$[5,1,5]_6$, $[2,3,2]_9$, $[1,9,1]_{10}$
	222	$[2,2,2]_{10}$, $[1,4,1]_{13}$
	232	$[2,3,2]_{10}$, $[1,10,1]_{11}$
	242	$[4,6,4]_7$, $[2,4,2]_{10}$
	282	$[3,4,3]_9$, $[2,8,2]_{10}$
	292	$[5,6,5]_7$, $[4,4,4]_8$, $[2,9,2]_{10}$
	313	$[3,1,3]_{10}$, $[1,11,1]_{13}$
	323	$[3,2,3]_{10}$, $[1,9,1]_{14}$
	333	$[5,1,5]_8$, $[3,3,3]_{10}$
	343	$[3,4,3]_{10}$, $[2,9,2]_{11}$, $[1,1,1]_{18}$
	353	$[3,5,3]_{10}$, $[2,1,2]_{13}$, $[1,6,1]_{16}$
	373	$[5,6,5]_8$, $[4,5,4]_9$, $[3,7,3]_{10}$
	414	$[6,3,6]_8$, $[4,1,4]_{10}$
	444	$[4,4,4]_{10}$, $[2,8,2]_{13}$
	454	$[4,5,4]_{10}$, $[3,8,3]_{11}$
	464	$[5,6,5]_9$, $[4,6,4]_{10}$, $[2,5,2]_{14}$
	484	$[4,8,4]_{10}$, $[1,2,1]_{21}$

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Digits d	Integer N	Base b Representations
	494	$[4,9,4]_{10}$, $[1,12,1]_{17}$
	505	$[5,0,5]_{10}$, $[1,10,1]_{18}$, $[1,3,1]_{21}$
	545	$[5,4,5]_{10}$, $[1,15,1]_{17}$
	555	$[6,7,6]_{9}$, $[5,5,5]_{10}$, $[3,10,3]_{12}$
	565	$[5,6,5]_{10}$, $[4,7,4]_{11}$
	575	$[5,7,5]_{10}$, $[3,5,3]_{13}$
	595	$[5,9,5]_{10}$, $[1,15,1]_{18}$, $[1,5,1]_{22}$
	616	$[6,1,6]_{10}$, $[4,3,4]_{12}$
	626	$[6,2,6]_{10}$, $[2,7,2]_{16}$, $[1,0,1]_{25}$
	646	$[7,8,7]_{9}$, $[6,4,6]_{10}$
	656	$[8,0,8]_{9}$, $[6,5,6]_{10}$
	666	$[6,6,6]_{10}$, $[3,12,3]_{13}$, $[1,16,1]_{19}$
	676	$[6,7,6]_{10}$, $[5,6,5]_{11}$, $[4,8,4]_{12}$, $[1,2,1]_{25}$
	686	$[6,8,6]_{10}$, $[2,2,2]_{18}$
	717	$[7,1,7]_{10}$, $[3,9,3]_{14}$
	727	$[7,2,7]_{10}$, $[1,11,1]_{22}$
	737	$[7,3,7]_{10}$, $[5,1,5]_{12}$, $[1,9,1]_{23}$
	757	$[7,5,7]_{10}$, $[1,15,1]_{21}$, $[1,1,1]_{27}$
	767	$[7,6,7]_{10}$, $[2,11,2]_{17}$
	787	$[7,8,7]_{10}$, $[6,5,6]_{11}$, $[3,1,3]_{16}$
	797	$[7,9,7]_{10}$, $[5,6,5]_{12}$, $[4,9,4]_{13}$
	818	$[8,1,8]_{10}$, $[2,14,2]_{17}$
	828	$[8,2,8]_{10}$, $[3,10,3]_{15}$
	838	$[8,3,8]_{10}$, $[2,6,2]_{19}$, $[1,4,1]_{27}$
	848	$[8,4,8]_{10}$, $[2,11,2]_{18}$
	858	$[8,5,8]_{10}$, $[4,5,4]_{14}$, $[3,12,3]_{15}$
	888	$[8,8,8]_{10}$, $[3,14,3]_{15}$
	898	$[8,9,8]_{10}$, $[7,4,7]_{11}$, $[1,16,1]_{23}$
	909	$[9,0,9]_{10}$, $[7,5,7]_{11}$
	919	$[9,1,9]_{10}$, $[4,1,4]_{15}$, $[1,7,1]_{27}$
	929	$[9,2,9]_{10}$, $[1,3,1]_{29}$
	949	$[9,4,9]_{10}$, $[4,3,4]_{15}$
	979	$[9,7,9]_{10}$, $[4,5,4]_{15}$, $[3,13,3]_{16}$
	989	$[9,8,9]_{10}$, $[3,7,3]_{17}$, $[2,5,2]_{21}$, $[1,12,1]_{26}$
	999	$[9,9,9]_{10}$, $[5,1,5]_{14}$
4	3663	$[7,1,1,7]_{8}$, $[3,6,6,3]_{10}$
	6776	$[6,7,7,6]_{10}$, $[3,1,1,3]_{13}$
	8008	$[8,0,0,8]_{10}$, $[4,7,7,4]_{12}$
	8778	$[8,7,7,8]_{10}$, $[3,12,12,3]_{13}$
5	13131	$[3,1,5,1,3]_{8}$, $[1,3,1,3,1]_{10}$
	13331	$[3,2,0,2,3]_{8}$, $[1,3,3,3,1]_{10}$
	16561	$[6,6,1,6,6]_{7}$, $[1,6,5,6,1]_{10}$
	25752	$[3,8,2,8,3]_{9}$, $[2,5,7,5,2]_{10}$
	26462	$[6,3,5,3,6]_{8}$, $[2,6,4,6,2]_{10}$
	26662	$[6,4,0,4,6]_{8}$, $[2,6,6,6,2]_{10}$
	26962	$[2,6,9,6,2]_{10}$, $[1,9,2,9,1]_{11}$
	27472	$[4,1,6,1,4]_{9}$, $[2,7,4,7,2]_{10}$
	30103	$[7,2,6,2,7]_{8}$, $[3,0,1,0,3]_{10}$
	30303	$[7,3,1,3,7]_{8}$, $[3,0,3,0,3]_{10}$
	35953	$[3,5,9,5,3]_{10}$, $[1,8,9,8,1]_{12}$
	38183	$[3,8,1,8,3]_{10}$, $[1,8,9,8,1]_{11}$
	39593	$[3,9,5,9,3]_{10}$, $[1,0,6,0,1]_{14}$
	40504	$[4,0,5,0,4]_{10}$, $[2,8,4,8,2]_{11}$
	42324	$[6,4,0,4,6]_{9}$, $[4,2,3,2,4]_{10}$

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Digits d	Integer N	Base b Representations
	43934	$[4,3,9,3,4]_{10},\ [2,1,5,1,2]_{12}$
	49294	$[4,9,2,9,4]_{10},\ [3,4,0,4,3]_{11}$
	50605	$[7,6,3,6,7]_{9},\ [5,0,6,0,5]_{10}$
	52825	$[5,2,8,2,5]_{10},\ [3,6,7,6,3]_{11}$
	56265	$[5,6,2,6,5]_{10},\ [1,12,7,12,1]_{13}$
	59095	$[5,9,0,9,5]_{10},\ [1,7,7,7,1]_{14}$
	60106	$[6,0,1,0,6]_{10},\ [1,2,12,2,1]_{15}$
	63936	$[6,3,9,3,6]_{10},\ [4,4,0,4,4]_{11}$
	67576	$[6,7,5,7,6]_{10},\ [1,5,0,5,1]_{15}$
	75157	$[7,5,1,5,7]_{10},\ [5,1,5,1,5]_{11}$
	88888	$[8,8,8,8,8]_{10},\ [4,3,5,3,4]_{12}$
	90209	$[9,0,2,0,9]_{10},\ [1,6,0,6,1]_{16}$
	94049	$[9,4,0,4,9]_{10},\ [1,6,15,6,1]_{16}$
	94249	$[9,4,2,4,9]_{10},\ [1,2,3,2,1]_{17}$
	96369	$[9,6,3,6,9]_{10},\ [1,7,8,7,1]_{16}$
	98689	$[9,8,6,8,9]_{10},\ [1,8,1,8,1]_{16}$
6	207702	$[6,2,5,5,2,6]_{8},\ [2,0,7,7,0,2]_{10}$
	546645	$[5,4,6,6,4,5]_{10},\ [1,0,3,3,0,1]_{14}$
	646646	$[6,4,6,6,4,6]_{10},\ [2,7,2,2,7,2]_{12}$
7	1496941	$[5,5,5,3,5,5,5]_{8},\ [1,4,9,6,9,4,1]_{10}$
	1540451	$[2,8,0,7,0,8,2]_{9},\ [1,5,4,0,4,5,1]_{10}$
	1713171	$[3,2,0,1,0,2,3]_{9},\ [1,7,1,3,1,7,1]_{10}$
	1721271	$[3,2,1,3,1,2,3]_{9},\ [1,7,2,1,2,7,1]_{10}$
	1828281	$[3,3,8,5,8,3,3]_{9},\ [1,8,2,8,2,8,1]_{10}$
	1877781	$[3,4,7,1,7,4,3]_{9},\ [1,8,7,7,7,8,1]_{10}$
	1885881	$[3,4,8,3,8,4,3]_{9},\ [1,8,8,5,8,8,1]_{10}$
	1935391	$[7,3,0,4,0,3,7]_{8},\ [1,9,3,5,3,9,1]_{10}$
	1970791	$[7,4,1,1,1,4,7]_{8},\ [1,9,7,0,7,9,1]_{10}$
	2401042	$[4,4,5,8,5,4,4]_{9},\ [2,4,0,1,0,4,2]_{10}$
	2434342	$[4,5,2,0,2,5,4]_{9},\ [2,4,3,4,3,4,2]_{10}$
	2442442	$[4,5,3,2,3,5,4]_{9},\ [2,4,4,2,4,4,2]_{10}$
	2450542	$[4,5,4,4,4,5,4]_{9},\ [2,4,5,0,5,4,2]_{10}$
	2956592	$[2,9,5,6,5,9,2]_{10},\ [1,7,3,10,3,7,1]_{11}$
	2968692	$[2,9,6,8,6,9,2]_{10},\ [1,7,4,8,4,7,1]_{11}$
	3106013	$[5,7,5,3,5,7,5]_{9},\ [3,1,0,6,0,1,3]_{10}$
	3114113	$[5,7,6,5,6,7,5]_{9},\ [3,1,1,4,1,1,3]_{10}$
	3122213	$[5,7,7,7,7,7,5]_{9},\ [3,1,2,2,2,1,3]_{10}$
	3163613	$[5,8,5,1,5,8,5]_{9},\ [3,1,6,3,6,1,3]_{10}$
	3171713	$[5,8,6,3,6,8,5]_{9},\ [3,1,7,1,7,1,3]_{10}$
	3192913	$[3,1,9,2,9,1,3]_{10},\ [1,0,9,11,9,0,1]_{12}$
	3262623	$[3,2,6,2,6,2,3]_{10},\ [1,9,2,9,2,9,1]_{11}$
	3274723	$[3,2,7,4,7,2,3]_{10},\ [1,9,3,7,3,9,1]_{11}$
	3286823	$[3,2,8,6,8,2,3]_{10},\ [1,9,4,5,4,9,1]_{11}$
	3298923	$[3,2,9,8,9,2,3]_{10},\ [1,9,5,3,5,9,1]_{11}$
	3303033	$[6,1,8,3,8,1,6]_{9},\ [3,3,0,3,0,3,3]_{10}$
	3360633	$[6,2,8,1,8,2,6]_{9},\ [3,3,6,0,6,3,3]_{10},\ [1,9,9,5,9,9,1]_{11}$
	3372733	$[3,3,7,2,7,3,3]_{10},\ [1,9,10,3,10,9,1]_{11}$
	4348434	$[4,3,4,8,4,3,4]_{10},\ [2,5,0,0,0,5,2]_{11}$
	4410144	$[4,4,1,0,1,4,4]_{10},\ [2,5,4,2,4,5,2]_{11}$
	4422244	$[4,4,2,2,2,4,4]_{10},\ [2,5,5,0,5,5,2]_{11}$
	4581854	$[4,5,8,1,8,5,4]_{10},\ [2,6,4,10,4,6,2]_{11}$
	4593954	$[4,5,9,3,9,5,4]_{10},\ [2,6,5,8,5,6,2]_{11}$
	5641465	$[5,6,4,1,4,6,5]_{10},\ [1,10,8,0,8,10,1]_{12}$
	5643465	$[5,6,4,3,4,6,5]_{10},\ [3,2,0,5,0,2,3]_{11}$

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Digits d	Integer N	Base b Representations
	5655565	$[5,6,5,5,5,6,5]_{10}$, $[3,2,1,3,1,2,3]_{11}$
	5667665	$[5,6,6,7,6,6,5]_{10}$, $[3,2,2,1,2,2,3]_{11}$
	5741475	$[5,7,4,1,4,7,5]_{10}$, $[3,2,7,1,7,2,3]_{11}$
	7280827	$[7,2,8,0,8,2,7]_{10}$, $[4,1,2,3,2,1,4]_{11}$
	7292927	$[7,2,9,2,9,2,7]_{10}$, $[4,1,3,1,3,1,4]_{11}$
	8364638	$[8,3,6,4,6,3,8]_{10}$, $[2,9,7,4,7,9,2]_{12}$
	8710178	$[8,7,1,0,1,7,8]_{10}$, $[4,10,0,10,0,10,4]_{11}$
	8722278	$[8,7,2,2,2,7,8]_{10}$, $[4,10,1,8,1,10,4]_{11}$
	8734378	$[8,7,3,4,3,7,8]_{10}$, $[4,10,2,6,2,10,4]_{11}$
	8746478	$[8,7,4,6,4,7,8]_{10}$, $[4,10,3,4,3,10,4]_{11}$
	8758578	$[8,7,5,8,5,7,8]_{10}$, $[4,10,4,2,4,10,4]_{11}$
	8820288	$[8,8,2,0,2,8,8]_{10}$, $[4,10,8,4,8,10,4]_{11}$
	8832388	$[8,8,3,2,3,8,8]_{10}$, $[4,10,9,2,9,10,4]_{11}$
	8844488	$[8,8,4,4,4,8,8]_{10}$, $[4,10,10,0,10,10,4]_{11}$
	8864688	$[8,8,6,4,6,8,8]_{10}$, $[1,10,11,4,11,10,1]_{13}$
	9046409	$[9,0,4,6,4,0,9]_{10}$, $[1,2,11,6,11,2,1]_{14}$
	9578759	$[9,5,7,8,7,5,9]_{10}$, $[1,3,11,4,11,3,1]_{14}$
	9813189	$[9,8,1,3,1,8,9]_{10}$, $[1,4,3,6,3,4,1]_{14}$
	9963699	$[9,9,6,3,6,9,9]_{10}$, $[3,4,0,6,0,4,3]_{12}$
9	130535031	$[7,6,1,7,4,7,1,6,7]_{8}$, $[1,3,0,5,3,5,0,3,1]_{10}$
	167191761	$[3,7,8,5,3,5,8,7,3]_{9}$, $[1,6,7,1,9,1,7,6,1]_{10}$
	181434181	$[4,1,8,3,5,3,8,1,4]_{9}$, $[1,8,1,4,3,4,1,8,1]_{10}$
	232000232	$[5,3,4,4,8,4,4,3,5]_{9}$, $[2,3,2,0,0,0,2,3,2]_{10}$
	356777653	$[3,5,6,7,7,7,6,5,3]_{10}$, $[1,7,3,4,3,4,3,7,1]_{11}$
	362151263	$[3,6,2,1,5,1,2,6,3]_{10}$, $[1,7,6,4,7,4,6,7,1]_{11}$
	382000283	$[8,7,7,7,1,7,7,7,8]_{9}$, $[3,8,2,0,0,0,2,8,3]_{10}$
	489525984	$[4,8,9,5,2,5,9,8,4]_{10}$, $[2,3,1,3,6,3,1,3,2]_{11}$
	492080294	$[4,9,2,0,8,0,2,9,4]_{10}$, $[2,3,2,8,4,8,2,3,2]_{11}$
	520020025	$[5,2,0,0,2,0,0,2,5]_{10}$, $[1,2,6,1,10,1,6,2,1]_{12}$
	537181735	$[5,3,7,1,8,1,7,3,5]_{10}$, $[2,5,6,2,5,2,6,5,2]_{11}$
	713171317	$[7,1,3,1,7,1,3,1,7]_{10}$, $[1,7,10,10,0,10,10,7,1]_{12}$
	796212697	$[7,9,6,2,1,2,6,9,7]_{10}$, $[1,10,2,7,9,7,2,10,1]_{12}$
	952404259	$[9,5,2,4,0,4,2,5,9]_{10}$, $[1,2,2,4,1,4,2,2,1]_{13}$
	998111899	$[9,9,8,1,1,1,8,9,9]_{10}$, $[4,7,2,4,5,4,2,7,4]_{11}$
	999454999	$[9,9,9,4,5,4,9,9,9]_{10}$, $[4,7,3,1,9,1,3,7,4]_{11}$
11	39276067293	$[3,9,2,7,6,0,6,7,2,9,3]_{10}$, $[1,5,7,2,5,3,5,2,7,5,1]_{11}$
	39453235493	$[3,9,4,5,3,2,3,5,4,9,3]_{10}$, $[1,5,8,0,6,3,6,0,8,5,1]_{11}$
	42521012524	$[4,2,5,2,1,0,1,2,5,2,4]_{10}$, $[1,7,0,4,0,0,0,4,0,7,1]_{11}$
	73183838137	$[7,3,1,8,3,8,3,8,1,3,7]_{10}$, $[1,2,2,2,5,1,5,2,2,2,1]_{12}$
13	1400232320041	$[4,8,5,5,2,1,7,1,2,5,5,8,4]_{9}$, $[1,4,0,0,2,3,2,3,2,0,0,4,1]_{10}$
	2005542455002	$[7,0,8,1,5,8,0,8,5,1,8,0,7]_{9}$, $[2,0,0,5,5,4,2,4,5,5,0,0,2]_{10}$
	2024099904202	$[7,1,4,4,5,0,0,0,5,4,4,1,7]_{9}$, $[2,0,2,4,0,9,9,9,0,4,2,0,2]_{10}$
	2081985891802	$[7,3,3,0,8,6,4,6,8,0,3,3,7]_{9}$, $[2,0,8,1,9,8,5,8,9,1,8,0,2]_{10}$
	4798641468974	$[4,7,9,8,6,4,1,4,6,8,9,7,4]_{10}$, $[1,5,9,0,1,0,2,0,1,0,9,5,1]_{11}$
15	101904010409101	$[4,4,0,7,2,7,0,5,0,7,2,7,0,4,4]_{9}$,

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continued from previous page		
Digits d	Integer N	Base b Representations
		$[1, 0, 1, 9, 0, 4, 0, 1, 0, 4, 0, 9, 1, 0, 1]_{10}$
	149285434582941	$[6, 4, 6, 5, 1, 5, 7, 1, 7, 5, 1, 5, 6, 4, 6]_9$,
		$[1, 4, 9, 2, 8, 5, 4, 3, 4, 5, 8, 2, 9, 4, 1]_{10}$
	149819212918941	$[6, 4, 8, 4, 1, 6, 5, 1, 5, 6, 1, 4, 8, 4, 6]_9$,
		$[1, 4, 9, 8, 1, 9, 2, 1, 2, 9, 1, 8, 9, 4, 1]_{10}$
	463906656609364	$[4, 6, 3, 9, 0, 6, 6, 5, 6, 6, 0, 9, 3, 6, 4]_{10}$,
		$[1, 2, 4, 8, 10, 6, 7, 8, 7, 6, 10, 8, 4, 2, 1]_{11}$
17	11111059395011111	$[5, 8, 8, 6, 1, 8, 8, 6, 3, 6, 8, 8, 1, 6, 8, 8, 5]_9$,
		$[1, 1, 1, 1, 1, 0, 5, 9, 3, 9, 5, 0, 1, 1, 1, 1, 1]_{10}$
	11199701210799111	$[6, 0, 3, 5, 0, 7, 5, 8, 3, 8, 5, 7, 0, 5, 3, 0, 6]_9$,
		$[1, 1, 1, 9, 9, 7, 0, 1, 2, 1, 0, 7, 9, 9, 1, 1, 1]_{10}$
	13577478487477531	$[7, 2, 8, 4, 4, 7, 6, 7, 1, 7, 6, 7, 4, 4, 8, 2, 7]_9$,
		$[1, 3, 5, 7, 7, 4, 7, 8, 4, 8, 7, 4, 7, 7, 5, 3, 1]_{10}$
	14802554345520841	$[7, 8, 8, 0, 4, 4, 4, 1, 1, 1, 4, 4, 4, 0, 8, 8, 7]_9$,
		$[1, 4, 8, 0, 2, 5, 5, 4, 3, 4, 5, 5, 2, 0, 8, 4, 1]_{10}$
	54470642224607445	$[5, 4, 4, 7, 0, 6, 4, 2, 2, 2, 4, 6, 0, 7, 4, 4, 5]_{10}$,
		$[1, 2, 0, 4, 9, 0, 3, 0, 7, 0, 3, 0, 9, 4, 0, 2, 1]_{11}$
	56681764446718665	$[5, 6, 6, 8, 1, 7, 6, 4, 4, 4, 6, 7, 1, 8, 6, 6, 5]_{10}$,
		$[1, 2, 6, 2, 9, 6, 1, 4, 3, 4, 1, 6, 9, 2, 6, 2, 1]_{11}$
	56831729892713865	$[5, 6, 8, 3, 1, 7, 2, 9, 8, 9, 2, 7, 1, 3, 8, 6, 5]_{10}$,
		$[1, 2, 6, 7, 2, 3, 8, 2, 3, 2, 8, 3, 2, 7, 6, 2, 1]_{11}$
	62712119691121726	$[6, 2, 7, 1, 2, 1, 1, 9, 6, 9, 1, 1, 2, 1, 7, 2, 6]_{10}$,
		$[1, 4, 0, 1, 6, 0, 1, 7, 6, 7, 1, 0, 6, 1, 0, 4, 1]_{11}$
	64224652625642246	$[6, 4, 2, 2, 4, 6, 5, 2, 6, 2, 5, 6, 4, 2, 2, 4, 6]_{10}$,
		$[1, 4, 4, 1, 3, 10, 5, 4, 2, 4, 5, 10, 3, 1, 4, 4, 1]_{11}$
19	6411682614162861146	$[6, 4, 1, 1, 6, 8, 2, 6, 1, 4, 1, 6, 2, 8, 6, 1, 1, 4, 6]_{10}$,
		$[1, 1, 7, 5, 9, 10, 6, 7, 4, 6, 4, 7, 6, 10, 9, 5, 7, 1, 1]_{11}$
	7861736017106371687	$[7, 8, 6, 1, 7, 3, 6, 0, 1, 7, 1, 0, 6, 3, 7, 1, 6, 8, 7]_{10}$,
		$[1, 4, 6, 1, 0, 4, 5, 4, 1, 5, 1, 4, 5, 4, 0, 1, 6, 4, 1]_{11}$
21	104618510424015816401	$[8, 5, 4, 0, 1, 3, 3, 4, 0, 4, 1, 4, 0, 4, 3, 3, 1, 0, 4, 5, 8]_9$,
		$[1, 0, 4, 6, 1, 8, 5, 1, 0, 4, 2, 4, 0, 1, 5, 8, 1, 6, 4, 0, 1]_{10}$
	686833076121670338686	$[6, 8, 6, 8, 3, 3, 0, 7, 6, 1, 2, 1, 6, 7, 0, 3, 3, 8, 6, 8, 6]_{10}$,
		$[1, 0, 2, 5, 9, 5, 4, 1, 7, 7, 4, 7, 7, 1, 4, 5, 9, 5, 2, 0, 1]_{11}$
	771341832818238143177	$[7, 7, 1, 3, 4, 1, 8, 3, 2, 8, 1, 8, 2, 3, 8, 1, 4, 3, 1, 7, 7]_{10}$
		$[1, 1, 6, 8, 0, 7, 1, 1, 3, 10, 1, 10, 3, 1, 1, 7, 0, 8, 6, 1, 1]_{11}$
	903253059636950352309	$[9, 0, 3, 2, 5, 3, 0, 5, 9, 6, 3, 6, 9, 5, 0, 3, 5, 2, 3, 0, 9]_{10}$
		$[1, 3, 8, 5, 0, 4, 6, 6, 7, 10, 9, 10, 7, 6, 6, 4, 0, 5, 8, 3, 1]_{11}$
23	89403957605050675930498	$[8, 9, 4, 0, 3, 9, 5, 7, 6, 0, 5, 0, 5, 0, 6, 7, 5, 9, 3, 0, 4, 9, 8]_{10}$,
		$[1, 1, 0, 9, 9, 0, 10, 6, 6, 10, 6, 2, 6, 10, 6, 6, 10, 0, 9, 9, 0, 1, 1]_{11}$
25	9986831781362631871386899	$[9, 9, 8, 6, 8, 3, 1, 7, 8, 1, 3, 6, 2, 6, 3, 1, 8, 7, 1, 3, 8, 6, 8, 9, 9]_{10}$,
		$[1, 0, 1, 7, 5, 8, 7, 5, 2, 10, 9, 3, 3, 3, 9, 10, 2, 5, 7, 8, 5, 7, 1, 0, 1]_{11}$

5. Future Directions

There are a few papers in the literature which focus on palindromes in different base systems. For example, [2] considers those palindromes which are perfect squares. Article [1] generalizes the question by considering those which are perfect powers. Article [3] presents some results on the number of ways an integer can be expressed as a palindrome in different bases. In fact, we present the following problem:

> *What is the largest list of bases b for which an integer $N \geq 10$ is a d-digit palindrome base b for every base in the list?*

If one chooses $N = 66, 88, 676, 989$, it is easy to see that there exists a d-digit palindrome base 10 that has at least four different bases b for which it is a d-digit palindrome base b. It is unclear whether this is an upper bound on the number of different bases.

References

[1] Santos Hernández Hernández and Florian Luca. Palindromic powers. *Rev. Colombiana Mat.* **40** (2006), 81–86.

[2] Ivan Korec. Palindromic squares for various number system bases. *Math. Slovaca* **41** (1991), 261–276.

[3] Helena Kresová and Tibor Šalát. On palindromic numbers. *Acta Math. Univ. Comenian.* **42/43** (1984), 293–298.

[4] Florian Luca and Alain Togbé. On binary palindromes of the form $10^n \pm 1$. *C. R. Math. Acad. Sci. Paris* **346** (2008), 487–489.