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EXISTENCE, UNIQUENESS, AND ANGLE COMPUTATION FOR THE LOXODROME ON AN ELLIPSOID OF REVOLUTION

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Abstract. We summarize a proof for the existence and uniqueness of the loxodrome joining two distinct points p_o and p_1 on an open half of an ellipsoid of revolution. We also compute the unique angle $\alpha \in [0, 2\pi)$ which the loxodrome makes with the meridians intersecting the loxodrome.

1. Introduction

A loxodrome on an ellipsoid of revolution is a curve that traverses all the meridians along its way at a constant angle. Since the earth is modeled as an ellipsoid of revolution, understanding loxodromes plays an important role in the science of navigation; see, e.g., [4–6, 9]. The existence and uniqueness of a loxodrome on an ellipsoid of revolution and a formula for its angle are known results; see, e.g., [4, 9].

Typically, the existence of a loxodrome on an ellipsoid of revolution is proved by constructing a one-to-one conformal map (is a continuously differentiable map that preserves angles between curves) Ψ from the open square $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \times \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \subset \mathbb{R}^2$ onto an open connected subset of the ellipsoid of revolution which contains the points p_o and p_1 that are meant to be joined by a loxodrome. The map Ψ is such that every vertical straight line segment in the open square $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \times \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ is mapped onto a meridian of the ellipsoid of revolution. Thus, if $q_o = \Psi^{-1}(p_o)$ and $q_1 = \Psi^{-1}(p_1)$, then q_o and q_1 can be joined by a straight line segment in the open square $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \times \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (which makes a constant angle with all vertical straight line segments on its way) and the image of that straight line segment joining q_o to q_1 under Ψ will be a curve on the ellipsoid of revolution joining p_o to p_1 which makes a constant angle with all meridians on its way (because Ψ is conformal). Moreover, the constant angle that the loxodrome makes with all the meridians is typically computed by using the technique of "infinitesimals"; see, e.g., [5, 6].