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GREEN'S FUNCTION, WAVEFUNCTION AND WIGNER FUNCTION OF THE MIC-KEPLER PROBLEM

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Abstract. The phase-space formulation of the nonrelativistic quantum mechanics is constructed on the basis of a deformation of the classical mechanics by the $*$-product. We have taken up the MIC-Kepler problem in which Iwai and Uwano have interpreted its wave-function as the cross section of complex line bundle associated with a principal fibre bundle in the conventional operator formalism. We show that its Green's function, which is derived from the $*$-exponential corresponds to unitary operator through the Weyl application, is equal to the infinite series that consists of its wave-functions. Finally, we obtain its Wigner function.

1. Introduction

We come to the reluctant conclusion that in our previous paper [5] we obtained only a piece of the local expression of the Green's function for the MIC-Kepler problem. There (Theorem 12) we have presented two expressions denoted by $G_+(\boldsymbol{r}_f,\, \boldsymbol{r}_i\,;\, E)$ and $G_-(\tilde{\boldsymbol{r}}_f,\, \tilde{\boldsymbol{r}}_i\,;\, E)$ where $\boldsymbol{r} = \tilde{\boldsymbol{r}}$ means the position vector $\boldsymbol{x}$ in $\dot{\mathbb{R}}^3 = \mathbb{R}^3\backslash\{0\}$ i.e., $\boldsymbol{r} = (x,\, y,\, z)$. However, $G_-(\tilde{\boldsymbol{r}}_f,\, \tilde{\boldsymbol{r}}_i\,;\, E)$ is actually identical with $G_+(\boldsymbol{r}_f,\, \boldsymbol{r}_i\,;\, E)$ because the transition function is constant (independent of $\boldsymbol{x}$) and therefore, despite the difference in appearance, τ_- is essentially the same local trivialization as τ_+. This is the reason why $G_-(\tilde{\boldsymbol{r}}_f,\, \tilde{\boldsymbol{r}}_i\,;\, E)$ became equivalent to $G_+(\boldsymbol{r}_f,\, \boldsymbol{r}_i\,;\, E)$ in the case of iii). After that we have succeeded in obtaining the other piece of the local expression denoted by $G_-(\boldsymbol{x}_f,\, \boldsymbol{x}_i\,;\, E)$ via of finding another local trivialization τ_- which is transformed into τ_+ by the transition function of principal S^1 bundle varying with the position (more precisely, the longitudinal angle) of point $\boldsymbol{x}$ (see [4]). We have found, in addition, the wave-function of the MIC-Kepler problem. In this paper, by turning the right-hand system of orthogonal curvilinear local coordinates on U_- into the left-hand one, we obtain the Green's function and wave-function in a new form. In this way we end up with two left-handed coordinate systems bringing the two local trivializations which are transformed into each other by the transition function of the principal S^1 bundle. Thus it becomes possible to obtain its Wigner function on $T^*(U_+ \cap U_-) \subset T^*\dot{\mathbb{R}}^3$.

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