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POISSON, THE PROBABILITY CALCULUS, AND PUBLIC EDUCATION

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Abstract. We examine Poisson's personal contribution to the probability calculus, placing it in the mathematical and social context of the beginning of the 19th century (§1). Then we look briefly at Poisson's administrative work in the Royal Council for Public Education from 1820 to 1840 (§2).

1. The Probability Calculus

At first glance, Poisson appears to have come to probability relatively late. It was at the age of 38, on March 13, 1820, that he read his first memoir on a question in probability to the Academy of Sciences, and the question seemed as anodyne as could be: how to calculate the house's advantage in the game of thirty and forty [71].

The decree of June 24, 1806, tolerated public games under certain conditions in the spa towns and in Paris. The ordinance of August 5, 1812, even conceded to the city of Paris the right to establish casinos and to derive from them proceeds that provided special funds for the police during the entire period of the Restoration. The most popular game at the time was thirty and forty, also known as "Red and Black."[1] Gamblers spent more than 230 million in 1820 francs on this game alone each year. So we can understand that the problem of calculating the house's advantage in advance came up.

Because we have to start somewhere and we are talking about probability and Poisson, we will start by briefly reviewing Poisson's calculation of the house's probability of winning in thirty and forty. He presented a simplified version of the problem as follows:

> An urn contains x_1 balls marked 1, x_2 balls marked 2, ... finally x_i balls marked i, the largest number on any of the balls. We successively draw one, two, three, ... balls, without putting them back in the urn after taking them out. This sequence of draws continues until the sum of the numbers on the balls drawn out equals or exceeds a given number x. What is the probability this sum will equal x? [71, pp. 176–177]

We set $x_1 + x_2 + \cdots + x_i = s$.

If the balls were put back into the urn after they were drawn, the solution of the problem would be simple. Indeed, it was known since the beginning of the 18th century that the

This article originally appeared in French as "Poisson, le calcul des probabilités et l'instruction publique," on pp. 51–94 of *Siméon Denis Poisson et la science de son temps*, Michel Métivier, Pierre Costabel, and Pierre Dugac, eds., Editions de l'Ecole Polytechnique, Palaiseau, 1981.

[1] The author uses quotations marks freely, and the passages and terms quoted are usually in French. We translate what is quoted into English but usually retain the quotation marks. We also translate names of institutions into English; occasionally we add the French name in parentheses. We leave names of books and periodicals in French. It is also interesting to mention that the subject of the present paper has also been extensively discussed in [39] and [87].

probability of obtaining the total x in m draws is given by the coefficient of t^x in the expansion of the polynomial

$$\left(\frac{x_1}{s}t + \frac{x_2}{s}t^2 + \cdots + \frac{x_i}{s}t^i\right)^m = \frac{T^m}{s^m},$$

and it follows that the probability $Z_{x,x_1,x_2,\ldots,x_i}$ of getting the same total x in any number of draws is the coefficient of t^x in the expansion of the series

$$1 + \frac{T}{s} + \frac{T^2}{s^2} + \cdots + \frac{T^m}{s^m} + \cdots,$$

i.e., in the expansion of $(1 - T/s)^{-1}$ [71, p. 197].

Called the method of generating functions, this method led Lagrange and Laplace quite naturally, by "passage from the finite to the infinitely small," to the idea of the Laplace transform. Indeed, because the probability $Z_{x,x_1,x_2,\ldots,x_i}$ is obviously the solution of the finite-difference equation

$$Z_{x,x_1,x_2,\ldots,x_i} = \frac{x_1}{s}Z_{x-1,x_1-1,x_2,\ldots,x_i} + \frac{x_2}{s}Z_{x-2,x_1,x_2-1,\ldots,x_i} + \cdots + \frac{x_i}{s}Z_{x-i,x_1,x_2,\ldots,x_i-1},$$

which describes the step from the next-to-last to the last draw, we see that the preceding method gives a solution to this type of equation and, by passage from the finite to the infinitely small, permits us to obtain solutions of certain partial differential equations as "definite integrals" – i.e., as Laplace transforms. Laplace presented this theory in his 1782 memoir [47].

Poisson undertook to adapt the argument to the case of drawing without replacement, where we no longer have convolution formulas that we can transform into products by interposing generating functions. It would be fastidious to give the details of the calculation, but we can certify its ingenuity. Poisson's result was that in the case without replacement, $Z_{x,x_1,x_2,\ldots,x_i}$ is the coefficient of t^x in the expansion of the integral

$$(s+1)\int_0^1 (1-y+yt)^{x_1}(1-y+yt^2)^{x_2}\cdots(1-y+yt^i)^{x_i}dy.$$

If the exponents $x_1, x_2, \ldots, x_i$ are large enough, we can apply the method that Laplace had perfected some forty years earlier [48] to this integral. Poisson showed in this way that the integral approximates the series obtained in the classical case, $(1 - T/s)^{-1}$. We suspected this might happen.

We might be surprised that such analytical virtuosity should be put to the service of so prosaic a question. But this is very much the style Poisson inherited from Laplace: subordinate methods to applications to the point that the generality and elegance of the methods disappear in favor of the specificity and bad taste of the applications. This Sulpician style[2] is surely one of the reasons that Poisson's mathematical contribution, though considerable, has so often been undervalued relative to that of contemporaries such as Fourier, who always seemed to hone in on the essential, Poinsot, who always sought elegance and generality, or Cauchy, whose torrential production did not pause over such profane considerations.

It was certainly not on the occasion of this memoir that Poisson discovered the probability calculus. Ten years earlier, as mathematics editor of the *Bulletin de la Société Philomatique*, he had published summaries of Laplace's two great memoirs of 1810 and 1811, memoirs that led Laplace, as we know, to undertake the monumental *Théorie analytique des probabilités*. Given

[2]The Sulpicians, a Roman Catholic order, are known for their elegance and high moral tone, but in France their name also evokes the religious trinkets, thought tacky by some, traditionally sold in the neighborhood of the church of Saint Sulpice in Paris.

these memoirs' central role in the development of probability and their influence on Poisson's work, it is appropriate to talk about them now.

To simplify, we begin in 1776. Two memoirs appearing that year, one by Lagrange [42] and the other by Laplace [44], both set for themselves, for quite different reasons [33, §6], the task of finding the probability distribution of the sum (or arithmetic average) of a large number n of random variables with the same known density ϕ – i.e., of evaluating the nth convolution ϕ^{*n} of ϕ.

Lagrange, inspired by a memoir by Simpson that had appeared 20 years earlier [84], used for this purpose a curious formula for inverting the "Laplace transform" that is valid for some functions ϕ. Laplace, taking up the problem again in 1777 [34], gave the now classic integral formula for the convolution of an arbitrary function ϕ that is zero outside an interval. But he acknowledged that the numerical calculation is impractical when n is too large and admitted that he could not obtain an asymptotic evaluation of

$$1_{[0,100]}^{*100},$$

which was indispensable for the application he had in mind. (Here $1_{[0,100]}$ designates the function whose value is 1 in the interval $[0, 100]$ and 0 elsewhere.)

During the following thirty years, Laplace would be led to calculate a very large number of asymptotic expansions of definite integrals containing large powers – see, for example, [47, 48, 49]. But the method he used did not apply to convolution formulas, which "needing to be halted when the variable becomes negative," do not lead directly back to products. On the other hand, as both Laplace and Lagrange knew, the Laplace transform, which changes a convolution into a product, inverts poorly in the real domain. And the technique of "passing from the real to the imaginary," already used audaciously by Laplace for calculating real definite integrals with the help of an imaginary change of variables, did not yet have the depth and flexibility Cauchy would give it.

Yet on April, 9, 1810, thirty-four years after he had first clearly posed the problem, Laplace announced to the Academy of Sciences the solution we now know: for an even function ϕ with compact support and for x of order $\sqrt{n}$,

$$\phi^{*n}(x) \simeq \frac{1}{\sqrt{2\pi c_n}} e^{-s^2/2c_n},$$

where $c_n = n \int_{-\infty}^{\infty} x^2 \phi(x)dx$. An interesting case of mathematical stubbornness. As Fourier wrote [29]: "An imperturbable consistency in viewpoint was always the main feature of Laplace's genius."

We have a right to ask about the reasons for Laplace's late success, obtained after he had abandoned the problem twenty years earlier [34, p. 265]. All the more so because he did not stop there; the following year he used the results of this initial memoir to give the first satisfying probabilistic theory of the method of least squares [54], and he never again abandoned the field of the probability calculus. The following argument could be made as a partial response to the question.

In 1807, responding to a question posed by the Academy of Sciences, Fourier, then the prefect of Isère and generally considered lost to science, derived the heat equation and solved it in the particular case of a torus with a given initial distribution of temperature. To do this, he remarked that the solution is trivial when the distribution is sinusoidal and then derived the case of an arbitrary initial distribution by developing it in a "Fourier" series.

Fourier's 1807 manuscript, of which Poisson published a summary in the *Bulletin de la Société Philomatique* in 1808, seems to have been badly received in the Parisian scientific community, particularly by Lagrange [35]. For his part, Laplace criticized the argument's physical assumptions in his 1808 memoir [50], where he gave what he considered "the true foundations of the heat equation" but said nothing about the theory of Fourier series. Looking for the solution of a differential equation in the form of a series, even a trigonometric series, would not have been particularly remarkable in his eyes. Laplace himself had already used, in his 1785 memoir [48, §XXIII], an artifice similar to the one Fourier gave for calculating the (Fourier) coefficients of a function. Perhaps also Laplace had simply not read Fourier's manuscript, which was deposited at the Academy in 1807 but not published until 1821.

During the summer of 1809 [35, p. 443ff], Fourier left Grenoble to stay in Paris for nearly a year, during which he completed editing the *Description de l'Egypte*. He was then named baron by the emperor, arriving at the apex of civil honors. Laplace, known not to be insensitive to the vanities of titles, received him in his estate at Arcueil, then the uncontested center of world science.

Neither Laplace nor Fourier wrote about these meetings at Arcueil at the end of 1809. We have only Fourier's late testimony [29]. Speaking of the visitors at Arcueil, he wrote, "Some were beginning their careers; others would soon have to finish theirs. Laplace treated them all with extreme politeness. He went so far that he would have given those who did not yet understand the full extent of his genius reason to believe that he himself could reap some benefit from the conversations." A clever sentence. How could you better say that you had been an inspiration for someone to whom you are required to pay academic homage?

Whatever the exact influence of those meetings, it is undeniable that from that point on Laplace's and Fourier's styles, while each remaining inimitable, begin more to resemble each other.

In his 1811 memoir, Fourier used the passage from the finite to the infinitely small that Laplace cherished to solve the problem of heat propagation with given initial temperatures in an infinite rod, a problem that had apparently brought him up short in 1807. It is enough to describe the function giving the initial temperatures at each point of the rod as a Fourier transform rather than a Fourier series.

As for Laplace, we will not speak about the 1809 memoir, in which Fourier's influence is clear. But in 1810, as we have already said, he published his "Memoir on approximations for formulas that are functions of very large numbers and on their application to probabilities" [35], in which he gave the solution to the 1776 problem. It is not easy to grasp the "new point of view" at the root of Laplace's solution, for he used nonstandard analysis freely, treating definite integrals as sums when he felt the need. Poisson himself does not seem to have grasped it immediately in his summary [67]. He gave greater emphasis to the end of Laplace's memoir, where Laplace treated the problem of evaluating $1_{[0,100]}^{*100}$ using two other more traditionally Laplacian methods, "integration by approximation of equations involving finite and infinitely small differences" and "reciprocal passage from imaginary to real results."

In 1824, however, Poisson would be the first to publish a comprehensible and rigorous version of Laplace's general demonstration [74]. In "modern" language, the result can be written as follows:

Let ϕ be an even positive function with compact support satisfying

$$\int_{-\infty}^{\infty} \phi(x)dx = 1 \quad \text{and} \quad \int_{-\infty}^{\infty} x^2\phi(x)dx = \sigma^2.$$

Write $\hat{\phi}$ for the Fourier transform of ϕ;

$$\hat{\phi} = \int_{-\infty}^{\infty} e^{itx}\phi(x)dx.$$

Then

$$\phi^{*n}(\sqrt{n}x) = \frac{1}{\pi}\int_{-\infty}^{\infty} e^{-it\sqrt{n}x}(\hat{\phi}(t))^n dt \tag{1.1}$$

$$= \frac{1}{\sqrt{n}\pi}\int_{-\infty}^{\infty} e^{-iux}(\hat{\phi}(\frac{u}{\sqrt{n}}))^n du \simeq \frac{1}{\sqrt{2\pi n\sigma^2}}e^{-\frac{x^2}{2\sigma^2}}.$$

We recognize equation (1.1), our modern rendering of equation (o′) in Laplace's memoir, as Fourier's inversion formula, which appeared in the 1811 manuscript and would be published in 1821.[3] But this has little importance, because Laplace considered the equality a trivial consequence of the formula for Fourier coefficients, which appeared in Fourier's 1807 manuscript. Cauchy, who published the first known demonstration of Fourier's formula in 1817 [14], attributed it to Poisson, who had in fact used it in his work on wave theory in 1816. Fourier having claimed the paternity of the formula, Cauchy quite willingly restored it to him in his next memoir [15].

Because we know that Laplace never acknowledged the least direct influence, we can hardly be surprised that he cited Fourier neither in his 1810 and 1811 memoirs nor in his *Théorie analytique*. It is nevertheless notable that he used his immense scientific authority in support of Fourier beginning in this period [35].

Finally, let us note that paragraph VIII of Laplace's 1810 memoir contained the first use of the Fourier transform to solve a differential equation. Poisson and Cauchy seized on this method very quickly. Like the Laplace transform, the Fourier transform gave solutions of equations in the form of definite integrals that could subsequently be calculated or, if one could not calculate them explicitly, evaluated by a quickly convergent series. Already in 1809 Laplace gave the first notable Fourier transform [51]:

$$\int_{-\infty}^{\infty} e^{itx}e^{-\frac{x^2}{2}}dx = \sqrt{2\pi}e^{-\frac{t^2}{2}}.$$

In 1811 Laplace [53] and Poisson [70] showed that

$$\int_{-\infty}^{\infty} e^{itx}\frac{dx}{1+x^2} = \pi e^{-t} \quad \text{if} \quad t \geq 0.$$

This integral is often attributed to Cauchy; we will soon encounter it again.

As we have seen in this introduction, Poisson was heavily involved with probability starting in 1810, at the very moment when the theory, after a twenty-year intermission, returned to the forefront of the scientific scene. In his 1810 summary, Poisson announced his intention to look for a "direct" demonstration of Laplace's theorem. We know he did not succeed, but until the end of his life he devoted himself to clarifying and simplifying Laplace's asymptotic theory. His *Recherches sur la probabilité des jugements*, published in 1837, is merely a window on this enormous work. We propose to examine some of its aspects in the following.

[3] In 1811, Fourier wrote a second paper on the heat equation. Though it won a prize, it joined his 1807 paper in the drawer; neither was published until the 1820s. Here the author calls the 1811 paper a "manuscript," even though he had earlier called it a "memoir."

1.1. **Laplace's theorem and the theory of errors.** Poisson's results on Laplace's theorem, which we have already mentioned, were published in 1824 and 1829 in the *Additions* to the *Connaissance des temps* [74, 75], one of the publications of the Bureau of Longitudes. (Poisson was adjunct geometer at the Bureau starting in 1808 and then, after Laplace's death in 1827, chief geometer.) It was a matter, Poisson said [74], of simple remarks designed to facilitate the reading of Chapter IV of Book II of Laplace's *Théorie analytique*. In fact, Poisson took up and clarified all Laplace's results, making explicit so far as possible their conditions of validity and proposing counter-examples, something not common in the literature of the period.

In the 1824 memoir, Poisson used Fourier's inversion formula to give the first clear and concise theory of the exact calculation of the probability law for the sum of a fixed number of "errors of observation." He gave a complete treatment for the uniform law $\mathbb{I}_{[A,B]}$, the so-called Gaussian law with e^{-x^2}, and the so-called Cauchy law with $1/(1+x^2)$, for which he had calculated Fourier transforms thirteen years earlier. He noted in particular that the law of the average error in the case of the Cauchy law is independent of the number s of observations, "from which it follows that in this particular example, the average error will not converge to zero or any other fixed quantity as the number s increases. No matter how large the number of observations, there will always be the same probability for the average anticipated error falling between given limits." This result is usually attributed to Cauchy, who actually obtained it about thirty years later [16]. Paul Lévy, who first proposed naming the function $1/\pi(1+x^2)$ after Cauchy, attributed responsibility for this erroneous reference to Polya [59, p. 78].

Poisson then extended Laplace's theorem to probability densities ϕ that satisfy $\int_{-\infty}^{\infty} x^2\phi(x)dx < \infty$. His demonstration, essentially rigorous, is sometimes attributed to Cauchy, who never even considered the question.

In the second part of his memoir, Poisson undertook, following Laplace, to evaluate the distribution of a linear combination $\sum_{i=1}^{s}\gamma_i\varepsilon_i$ of errors $\varepsilon_1,\ldots,\varepsilon_s$. Assuming that the ε_i are identically and symmetrically distributed, Laplace had of course found that $\sum_{i=1}^{s}\gamma_i\varepsilon_i$ is asymptotically normal and centered, with variance $\sum_{i=1}^{s}\gamma_i^2\mathrm{Var}\varepsilon_i$. Poisson gave several examples to show that this result may fail. One example is where ε_i has a symmetrized exponential distribution, with density $e^{-2|x|}$, and $\gamma_i = 1/i, i \geq 1$; we then have

$$P\left(\left|\sum_{i=1}^{s}\gamma_i\varepsilon_i\right| \leq c\right) \approx \frac{1-e^{-2c}}{1+e^{2c}}. \quad \text{[74, p. 290]}$$

Fourier considered Poisson's examples artificial [27], but Bienaymé later observed that the effect of compound interest on the profits of insurance companies results from this sort of "abnormal" behavior [6].

On the other side, Poisson showed that Laplace's result extends to the case where errors are not identically distributed under the general though not very precise condition that the product of the Fourier transforms vanishes rapidly far from the origin. Poisson later returned to his proof [78, chap. IV]; he then "rigorously" proved that if $\gamma_i = 1$ and if the ε_i take two values 0 and 1 with probabilities $1-p_i$ and p_i, the "necessary and sufficient condition" for the sum $\sum\varepsilon_i$ to be asymptotically normal is that the series $\sum p_i(1-p_i)$ diverges. Poisson has been credited with this result, but actually it is implicit in Chapter IX of Book II of the *Théorie analytique*. After Chebyshev, the St. Petersburg school sharpened Poisson's results and obtained contemporary versions of the central limit theorem (though not of the law of large numbers, which is much weaker than Poisson's theorem in its weak form and much too strong in its strong form).

In the last part of [74], Poisson takes up Laplace's theory of least squares, developing the technique Laplace had proposed in the first supplement to the *Théorie analytique* (1816) for determining the unknown parameters involved in the asymptotic formulæ; see [83] for a more detailed analysis. This is the starting point for Poisson's second memoir [75], remarkable not so much for its originality as for its presentation. As the problem plays a certain role in the following considerations, we present it in a simplified version.

Consider a large number of errors in observations $\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n$ with a common centered distribution ϕ. Laplace's theorem gives an asymptotic evaluation of the probability density for the arithmetic mean, namely

$$\frac{\sqrt{n}}{\sqrt{2\pi}\sigma} e^{-\frac{nx^2}{2\sigma^2}} \text{ with } \sigma^2 = \int_{-\infty}^{\infty} x^2 \phi(x) dx.$$

But because the density ϕ is unknown a priori, the parameter σ is as well. So in the first supplement, Laplace proposed to replace σ^2 by the mean of the squares of the observed errors, $\frac{1}{n}\sum_{i=1}^{n} \varepsilon_i^2$.

Poisson, for his part, proposes to justify this method. To this end, he computes the Fourier transform of the distribution of

$$\frac{X(\varepsilon_1) + \cdots + X(\varepsilon_n)}{n},$$

where X is an arbitrary "increasing" function. He thereby shows directly that this expression is asymptotically normal with mean $\int_{-\infty}^{+\infty} X(u)\phi(u)du$, so that we may write

$$\frac{1}{n}\sum_{i=1}^{n} X(\varepsilon_i) \approx \int_{-\infty}^{\infty} X(u)\phi(u)du + \frac{\alpha g}{\sqrt{n}},$$

where g is a constant and α a random quantity with the standard normal distribution:

$$P(\mid \alpha \mid \leq a) = \sqrt{\frac{2}{\pi}} \int_0^a e^{-\frac{x^2}{2}} dx.$$

So if we neglect quantities of order $1/\sqrt{n}$, we can replace σ^2 by $\frac{1}{n}\sum_{i=1}^{n} \varepsilon_i^2$ with an arbitrarily large probability.

The reader will have decided that this result is gratuitous – that it is already contained in Laplace's theorem applied to the sequence $X(\varepsilon_1), X(\varepsilon_2), \ldots, X(\varepsilon_n)$. But how could Poisson assert it a priori, without a clear theory or even an approximate definition of what a "random variable" is? Laplace seems never to have worried about this conceptual difficulty, no more than he ever worried about explaining what he meant by the word "function"; Laplace's theorem was concerned simply with a sequence of "errors with the same law of facility." We can agree that a sum of errors is still an error, but is an increasing function of an error still an error, and how does the law of facility change? This is the kind of difficulty that Poisson brought to light in this second memoir. In the *Recherches* [78, p. 140] he also became the first, as Sheynin quite properly emphasizes [83], to give a "definition" of the notion of a random quantity. Above and beyond the theory of errors developed by Laplace and Gauss [55], Poisson was already looking for a theory of random variables. After Poisson, Cournot would propose a remarkable practical theory of random variables [19, chap. VI] that anticipated Kolmogorov's axiomatic theory.

1.2. **Statistics of births and Poisson's theory of inference.** Poisson's first statistical study, devoted to the "proportion of births of girls and boys," appeared in 1824 in the almanac (*annuaire*) of the Bureau of Longitudes [72]. It was repeated in the lengthy memoir read on February 8, 1829, at the Academy of Sciences [73] and again in the *Recherches*.

The Bureau of Longitudes had been created by the Convention (law of the 7th of Messidor, year III of the Republic, or June 25, 1795) to "improve the different branches of astronomical science and their application to geography, navigation, and the physics of the earth." Aside from the annual astronomical tables, the "knowledge of times," the Bureau also had to publish each year an almanac "suitable for setting straight those of the whole Republic." François de Neufchâteau, known for his essential role in the development of national statistics, had designed the almanac as interior minister, and he had included vital statistics. So Laplace, geometer along with Lagrange at the Bureau, inevitably found himself supervising the publication of the statistics of France, and so it went also with Poisson, his adjunct and later his successor.

Reworking the political arithmetic of the 17th and 18th centuries, Laplace had already applied his analytical methods to determining the "possibility of an event,"[4] such as the birth of a boy, on the basis of many observations. We could say confidently that the whole Laplacian theory of inference derives from the single example of the proportion of births of girls and boys. Let us briefly review the problem. Since the first compilations of civil registers and Arbuthnot's famous memoir [2], it was known that the proportion between the numbers of births of girls and boys was relatively stable from one year to the next, the number of boys always being higher. Since Nicolas Bernoulli [62], it was also known that this relative stability was very much like that observed in the proportions of heads and tails in coin tossing. The mathematical problem was therefore to study precisely the variation of these very strangely stable proportions, one of the goals being to decide whether the observed deviations from one year to the next, or one country to another, remain within theoretical limits or not. And as Laplace indicated in 1780 [46], "this subject is one of the most interesting to which we can apply the probability calculus."

If the probability of the birth of a boy were given a priori, the problem just posed could be solved by Bernoulli's theorem, as made more precise by De Moivre (and Laplace [55]): If p is the probability of a birth of a boy and N_n the number of boys observed in n births, one has (almost in Poisson's notation)

$$\frac{N_n}{n} - p \approx \alpha\sqrt{\frac{p(1-p)}{n}} \text{ with } P(\mid \alpha \mid \leq a) = \sqrt{\frac{2}{\pi}}\int_0^a e^{-\frac{x^2}{2}}\,dx. \tag{1.2}$$

The difficulty with this equation is that the parameter p appears on both sides. If it is unknown, as it generally is, the equation gives little information about the fluctuations of N_n/n.

Laplace first resolved this technical difficulty using Bayes's method [43], which, as we cannot say too often, owes Laplace everything but its name. In this method, we suppose that every value of p is "a priori equally probable." We then show that, if we have observed m births of boys among n births, the fluctuations in the number $N_{n'}$ of births of boys that will be observed among n' new births is governed by (1.2) with p replaced by m/n. For large numbers, that is to say,

$$\frac{N_{n'}}{n'} \approx \frac{m}{n} + \alpha\sqrt{\frac{m(n-m)}{n^2 n'}}, \tag{1.3}$$

[4]The French word for probability is *probabilité*, but Laplace often used *possibilité* to refer to what we might call an objective probability.

with α as before. After a first unsuccessful attempt in 1780 [46], Laplace obtained (1.3) in 1786 [49] using his method (not by chance called Laplace's method!) for evaluating integrals containing factors with large exponents [48]. Poisson was to give the first clear account of Laplace's method and its application to (1.3) in his 1829 memoir [73].

Using this formula, Laplace concluded that

(1) The possibility of a birth being a boy in London is greater than in Paris.
(2) The same is true for the Kingdom of Naples, but to a lesser extent.
(3) "Over a century," we can "bet almost two to one that more boys than girls will be born every year."

Formula (1.3) naturally suggested a second method, called the "inverse-Bernoulli" method by Todhunter [89], that Laplace used systematically starting in 1816; we simply replace the unknown p by the observed value N_n/n in the error term

$$\alpha\sqrt{\frac{p(1-p)}{n}}$$

of (1.2). As we have seen, Poisson had given an asymptotic justification for this non-Bayesian method in the last memoir we analyzed [75].

Poisson took up Laplace's investigation again in [73], spelling out proofs as usual. He also proved, as Laplace apparently did not explicitly do, that if we count s boys out of m births in one population and s' out of m' in another, and we assume that all values of the respective possibilities p and p' for a boy are equally probable, then the a posteriori distribution of the difference $p - p'$ is asymptotically normal. We have

$$p - p' \approx \frac{s}{m} - \frac{s'}{m'} + \alpha\sqrt{\frac{s(m-s)}{m^3} + \frac{s'(m'-s')}{m'^3}}, \tag{1.4}$$

which gives very precise information about $p - p'$ and allows us to decide whether p and p' are significantly different. Poisson draws these conclusions (quoted verbatim):

(1) The ratio of births of boys to girls is 16/15, instead of 22/21, as previously believed.
(2) This ratio is almost the same for the south of France as for the whole of France, appearing to be independent of variation in climate, at least in our country.
(3) Its value for illegitimate births, approximately 21/20, is significantly less than for legitimate births.

Poisson avoided offering the least opinion based on these results, but we can be assured that this was not true of his successors; on this point see particularly Quételet's treatise *Sur l'homme* [79]. Arago, commenting on Poisson's third conclusion, offered this opinion: "One sees how important it would be to make the same calculations for places where polygamy occurs; but unfortunately we have no data."

As we have just seen, Poisson's 1829 statistical memoir is purely Bayesian. But like Laplace at the end of his life, Poisson thought that asymptotic methods, using large numbers, should allow us to do without any a priori hypothesis, such as that of a uniform probability distribution for the parameter. This is why, when he returns to the problem in the *Recherches*, he proposes a third method. Let us go back to the "inverse-Bernoulli" version of equation (1.3):

$$\frac{N_n}{n} \approx p + \alpha\sqrt{\frac{N_n(n-N_n)}{n^3}}. \tag{1.5}$$

It gives a "confidence interval" for p:

$$P_p\left(\left|\frac{N_n}{n}-p\right|\leq a\sqrt{\frac{N_n(n-N_n)}{n^3}}\right)\approx\sqrt{\frac{2}{\pi}}\int_0^a e^{-\frac{x^2}{2}}dx. \tag{1.6}$$

Notice, while we are here, that if we obtain n' new observations, we then have, with obvious notation,

$$\frac{N'_{n'}}{n'}\approx p+\alpha'\sqrt{\frac{N'_{n'}(n'-N'_{n'})}{n'^3}}. \tag{1.7}$$

Taking differences and making an orthogonal change of variables, Poisson obtains from (1.5) and (1.7) the relationship

$$\frac{N_n}{n}-\frac{N'_{n'}}{n'}\approx\alpha\sqrt{\frac{N_n(n-N_n)}{n^3}+\frac{N'_{n'}(n'-N'_{n'}))}{n'^3}},\quad [78, p. 223] \tag{1.8}$$

which also allows us to decide whether the two observed frequencies are significantly different or not. If the possibility p is the same for the two sequences, one should have

$$\left|\frac{N_n}{n}-\frac{N'_{n'}}{n'}\right|\leq a\sqrt{\frac{N_n(n-N_n)}{n^3}+\frac{N'_{n'}(n'-N'_{n'}))}{n'^3}}, \tag{1.9}$$

a being chosen so that $\sqrt{\frac{2}{\pi}}\int_0^a e^{-\frac{x^2}{2}}dx$ is as close to 1 as desired.

But let us go back to (1.5). It can be written

$$p\approx\frac{N_n}{n}+\alpha\sqrt{\frac{N_n(n-N_n)}{n^3}}. \tag{1.10}$$

Without any a priori hypothesis, equation (1.10) gives the "probability law" for p after we have observed the actual number N_n of boys out of n births: if we have observed that $N_n=m$, the "a posteriori probability law" for p is the normal distribution with mean m/n and variance $m(n-m)/n^3$. In the case where (1.10) is exact, not asymptotic as here, Fisher cunningly called this "fiducial" reasoning [26].

From here, Poisson easily recovers the Bayesian formulæ of his memoir on births and in particular formula (1.4): it is sufficient to compute the distribution of the difference of two independent normal variables p and p' [78, p. 227, formula (26)], and Poisson does this with the classical orthogonal change of variables (already present in [55, II, no. 27]).

So does this make Poisson the first fiducialist anti-Bayesian statistician? The question is actually meaningless. Poisson was not looking for opportunities for academic battle in the proliferation of methods and points of view. Rather, like Laplace, he was looking for confirmation of probability theory as a whole. He must have thought, like Condorcet, that we never attain truth but can approach it as closely as we want, with an arbitrarily large probability, by accumulating partial truths in the best possible way. Perhaps he also was beginning to think, like Cournot, that so remarkable a concordance of results obtained by methods so independent could hardly come from blind chance, and that it reinforced the "philosophical probability" of the whole theory of chance and its correspondence with nature (see [78, p. 103] for a hint in this direction).

Before leaving the question of births, we should mention the "Poisson distribution" that appears in the 1829 memoir on pages 261 and 262. Most of Poisson's fame as a probabilist is associated with this distribution, but the well-informed reader knows that traces of the distribution can be found well before 1829, for example in the first edition of the *Doctrine of Chances* by De Moivre in 1718 [61, p. 45] [38].

Poisson, reworking Laplace's proof of (1.2), observes correctly that it is valid only if the products np and $n(1-p)$ are both very large. So he studies separately the case where one of them, say np, remains small for a long time. Setting $np = \omega$, we have

$$P(N_n \leq x) = \sum_{k=0}^{x} \binom{n}{p} p^k (1-p)^{n-k} = e^{-\omega}\left(1 + \frac{\omega^2}{2} + \cdots + \frac{\omega^x}{x!}\right) + O\left(\frac{1}{n}\right).$$

This is Poisson's formula. Reproduced in the *Recherches* on page 205, it will turn out to have a remarkable career. Already highly recommended by Cournot for use in the theory of insurance [19, p. 331], it owes the kernel of its success to von Bortkiewicz at the end of the 19th century, who found it in every enumeration of rare events. It is the preeminent law of small numbers, to which even the Prussian cavalry is subject [9].

Even then, the Poisson distribution would not have been more than statistical folklore but for the well-known independent rediscoveries, at the beginning of the 20th century, by Erlang [25] and Bateman [3], the former in the context of telephone calls and the latter in the context of atomic decay. And it was Paul Lévy who placed the distribution on the theoretical summit where it now stands.

By an irony of fate, Poisson himself obviously never attached the least importance to the expression $e^{-\omega}\omega^k/k!$. What is more, this embarrassing formula might just as well have been attributed to Fourier. Fourier estimates the first terms of the binomial development of $(\alpha+\beta)^p$ when β/α is of order $1/p$ in notes for a class on the probability of testimony that we find in the manuscript (National Library MFF 22515 pp. 36–41) of the probability course he taught in 1818 at the Atheneum (*Athénée*). Curiously enough, he gets a bit lost in the calculations, but we may hope that his oral presentation was better than his notes. In any case, he concludes that when $\beta/\alpha = 1/p$, "the respective probabilities of losing less than 1 or 2 or 3 or 4 or 5 are approximately represented by the numbers

$$1,\ 1+\frac{1}{2},\ 1+\frac{1}{2}+\frac{1}{2.3},\ 1+\frac{1}{2}+\frac{1}{2.3}+\frac{1}{2.3.4}\ldots\text{"}$$

Fourier returns to this problem later in the context of installment payments (page 62 of the same manuscript); he then evaluates the initial terms of the binomial expression $(\frac{n-1}{n}+\frac{1}{n})^m$.

We know that Fourier intended to write a treatise on the probability calculus; he had even published chapters of it as lead articles in the first issues of the *Recherches statistiques de la ville de Paris* [27],[5] for which he was editor. His accidental death in 1830 prevented the completion of a project that would surely have overshadowed Poisson's book, for reasons we explain in the next section.

So a puff of air would have been enough for Poisson processes to have been called Fourier processes. Certainly De Moivre did not stand a chance; who had read De Moivre in the 19th century, except Todhunter [89], who had not seen Poisson's formula there? But would this have removed Poisson altogether from the probabilistic scenery? Certainly not, for he is the inventor of one of the most arresting slogans of probability theory, "the law of large numbers," to which we now turn.

1.3. **The law of large numbers and the probability of judgements.** In the previous section, we considered the problem of applying probability to natural phenomena by means of a statistical apparatus that provides a numerical result no matter what happens and no matter

[5]See also the present issue of the *Electronic Journal for History of Probability and Statistics.*

what method one uses. As we might expect, the real difficulty begins only afterwards: just what significance, if any, might the result have?

This problem appeared at the very beginning. One may find it already in the correspondence between Bernoulli and Leibniz in the winter of 1703–1704 [58, pp. 72–89]. To Bernoulli, who had claimed in his letter of October 3, 1703, to have a method for obtaining the absolute probability of an event a posteriori, with the same certainty as if it were known to us a priori, Leibniz answers on December 3, 1703: "Nature has its own habits, born from recurrence of causes, but only in general. So who can say that a new experiment will not deviate from the rule of preceding experiments because of the great variability of things? New illnesses pour down constantly on humankind. If you had carried out as many experiments as you wanted on the causes of mortality, you would not have arrived at the limits of the world, beyond which these causes could not change in the future." To this objection, Bernoulli answered (April 20, 1704) that it is then necessary to obtain more observations, as "it is clear that the old observations cannot apply to the new ones." (See also [4, pp. 227–228].)

We find this problem of the variability of chances [19, chap. 7] intact at the beginning of the 19th century, where it will be one of the statistical themes in fashion. Fourier, Poisson, Bienaymé, and Cournot will study it before it is taken up again by the "continental" school at the end of the century [41] and then integrated into the theory of spatial and temporal processes at the beginning of the 20th century.

Until about 1820, especially during the Laplacian period, the available statistical data contained so little truth that the conclusions one could obtain from them using the probability calculus could not substantially worsen their truthlessness, even when the hypotheses, generally not spelled out, on which these conclusions were based had the least possible justification. On the contrary, the ingenuity and the power of the methods used could bring the data back to its truth, or at least as close to it as desired, with a probability as close to one as desired. In the limit, the data are no longer indispensable; it is enough to have men sufficiently enlightened to show the best way to do without them. The most interesting example of this alliance of Enlightenment philosophy and analytical methods is evidently the *Essai sur l'Application de l'Analyse à la probabilité des décisions rendues à la pluralité des voix* by Condorcet [17] [78, p. 2].

This explains in part how it can be that Laplace, who improved or even created from whole cloth most of today's statistical techniques [86, for example], never obtained really important statistical results, despite the impressive number of decimals with which he usually embellished them. Aside from the proportion of births, already mentioned, one can give as an example the discussion of the mass of Jupiter [19, p. 242].

Just the same, statistics was developing at the beginning of the 19th century as an autonomous science, with its proper methods of numeration and comparison, driven by great administrators in service to the state and humanists in service to mankind and industry. The Latinist Alfred de Wailly, Poisson's son-in-law, celebrated them in the person of Montyon [90]:

Thanks to you, Montyon, exercised hands
Construct for our beautiful country tables
Which present to the eyes of the young magistrate
The state and the customs of an entire province,
Its arts, inhabitants, needs, and sorrows,
The products proving the wealth of the soil,

In a word, the good that must be encouraged,
And above all the abuses that ought to be corrected.[6]

The first statistical data worthy of the name appearing in France were the *Recherches statistiques sur la ville de Paris* starting in 1821, the *Comptes généraux de l'administration de la justice criminelle* starting in 1825, and the first real censuses starting in 1836 (the earlier ones being quite rudimentary).

The geometers' enlightenment would no longer suffice; it was necessary to take the data into account. So Leibniz's objection to Bernoulli regained all its pertinence. Some pure statisticians were to go so far as to assert that the data are sufficient, the geometers' intervention serving only to obscure them (see [36] for numerous references on this subject).

So it was a matter of reconquering the market for statistics. Of course, as Bernoulli admitted, causes and therefore chances can vary by time and place, but only the probability calculus permits us to identify them [78, p. 8]. For this, it is sufficient, as Fourier recommended [27, vol. 3], to break the totality of the observations into groups of adequate size and to compare the frequencies with which results appear; as we have said, Poisson's formulæ (1.4) or (1.8) allow us to decide whether the frequencies are significantly different. If the frequencies stay within limits tolerated by the formulæ, we may conclude with high probability that there has been no variation of causes. Poisson called this certified stability of frequencies, which was applicable to "all sorts of things," the "law of large numbers" [78, p. 143].

Now appears a new difficulty, the source of much confusion. For simplicity, let us go back to the problem of births studied in the previous section. Poisson observes [73, no. 17] that the probability p for a boy's birth must vary from one family to another. So what does the stability of frequency mean?

After first trying to explain the matter in [73], Poisson returns to the problem in the *Recherches*. He supposes that each observed family belongs to one of ν well-defined types of families. For each type, there is a probability $c_i, 1 \leq i \leq \nu$, for the birth of a boy. For simplicity, suppose a particular family has equal chances of belonging to one or another of these types. (Poisson actually considers a slightly more general case [78, p. 220].) Let $p_1, p_2, \ldots, p_n, \ldots$ be the sequence of (random) chances that the observed families have for getting a boy. Applied to this sequence, Laplace's theorem (§1.1) can be written as

$$\frac{1}{n}(p_1 + \cdots + p_n) \approx \frac{1}{\nu}(c_1 + \cdots + c_\nu) + \frac{\alpha\sqrt{g}}{n}$$

with

$$g = \frac{1}{\nu}\sum_{i=1}^{\nu} c_i^2 - (\frac{1}{\nu}\sum_{i=1}^{\nu} c_i)^2 \quad \text{and} \quad P(\mid \alpha \mid \leq a) = \sqrt{\frac{2}{\pi}}\int_0^a e^{-\frac{x^2}{2}}dx.$$

[6] *Grâce à toi, Montyon, par des mains exercées,*
Pour notre beau pays des tables sont tracées,
Qui présentent aux yeux du jeune magistrat
D'une province entière et les mœurs et l'état,
Ses arts, ses habitants, leurs besoins, leur détresse,
Les produits qui du sol attestent la richesse,
En un mot, et le bien qu'il faut encourager,
Et les abus surtout qu'il faudrait corriger.

The generalization of Laplace's theorem Poisson obtained then shows that the number N_n of boys born in the first n observed families satisfies

$$\frac{N_n}{n} \approx \frac{1}{n}\sum_{i=1}^{n} p_i + \alpha' \frac{\sqrt{\gamma_n}}{\sqrt{n}}$$

with

$$\gamma_n = \sum_{i=1}^{n} p_i(1-p_i) \quad \text{and} \quad P(\mid \alpha' \mid \leq a) = \sqrt{\frac{2}{\pi}} \int_0^a e^{-\frac{x^2}{2}} dx.$$

One finally obtains

$$\frac{N_n}{n} \approx \frac{1}{\nu}\sum_{i=1}^{\nu} c_i$$

up to a controllable error.

So frequencies are stable. This is the proof of the "law of large numbers" that Poisson announced to the Academy of Sciences at the beginning of 1836 and to which he admitted attaching "a great importance" [77, p. 396].

The careful reader has surely noticed that this proof, as remarkable as it may be, is unneeded, because the two-step scheme Poisson proposes reduces trivially to Bernoulli's scheme relative to the constant probability $p = \frac{1}{\nu}\sum_{i=1}^{\nu} c_i$. Bienaymé observed this immediately but published his explanation of it only in 1855, out of respect for the memory of Poisson, for whom he had great esteem [8].

But this is not the important point, for Poisson's aim is to show that "the universal law of large numbers is the basis of all applications of the probability calculus" [78, p. 12]. Poisson does not assimilate the law of large numbers to the law of "enormous numbers" that satisfy the condition of stability of averages by simple accumulation [19, p. 208]. His proof itself shows that he attaches the highest value to the preliminary investigation of the causes governing the data studied. His proposed scheme, variable causes extracted from an invariant set of possibilities, is indeed essentially trivial, but it has the fundamental advantage of existing. More or less in the same period [5], Bienaymé would propose more complex schemes of causes that would allow one to account somewhat for differences between frequencies much larger than those given by Poisson's formula (1.8), such as the differences already observed by Fourier in 1821 [27, vol. 1, p. 40].

Cournot developed this scheme of extended causes with great clarity in Chapter 9 of his book [19], concluding, "The main aim of statistics is the investigation of causes that govern physical and social phenomena... to attain this aim, it is now necessary... to decompose chances that are piled one on the other, to somehow purify the conditions of fate, so that individual cases are being accumulated in series only for the purpose of averaging out the effects of chance." In 1845 [7], Bienaymé would propose the first "decomposition of chances" not directly issuing from Bernoulli's scheme: the cascade scheme that anticipated Galton and Watson's work, as Heyde and Seneta have clearly demonstrated [41], by some 30 years.

Poisson is therefore at the origin of a movement that only recently attained its full maturity [63] and consists in building, for each phenomenon, a model (Neyman calls it a "chance mechanism") that gives an account of the different observed frequencies (the "law of large numbers" permitting us to certify the adequacy of the model). By proposing the asymptotic theory of Laplace (and Poisson) as the basis for applications of probability, in place of the "decision" theory of Condorcet (and Pascal), Poisson also placed himself at the origin of one of the most important schools of mathematical statistics.

This, we think, is the meaning that should be attributed to the sentence recalled above: "the law of large numbers is the basis of all applications of the probability calculus."

Poisson applied this law, as we know, principally to research on the probability of judgements. Something new had actually been observed in the 1820s. Guerry and Quételet had noticed, independently (or not), that the numbers of crimes and misdemeanors published in the *Comptes généraux* were stable from one year to the next. Even more curiously, this was also true for the proportion of defendants found guilty in each jurisdiction. The law of large numbers thus extended its empire to facts in the moral domain. Here was an unhoped-for occasion to revise Condorcet's and Laplace's work on the probability of judgements radically, replacing decision theory with asymptotic theory while retaining some of Condorcet's "models."

This important work of Poisson's is too long to be detailed here. Modern statisticians have recognized its value [31], but it encountered a tragic destiny. All the currents of thought that then divided France formed a coalition to ridicule Poisson's book, naturally without having read the first line of it [36]. Poisson's innumerable efforts to defend his vision of probability, in all the positions he held for over twenty years, were thus thrown into doubt by the very excess of his zeal. The tardy triumph of French positivism, whose hostility to the probability calculus is well known, later accelerated the process. In 1881, who would have thought to celebrate Poisson's centennial? And we know that this state of affairs has survived in some circles right up to the present, for a variety of reasons.

Nevertheless, one cannot reproach Poisson for lack of consistency or lack of aptitude. We will now briefly try to demonstrate this, in the hope of placing one last item in the dossier for Poisson's rehabilitation.

2. Public Education

Poisson concluded his speech at Laplace's funeral on March 7, 1827, with these words: "His ardent love for the sciences was his life, and it ended with him. Who will now give them the impetus they received from the activity of his spirit? Where will those who cultivate them find so flattering an approbation and such noble encouragements? Thinking of the way he welcomed my youth, of the signs of warm friendship he so often gave me, of the communications of his thought that have enlightened my thinking on so many different subjects, I feel acutely my inability to express in this last farewell all the love I had for him and all the gratitude I owe him" [76].

It is not so common to declare one's love during an official funeral, and Poisson's sincerity is beyond question. All testimonies [1, 22, 23, 60] agree that Poisson had the greatest possible passion for mathematics, at least for mathematics in the tradition of Galileo, Newton and d'Alembert, which sought to subdue nature by submitting itself to nature. This was the mathematics Laplace personified so perfectly at the beginning of 19th century.

One feels that Poisson was directing his exhortations to himself. Will he know how to develop Laplace's legacy, one of the weightiest parts of which he had already assumed? The French scientific community was in fact undertaking a new responsibility, to which it had paid little attention before: public education.

Until 1789, the task of teaching the nation and forming its elites had been filled by clergymen, over which parliaments and even the king had little control. The French universities, to which the Council of Trent had given the exclusive right to grant diplomas in theology and philosophy, subject to the authorization of the state (Edict of Blois, May 1579), had seen their privileges gradually eroded by the Jesuits, who had managed to take over the monopoly on education in the 18th century, in France as well as in Spain, Italy, and the Catholic part of Germany [21]. The Jesuits' teaching was classical, the practice of Latin verse then being considered less subversive than that of mathematics.

The University of Paris, in reaction, had become Jansenist, adopting most of the Port-Royal doctrines on education, which where often modernist. But the expulsion of the Jesuits in 1762, on the eve of Revolution, came too late for the university to renew itself and reconquer power and influence, as the German universities managed to do during the same period.

The French Revolution suppressed the teaching congregations, dissolved the universities, and closed all the colleges. The Convention decided to create a "central school" (*école centrale*) in each department,[7] devoted to educating the youth in the new republican, philosophical, and scientific spirit. The obvious difficulty for this first great reform of the educational system was that the Republic had no one to implement it. On the 3rd of Brumaire, Year III, Lakanal could only proclaim at the tribune of the Convention, "Nothing has been done for education for five years. Are there in France, are there in Europe, are there on Earth two or three hundred men ready to teach the useful arts and the necessary knowledge. . . ?" So the normal school (*école normale*) of Year III was created. Charged with using new methods to prepare 1500 future teachers in four months, it failed spectacularly in spite or because of the exceptional personality of its staff of professors, which included Lagrange, Laplace, Monge, Vandermonde, Daubenton, Haüy, Berthollet, and Thouin for the sciences and Mentelle, Volney, Bernardin de Saint Pierre, Sicard, Garat and Laharpe for literary studies (see [36] or the *Moniteur Universel* for 1795 for delightful details).

So it was necessary to fall back on those members of the old teaching congregations who had shown satisfactory proof of revolutionary spirit by getting married, or on local autodidacts like Ampère, who taught at the central school in Bourg-en-Bresse. And for the first time, private teaching filled gaps in state education. The habit caught on quickly.

Bonaparte replaced the central schools with lycées, less numerous but better financed through a system of state grants, and also better staffed, the first general inspectors having selected the best teachers from the central schools. At one point, apparently, Napoleon wanted to restore some of the teaching congregations, but he eventually decided to create a corps of secular teachers, the University, which was charged with diffusing and developing knowledge at every level, from kindergarten to the university.

To staff the University, he instituted in 1808 the faculties, as we knew them until quite recently, giving them a monopoly on conferring degrees (except for theology, which would have required difficult negotiations with the pope), and a new normal school. The normal school was conceived on the model of the seminaries: students had to follow lectures in the faculties, and from the normal school itself they received additional instruction and an environment designed to develop team spirit, a sense of discipline, and devotion to the higher interests of public education. The resemblance to seminaries was pushed quite far: we find, for example, a leaflet from November 16, 1812, that forbids women attending lectures at the Paris Faculty of Sciences so as to avoid troubling the spirits of the normal school students (National Archives

[7]The Revolution had replaced the provinces with smaller administrative units called departments. For those unfamiliar with French political history, it may also be useful to recall that the revolutionary regime evolved into Napoleon Bonaparte's empire, followed by the restoration of the Bourbon monarchy in 1815. The author also alludes to the ultraroyalists who came to power in 1820, the constitutional monarchy that emerged from the revolution of 1830, the second republic that followed the revolution of 1848, and the second empire that emerged in 1852.

AJ.16 5216). This suggests that there actually were women studying science in Paris between 1809 and 1812.

The Napoleonic faculties were set beside the old regime's applied schools the Convention had kept, the polytechnic (*école polytechnique*) it had created, and the College of France (*Collège de France*), the traditional center of high-level teaching. The Paris faculties eventually acquired complete responsibility for educating the teaching profession, while the provincial faculties had no role at all at this time, because all future teachers had to pass through the Paris normal school. But surely it is not a bad thing to create institutions without their own purpose, for this allows one to see what they can really do.

As early as 1802, Bonaparte summarized the teaching program of the lycées in a laconic formula that reconciled the old and new regimes: "In the lycées one will teach mainly Latin and mathematics" (decree of December 10, 1802 [21]). So the normal school had two sections, letters and sciences.

The whole of the Imperial University was placed under the authority of a Great Master, the poet Fontanes, assisted by a council in which the scientific disciplines were represented by Delambre, Legendre, Cuvier, and Jussieu. But Laplace or Monge, who alternated as president of the Senate, clearly intervened directly in important decisions such as appointments or the specification of responsibilities for chairs in the Faculty of Sciences. Poisson owed his appointment to the chair of mechanics in 1809 to Laplace.

"Upon the return of the Bourbons," Cournot reports [21], "no institution was more criticized, more attacked, more calumniated than the Imperial University, but the difficulty was to improvise another system, and neither the clergy nor the Jesuits who had reappeared felt in a position to take over altogether from the university militia." The Bourbons were in the same situation as the Republic 20 years before. As Bishop Frayssinous himself admitted [40], there was a shortage of 15,000 parish priests at the beginning of the Restoration, and the large teaching congregations were disorganized and emptied out.

While the shortages were being addressed, the University was placed under a commission with five members: Cuvier, Royer-Collard, Silvestre de Sacy, Gueneau de Mussy, and Abbé Frayssinous. Silvestre de Sacy, Royer-Collard and Gueneau de Mussy represented the Jansenist sprit of the old Paris University, whose model was Rollin's *Traité des Etudes* [81]: "The University offers three major subjects: science, morals, and religion." Their "moderate royalism" and their traditional suspicion of bishops and the clerical spirit made them objective allies of the secular and "liberal" university, as it began to define itself under the influence of the young Turks of the literary section of the normal school. Cuvier, as Cournot so justly wrote, represented "high scientific fame together with political flexibility" [21]. Finally, Abbé Frayssinous represented the Sulpician spirit that Renan described so well [80], which then constituted the mystical component of the clerical and monarchist movement formed at the very beginning of the 19th century under the name "congregation" [32], which would gradually take over King Louis XVIII's court and all the key positions and be accused, for that reason, of "crypto-Jesuitism," and which was irreducibly hostile to the "antireligious education dispensed by the University."

The lycées were renamed royal colleges, Abbé Eliçagaray, a member of the congregation, replaced Abbé Frayssinous at the commission, but under the presidency of Royer-Collard and Cuvier, the university did not undergo other changes until 1820, when the "pure royalists" came to power with the new Villèle ministry. One of its avowed objectives was to purify the administration of its "revolutionary and regicidal" elements. The commission for public education, renamed the "Royal Council for Public Education" (*Conseil Royal de l'Instruction Publique*), was then placed under the presidency of Interior Minister Corbière. It was enlarged to seven members, and Poisson and Ambroise Rendu were brought in (July 22, 1820). It would be a mistake, though, to think that Poisson and Rendu were Villèle's men. They owed their appointment mainly to technical considerations. Rendu, general inspector, had been a specialist in the administration of the university since its creation; moreover, he had Jansenist tendencies. As for Poisson, the Commission had called on him regularly since 1815 to deal with problems touching on the exact sciences, which were not directly represented on the Commission.

Poisson had been a republican during his youth, hostile to the Emperor [1], and naturally became royalist at the Restoration. Cournot, who always speaks of Poisson with the highest esteem, wrote in his memoirs that "Mr. Poisson added a great refinement of spirit to his very smooth courtesy, a great stock of common sense and tolerance that inclined him towards conservative ideas. The royalist party had mistaken this conservative spirit for a royalist spirit, and Poisson welcomed the confusion. Abbés Frayssinous and Nicolle and their Jansenist colleagues lived with him on the footing of a worldly friendship, as with a man too hard to convert but who would let them sow the good doctrine, even if he did not profit from it himself."

Nevertheless, his "well known skepticism about faith and dogma" [1] could have prevented him from entering the teaching profession, which was to follow a "openly Christian and monarchist direction" starting in 1821. Abbé Eliçagaray and the mathematician Dinet, a "great supporter of the Jesuits and absolute power" [37] and one of Poisson's oldest friends [60], were charged that very year with an inspection of all the royal colleges. "A climate of spying and denunciation set in" [37, September 9, 1830]. During his visit to the college of Marseille, closed soon afterwards, Abbé Eliçagaray supposedly declared [24]: "Gold medals will be distributed to professors who distinguish themselves in the accomplishment of their functions. Zeal is needed, and fervor; when you would have all the education of Rollin, if you have not his piety, you will not have a medal. Do what is necessary." Later: "It is not sufficient to have religious feelings, one must then express them by scrupulously following one's religious duties."

These quotations must be taken with a grain of salt, for they have been used too often in the school conflicts to convince anyone any longer. But we may observe that Abbé Frayssinous, who afterwards became Bishop of Hermopolis and was named Minister of Public Education and Ecclesiastic Affairs in 1822 (the first national education minister in France) declared himself to be directed by two thoughts: "first that education is something more moral and religious than literary and scientific; second that for piety and good morals to flourish in institutions of public education, the zeal and efforts of the civil servants in the University must find support in the influence of the bishops." And he concluded: "Through the efforts of the bishopric and the University, public education will produce more educated and virtuous subjects, an immense benefit for religion as well as society." The picture Stendhal painted of French society under Charles X in [85] may be too dark, but if it is at all faithful, it is hard to imagine just how Bishop Frayssinous's principles were put into practice. But his character, his saintliness, and the grandeur of his vision are not in doubt [40].

In any case, the normal school, whose "apparent order hid a too real corruption," was dissolved in 1822, and the most militant Jansenists were removed from the council.

How then can one explain that Poisson, notorious unbeliever, member of the Arcueil society, constant and devoted friend of Laplace, who had just upheld in the *Essai philosophique* [56] that the probability of religion diminishes with time, should have gained the confidence of the Jansenist party as well as that of the congregation before gaining that of the liberal party after 1830? Should not Cauchy, congregationist since 1804, or Binet, congregationist since

1806, have been the designated successor to Poisson at the royal council? Yet Poisson was maintained in the function after 1822 and even took over the position of treasurer in that year; he was even made Baron in 1825. Arago and Libri report, on this topic, that he never picked up the certificate, perhaps from fidelity to his father, "a convinced republican of modest origins."

One can only conclude that Poisson had quickly learned how to be indispensable to the management of the University and that he must have considered his margin of maneuver in the council sufficient to continue managing it without giving up his principles.

Poisson possessed, in fact, the greatest refinement of the subtle art of indirect management. The decisions of the royal council, whose president was the minister, had be taken unanimously. So for every question (appointments, creation of positions, curriculum, sanctions to be taken, etc.), it was necessary to work out the least poor solution that could gain a consensus, which obviously varied with the composition of the council.

Poisson never made mistakes. According to Laplace, Poisson "arranged" all the elections to the Academy of Sciences starting as soon as he entered it in 1812, whereas Laplace himself erred every time [1] [60, p. 435]. Poisson also had a considerable capacity for work. The task of members of the royal council was enormous. The council, which met "at least" twice a week, took the final decisions on all questions of public education, down to the tiniest details. As treasurer, Poisson had to personally examine the accounts of all the royal colleges to search out abuses and dissimulations. The Court of Accounts (*Cour des Comptes*), though it had been instituted in 1807, did not start examining the University until at least 1828. So it is so not incomprehensible that Poisson could keep his positions until his death, even after the arrival in power in 1830 of the liberal university representatives Guizot, Cousin, and Villemain, whom Bishop Frayssinous and his council had not treated well.

But what about his convictions? Why did he accept so weighty and thankless a task? Some authors with bad intentions [37, 82] have claimed that he was mainly concerned with keeping his titles and that he used his personal skills and positions of power to serve himself. Even a very superficial study of his administrative work shows that this cannot be the case and that his single goal, to which he sacrificed part of his personal production and his health, was the defense and glory of mathematics, a cause worth something after all.

Since we cannot give a complete analysis of the some 1800 sessions of the royal council in which Poisson participated between 1820 and 1840 (their minutes fill about 40 boxes in the National Archives in Paris), we restrict ourselves to a few examples.

2.1. **Appointments.** The normal school, dissolved in 1822, had been replaced by a system of scholarships for use at a number of Parisian royal colleges (the *écoles normales partielles*), which quickly proved ineffective. So in 1826 it was decided to create a "preparatory school" on the model of the old normal school. The teaching staff and the chaplains were to be chosen very carefully to avoid a return to the transgressions of the past. The newspaper *Le Globe* of November 18, 1829, credits Bishop Frayssinous's Gallicanism for the creation of the preparatory school, a potential machine of war against the ultramontanes. This it did become, but it did not spare the Gallicans either.[8]

In October 1826, Poisson proposed Leroy and Abbé Pinault to be lecturers (*maîtres de conférences*) in mathematics and physics (National Archives, Leroy dossier). Leroy's "monarchist and religious feelings" were apparently known. As a lecturer in the old normal school, he

[8]The ultramontanes emphasized the supremacy of the pope, while the Gallicans favored more autonomy for the French church.

had helped teach Poisson's course in mechanics, and Poisson had been able to appreciate his merit. Alexis-Marie Pinault, born in 1794, had been Poisson's student at the polytechnic in 1813; after a period in the normal school (1814), he had taught mathematics in Nîmes (1816), Pau (1817), and Limoges (1819). Called back to Paris in 1821 as "adjunct to the examination commission for the literary baccalaureate," he entered the Saint-Sulpice Seminary.

Proposing a Sulpician to lecture at the normal school while Bishop Frayssinous was minister might look like a very clumsy maneuver, from which the exact sciences could hardly benefit. But when we realize that Abbé Pinault was also a remarkable scientist, as his books [64, 65] amply demonstrate, we begin to understand the system Poisson followed on this occasion and throughout his lengthy administrative career. As a permanent examiner at the polytechnic, president of the competition for *agrégation*, and a member of the Institute of France and numerous scientific societies,[9] Poisson actually knew personally almost all French mathematicians, from the most obscure to the most distinguished. He had an opinion about each one, and he also knew what opinions his fellow council members might have. So all he had to do was reach into his sack and pull out the best possible candidate for mathematics, taking into account the circumstances and the council's proclivities. Sometimes he did not even bother to warn the candidate himself. This is how Cournot was named general inspector against his will.

To be done with Abbé Pinault, let us mention that he resigned from the normal school in 1828 as Bishop Frayssinous left office, victim of the liberal-Jansenist coalition that brought down the Villèle ministry. He was named professor of "physics" at the Issy seminary, where he was still teaching when Renan studied philosophy there. The portrait Renan paints of the abbé is so striking that we cannot resist the pleasure of quoting from it: "Mr. Pinault's course was the strangest thing in the world. He did not hide his contempt for the sciences he taught and the human spirit in general. Sometimes he almost fell asleep while teaching. He pointed his students diametrically away from the subject. And yet he still had in himself aspects of the scientific spirit he had been unable to destroy. Sometimes he had surprising insights. Some of the lessons he taught us on natural history have been one of the foundations of my philosophical thought." And later: "The scientific spirit was the essence of my nature. Mr. Pinault would have been my true master had he not, by the strangest of faults, indulged a mania to hide and defile the most beautiful parts of his genius. I understood him in spite of himself, and better than he would have wished" [80, chap. IV]. It is interesting that while showing utter contempt for Laplace in his books, Abbé Pinault had a definite fondness for Poisson.

As we have mentioned, Poisson was a tolerant man in a quite intolerant century. While it was not in his nature to try to force destiny, he always tried to repair the injustices in which he participated against his will. So it was that after 1830, he called back almost all those he had been obliged to abandon during the Restoration. We may cite the example of Abélard Lévy; this brilliant student of the normal school of 1813 was excluded from teaching in 1816 because of his religion, as was his fellow student Maas, who later headed the office of the first French life insurance company, founded in 1819. After many difficulties, Lévy had become reader (*lecteur*) at the new University of Liège. In October 1830, Poisson had him named lecturer at the normal school, where he taught, not surprisingly, the probability calculus. Poisson arranged appointments for more or less all the members of the old normal school after 1830, at least all who had not become socialists or revolutionaries like Saigey. Thus Cournot, who had been expelled from teaching in 1822 for "lack of tender piety," was successively named professor in the Lyon faculty of sciences, rector of Grenoble University, and general inspector by Poisson.

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[9]The French *agrégation* is the highest teaching license at the secondary level. The *Institut de France* consists of several academies, including the Academy of Sciences.

Some will point out that the "Poisson system" obviously had its limits. Together with the whole royal council, Poisson expelled Evariste Galois from the normal school. But could he have done otherwise? Had not Galois just publicly ridiculed Guigniaut, director of the school, at the very moment when it was coming back into its own? We know that Poisson was aware of Galois's mathematical talents, even if he had not immediately recognized all his genius, and it is absolutely certain that Poisson would have later supported Galois's career if Galois had lived long enough to have one and the opportunity had arisen.

We could pursue this theme at length, mentioning Binet's appointment to the College of France in 1823, Demonferrand's appointment as general inspector in 1839, or Dinet's mission in 1821, but no doubt it is better to leave it at that.

2.2. **Curricula.**

2.2.1. *Colleges.*[10] As we mentioned, Napoleon wanted to educate the nation's elites by teaching them Latin and mathematics. Corbière's ordinance of February 17, 1821, which returned to the old regime's curricula, presumed, for its part, that Latin was quite sufficient. Mathematics, needed all the same for building bridges or aiming cannon balls, should be reserved exclusively for candidates for the polytechnic. This ordinance was never put into effect by the royal council.

We have no details of the discussions of Corbière's ordinance inside the council, but it is clear that Poisson had to make firm use of all his art of compromise. In fact, Poisson had fought all his life in defense of the Napoleonic idea of a core curriculum of Latin and mathematics in all the lycées. He merits some credit for this, because he himself, educated at the Fontainebleau central school where Latin had been banished, knew nothing more of this beautiful language than the bits absolutely necessary for reading Euler. So Poisson accepted Latin on condition that he be granted mathematics.

He finally succeeded. The decree of September 16, 1826, gave up the principle of separation between sciences and literature. He similarly succeeded in containing the efforts of literary liberals, who tried to restore the separation after 1830. At the end of the summer of 1830, they mounted a frontal attack against Poisson. *Le Lycée*, the liberal normalian paper that opposed the conservative *Gazette des Ecoles* before disappearing in 1832, declared on Thursday, August 19, 1830, that Poisson "seems to be the author of these half-baked plans, which have prescribed the study of mathematics for our young literary students and brought confusion into our colleges."

According to Arago, who claimed credit for keeping Poisson in office, he was close to being dismissed from the council at this time [1]. But Poisson had seen it before. He resisted the liberals as he had resisted the Jesuits. The core was preserved through the whole curriculum, even in rhetoric.

Poisson overhauled the college mathematical curriculum several times. The last version, which he brought to the royal council on September 21, 1838, can be summarized in five points:

(1) In the forth form, there will be a two-hour lecture each week, and no more than two additional one-hour lectures.

[10]In French usage, a college (*collège*) is usually a secondary school. The curriculum discussed here lasted four years. Beginning students were in the fourth form, which was followed by the third and second forms. The final year was called rhetoric.

(2) In the third form, there will be a two-hour lecture on arithmetic (following Bézout's book) and a two-hour lecture on plane geometry ("based on the method of infinitely small elements ... which allows one to avoid incommensurable numbers").
(3) In the second form, there will be a two-hour lesson on logarithms and the geometry of solids.
(4) In rhetoric, there will be a two-hour lesson on cosmography.
(5) "When the arithmetic or geometry lesson falls on a holiday, it will be moved to another day of the same week, replacing a lesson in grammar or the humanities."

The decree of September 28, 1838, which repeats Poisson's conclusions, is signed by Villemain, Cousin, and Salvandy, the minister. What a triumph for Poisson!

The triumph did not outlive him long. During the Second Empire, in fact, Fortoul put into place the so-called "bifurcated" system, which long separated literary from scientific studies, despite the efforts of Cournot, a resolute supporter of Poisson's doctrines [21].

2.2.2. *Faculties.* The curriculum of the Faculties of Sciences had been fixed in 1808, when the topics for chairs were specified. In mathematics, there were chairs for "differential and integral calculus," "mechanics," and "astronomy," to which Monge managed to add "descriptive geometry" by making Hachette an "adjunct professor." This division accurately reflected the situation of the exact sciences in 1808. But after that date, as we have tried to show, Fourier, Laplace, and Poisson (not to mention Cauchy) substantially developed the probability calculus and mathematical physics. It was up to Poisson to incorporate this new reality in new curricula. Here is how he did it. Hachette, in charge of the lectures in descriptive geometry (and mathematical instruments) died in 1834. Against the will of some of his colleagues in the Faculty of Sciences, Poisson proposed that the royal council create a chair in the probability calculus or mathematical physics. The minister of public education, then Guizot, decided in favor of the former designation, and following Poisson's system, the chair was awarded to Libri, a personal friend of Guizot, even though his probabilistic work was imperceptible. During the academic year 1836–1837, Poisson, who was writing his *Recherches*, traded teaching duties with Libri, lecturing on probability instead of mechanics. This explains why Poisson's book, with such a specialized title, contains four chapters, totaling 300 pages, on "the general rules of the probability calculus" and "the calculation of probabilities that depend on very large numbers." This is the course the Faculty of Sciences inherited from Poisson. Its clarity of exposition and its choice of examples makes it a model even today. This makes it all the more regrettable that Poisson's book has become a rarity for book lovers.[11]

Anticipating the creation of a chair of mathematical physics he intended to obtain, Poisson similarly spent the last years of his life writing a large treatise on mathematical physics, of which only some chapters (the theory of capillary action, the theory of heat, and the beginning of the theory of light, for example) have survived.

After Libri lost his position in 1848 (too long an affair to discuss here), and no doubt because probability was beginning to be seen as not very serious, the chair in probability was combined with mathematical physics. Thus was established, in 1850, the chair of "probability calculus and mathematical physics," the quintessentially Poissonian chair, in which Lamé, Boussinesq and Poincaré won fame.

Could Poisson have dreamt of a better legacy? Much later, probability, long in stagnation, also blossomed from the chair Poisson had created, under the influence of Borel, who occupied

[11]The book was reprinted by J. Gabay, Paris, in 2003.

the chair for so long. Recall that today's Laboratory of Probabilities and Stochastic Models at the University of Paris VI can be considered the last trace of the chair of 1834, after May 1968 and the minister Edgar Faure decided to sever the line of descent, or at least to pretend to do so.[12]

Poisson devoted himself almost exclusively to defending Laplace's legacy, which remained within d'Alembert's picture of "mixed mathematics." This picture included "mechanics, geometric astronomy, optics, acoustics, pneumatics (?), the art of conjecture and the physical-mathematical sciences." Some may protest that this produced two victims: "experimental physics" (Poisson even kept the *agrégation* competition for physics and mathematics unified until his death) and "pure mathematics," which was beginning to flower so brilliantly outside France and really penetrated France only after Poisson's death. But is this accusation, already brought by Saigey starting in 1830 [82], true? And should Poisson alone be charged? "Despite his preferences," Libri noted, "Poisson "never stopped following advances in pure analysis, as he proved when he presented to the Institute his beautiful report, known to all geometers, on Mr. Jacobi's work on elliptic transcendentals" [60, p. 431]. Should we not lay the charge instead at the door of the excessive centralization of scientific institutions in France? In the middle of the 19th century, science became so diverse that concentrating power in the hands of a single man, be he the best man possible, could only be a cause of decline. Paris, which held a genuine world monopoly on science during the Revolution and the Empire, was a victim of its own centralization, not merely a victim of Poisson, whose devotion to the cause of mathematics was exemplary, and whose genius was incomparable, as we now hope to have demonstrated.

References

[1] ARAGO F., Œuvres complètes, tome 2, Paris, 1854.
[2] ARBUTHNOT J., "An argument for divine providence, taken from the constant regularity observed in the births of both sexes," Phil. Trans. Roy. Soc., London 27, 1710, 186–190.
[3] BATEMAN H., "On the probability distribution of particles," Phil. Mag. and J. of Science, 6th series, 20, 1910, 704–707.
[4] BERNOULLI J., Ars Conjectandi, Bâle, 1713.
[5] BIENAYME, I. J., "Sur la probabilité des résultats moyens lorsque les causes sont variables durant les observations," L'Institut, 284, 1839, 187–189.
[6] BIENAYME, I. J., "Effets de l'intérêt composé," L'Institut, 286, 1839, 208-209.
[7] BIENAYME, I. J., "De la loi de multiplication et de la durée des familles," L'Institut, 589, 1845, 131–132.
[8] BIENAYME, I. J., "Sur un principe que M. Poisson avait cru découvrir et qu'il avait appelé Loi des grands nombres," C.R. Acad. Sci. Morales et Politiques, 11, 1855, 379–389.
[9] BORTKIEWICZ L. von, Das Gesetz der kleinen Zahlen, Leipzig, 1898.
[10] BOSSUET, J.B., Traité de Concupiscence, Meaux, 1694.
[11] BROCKMEYER E., HALSTROM H. L. and JENSEN E., The life and works of A.K. Erlang, Copenhagen, 1948.
[12] BURNICHOU J., La Compagnie de Jésus en France, tome 1, 1814–1830, Paris, 1914.
[13] CALLENS S., Séminaire M. Foucault, Paris, Collège de France, 8 mars 1981.
[14] CAUCHY A. L., "Sur une loi de réciprocité qui existe entre certaines fonctions," Bul. Soc. Philomatique, 1817, 121–124.
[15] CAUCHY A. L., "Seconde note sur les fonctions réciproques," Bul. Soc. Philomatique, 1817, 180–181.
[16] CAUCHY A. L., "Théorie des erreurs" (7 notes), C.R. Acad. Sci. Paris 1853, 36, 1114–1122; 37, 64–69, 100–109, 150–162, 198–206, 264–272, 326–334.
[17] CONDORCET M. J. A. de Caritat de, Essai sur l'application de l'analyse à la probabilité des décisions rendues à la probabilité des voix, Paris, 1785.
[18] COUMET E., Communication personnelle, Paris, 1981.

[12] Faure's reorganization of the University of Paris was one eventual outcome of the demonstrations by students and workers in May 1968.

[19] COURNOT A. A., Exposition de la théorie des chances et des probabilités, Paris, 1843.
[20] COURNOT A. A., Essai sur les fondements de nos connaissances et sur les caractères de la critique philosophique, Paris, 1851.
[21] COURNOT A. A., Des institutions d'instruction publique en France, Paris, 1864.
[22] COURNOT A. A., Souvenirs 1760–1860, Paris, éd. Bottinnelli, 1913.
[23] COUSIN V., Discours prononcé aux obsèques de Poisson, Paris, 1840.
[24] ELICAGARAY D., Discours prononcé à sa première visite au Collège de Marseille, Carcassonne, 1821.
[25] ERLANG A. K., "Calcul des probabilités et conversations téléphoniques (Suédois)," Nyt Tidsskift for Mat. B, 20, 1909, 33–39.
[26] FISHER R. A., "Uncertain inference," Contributions to Mathematical Statistics, New York, Wiley, 1950.
[27] FOURIER J. B. J., Introduction des 4 premiers volumes des Recherches statistique sur la ville de Paris, volume I, 1821, volume II, 1823, volume III, 1826, volume IV, 1829.
[28] FOURIER J. B. J., Théorie analytique de la chaleur, Paris, 1822.
[29] FOURIER J. B. J., Eloge historique de M. le Marquis de La Place, Paris, 1829.
[30] FREUDENTHAL H., "A. L. Cauchy," Dictionary of Scientific Biography 3, New York, ed. C.C. Gillispie, 1971, 131–148.
[31] GELFAND A. E. Et SOLOMON H., "A study of Poisson's models for jury verdicts in criminal and civil trials," J. Amer. Stat. Assoc. 68, 1973, 271–278.
[32] GEOFFROY de GRANDMAISON C. A., La congrégation, 1801–1830, Paris, 1889.
[33] GILLISPIE C. C., "Laplace, Pierre-Simon," Dictionary of Scientific Biography, Supplément I, New York, 1978, 273–403.
[34] GILLISPIE C. C., "Mémoires inédites ou anonymes de Laplace sur la théorie des erreurs, les polynômes de Legendre et la philosophie des probabilités," Revue Hist. Sci. 32, 1979, 223–280.
[35] GRATTAN-GUINNESS I., Joseph Fourier 1768–1830, M.I.T., 1972.
[36] GUERRY A. M., Statistique morale de l'Angleterre comparée à la statistique morale de la France... avec une introduction contenant l'histoire de l'application des nombres aux sciences morales, Paris, 1864.
[37] GUILLARD A. P., éd. La Gazette des Ecoles, année 1830.
[38] HAIGHT F. A., Handbook of the Poisson Distribution, New York, 1967.
[39] HALD, A., A History of Mathematical Statistics from 1750 to 1930, New York, John Wiley & Sons, 1998.
[40] HENRION M. R. A., Vie de M. Frayssinous, évêque d'Hermopolis, Paris, 1844.
[41] HEYDE C. C. et SENETA E., I. J. Bienaymé, Springer-Verlag, 1977.
[42] LAGRANGE L., "Mémoire sur l'utilité de la méthode de prendre le milieu entre les résultats de plusieurs observations," Œuvres complètes II, 1776, 173–234.
[43] LAPLACE P. S., "Mémoire sur la probabilité des causes par les événements," Œuvres complètes VIII, Paris, 1774, 27–65.
[44] LAPLACE P. S., "Mémoire sur l'inclinaison moyenne des orbites des comètes, sur la figure de la terre et sur les fonctions," Œuvres complètes VIII, Paris, 1776, 279–321.
[45] LAPLACE P. S., "Mémoire sur l'intégration des équations différentielles par approximation," Œuvres complètes IX, Paris, 1780, 357–379.
[46] LAPLACE P. S., "Mémoire sur les probabilités," Œuvres complètes IX, Paris, 1780, 383–485.
[47] LAPLACE P. S., "Mémoire sur les suites," Œuvres complètes X, Paris, 1782, 1–89.
[48] LAPLACE P. S., "Mémoire sur les approximations des formules qui sont fonctions de très grands nombres," Œuvres complètes X, Paris, 1785, 209–291.
[49] LAPLACE P. S., Suite du mémoire précédent, Œuvres complètes X, Paris, 1785, 295–338.
[50] LAPLACE P. S., "Mémoire sur le mouvement de la lumière dans les milieux diaphanes," Œuvres complètes XII, Paris, 1809, 267–298.
[51] LAPLACE P. S., "Mémoire sur divers points d'analyse," Œuvres complètes XIV, Paris, 1809, 178–214.
[52] LAPLACE P. S., "Mémoire sur les approximations des formules qui sont fonctions de très grands nombres et sur leur application aux probabilités," Œuvres complètes XII, Paris 1810, 301–353.
[53] LAPLACE P. S., "Sur les intégrales définies," Bul. Soc. Philomatique 43, avril 1811, 262–266.
[54] LAPLACE P. S., "Mémoire sur les intégrales définies et leur application aux probabilités et spécialement à la recherche du milieu qu'il faut choisir entre les résultats des observations," Œuvres complètes XII, Paris, 1811, 357–412.
[55] LAPLACE P. S., "Théorie analytique des probabilités," Œuvres complètes VII, Paris, 1812, 1–493.
[56] LAPLACE P. S., "Essai philosophique sur les probabilités," Œuvres complètes VII, Paris, 1814, V–CLIII.
[57] LAPLACE P. S., "Premier supplément à la théorie analytique des probabilités," Œuvres complètes VII, Paris, 1816, 497–530.

[58] LEIBNIZ G. W., Œuvres complètes, ed. G.H. Pertz, 3e serie, III, Halle, 1855.
[59] LEVY P., Quelques aspects de la pensée d'un mathématicien, Paris, A. Blanchard, 1970.
[60] LIBRI-CARRUCCI DELLA SOMMAIA G., "Lettres à un américain sur l'état des sciences en France, III, Poisson," Revue des deux mondes 23, 4e série, 1840, 410–437.
[61] MOIVRE A., The doctrine of chances, 3rd ed., London, 1756.
[62] MONTMORT P. R. de, Essai d'analyse sur des jeux de hasard, 2e ed., Paris, 1713.
[63] NEYMANN J., "Indeterminism in science and new demands on statisticians," J. Amer. Stat. Assoc. 55, 1960, 625–639.
[64] PINAULT A. M., Eléments et compléments de mathématiques, Paris, 1847.
[65] PINAULT A. M., Eléments et compléments de physique, Paris, 1852.
[66] POISSON S. D., "Extrait du mémoire de M. Fourier sur la propagation de la chaleur," Bul. Soc. Philomatique 4, Mars 1808.
[67] POISSON S. D., "Extrait d'un mémoire de Laplace sur les approximations des formules qui sont fonctions de très grands nombres," Bul. Soc. Philomatique 35, août 1810, 132–136.
[68] POISSON S. D., "Sur les intégrales définies," Bul. Soc. Philomatique 42, mars 1811.
[69] POISSON S. D., "Extrait d'un mémoire de Laplace sur les intégrales définies," Bul. Soc. Philomatique 49, octobre 1811.
[70] POISSON S. D., "Sur les intégrales définies," Bul. Soc. Philomatique 50, novembre 1811.
[71] POISSON S. D., "Sur l'avantage du banquier au jeu de trente-et-quarante," Ann. Math. pures et appl. 16, 1820, 173–208, (1825–1826).
[72] POISSON S. D., "Observations relatives au nombre de naissances des deux sexes," Annuaire du bureau des longitudes pour 1825, 1824, 98–99.
[73] POISSON S. D., "Mémoire sur la proportion des naissances des filles et des garçons," Mém. Acad. Sci. Paris, 8, 1829, 239–308.
[74] POISSON S. D., "Sur la probabilité des résultats moyens des observations," Conn. des temps pour 1827, 1824, 273–302.
[75] POISSON S. D., Suite du mémoire précédent, Conn. des temps pour 1832, 1829, 3–22.
[76] POISSON S. D., "Discours prononcé aux obsèques de . . . Laplace," Conn. des temps pour 1830, 1827, 19–22.
[77] POISSON S. D., "Note sur le calcul des probabilités," C.R. Acad. Sci. Paris 2, 1836, 395–400.
[78] POISSON S. D., Recherches sur la probabilité des jugements en matière criminelle et en matière civile précédées des règles générales du calcul des probabilités, Paris, 1837.
[79] QUETELET A. L. J., Sur l'homme et le développement de ses facultés, ou essai de physique sociale, Paris, 1835.
[80] RENAN E., Souvenirs d'enfance et de jeunesse, Paris, 1883.
[81] ROLLIN C., De la manière d'enseigner et d'étudier les belles lettres, Paris, éd. Gueneau de Mussy et Rendu, 1805.
[82] SAIGEY J. F., "Poisson," Biographie universelle et portative des contemporains, Paris, éd. Rabbe, Vieilh de Boisjolin et Sainte-Beuve, 1834.
[83] SHEYNIN O. B., "S. D. Poisson's work in probability," Arch. Hist. Ex. Sci., 18, no. 3, 1978, 245–300.
[84] SIMPSON T., "An attempt to show the advantage arising by taking the mean of a number of observations in practical astronomy," Miscellaneous Tracts, London, 1752, 64–75.
[85] STENDHAL M. H. BEYLE (dit), Le rouge et le noir, Paris, 1831.
[86] STIGLER S. M., "Laplace, Fisher and the discovery of the concept of sufficiency, Biometrika 60, 1973, 439–445.
[87] STIGLER S. M., The History of Statistics. The Measurement of Uncertainty before 1900, Cambridge, The Belknap Press of Harvard University Press, 1986
[88] TATON R., "Les relations d'Evariste Galois avec les mathématiciens de son temps," Revue Hist. Sci. 1, 1947, 114–130.
[89] TODHUNTER I., A history of the mathematical theory of probability, from the time of Pascal to that of Laplace, Cambridge, 1865.
[90] WAILLY A. (de), Epitre à J. J. Rousseau, Paris, 1826.
[91] WHITAKER Lucy, "On the Poisson law of small numbers," Biometrika 10, 1914, 36–76.