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Jacob Bernoulli, teacher and rival of his brother Johann

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Abstract

In this paper, we brush a portrait of Jacob Bernoulli as seen by his youngest brother Johann. To associate both brothers is a long habit in historiography (Comte, Mach, Spiess), which will not be completely lost here. After having recalled what we know about the training of the two brothers, we will shortly describe the works done in common, especially those concerning the Leibnizian differential and integral calculus. The rivalry between the brothers is at the origin of a number of statements and judgements in the correspondences of the Bernoulli family. They give us an opportunity to better understand the person and mathematician Jacob Bernoulli, his relations with his brother and also what their competition owes to the nature of mathematical practices at that time.

I. INTRODUCTION : THE BERNOULLI BROTHERS, A HISTORIOGRAPHICAL CHIMERA

In the history of analysis, Jacob and Johann Bernoulli are intrinsically linked, forming a twosome, that has contributed to the progress of differential calculus such as it was formulated by Gottfried Wilhelm Leibniz. Indeed, some parts of their achievements-precisely those which were concerned with developing the first applications of Leibnizian calculus-make it natural to consider them together, since it was by joint effort that the two brothers acquired, in Basel in the late 1680's and the beginning of the 1690's, a thorough understanding of this calculus.

"In the early days of infinitesimal analysis, the most famous mathematicians such as the illustrious brothers Johann and Jacob Bernoulli, rightfully attributed a greater importance to extending and developing the immortal discovery of Leibniz and to explore its multitude of applications, than to rigorously establishing the logical foundations of this new method of calculus. For a long period, their sole response to the pronounced skepticism of lesser mathematicians, against the principles of

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the new calculus, consisted in providing solutions one had not dared hope for to the most difficult problems”.[2] This is how Auguste Comte describes the two brothers' contribution to analysis. In association, they are considered the foremost mathematicians of their time, in virtue of having developed the calculus discovered by Leibniz. While it is true that they had not been able to provide a rigorous foundation for it, responding to critics solely by displaying how fruitfully the new calculus could be used to solve very difficult problems, they had nevertheless, by multiplying the applications of someone else's discovery, together set themselves apart from the crowd of "lesser mathematicians" most of whom were unable to grasp the full significance of the calculus introduced by Leibniz.

Based on Comte, whom he quotes, Joachim Otto Fleckenstein, an expert on the works of the Bernoulli family, editor of a volume of Jacob Bernoulli’s works [Jacob Bernoulli, Werke 1] and the author of a joint biography of the brothers, takes the argument suggested by the positivist philosopher even further by founding what I will refer to as the chimera of "the Bernoulli brothers". Their contributions have become inseparable and indistinguishable. This illusory twosome, "the Bernoulli brothers" has converted the Leibnizian method of calculus into an analytical tool, which became unexpectedly powerful in the hands of Euler. Thus, the Bernoulli brothers painstakingly join the ranks of Leibniz and Euler, on condition that their works are not separated: « The historical importance of the Bernoulli brothers...is indeed almost on a par with the memorable achievements of the great classics of mathematical science, as long as the two brothers' contributions are considered together »[3].

Ernst Mach had already expressed an analogous judgment in his *Mechanik* [Mach, 1883, Kap.4]. Speaking of the two brothers, he wrote « the genius of one and the depth of the other came to the most fertile use through the influence that their solutions had on Euler and Lagrange » [Mach, *ibid.*]. Each brother had his distinct psychological traits-genius *versus* depth-but their combined achievements were a vital source of inspiration to later generations. Mach chose the Bernoulli brothers as an example that illustrated his vision of scientific genius, which, according to him, has two sides: creative imagination and critical depth. When found in the same person, these qualities make for a great scientist. "epochemachend" as Fleckenstein would say, such as Galileo or Newton. When they are separated between two individuals, they may cause a clash that culminates in an open struggle. According to the analysis of Mach, this was the case for the Bernoulli brothers. The intuitive imagination of Johann the artist and Jacob's critical rigor entered into conflict and were at the origin of a number of regrettable quarrels between the brothers, but together these qualities bore the "most beautiful fruit". In order to possess the force of genius, wrote Mach, the brothers must remain together, two sides of the same coin and collaborate, since each brother, on his own, embodied only one of the two traits required for scientific brilliance. To provide what is lacking, the brothers are thus condemned to be united and at the same time to fight each other.

However, Jacob and Johann Bernoulli do not form this illusory and terrifying assembly by which historiography depicts them, since they possess distinct personalities as well as a scientific production that is not limited to analysis, the area that became the battlefield of their fierce competition. Although, it is my wish, in this article, to distance myself from this chimerical construction, the basic assumptions of which can be traced back to the XIX century, I am still, in

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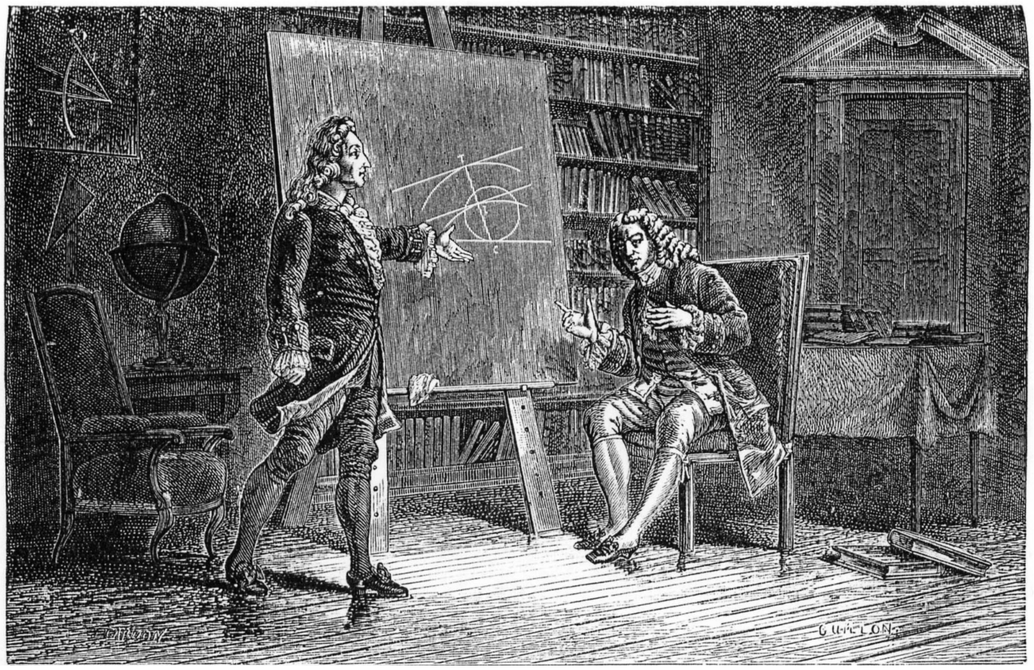
[2] « Dans les premiers temps de l’analyse infinitésimale, les géomètres les plus célèbres tels que les deux illustres frères Jean et Jacques Bernoulli, attachèrent, avec raison, bien plus d’importance à étendre, en la développant, l’immortelle découverte de Leibniz, et à en multiplier les applications, qu’à établir rigoureusement les bases logiques sur lesquelles reposaient les procédés de ce nouveau calcul. Ils se contentèrent pendant longtemps de répondre par la solution inespérée des problèmes les plus difficiles à l’opposition prononcée de la plupart des géomètres du second ordre contre les principes de la nouvelle analyse … » [Comte, 1864, vol.I, 6th lesson, 178].

[3] « Die historische Bedeutung der beiden Brüder Bernoulli … reicht in der Tat fast an die epochemachenden Taten der Klassiker der mathematischen Wissenschaften heran, wenn man die Leistungen der beiden Brüder zusammennimmt » [Fleckenstein, 1949, 2].

the XXI century, unable to perform this separation. This is simply due to the fact that my research is concerned with Johann Bernoulli and his contribution to the development of the new calculus. I will however attempt to enter new ground and to add a perspective by adopting Johann's view of his brother. The portrait of Jacob that I will sketch in what follows, relies on Johann's statements, that appear throughout his many correspondences, on his hasty judgments, made in outbursts of anger or while latently or overtly in conflict with his brother. Necessarily, such a portrait, tainted by animosity, must be incomplete, partial and distorted. Nevertheless, it allows us, in particular, to gain insight into the nature of the relationship between Jacob and Johann and to understand what their rivalry owes to the mathematical practices of their time.



Portrait of the chimerical «Bernoulli brothers » [Figuier, 1870]

II. JACOB BERNOULLI AND HIS YOUNGER BROTHER JOHANN

1. Jacob Bernoulli's training

1.1. Philosophical and theological studies

Let us recall briefly, what we know of Jacob Bernoulli's education before he obtained, at the age of 34, the chair in mathematics in his hometown Basel. Jacob was born in this city on December 27th,1654 (according to the old calender) in a protestant family of spice traders who had fled the Spanish low lands after the fall of the Duke of Alba. Complying with the wish of his father Nicolas Bernoulli, a state adviser and magistrate, Jacob studied philosophy and then theology until 1676. As was common at the time, he chose a motto. His came from Phaeton who drew the solar carriage « *Invito patre sidera verso* » which may be translated by " Despite my father, I am among the stars". Rather than exaggerated modesty, this motto was a proud affirmation of superiority.

The young Jacob fully benefited from what Daniel Roche calls "culture de la mobilité" [Roche, 2003, 10] promoted in the second half of the XVII century by new institutions, which facilitated the movement of individuals and the spread of knowledge. Starting in August 1676, he traveled by horse to Geneva where he remained for twenty months [Battier, 1705] preaching, instructing a blind young girl, Elisabeth von Waldkirch, and serving as an opponent during the theological *disputationes* [Merian,1860]. He relates his experience teaching mathematics to the blind in an article published in the *Journal des savants* in 1685 [Jacob Bernoulli, Opera I, 209-210]. This article is probably a reaction to an account by Spon published in the same journal in 1680, in which the author attributes to the father of the blind girl the writing system that was in fact developed by Jacob [Jacob Bernoulli, Werke, 1, 237]. It is here that Jacob meets Nicolas Fatio de Duillier a life long friend who recalled in a letter dating from July 22nd, 1700 [Jacob Bernoulli, Briefwechsel, 164 et 168], that he had seen Jacob play court tennis in Geneva, a game on which Jacob later wrote a famous letter to a friend [Meusnier, 1987, 97-131] recently translated by Edith Sylla [Jacob Bernoulli, 2006]. In June 1678, Jacob continues his extensive traveling in France, residing in the Limousin (in Nède with the marquis de Lostanges, where he constructs two sundials in the castle courtyard), then in Bordeaux and a few weeks in Paris. During this journey, he begins, in 1677, to write his mathematical journal, *Meditationes, annotationes, animadversiones theologicae et philosophicae*, which contains 236 articles, the first of which have been described by Silvia Roero as simple exercises [Jacob Bernoulli, Werke 2, 15]. The journal is a precious testimony from this early phase of Jacob's scientific training which only really began when he encountered the Cartesian environment, initially in France, later mainly in the Netherlands (Amsterdam and Leiden) and in England during a second journey (April 1681-October 1682). In August 1682, Jacob attended a meeting of the *Royal society* [Merian, 1860] in London. Jacob started out by acquainting himself with the Cartesian philosophy of nature after which he turned to geometry. This can be seen from two works *Conamen novi systematis cometarum* (1682) and *Dissertatio de gravitate aetheris* (1683), published in Amsterdam. According to Joachim Otto Fleckenstein, editor of the volume on astronomy and natural philosophy of Jacob's collected works [Jacob Bernoulli, Werke 1], a third of Jacob's work was dedicated to topics of natural philosophy and logics.

1.2. The choice of mathematics

After his return to Basel in 1682, Jacob gave up the idea of a career in the clergy and decided to devote himself to mathematics. At the University of Basel he gave courses in experimental physics, as can be gathered by a pamphlet printed in Basel in 1686 [Jacob Bernoulli, Opera I, 251-276]. From 1682 on, he also submitted short articles to the *Journal des savants*-reactions to the works of others that he presented or criticized-initially in the area of natural philosophy (machines for breathing under water, to elevate water, to weigh air, oscillation center), then from 1685, in mathematics. It is noteworthy that one of the first problems that he brought up, concerned a game of dice, the solution of which he gave himself in the *Acta eruditorum* of 1690 and which he included in l'*Ars conjectandi* [Jacob Bernoulli, 1713, Pars 1, Append., probl.1, 49-57].

Silvia Roero has described, in her introduction to the young Jacob's works in arithmetic, synthetical geometry, and algebraic geometry [Jacob Bernoulli, Werke 2], how he slowly acquired a knowledge of mathematics, at first through his readings of the second Latin edition of Descartes' *Géométrie*, [Descartes, 1659-1611], later that of Arnauld and his *Logique*, Malebranche and Prestet. Jacob spent five or six years trying to solve, by Cartesian methods, problems that he started to publish in 1686 almost exclusively in *Acta eruditorum*. At the University of Basel, which he made a point of not neglecting, he presented, in particular, theses of logic published as brochures. A first work by both brothers, entitled *Parallelismus ratiocinii logici et algebraici* [Jacob Bernoulli, Opera I, 211-224] was presented in Basel on September 9th, 1685. It is a *specimen*, a *disputatio* exercise, two of which must be presented to obtain the title *magister artium*. Johann, who obtained this title on the 8th December that same year, was mainly a respondent and later refrained from including this work in his *Opera*. The theses are of two types; seventeen theses concerning the parallelism between logical and algebraic reasoning, as the title announced, and twenty-seven mixed theses, juxtapositions of phrases stating the obvious such as « *Risibilitas est risibilis proprietas* » (thesis 5) and abrupt judgments about previous publications of the *Journal des savants* such as: « Professor Le Montre's arguments against my comet system which appeared in the French journal *des savants* in 1682 is of no value » (thesis 23) or « Abbé Catelan was also mistaken concerning the oscillations of the pendulum » (thesis 24)[4]. However, we also find problems in probability such as the one concerning the marriage contract between Titus and Caia (thesis 21), which was studied in the article 77 of *Mediationes*, and included in l'*Ars conjectandi* [Jacob Bernoulli, 1713, 1^{e} partie, probl.5] and studied by Norbert Meusnier [Meusnier, 1987, 134-152]. This problem might be connected to the financial transactions that accompanied Jacob's marriage with Judith Stupan in 1684.

Infinitesimal analysis enters Jacob's journal in the form of notes from his readings of Wallis and Barrow. We can see that Jacob used, and critically analyzed, methods from Descartes, Hudde and Fermat to solve problems of tangent and curvature. The problems *de maximis et minimis* that are solved by methods from Hudde in the *Meditationes* 96 bis and 100 and which Silvia Roero date to 1686/87, were later treated using differential calculus in the *Lectiones calculi differentialis* (1691-1692) of Johann Bernoulli, to which we shall return later on. These works clearly attest the joint efforts that the brothers made to understand Leibniz method. However, we lack documents about the actual encounter with this method. From 1688, Jacob begins to formulate a critique of Descartes' *Géométrie*. This results in a new edition of Descartes' Latin geometry commented by Jacob [Descartes, 1695]. At this time, Jacob starts to take an interest in the problem of classifying curves, in particular third degree curves.

1.3 Jacob Bernoulli's cognitive approach

Relying on a solid knowledge of the *meditationes* from this period, Silvia Roero has attempted to characterize the mathematical work of the young Jacob, his way of proceeding and his style. According to her, Jacob is confronted with precise problems, often stemming from the area of applied mathematics. Solving these, leads him to discover general methods. He begins by a thorough study of previous works, which will serve him as a springboard to make further headway and produce new results. On several occasions, Jacob voices the opinion that it is necessary to base one's own progress on the knowledge of what has been done in the past. Accordingly, in the memoir entitled « *Solutionem tergemini problematis arithmetici, geometrici et astronomici* » [Jacob Bernoulli, Werke 2, 77-120], presented on February 4th, 1684, in order to obtain the mathematics chair in Basel, he describes his own way of proceeding in the following way : « In reality, he who embraces a career as a mathematician is not the one who copies the inventions of others, remembers them and recites them on occasion, but the one who is truly innovative and is able to invent by using the divine algebra and thus to revolutionize what has been studied by others »[5]. His entire epistemology, such as he expresses it for instance in the 1695 *Acta eruditorum*, relies on the idea that knowledge is constructed by small steps starting from what is previously established :
« Indeed, in the sciences like in nature, there are no leaps, knowledge, like natural quantities grows element by element and progresses only slowly; thus, to pass from one state to the next one, an infinitely small jump so to say, is sufficient; this ensures that those who proceed in an orderly manner and who have understood the previous parts will not be stopped and that they will by their own means manage to take the next step »[6].

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[4] « Rationes Professoris Montræi adversus systema meum Cometicum allatae, & *Ephemeridibus Erudit. Gall.* anni 1682 insertae, nullius sont pretii » [Jacob Bernoulli, Opera I, 223] et « Etiam Abbas Catelanus, circa doctrinam de oscillationibus funependulorum fallitur » [*ibid.*].

[5] « Mathematici namque partibus defungitur, non qui aliorum inventa exscribere, memoria tenere, aut recitare data occasione potest ; sed qui ab aliis proposita, divinae ope Algebrae, invenire et eruere novit ipse ». I quote Silvia Roero [Jacob Bernoulli, Werke 2, 260].

[6] « Quemadmodum enim in Natura nuspiam, ita nec in Scientiis saltus datur, sed omnis nostra cognitione, more quantitatum, crescit per elementa, atque ita pedetentim augetur, ut ab uno ejus gradu ad gradum proxime sequentem

2. Johann Bernoulli's education

Johann, the tenth child in the family, was born in Basel on July 27th, 1667, more than twelve years later than Jacob. He was nine years old at the time Jacob left home and he was an adolescent who had just enrolled at the university of Basel when Jacob returned in the autumn 1682. At this point, it was Johann who departed, since his father who intended for him a career in trade had placed him as an apprentice for a year (1682-1683). Despite his lengthy absence from the university, Johann, as we have seen, obtained his *magister artium* in December 1685 with the philosophy professor Nicolaus Eglinger.

Having refused to make his living as a merchant, as was his father's wish, Johann had not been able to persuade the latter to let him study mathematics, "which excited him in a singular way "[7], but he did obtain the permission to enroll in medical studies. In 1690, he obtained the *licence* after a public *disputatio* chaired by the aforementioned Nicolaus Eglinger who had become a professor of medicine. On this occasion a brochure entitled *De effervescentia et fermentation* [Johann Bernoulli, Opera I, 1-44] was published. It gave rise to an anonymous summary in the February 1691 issue of *Acta eruditorum*-in fact written by Leibniz who had recognized the brother of the famous Bernoulli. This was how Johann was introduced to the scholarly community. A few months later, Johann published his first work in mathematics entitled « *Solutio problematis funicularii* », to which I will return later, in the same *Acta* in Leipzig. Johann interrupted his studies and left for Geneva, at the end of December 1690, and stayed there eight months with Daniel Leclerc. He became friends with Jean-Christophe Fatio, Nicolas' older brother who was a fortification engineer and whom Johann taught « advanced mathematics ». He then continued to Paris where he remained from the end of 1690 to November 1691. At the philosopher Nicolas Malebranche's, he met with the marquis de l'Hôpital whom he instructed in differential and integral calculus. These lessons provided the material and the intellectual foundation for the *Analyse des infiniment petits* [L'Hôpital 1696], the first treatise on differential calculus, published anonymously. This event has been thoroughly analyzed by Otto Spiess who has been able to establish with certainty, using the handwritten lessons and the correspondence between Johann and the marquis, how much the latter owed to the exchange with his young friend. Upon his return to Basel, Johann began an epistolary exchange with l'Hôpital, Pierre Varignon and from the end of 1693, Gottfried Wilhelm Leibniz. In the beginning of 1694, he was awarded the title of doctor of medicine with a work[8] *De motu musculorum* in which he applies differential calculus to muscular contractions and which was as much a work in mathematics as in medicine. One week later, on March 26th, 1694, he married Dorothea Falkner, the daughter of one of the foremost magistrates in the Basel republic, after having accepted a position as a land surveyor.

III. FROM «STIMULATING COMPETITION » TO « BLIND ENVY »

> « J'oubliois de vous dire que le mot d'émulation dans notre langue ne signifie point jalousie comme vous le pensez ; mais une noble ardeur d'excéler en quelque chose, & d'y surpasser tous ceux qui s'en

non nisi saltus, ut sic dicam, requiratur infinite parvus ; ut nemo tam sit hebes, qui si modo ordine incedere velit, ac praecedentia intellexerit, non proprio marte pergere & ad sequentia transire possit » [Jacob Bernoulli, Opera 1, 662].

[7] « darzu ich eine sonderbahre lust bey mir verspühret » [Bernoulli, Gedenkbuch, 1922, 83].

[8] For a modern and commented copy, see *Dissertations on the mechanics of effervescence and fermentation and On the mechanics of the movement of the muscles by Johann Bernoulli*, ed. and translated by Paul Maquet assisted by August Ziggelaar, with an introduction by Troels Kardel, Transactions of the American Philosophical Society, vol.87, Pt.3, Philadelphia 1997.

mêlent, sans chagrin, cependant que les autres y réucissent, comme l'excite la jalousie »
(Pierre Varignon à Johann Bernoulli, [Johann Bernoulli, Briefe 2, 156])

1. Jacob Bernoulli, teacher of his brother Johann

It was in the mid 1680's that the brothers, both active at the University of Basel, Jacob as *magister artium* giving a course in experimental physics and Johann as a medical student, began to work together. This can be seen from the theses from 1685 signed by both brothers, of which we spoke previously. Although we have little information about this period, most historians[9] agree that Jacob guided his younger brother, aged twenty at the time, in his mathematics studies. In his autobiography[10] written in French, Johann claims : "it was during that time, imitating the interests of my late brother Jacob… that I seriously engaged in the study of mathematics". Somewhat further, he pursues : "In less than two years, not only had I become familiar with all the classic authors who wrote on mathematics but also with the modern ones, such as the geometry of Descartes and his commented algebra". Here Johann depicts himself as an autodidact who had merely followed in his older brother's footsteps, whereas Jacob had always considered his brother as his student. We can cite, for example, a letter to Leibniz from March 4th, 1696, where he clearly states that Johann had learned the foundations of (mathematical) science from him[11].

Towards 1687, when Jacob had just obtained the mathematics chair in Basel[12], the two brothers discovered, in the *Acta eruditorum* from 1684 the « *nova methodus* » that is Leibniz's algorithm of differential calculus. The article is obscure, the explanations very brief and disfigured by a number of typographical errors, but according to Johann's autobiography: "this was enough to allow us in a few days time to learn the whole secret".[13]

What we know is that on December 15th, 1687, Jacob addressed a letter to Leibniz [Jacob Bernoulli, Briefwechsel, 47-51], asking him for further explanations. He does not mention his brother. What is less known, is that it was a craftsman from Basel who provided him with the pretext, having asked him for advice about the best choice of shape for the beam of a scale. This example shows that Jacob, who had acquired, perhaps through his *Collegium experimentale*, a solid reputation as a good mechanic, was sufficiently implicated in the daily life of the city of Basel for a craftsman to come to him for advice. The question of the craftsman had led to an elasticity problem which had been partially solved in the *Acta* of 1684. While Leibniz had examined the case where a balk gave in under its own weight, Jacob was unable to find an equation for the case where the balk was deformed by a weight hung on its unattached end. In his letter to Leibniz, he supposed that some higher level geometry, « *sublimior quaedam Geometria* », would be required. We make note of the fact that the elastic curve would become a central theme in Jacob's later research.

As we know, this letter reached Leibniz only three years later. This delay gave the Bernoulli brothers the time to absorb the ideas of Leibniz, such as they were presented in the *Acta* of 1684

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[9] Cf. [Fleckenstein, 1958] for instance.

[10] There are two autobiographies of Johann Bernoulli, one in German [Bernoulli, Gedenkbuch 1922, 81-103] and the other one in French, published by Rudolf Wolf [Wolf, 1859, 71-104]. Johann writes : « ce fut pendant ce temps là, qu'à l'imitation et l'inclination de feu mon frère Jacques, …, je commençai à m'appliquer à l'étude des Mathématiques. … en moins de deux ans non seulement je m'étais rendu familier presque touts les anciens auteurs qui ont écrit sur les Mathématiques, mais aussi les modernes, comme la géométrie de Descartes et son Algèbre avec ses Commentaires » [Wolf, 1859, 72].

[11] Speaking of his brother and of applying mathematics to medecine, Jacob wrote : « illum stimulavi, ut principia Scientiae, quam a me didicerat, huc applicaret » [Jacob Bernoulli, Briefwechsel, 77].

[12] He was invited to take the chair the 15th February 1687. He inaugurated it the 11th March with a dissertation on the origin and the progress of mathematics [Merian, 1860].

[13] « c'en était assez pour nous, pour en approfondir en peu de jours tout le secret » [Wolf, 1859, 72]

and 1686. In an article from May 1690, Jacob had managed to solve the problem of the *curva aequabilis descensus*, or isochronic curve that Leibniz had proposed in the *Nouvelles de la République des lettres* of September 1687: "To find a line of descent, on which a heavy body descends uniformly & approaches the horizon by equal distances in equal time intervals"[14]. The semi-cubic parabola possesses this property, as was shown by Huygens, then by Leibniz without using the new analysis. Jacob was able to provide a solution based on differential calculus. In the same dissertation, Jacob introduces the term « integral » which Johann has always claimed as his, and suggested at the end of it, the problem of the catenary curve. Galileo had been interested in this curve formed by a chain or a cord whose endpoints were attached at two fixed points, and had suggested that it was a parabola. It is in fact a transcendental curve. The brothers worked away on the problem until they learned that Leibniz had solved it. Johann was able to find a solution to the problem by reducing it to a rectification of the parabola and to the squaring of the hyperbola [Johann Bernoulli, Opera I, 48-51]. Thanks to this achievement, made public in the *Acta* in June 1691, he suddenly joined the ranks of the foremost mathematicians in Europe along with his brother, Leibniz and Huygens.

This is how Johann, more than a quarter of a century later, on September 29th, 1718, describes his discovery to Pierre Rémond de Montmort : « My brother was not successful in his efforts...as for me I was in greater luck, since I found a clever trick(I say this without wishing to boast, why should I hide the truth?) that allowed me to solve it completely and to reduce it to the rectification of a parabola. It is true that contemplating the problem, I lost a full night of sleep; at that time, when I was young and inexperienced it seemed like a lot, but the next day, I ran exalted, to see my brother who was still struggling to no avail with that Gordian knot, since he still suspected, like Galileo, that the catenary curve was a parabola; give it up! give it up! I told him, do no longer attempt vainly to identify the catenary curve with the parabola for they are not the same. One can be used for constructing the other, but these curves are as different as an algebraic and a transcendental one. Having intrigued him, I showed him my solution and discovered the method which had led me to it".[15] In this vivid and touching account, Johann contrasts, as we will often see him do, his own shrewdness with the slow labor of Jacob. But the naive spontaneity with which Johann had said that he rushed to see his brother after his wake, which Jacob would later make fun of, to inform him of the solution, shows that the brothers still got along and trusted each other.

2. **Closely intermingled works**

On December 21st, 1690, Johann left Basel, to embark on what is commonly referred to as his « *peregrinatio academica* ». Separated, the two brothers wrote each other about twenty letters of which only four of Johann's drafts have been kept. Otto Spiess, the editor of volume 1 of the correspondence of Johann Bernoulli, has reconstituted the contents of the other letters, based on the *meditationes* and the published memoirs. This correspondence, albeit incomplete, gives some slight indications about the brothers' collaboration at the moment when Johann's talent blossomed and he tried to find his own path.

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[14] « Trouver une ligne de descente, dans laquelle le corps pesant descende uniformément, & approche également de l'horison en temps égaux ».

[15] « Les efforts de mon frere furent sans succès, …, pour moi, je fus plus heureux, car je trouvai l'adresse (je le dis sans me vanter, pourquoi cacherois-je la vérité ?) de le resoudre pleinement et de le reduire à la rectification de la parabole. Il est vrai que cela me couta des meditations qui me deroberent le repos d'une nuit entiere ; c'etoit beaucoup pour ce tems là et pour le peu d'age et d'exercice que j'avois, mais le lendemain, tout rempli de joie, je courus ches mon frere, qui luttoit encore miserablement avec ce noeud Gordien sans rien avancer, soupçonnant toujours comme Galilée que la chainette etoit une parabole ; cessés ! cessés ! lui disje ne vous tourmentés plus à chercher l'identité entre la chainette et la parabole, là où il n'y en a point. Celle-ci aide bien à construire l'autre, mais ce sont deux courbes aussi differentes que peuvent l'etre une courbe algebrique et une transcendante, j'ai développé tout le mystere ; ayant dit cela je lui montrai ma solution et decouvris la methode qui m'y avoit conduit » [29.9.1718, UB Basel, LIa 665]

Before departing, Johann confided to Jacob an article on caustic that the latter was supposed to send to Leipzig for publication in the *Acta eruditorum*. But Jacob, who saw a possible generalization, added a paragraph on his own initiative[16], where he spoke of himself in the third person. Published in the January 1692 issue, the article was followed in March by another work by Jacob [Jacob Bernoulli, Werke 5, 350] on the osculating circle and the nature of the contact.

Similarly, Jacob followed up his memoir dating from May 1692, on cycloids by an addition that appeared in June and the contents of which is said to come from a letter he received from his brother after having sent his manuscript to Leipzig. We are able to conclude that the competition between the brothers was profitable for both of them and gave rise to a number of results from one or the other. They attacked the same problems, often stated by Jacob, more or less simultaneously but never wrote any articles together. Their published results were always clearly attributed to one of them. However, it was systematically the older brother who acted as the intermediary with the *Acta* of Leipzig, sometimes without keeping his younger brother duly informed.

Thus, Jacob speaks to Johann of his work on the oscillation center only after having sent it to Leipzig. Johann reacts to this, on June 17th, 1691, in the following way: «As for our mathematics business, I am upset that you did not wait a bit longer to share your inventions concerning the oscillation center, because I think that I have found, before receiving your letter, a beautiful method for finding the center which I believe to be more general than yours which can not be adapted to the bodies nor to the planes which vibrate around the axis when it is perpendicular to the planes; perhaps you believe that it is from your ideas that I have gathered some insights as not to give the impression that I do not understand this area of research; nothing could be further from the truth, I assure you, and Mr. Fatio is my witness that when I was in Duillier I had already found the method of which I speak;…”.[17] We see that the trust between the brothers is still intact in the summer of 1691. Johann who already seems very sure of himself and his abilities expects his brother Jacob to publish their results together. But his reaction also reveals what will soon become a problem for him: how to become independent? How to make a name for himself? In fact Johann's solution for finding the oscillation center, which turns out to be false, is based on an idea identical to the one used by Huygens in 1673, as Jacob points out. Johann's reply dating from September 29th, 1691 is symptomatic : « that is possible, but I assure you that I have never read the treatise of Mr. Huygens, nor was I aware of your contesting this author and Catelan, and thus, (as you once claimed in a public *disputatio*) I am not less worthy of praise than if I had been the first to find it”.[18] If the mathematical community only retains the name of the one who finds a result first, what recognition can the second discoverer hope for? Does he merely « *ova post prandium apponere* »[19] as one said at that time? This question that the younger Bernoulli brother dwelled on, had apparently been the subject of a public disagreement between the brothers. It is this search for independence, crucial to Johann, which would cause the first public quarrels.

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[16] The paragraph in question begins : « Caeterum animadvertit Clarissimus Frater, methodum hanc posse generalem effeci » [Jacob Bernoulli, Opera I, 471]. We note that this memoir of Johann was published with others in Jacob's *Opera*.

[17] « Quant à nos affaires Mathematiques, ie suis bien faché de ce que Vous n'avez pas differé encore pour quelque temps Vos inventions touchant le centre d'oscillations, car i'ay trouvé avant avoir reçûe Vôtre lettre une belle methode pour chercher ce centre qui ce me semble est plus generale que la Vôtre laquelle ne peut s'accommoder ni pour les corps ni pour les plans faisans leurs vibrations autour de l'axe quand il est perpendiculair à ces plans ; vous croirez peutétre, que ce sont Vos intentions dont i'aye recueilli quelque chose pour ne pas paroitre comme si cette recherche surpassoit ma portée ; mais bien loin de là, ie vous asseure et ie prens Mr. Fatio à témoins, qu'étant à Duillier j'ay déjà trouvé cette methode dont je vous parle ; … » [Johann Bernoulli, Briefe 1, 109].

[18] : « cela se peut, mais ie vous asseure, que ie n'ay iamais lû ni le traitté de Mr. Huygens ni la contestation entre Vous, cet Autheur et Catelan, de sorte que ie puis dire que ie n'ay pas merité moins de louange (à ce que Vous soutintes une fois dans une dispute publique) que si ie l'eusse trouvé le premier » [Johann Bernoulli, Briefe 1, 115].

[19] That is « to serve eggs after breakfast » (see for example [Leibniz, Math. Schriften 2, 270]).

3. The problem of the curvature of the sail exposed to the wind, sources of conflict revealed

The history of the *velaria*[20] is well documented in the literature. For me, it is an occasion to examine the statements that Johann made about his brother and to confront them with those of Jacob. The two brothers start to take interest in the problem of the shape of a sail exposed to the wind (*velaria*) in January 1691. In a letter that has not been kept, Jacob sends his brother the differential equation for the curve in the form $d^2x : dx = dy^3$ and asks to derive from it a literal or algebraic equation which expresses the nature of the curve, or at least to determine its points by some construction [Johann Bernoulli, Briefe 1, 100]. According to Jacob, Johann had proposed several hypotheses, all of which he had rejected. Initially Johann believed that the curve of the sail could be identified with a funicular curve for which the weight *ds* is proportional to *dx*, thus[21] to a parabola. At the end of April, Johann believed that the sail had the shape that it would take on if it was filled with liquid, and he tried to calculate its curvature.

Jacob considered several cases, and seems to have had a greater understanding of how the wind can act on a sail[22]. Johann obstinately refused to understand the distinction between the different cases and wrote on June 17th, 1691 to Jacob : « what surprises me the most is that you claim that the sail is partially the periphery of a circle, partially some other curve, I shall never be able to understand how the same cause could produce two different curves. You are like the ancients who believed that the trajectory of a cannon ball made three different lines, read Sventer and you will see".[23] On September 29th, he overtly mocks his brother : « Unfortunately I am afraid that I am incapable of understanding your sail and its two curves, when I want to behold one of them, it shows me the other one, if it is not completely illusory, it must at least be a regular Protheus".[24] Here Johann accuses his brother, in a rather unpleasant manner, of following the methods of the ancients and refers to Daniel Schwenter, a mathematics professor in Altdorf, author among other works of a *Deliciae physico-mathematiae* (1651) where he reports that in old times, the trajectory of a cannon ball was believed to be composed of straight lines and curves. Then, using the image of Protheus, a sea world god, who possessed powers of divination, but eluded all questions by metamorphosing incessantly, Johann makes fun of a versatile and intangible curve that takes one form after another.

While Johann was on his journey to Paris, Jacob found the remarkable result that the curve was a catenary. At the end of 1691, he informs Johann of this result in the following manner: *Sumptibus aequalibus curvae portiunculis, Cubi ex primis differentiis ordinatarum sunt proportionales secundis differentiis abscissarum* », that is $adsddx = dy^3$ (*). He does not give the least indication as to his method nor of the curve, that satisfies the equation (*). Johann immediately discovers that it is a catenary, and replies in a letter that « *curvam huius aequationis eandem esse cum catenaria* », while suspecting that Jacob does not know what kind of curve it is. He suggests this in an article in the *Journal des savants*, of April 28th, 1692 (p.189) where he speaks of Jacob in the following terms : "once more he forces me to complete the solution that he has begun and

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[20] We can follow it in the account of Otto Spiess in [Johann Bernoulli, Briefe 1]. See also [Hofmann, 1956] and Sybille Ohly's Ph.D. Thesis directed by H. N. Jahnke [Ohly 2001].

[21] According to a theorem known by both brothers and published in « Specimen alterum calculi differentialis » [Jacob Bernoulli, Opera I, 442-453].

[22] See the short history of the sail that Jacob published in 1695 in the *Acta eruditorum* [Jacob Bernoulli, Opera I, 652-655].

[23] « ce qui m'etonne le plus est que vous dites que la voiliere est partie une peripherie du cercle, partie une autre courbe, ie ne saurois jamais comprendre, comment une méme cause peut produire deux courbes differentes. Vous faites comme les Anciens, qui ont cru que le jet d'un boulet de canon fasse trois lignes, lisés dans Sventer et vous le verrez ; … » [Johann Bernoulli, Briefe 1, 111].

[24] « Hélas ! que je suis malheureux de ne pouvoir point comprendre votre voiliere bicourbe, aussitôt que ie veux regarder une de ses courbures, voicy l'autre qu'elle me montre, enfin si elle [n'est] pas chimere, du moins sera [-t-] elle le veritable Protheus » [Johann Bernoulli, Briefe 1, 115].

developed until this equation [equation(*)], after which he apparently gave up".[25] Unfortunately, for him, Jacob had sent his solution to Mencke on March 9th, 1692 and it was published in the *Acta eruditorum* in May 1692 [Jacob Bernoulli, Opera I, 481-490]. From then on, the relationship between Jacob and Johann gradually deteriorated. Jacob grew increasingly distant. In 1693, he still spoke in very neutral terms of the curvature of the sail that his brother had found after having guessed the trick that had led Jacob to his equation[26] ; nevertheless, he made it clear that he had first claims to the discovery and the method that his younger brother had followed.

3.1. Claiming to be the first discoverer

Undoubtedly chocked by the unfounded public accusation that Johann had made in the *Journal des savants*, Jacob decided to make an inventory of his discoveries to make it known that he had been first to have ideas that he had carelessly shared with Johann. Later, he expressed this intention in a letter to Leibniz dating from November 15th, 1702, saying that he had wanted in one of his first letters to Leibniz to tell the story of his and Johann's lives and of the mathematical achievements they had both made from their early adolescence. He claims for himself the merit of having been the first to penetrate the mystery of Leibniz's calculus and having shared it with Johann[27].

Thus, in June 1694, Jacob publishes his « *theorema aureum* » or golden theorem, an expression that appears for the first time in his *meditatio* CXCII, from the spring 1692. The name expresses the importance that Jacob accorded to this theorem, which was a source of pride to him, and the novelty and usefulness of which he emphatically boasted [Jacob Bernoulli, Opera I, 577]. It is in fact a formula for the radius of curvature that Jacob notes $z = ds^3/dxddy$ and which Johann used as a card for introducing himself when he traveled – a fact that Jacob did not fail to underline in the A*cta*. This beautiful result does indeed belong to Jacob alone, because it is possible to reconstruct from the *Meditationes*, the path that lead him to the famous formula[28]. Jacob also translates the formula into polar coordinates, qualifying this result as unknown even by my brother (« inconnu même de mon frère »). We note that the marquis de l'Hôpital included this theorem in his *Analyse des infiniment petits* [L'Hôpital, 1696,77] without any reference whatsoever to Jacob Bernoulli, nor for that matter to Johann.

3.2 From the first squabbles to the split

There is a brutal change of tone in an article that same month of June in 1694 on the paracentric isochrones, that is the curve on which a heavy body approaches by equal distances in equal time intervals to a given point [Jacob Bernoulli,Opera I, 601]. Jacob accuses Johann's inverse tangent method of being inefficient, lacking in generality and consisting only in a small trick that, he Jacob, would not dare call a method[29]. Johann was extremely hurt, and wrote in an outburst of anger a letter to l'Hôpital on January 12th, 1695 : "he is a misanthropist in general and does not even spare his own brother. ..He is filled with rage, hate, envy and jealousy against me. He holds grudges

[25] « il me pousse encore d'achever la solution qu'il avoit commencée, & conduite jusqu'à cette équation [l'équation (*)] ; ce qu'il tenoit apparemment pour desesperé » [Johann Bernoulli, Opera I, 60].

[26] « Quippe nec Frater meus, qui dum adhuc Parisiis versaretur Problema plene absolvit, detecto quod me ad aequationem $adsddx = dy^3$, [suppositis elementis curvae *ds* aequalibus] perduxerat artificio » [Jacob Bernoulli, Opera I, 562].

[27] « Animus fuerat olim, quam primum ad Te darem literas, in mei justificationem perscribi Tibi historiolam vitae et profectuum nostrorum, quos ambo a prima adolescentia in Mathesi fecimus (ubi inter alia vidisses, non ipsum, sed me calculi Tui mysteria primum penetrasse ipsique impertivisse ...) sed mutavi sententiam, quia video nil profutura » [Jacob Bernoulli, Briefwechsel, 101].

[28] Martin Mattmüller has carefully studied it [Jacob Bernoulli, Werke 5, 331 et sq.].

[29] « At statim sensi, illas non continere nisi artificia quaedam particularia, quae methodum appellare non ausim » [Jacob Bernoulli, Opera I, 607].

against me because you have been kind to me, and he has persecuted me ever since you honored me with a rent, his vanity makes him believe that his reputation will suffer, he finds it unbearable that I the younger brother receive as much esteem as he the older one does, and he would take great satisfaction in seeing me miserable and reduced to humility. How unworthy of a brother, what atrocious pride".[30] He adds somewhat further on: "do not fear that I tell my brother what we write to each other, because I have not spoken to him for more than six months". It seems like the split had become definite[31] in the summer of 1694, just after Johann's marriage, which Jacob had attended. In the passage above, Johann insinuates that the source of the break was L'Hôpital's outspoken acclaim. Undoubtedly the real reasons were more complicated.

After Johann's departure for Groningen, Jacob took upon himself to send to his brother and to L'Hôpital, books which passed through Basel, among others the *Acta eruditorum*. This triggered a strong reaction from Johann, which he expressed in a letter, addressed to l'Hôpital, on April 21st, 1696 : "To what do we owe this sudden courtesy of my brother the professor, suddenly so eager to write, to help, to send you acts, in one word to woo, he who is usually so stoic, so misanthropist, so lowly not even to answer the letter that Mr. Leibniz wrote him several years ago, and which Mr. Leibniz complained about to me several times. You would not believe how much this brother, unworthy of the name, hates me, persecutes me and tries to destroy me, since I have the honor of being highly regarded by you; once more he gives me some fine examples from the acts of last December, where he tears me apart miserably and spews against me the worst outrage and lies, and even that which seems kind and gentle to me, contains a hidden dose of poison...".[32]

Here Johann refers to the « *brevis historiola* » or a brief history of the invention of the curve taken on by a sail exposed to the wind, published in the *Acta eruditorum* of December 1695. Still trying to clarify the situation by attributing to each one his results, Jacob had been led to reconstruct in great detail, based on the correspondence with his brother, the story[33] of the identification of the curvature of the sail exposed to the wind with the catenary. He is ruthless with his younger brother, revealing the lack of understanding of the latter for the mechanical phenomena related to the pressure of a fluid. Returning to the paracentric isochrones, he reproaches Johann of not adding anything to his own discoveries. At the end of the article, he exposes his conception of scientific research, returning to the theme that the brothers had undoubtedly debated, about two individuals working in the same field of research. Usually they follow "different paths which are not equally adapted to the nature of the subject" (des voies différentes non également adaptées à la nature de la chose), which is something those who follow them cannot foresee from the start. Jacob compares them to two people who travel through unknown territories from which each one brings back what

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[30] « c'est un misantrope general qui n'epargne pas méme son frere, …, il créve de rage, de haine, d'envie et de jalousie contre moy, il m'en veut du mal à cause que vous m'en voulez du bien, il me persecuta dés le moment que vous m'avez fait l'honneur d'une pension, il croit que cela fait tort à sa vaine reputation, ne pouvant pas souffrir que moy qui suis le cadet soit aussy bien estimé que luy qui est l'ainé, enfin ce seroit avec le plus grand plaisir de me voir dans l'état le plus miserable et reduit à l'extrémité. Quelle indignité à un frere ! quel execrable orgueil ! … n'ayez pas peur, que je fasse part à mon frere de ce que nous nous ecrivons, car il y a plus de 6 mois que je ne luy ay parlé mot » [Johann Bernoulli, Briefe 1, 255].

[31] See Fritz Nagel's contribution in this issue. Also see what Johann wrote to Leibniz the 12th February 1695 [Leibniz, Math. Schriften 3, 163].

[32] « d'où vient cette nouvelle courtoisie de mon frere le professeur ? qu'il est si promt à écrire, à servir, à vous envoyer les actes ? en un mot à vous faire la cour, qui d'ailleurs est si stoïque, si misantrope, si vilain que de ne pas donner une seule reponse à la lettre que Mr. Leibnits luy a ecrite il y a plusieurs années, et dont Mr. Leibnits s'est plaint à moy deja souventefois. Vous ne sçauriés croire combien ce frere qui n'est pas digne de porter le nom de frere me hait, combien il me persecute et tache de m'abimer, depuis que j'ay l'honneur d'étre bien regardé auprés de vous ; il en a donné nouvellement un bel échantillion dans les actes du dernier décembre, où il me dechire miserablement et vomit contre moy des calomnies et faussetés epouvantables, et meme tout ce qui y paroit étre de plus doux et à mon avantage, est rempli de poison caché … » [Johann Bernoulli, Briefe 1, 317].

[33] What is usually refered to in the German litterature as the « Velaria-Bericht » published in [Jacob Bernoulli, Explicationes, annotationes et additiones ad ea quae in Actis … de curva elastica, isochrona paracentrica et velaria … leguntur, *Acta eruditorum* de décembre 1695, 546-547 [Jacob Bernoulli, Opera I, 652-655].

he can. And nevertheless, none of them can take everything the land of the other one produces[34]. Even if two people simultaneously attack the same problems, as was the case for the Bernoulli brothers for about a decade, and even if each one found the correct solution in his own way, neither one deserved to get all the credit. This seems to be Jacob's philosophy. However, as far as the sail exposed to the wind was concerned, he insinuated that he was the one who found a path on which Johann merely followed him. Here, he seems to deny his younger brother any originality.

Johann tries to take his public revenge as soon as he is on a par with his brother, i.e. as soon as he earns the title of professor and obtains the chair of mathematics at the university of Groningen.

4. Mathematical communication understood differently

On several occasions, Johann Bernoulli reproached his brother for his secrecy. After some uncontrolled outbursts to l'Hôpital in the beginning of 1695, Johann then complained more calmly to his friend Leibniz, about the hostile attacks of his brother. In a letter from February 12th, he contrasts his own character with Jacob's « zealously seeks to hide everything using anagrams (*logogriphis*) from which he derives a futile glory and admiration, something I cannot understand. That is why he persecutes me arduously (which he is ashamed to admit) with his secret hatred,..”[35].

When in 1718, in the middle of the quarrels about the first discovery of differential calculus, Montmort who wanted to write a history of geometry, incited Johann to return to the subject of the origins of differential and integral calculus, the latter wrote : « You naively admit that I and Mr. Leibniz have revealed early on to the marquis de l'Hôpital our secrets, which, as you have added, would have remained secret to all mathematicians until this day, if we had wanted to hide them as Mr. Newton did; who knows what would have happened, had I followed my brother's bizarre moods, he who was initially as secretive as Mr. Newton. I can show you some of the letters that he has written me, in which he complained about the fact that I so willingly shared our secrets and exhorted me to hide them”.[36] This is his flattered reaction to comments that Montmort had made to an English audience, Brook Taylor in particular. Indeed Montmort gave Leibniz and the Bernoulli brothers full credit for having developed and spread the use of the new methods of analysis : “they alone taught us the rules for differentiating and integrating... to convince oneself of this, it is sufficient to open the journals of Leipzig,..no one except Mr. de l'Hôpital, whom we may add to the ranks of these men, although he was the disciple of Mr. Jean Bernoulli, appeared with them on the scene until about 1700”.[37]. Johann is eager to claim as his this culture of open communication, which, according to him, was not shared either by Newton or by Jacob.

It is certainly true that Jacob, despite having published a great number of articles, almost all of which appeared in the *Acta eruditorum*, had not established as regular and extensive a letter-writing

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[34] « etiamsi duo eidem quaerendae rei mentem applicent, fieri plerunque solet, ut diversas vias ineant, naturae rei non aeque accomodas, quas tamen quo ducant initio praevidere non possunt ; similes duobus, qui pari quidem sagacitate Terras incognitas lustrant, amboque novis spoliis onusti domum redeunt ; sed neuter, quae alterius tantum Terra tulit, asportare potest » [Jacob Bernoulli, Opera I, 663].

[35] « Hac autem in parte frater meus omnino est contrariae naturae, quippe qui omnia summo studio celare et logogriphis suis involvere conatur, ex quo nescio quam vanam gloriolam et sui admirationem captat, meque propterea (quod pudet dicere) clandestino odio fervide prosequitur,… [Leibniz, Math. Schriften 2, 163].

[36] « Vous avoués ingenument que nous avons, Mr. Leibnits et moi, revelé de bonne heure à Mr. le M. de l'Hopital nos secrets, qui apparemment, ajoutés Vous, en seroient encore pour tous les Geometres d'aujourd'hui, si nous avions voulu les cacher à l'imitation de Mr. Newton ; que sçait on ce qui seroit arrivé si j'avois voulu suivre l'humeur bizarre de mon frère, qui au commencement etoit pour le moins aussi mysterieux que Mr. Newton ; je pourrois vous montrer quelques unes de ses lettres qu'il m'a ecrites lorsque j'etois à Paris, dans lesquelles il m'a grondé souvent de ce que j'etois si facile à communiquer nos secrets, et m'exhortois à les tenir cachés » [15.6.1719, UB Basel, LIa 665].

[37] « ce sont eux et eux seuls qui nous ont appris les regles de differentier et d'integrer, … il suffit pour s'en convaincre d'ouvrir les journaux de Leipsic, …, personne hors M. de l'Hopital, qu'on peut joindre en partie à ces Messieurs, quoiqu'il ait été disciple de Mr. Jean Bernoulli, n'a pas paru avec eux sur la Scene jusqu'en 1700 ou environ » [UB Basel, LIa 665].

network as Johann. The latter describes him as not « prompt à écrire » [Johann Bernoulli, Briefe 1, 317]. His correspondence, inventoried in *Der Briefwechsel von Jacob Bernoulli,* a thin volume of hardly 300 pages, has not been preserved. In addition, Jacob often neglected his correspondence, on the account of health problems "to which were added an innate sluggishness when writing and a remarkable laziness".[38] Thus, Jacob interrupted his correspondence with Leibniz on two occasions, between 1690 and 1695 and again from 1697 to 1702, when he suspected his correspondent of taking his brother's side in their conflicts.

Jacob did not have Johann's spontaneous way of sharing his own discoveries, as well as those of his brother. More thoughtful, he took his time. Aware of having an original approach in a field that was just opening up, he probably sought to protect his intellectual property by not widely spreading his unpublished results (except to his brother when they « walked *passibus aequis* and to his disciples) and by making sure that he was recognized to have been first. But unlike Newton, he did not seek to keep his results secret. Less capricious and communicative than his brother, Jacob shared his discoveries through a large number of articles carefully written in Latin, as can be seen from the two volumes of *Opera*.

IV. PUBLIC JOUSTS 1696-1700

The brotherly rivalries would be made public through mathematical challenges with deadlines and rewards. Once both brothers had established themselves professionally the conflict escalated rapidly, violently and irreversibly. The *Streitschriften*-that is the documents that both brothers wrote during their controversies-have been united in a volume of works of the Bernoulli brothers [Jacob & Johann Bernoulli, Streitschriften] which makes them easy to access. As before, I will concentrate on discerning in this correspondences the outraged statements that resulted from these conflicts and which allow us to get a clearer picture of Jacob Bernoulli and in particular of his scientific personality.

1. The challenge of the brachystochrone

Johann was the one who started the conflict by a first skirmish to which Jacob referred as « *velitatiuncula* » in a public letter to his brother [Jacob & Johann Bernoulli, Streitschriften, 471]. In the June edition of the *Acta eruditorum* from 1696, he proposed a new problem that he invited mathematicians to solve : Two points A and B being given in a vertical plane, determine the curve AMB in which a body M, starting from A, descends solely by its own weight and will reach the point B in the shortest possible time.

Convinced that this was an important problem, Johann made sure that it received a great deal of publicity, and established a fixed period of time within which the foremost mathematicians in the world were invited to propose a solution. He even printed, in Groningen, a poster that Jacob received in January 1697. By proposing this difficult problem, Johann intended to show the superiority of Leibniz's methods but it was also an outright provocation directed against his brother whom he counted "among those who proudly believe that they have penetrated the deepest mysteries of mathematics thanks to specific methods and that they have extended its reach by golden theorems which they imagine to be unknown to everyone, while they have in reality been published earlier by others"[39]. Jacob is not fooled, and writes to Leibniz : "At this very instant, I

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[38] « Cui si adjungas nativum meum ad scribendum lentorem ac segnitiem non mediocrem » [Jacob Bernoulli, Briefwechsel, 68].

[39] « … etiam inter illos ipsos qui per singulares quas tantopere commendant methodos, interioris Geometriae latibula non solum intime penetrasse, sed etiam ejus pomoeria Theorematis suis aureis, nemini ut putabant cognitis, ab aliis

lay my hands on a printed matter in which my brother urges in words filled with venom, and for the third time all the mathematicians in the world, and it seems, me in particular, to solve his problem".[40] Despite the well-organized publicity, there were few candidates for solving the problem. Leibniz announced that only Jacob, L'Hôpital and Hudde, if he resumed his work, would be capable of finding a solution. This was humoring L'Hôpital who had had to cheat to remain on the list [Peiffer, 1989].

The problem could not be solved by the usual *de maximis et minimis* methods. Indeed, one seeks to determine a curve among an infinity of possible ones with the same endpoints, on which the time of fall of an object dropped without initial speed, is the shortest possible. The time that should be minimized is expressed by an integral that contains not y, if the curve that is to be determined is given by $y=f(x)$, but y' its derivative. This situation pertains to what we call variational calculus, an area that was completely unexplored at that time. The solution curve is an arc of a cycloid with a horizontal basis, the origin in the highest of the given points, and the generating circle of which has a diameter that goes through the second point. Leibniz, Newton, Jacob and Johann Bernoulli, each provided a solution, which were published together, with an introduction by Leibniz, in the May issue of the 1697 *Acta eruditorum*.

Johann uses a shrewd analogy with optics, from which he finds the equation of the curve almost immediately. He identifies the brachystochrone with the curve that a ray of light would take when propagating in a medium whose density is inversely proportional to the speed that a heavy body acquires when falling pulled by its own weight. The curve according to which the light propagates in the shortest time must obey Fermat's principle in every point. Translating this principle into analytical terms, Johann was able to write down the differential equation for the curve, in which he recognized a cycloid. It is an *ad hoc* method, elegant but not possible to generalize at all. It has a touch of genius, which Jacob would call a trick that does not deserve to be called a method.

Jacob proceeded more systematically, showing initially that the extremal properties must be conserved in each part of the curve. Then he considers, for an element of the curve, a second curve used for comparison and equals the time of the fall on each of the two curves, which are supposed to be brachystrochone. His method can be generalized and applies to a class of (variational) problems, but it is longer to carry out and requires a number of long and painstaking calculations, a point that Johann never fails to bring up when writing of Jacob. Here is an example dating from 1697 : "This shows how fortune plays tricks on us: ever changing, she led him onto a rude and thorny path, while I was lucky enough to find a gentle and very short path, easy to take and on which I found even more than I looked for".[41]

2. The isoperimetric problem

Having replied to Johann's challenge on the brachistochrone, Jacob brought up some problems of even greater difficulty, which he had come to ponder [Jacob & Johann Bernoulli, Streitschriften, 275], among others an isoperimetric problem: "Of all isoperimetric curves on a given axis BN, we seek the one that like BFN does not contain the greatest surface, but which maximizes another one contained by the curve BZN, after having extended FP in such a way that PZ is any ratio multiplied or divided by PF or the arc BF, that is to say that PZ is any proportion of a given A and of the

tamen jam longe prius editis mirum in modum extendisse gloriantur » [Jacob & Johann Bernoulli, Streitschriften, 259-262, quote on p.261].

[40] Hac ipsa hora incidit mihii n manus ingens aliquod Programma typis excusum, quo frater jam tertium omnes totius orbis Geometras, & ut videtur me in specie, verbis jactantia & felle plenis, ad solutionem sui Problematis provocat » [Jacob Bernoulli, Briefwechsel, 94].

[41] « Cependant voici comme la Fortune se joüe des hommes ; cette inconstante ne lui ayant montré qu'un chemin très-rude & très épineux, m'a été si favorable qu'elle m'a mené par une voye douce, très-courte & très aisée, par laquelle j'ai même plus trouvé que je ne cherchois » [Jacob & Johann Bernoulli, Streitschriften, 289] .

distance PF or the arc BF" [42]. He addresses this challenge directly to his brother whom someone anonymously offers fifty silver ecus if he is able to find the solution in less than three months. Despite some bragging-he claims to have solved the problem in three minutes-, Johann was incapable of finding a correct solution to this problem. Upon this, followed a long and painful controversy that involved scientific journals in France and Holland as well as the Royal academy of science.[43]

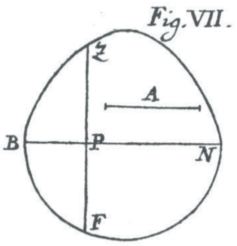

Diagram of the isoperimetric problem [Jacob & Johann Bernoulli, Streitschriften, 275]

This conflict reveals two very different personalities. Facing the subtle and hurtful irony of Jacob, Johann's only means of defending himself is by a straightforward attack. Johann is proud of the fact that he solves problems very quickly but he acts with precipitation and commits errors that Jacob takes a nasty pleasure in revealing. Thus, in a brief notice in the *Journal des savants* dated May26th, 1698 (p.240), he asks his brother on a mocking tone « to go through his latest (solution) once more, to examine all parts attentively and then tell us if everything is all right; declaring to him: when I will have given my solution, pretexts for rushing will no longer be accepted".[44]

Aware of Johann's interest in money, that he himself shared, Jacob did not hesitate to stake a sum that he ended up not having to pay. In a letter to Pierre Varignon also published in the *Journal des savants*, Jacobs acts with a slyness that enrages Johann : "When I suggested some problems to my brother in the Leipzig journal, I was mainly hoping that one day he would give us the solution. In this way I believed that we could partake in the glory of those who are able to excel in a science where we have only recently begun to advance; I also had reasons to hope that he could succeed and win a small reward that one of my friends had provided. I tell you this, sir, to make you understand the joy I felt when I was able to read the solution to my problem that you so kindly sent me, especially so, since I thought I saw a certain resemblance to my own solution which made me believe that he was on the right path. But this pleasure was short-lived. So soon, my hopes were

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[42] « D'entre toutes les courbes isopérimètres constituées sur un axe déterminé BN, on demande celle comme BFN, qui ne comprene pas elle-même le plus grand espace ; mais qui fasse qu'un autre compris par la courbe BZN soit le plus grand aprés avoir prolongé l'appliquée FP de sorte que PZ soit en raison quelconque multipliée ou soumultipliée de l'appliquée PF ou de l'arc BF, c'est-à-dire que PZ soit la tantiéme proportionele que l'on voudra d'une donée A & de l'apliquée PF ou de l'arc BF ». In Johann's translation published in the *Journal des savants* of December 2nd 1697, 458-465. Voir [Jacob & Johann, Streitschriften, 309].

[43] For a complete account of the controversy and especially its mathematical aspects, see for instance Goldstine's introduction to [Jacob & Johann Bernoulli, Streitschriften, 1-113].

[44] « de repasser tout de nouveau sur sa derniere [solution], d'en examiner attentivement tous les points, & de nous dire ensuite si tout va bien ; lui déclarant qu'aprés que j'aurai doné la miene, les prétextes de précipitation ne seront plus écoutez » [Jacob & Johann Bernoulli, Streitschriften, 354] .

deceived...".[45] Since the brothers know each other so well, they anticipate each other's reactions. Jacob even guesses the methods that his brother would use. The latter lashes out in an excessive reply, accusing Jacob of "an imagination greater that that of those so-called magicians who believe themselves to be physically in Sabat".[46] The difference in tone is striking. The older brother disperses a subtle poison, maintaining perfect control of what he says, while the younger brother throws out excessive insults.

Jacob, who continues this quarrel after both brothers have been admitted to the Royal Academy of Science, on the explicit condition that they put an end to the hostilities, expresses to Varignon that he has support in Basel, where Johann is perceived as ungrateful. On June 25th, 1700, Varignon cites a letter where Jacob writes : « Here where I am judged by what I have done for him, his reactions are considered so abominable that they should not be ignored. How can I help it, if the circumstances have forced me to do what I have done? Please, do not be upset with me; but rather reconsider your own attitude and your excessively high opinion of the abilities of my brother, who has made you publish all his writings".[47] This is not untrue! Varignon had become a pawn in the hands of Johann Bernoulli. But I do not believe that Jacob's surroundings had to force him to reply to Johann's massive attacks. In fact, he did not hesitate to ask Nicolas Fatio for help to turn the English against his brother. The fact that he was surrounded by family and disciples, in particular Jacob Hermann, whose strong attachment to Jacob, Johann recalled later, in 1718, certainly reinforced Jacob and made him feel supported in his struggle. As far as the isoperimetric problem was concerned, Nicolas Bernoulli who was also a student of his uncle Jacob, conscientiously criticized his other uncle Johann's solution and confessed on April 20th 1745 to Euler his regret that "Jacob had so unjustly been vilified by his brother".[48]

While Jacob could rely on the support from his family and his disciples in Basel, the younger brother was exiled in Holland. A professor at the University of Groningen, certainly, but far away from home and struggling to be rightfully acknowledged. Thus, he writes on December 24th, 1697 to Pierre Varignon : « my brother may be jealous, I am not.. You correctly believe that the bitterness that he shows for me on all occasions does not come merely from emulation, it is sufficient to say that it stems mainly from three fine qualities ambition, envy and avarice. If I sometimes stab him back, it is to show that I am not indifferent to the way he treats me, nor enough of a coward to let him keep me down; I manage without him, I do not depend on him in any way, and I owe him nothing".[49] There is no clearer way to express a recently conquered independence.

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45 « Lors que je proposai dans les Journaux de Leipsic à mon frere quelques problêmes de Geometrie, ce fut principalement dans la vuë & dans l'esperance qu'il nous en doneroit un jour la solution. Car outre que je considerois que nous pouvons avoir bone part à la gloire de ceux qui se rendent habiles dans une science, dont il n'y a pas long-temps que nous leur avons doné les premieres ouvertures ; j'avois encore des raisons particulieres pour souhaiter qu'il y pût réussir & gagner le petit prix qui y a été joint par un de mes amis. Ce que je dis, M. pour vous faire comprendre le plaisir que j'ai eu à lire la solution de mes problêmes dans le cahier du Journal que vous avez eu la bonté de m'envoyer, & plus encore à y remarquer d'abord quelque conformité avec la miene, laquelle me faisoit croire qu'il s'en étoit acquité en habile home. Mais que ce plaisir a duré peu ! Il a été bien-tôt suivi du chagrin de voir mon atente frustrée, … » [Jacob & Johann Bernoulli, Streitschriften, 356].

46 « une imagination plus forte & plus vive que celle de ces prétendus sorciers qui croyent se trouver corporelement au Sabat ». Extracted from a letter of M. Bernoulli…, published in the *Journal des savants* the 8th December 1698, p.477-480 [Jacob & Johann Bernoulli, Streitschriften, 376].

47 « Mais ici, où l'on en juge par raport à ce que j'ay fait à son égard, on trouve la piece tout à fait abominable, & qui ne pouvoit nullement demeurer sans replique. Qu'en puis-je donc, si j'ay été forcé & comme tiré par les cheveux à faire ce que j'ay fait ? Ne vous en prenez pas, je vous prie, à moy ; mais prenez vous en plus tost à vous même, & à la trop bonne opinion que vous avez de l'habileté de mon frere, qui vous avoit fait publier touttes ses pieces, &c. » [Johann Bernoulli, Briefe 2, 250].

48 « Jacob ait été injustement vilipendé par son frère » [Euler, Briefwechsel 2, 617 et 621].

49 « car si mon frere est jaloux, je ne le suis pourtant pas ; … Vous avez raison de croire que l'aigreur qu'il fait sentir en toute occasion contre moy ne vient pas seulement d'emulation, c'est assez que je vous dise qu'elle vient principalement de ses trois belles qualités d'*ambition*, d'*envie* et d'*avarice*. Si je luy donne quelques fois des coups de lame, c'est pour luy montrer que je ne suis pas ni insensible à ses traitements, ni si poltron à me laisser mettre le

What conclusions can we draw when reading about this lamentable quarrel? On one hand, we have the older brother, slow, poised and proposing general and carefully thought through methods, on the other hand a young desperado who throws himself at new problems, thinks quickly and bases his solutions on tricks, analogies or general intuition. As far as the challenges and brotherly duels in Leibnizian analysis are concerned, the above is a fair description of the brothers' behavior. But does this still hold true if we study the *Ars conjectandi* or Johann's later memoirs on mechanics? We would not necessarily reach the same conclusions.

3. A culture of challenge

In the context that we have described, the violent competition between the brothers was probably unavoidable. Jacob considered Johann as his pupil who as such owed gratitude and respect to his teacher. Johann needed to emancipate himself from his brother to be able to fully develop his talent and stand on his own two feet. But their rivalry was exacerbated by the mathematical praxis at the turn of the century, when the new analysis must be introduced and its superiority established. In 1706, Leibniz confirms this in the *Nouvelles de la République des lettres* : "The late Mr. Bernoulli who saw that a new field was opening up, asked me to consider whether this analysis could also be used for other problems, unsuccessfully manipulated by others, and in particular the problem of the curve formed by a chain that is supposed flexible in each point". [50] There were a multitude of challenges where only those who manipulated the new methods with the greatest dexterity could impose themselves. This practice leads to the emergence of a small elite, to which one sought to belong at any cost. It was not only a question of contributing to the progress in an area by solving problems, but to solve them in a determined period of time to prove one's superiority.

We have a valuable testimony as to this practice of challenges provided by Nicolas Fatio, whose ties to the English, Jacob had wanted to use to polemic against Johann. Fatio expresses repeatedly his refusal "to propose problems in public (and) to lead others onto one's own path without giving them any other recognition than that of having been able to follow someone else".[51] Fatio had, as most of the mathematical community, been excluded early on from the inner circle made up of those who were capable of solving the brachystochrone problem. This, in fact makes his testimony all the more interesting. He wrote to Jacob on March 22nd, 1701: « It is true that I became aware, when I had to withdraw, that a kind of tyranny, a sovereign authority had appeared among mathematicians, that programs were published, that everyone was interrupted and worried, that some of the decrees of this new tribunal began by a PLACUIT, that problems were proposed, that time limits were imposed and that sometimes new terms were added to modify the time given, that it was declared that only so and so were capable of solving the problems; It is true that I felt that I should speak up; that each man with some sensitivity should overtly protest and condemn these haughty manners. But Mr. Leibniz claims that I only defend my own cause, while pretending to defend that of the community. He is unable to see that these are so closely related that even I am not capable of separating them".[52] When being acknowledged for one's work, something everyone

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pied sur la gorge ; je subsiste sans luy, je ne suis point de sa dependance, aussy n'ay je rien de luy dont je luy puisse être redevable » [Johann Bernoulli, Briefwechsel 2,].

[50] « Feu Mr. Bernoulli voiant qu'un nouveau champ etoit ouvert, il me pria de penser, si par la meme analyse on ne pourroit arriver à des problemes plus difficiles, maniés inutilement par d'autres, et particulierement à la courbe qu'une chaine doit former, supposé qu'elle soit parfaitement flexible partout… ». Quoted by Johann Bernoulli in the same letter to Montmort the 29th Septembre 1718.

[51] « à proposer des problemes au public, [et] à faire marcher les autres sur ses propres pas sans qu'il y ait d'autre gloire à attendre pour ceux-là que d'avoir pu suivre ceux ci » [Jacob Bernoulli, Briefwechsel, 164].

[52] « Il est vrai quand j'ai vû, dans ma retraite, qu'il s'elevoit une espece de Tyrannie et d'autorité souveraine parmi les Mathématiciens, qu'on publioit des programmes, qu'on interrompoit et qu'on inquietoit tout le monde, que quelques uns des arrets de ce nouveau Tribunal commençoient par un PLACUIT, qu'on proposoit des problemes, qu'on limitoit des Jours et qu'on ajoutoit quelquesfois par grace de nouveaux termes pour le Temps de leur Solution qu'enfin on prononçoit que tels et tels seulement les avoient resolus et qu'on avoit bien prévû que tels et tels seuls les pourroient

could legitimately expect, is only obtained by entering on someone else's ground and beating him, it is not surprising that competition is relentless, especially in the case of two brothers who have been raised together and who collaborate in mathematics. Such practices do not only exacerbate competition but marginalize researchers who work quietly in the area of their choice. Furthermore, it excludes the larger number, who can only step aside and count the points as bystanders without being allowed to compete or to participate in any way whatsoever.

4. Enduring hostility

The two brothers were never reconciled. In 1705, under pressure from his in-laws, Johann returned to his hometown Basel as Jacob lay dying. He received the news of his brother's death during the journey and succeeded him in the mathematics chair as Jacob had foreseen in a premonitory letter written on June 3rd, 1705 to Leibniz : "If the rumor is true, my brother is certainly returning to Basel, not to take the Greek Chair, but rather mine (which he believes and correctly so that he will be able to take shortly), since I feel life slipping away completely".[53] Jacob died on August 1st, 1705, from complications of the gout.

After Jacob's death, the family relations were very tense, because Jacob's family prevented Johann from accessing his brother's *Nachlass*, including l'*Ars conjectandi*. The correspondence between Johann and Montmort, not yet published [UB Basel LIa 665], gives some indications about the climate between the two families. Johann suspects Jacob of having given instructions to prevent him from accessing his papers : "I have not seen any of his papers or the manuscripts he left behind; I believe that as he lay dying, he ordered by precaution, not to let me take part of any of his writings after his death".[54] Initially it was only Jacob Hermann who went through, read and even took some manuscripts. He was also the one who wrote the memoir[55] that Fontenellle used for his "*éloge*" on Jacob Bernoulli [Fontenelle, 1707]. Later, Jacob's son gave some of the documents to Nicolaus. This is how Johann reacted to this in a letter to Monmort : "at first, after my brother's death, Mr. Hermann had free access to his study whenever he wanted and took from it whatever writings of the deceased interested him, so that not a single paper of my brother's escaped from Mr. Hermann who was free to copy them, or keep them, whatever he found suitable: It is true that later, the son of the deceased gave my nephew from Padova a substantial part of his late father's manuscripts".[56] It is Nicolaus who transmits them to Gabriel Cramer to edit them in two volumes of *Opera,* published in 1744 by Marc-Michel Bousquet in Geneva.

CONCLUSION

Verbose, ambitious, greedy, secretive, misanthropist, envious, proud, and too imaginative... this is an edifying list of adjectives that Johann uses to qualify his brother Jacob. Without asking whether

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resoudre ; Il est vrai dis je qu'alors j'ai crû devoir sortir du Silence ; et tout homme de coeur s'opposera ouvertement à des manieres si hautaines et ne manquera pas de les condanner. Mais Mr Leibnitz dit que c'est ma cause seule que j'ai defendue, sous ombre de defendre celle du Public. Il ne veut pas s'apercevoir qu'elles sont si confondues qu'il lui est encore plus impossible qu'à moi meme d'en etablir la Separation » [Jacob Bernoulli, Briefwechsel, 183].

[53] « Si rumor vera narrat, redibit certe frater meus Basileam, non tamen Graecam (…) sed meam potius stationem (quam brevi cum vita me derelicturum, forte non vane, existimat) occupaturus » [Jacob Bernoulli, Briefwechsel, 150].

[54] « je n'ai jamais rien vû de ses papiers et des manuscripts laissés apres sa mort ; Je crois que se voyant mourir, il a pris les precautions en ordonnant que rien de ses ecrits ne me seroit communiqué, quand il seroit mort » [29.9.1718, UB Basel, LIa 665].

[55] Cf. [Johann Bernoulli, Briefwechsel 2, 178].

[56] « d'abord aprés la mort de mon frere, Mr. Herman eut autant de fois qu'il vouloit un libre acces dans sa bibliotheque, d'où il prit des ecrits du defunt, tout ce qui l'accommodoit, en sorte que pas un billet des papiers de mon frere a pû echapper des mains de Mr. herman, et qu'il ne tenoit qu'a lui de le copier ou de le garder, selon qu'il le trouvoit à propos : Il est vrai que du depuis le fils du defunt a donné à mon Neveu de Padoüe une bonne partie des manuscripts de feu son Pere » [29.9.1718, UB Basel, LIa 665].

it allows us to seize a reality that necessarily evades us, it does say something about the relationship between the two brothers and the ferocious competition between them. This competition comes to expression partially through mathematics. Johann explicitly expresses this fact in 1687: “My brother must be extremely conceited since he believes that I am incapable of solving the problems he has solved; but if I was in the mood to do the same to him, I could come up with questions so subtle and so unusual that he would spend his whole life on them to no avail, and yet I have solved them very easily”.[57]

To find a result, to be among the first to discover it or at least to have come upon it independently: this seems to have been the ambition of the turn of the century mathematician who actively participated in developing the new analysis. When two brothers are confronted, and one of them is also the other's teacher, this exacerbates competition and brings it into the personal sphere, as is reflected in the list above. The older brother Jacob has a hard time accepting that the one he has taught everything takes off on his own and may at any moment surpass his teacher. The younger brother, Johann tries to acquire some independence by proving that he is able to solve the most difficult problems quickly and ingeniously. This remains the aspect that Johann emphasizes in 1719 when he recalls to Montmort the conflict with his brother : “at the time when my brother and I were arguing, for quite some time and very heatedly, about the isoperimetric problem, and he accused me of having learned from him the very foundations of mathematics, I countered this attack by reminding him that he was indebted to me for things of greater importance, such as the first theory of catenary chains;etc”.[58]

Jacob, the more thoughtful of the two, had in fact reflected on the situation where two researchers work in the same area. Some traces of this remain in his works. His considerations are closely linked to his conviction that in science progress is made starting from a foundation of knowledge acquired earlier and that it is made through small steps that add up. Even when two people try to solve the same problem, they often do it in different ways. To rediscover a result that has already been established-disdainfully described in the letters of their time by the metaphor « *ova post prandium apponere* »- is worthy of praise if the research is made completely independently. One of the two discoverers should not completely overshadow the other one. This was finally, what happened. Jacob and Johann Bernoulli share the glory for having developed the differential calculus discovered by Leibniz, but at a high price; an artificially constructed twosome where one is barely distinguishable from the other, and yet Johann had struggled for years to break free from the grip of his brother and master. While Jacob had generously shared his knowledge of mathematics with his brother, the same cannot necessarily be said for sharing the recognition and the glory. His humor, which comes to its fullest expression in his letters to Nicolas Fatio, turned into biting irony, his thorough knowledge of Johann served to humiliate the latter and his capacity for criticism was employed to curb Johann's creativity. In vain!

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[57] « Il faut que mon frere soit boursoufflé d'une terrible suffisance, puisqu'il croit que je ne pourray pas resoudre ce qu'il a resolu ; mais si j'étois d'humeur de luy rendre la pareille, je luy proposerois des questions si subtiles et si peu communes, qu'il y crouperoit toute sa vie sans en pouvoir venir à bout, que j'ay pourtant le bonheur de resoudre fort facilement » [Johann Bernoulli, Briefe 2, 120].

[58] « etant autrefois en dispute avec mon frere, au sujet du probleme des Isoperimetres, laquelle duroit assez longtemps, avec beaucoup de chaleur de part et d'autre, sur ce qu'il m'avoit reproché d'avoir appris de lui les premiers commencements de la Geometrie, je lui retorquai ce reproche en le faisant souvenir, qu'il m'etoit redevable d'autres choses de plus grande importance, entre autre de la premiere theorie des chainettes ; etc. » [29.9.1718, UB Basel, LIa 665].

Bibliography

Handwritten documents

[UB Basel, LIa 665] Correspondance de Johann Bernoulli avec Pierre Rémond de Montmort (1703-1719), Universitätsbibliothek Basel, Handschrift LIa 665

Printed documents

[Battier, 1705] J.J. Battier, Vita Jacobi Bernoullii, Basel 1705 = [Jacob Bernoulli, Opera I, 7-34]

[Bernoulli, Gedenkbuch, 1922] Gedenkbuch der Familie Bernoulli zum 300. Jahrestage ihrer Aufnahme in das Basler Bürgerrecht, Basel 1922

[Jacob Bernoulli, 1713] Jacob Bernoulli : De arte conjectandi, Basel 1713

[Jacob Bernoulli, Briefwechsel] Der Briefwechsel von Jacob Bernoulli, bearbeitet und kommentiert von André Weil, mit Beiträgen von Clifford Truesdell und Fritz Nagel, Basel : Birkhäuser Verlag, 1993

[Jacob Bernoulli, Opera] Jacobi Bernoulli, Basileensis, Opera, t.I –II, Genevae, 1744

[Jacob Bernoulli, Werke] Die Werke von Jacob Bernoulli, 1-5, Basel 1969-1999

[Jacob Bernoulli, 1700] *Jacobi Bernoulli Prof. Basil. & design. Acad. Rect. Ad Fratrem suum Johannem Bernoulli Professorem Groninganum Epistola*, Basel 1700. Cette lettre publique n'a pas été incluse dans les *Opera* de Jacob, mais a été republiée dans [Jacob & Johann Bernoulli, Streitschriften, 471-484]

[Jacob Bernoulli, 2006] *The Art of Conjecturing*, together with « Letter to a Friend on Sets in Court Tennis », English translation by E.Sylla, Baltimore 2006

[Johann Bernoulli, Briefe] Der Briefwechsel von Johann Bernoulli, 1-3, Basel 1955-1992

[Johann Bernoulli, Opera] Johannis Bernoulli, …, Opera omnia, t.I-IV, Lausannae & Genevae, 1742

[Jacob & Johann Bernoulli, Streitschriften] Die Streitschriften von Jacob und Johann Bernoulli. Variationsrechnung, bearbeitet und kommentiert von Herman H. Goldstine, mit historischen Anmerkungen von Patricia Radelet-de Grave, Basel : Birkhäuser Verlag, 1991.

[Nicolaus Bernoulli, 1709] Nicolaus Bernoulli : De usu artis conjectandi in jure, Basileae, 1709

[Comte, 1864] A. Comte : Cours de philosophie positive, 2e éd. par E. Littré, Paris 1864

[Descartes, 1659-1661] Geometria, a Renato Des Cartes Anno 1637 Gallice edita ; postea … in Latinam linguam versa, & Commentariis illustrata, Opera atque studio Francisci a Schooten, Amstelaedami, 1659-1661

[Descartes, 1695] Renati des Cartes Geometria … Ex Revisione, & cum Notis tumultuariis Viri Cl. Jac. Bernoulli, Francfurti ad Moenum, 1695`

[Euler, Briefwechsel 2] L. Euler : Briefwechsel, Opera omnia, series quarta A, vol.II, hrsg. von E.A. Fellmann et G.K. Mikhajlov, Basel 1998

[Figuier, 1870] L. Figuier : Vie des savants illustres du XVIII^{e} siècle, Paris 1870

[Fleckenstein, 1949] J.O. Fleckenstein : Johann und Jakob Bernoulli, Basel 1949

[Fleckenstein, 1958] J.O. Fleckenstein : L'école mathématique bâloise des Bernoulli à l'aube du XVIIIe siècle, Conférence faite au Palais de la Découverte le 3 mai 1958, Université de Paris

[Fontenelle, 1707] B. le Bovier de Fontenelle : Eloge de M. Bernoulli, Mémoires de l'Académie royale des sciences pour l'année 1706, Paris 1707, Partie Histoire, p.139-150

[Hofmann, 1956] J.E. Hofmann : Über Jacob Bernoullis Beiträge zur Infinitesimalmathematik = Monographies de l'Enseignement mathématique 3, Genève 1956

[Leibniz, Math. Schriften] G. W. Leibniz : Mathematische Schriften 2-3, hrsg. von C.I. Gerhardt, Hildesheim-New York : Georg Olms Verlag, 1971 (Reprint de l'éd. originale, Halle 1855)

[L'Hôpital, 1696] G.F.A. de l'Hôpital : Analyse des infiniment petits pour l'intelligence des lignes courbes, Paris 1696

[Mach, 1883] E. Mach : Die Mechanik in ihrer Entwicklung historisch-kritisch dargestellt, Leipzig 1883

[Merian, 1860] P. Merian : Die Mathematiker Bernoulli. Jubelschrift zur vierten Säcularfeier der Universität Basel, Basel 1860

[Meusnier, 1987] N. Meusnier : Jacques Bernoulli & l'*ars conjectandi*. Documents pour l'étude de l'émergence d'une mathématisation de la stochastique, brochure de l'IREM de Rouen, 1987

[Meusnier, 1992] N. Meusnier : L'usage de l'art de conjecturer en droit. Traduction française avec des notes, Centre d'analyse et de mathématiques sociales, Paris, 1992

[Montmort, 1708] P. Rémond de Montmort : Essay d'analyse sur les jeux de hazard, Paris, 1708

[Ohly, 2001] S. Ohly : Johann Bernoullis mechanische Arbeiten 1690-1713, Dissertation der Universität Bielefeld unter der Leitung von Hans Niels Jahnke, 2001.

[Peiffer, 1988] J. Peiffer : L'école mathématique bâloise des Bernoulli, Préfaces 9, 101-115

[Peiffer, 1989] J. Peiffer : Le problème de la brachystochrone à travers les relations de Jean I Bernoulli avec L'Hôpital et Varignon, Studia Leibnitiana Sonderheft 17 : Der Ausbau des Calculus durch Leibniz und die Brüder Bernoulli (Symposium der Leibniz-Gesellschaft und der Bernoulli-Edition der Naturforschenden Gesellschaft in Basel, 15. bis 17. Juni 1987), hrsg. von Heinz-Jürgen Heß und Fritz Nagel, Stuttgart 1989, 59-81

[Peiffer, 1991] J. Peiffer : Le problème de la brachystochrone, un défi pour les méthodes infinitistes de la fin du XVIIème siècle", Sciences et techniques en perspective, Nantes 1991, 54-81

[Roche, 2003] D. Roche, Humeurs vagabondes, Paris, 2003

[Wolf, 1859] R. Wolf : Biographien zur Kulturgeschichte der Schweiz, 2. Cyclus, Zürich 1859