JGSP **35** (2014) 21–60

JOURNAL OF
Geometry and Symmetry in Physics
ISSN 1312-5192

HARMONIC ANALYSIS ON THE EINSTEIN GYROGROUP

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Communicated by Abraham A. Ungar

Abstract. In this paper we study harmonic analysis on the Einstein gyrogroup of the open ball of $\mathbb{R}^n$, $n \in \mathbb{N}$, centered at the origin and with arbitrary radius $t \in \mathbb{R}^+$, associated to the generalised Laplace-Beltrami operator

$$L_{\sigma,t} = \left(1 - \frac{\|x\|^2}{t^2}\right)\left(\Delta - \sum_{i,j=1}^{n} \frac{x_i x_j}{t^2} \frac{\partial^2}{\partial x_i \partial x_j} - \frac{\kappa}{t^2}\sum_{i=1}^{n} x_i \frac{\partial}{\partial x_i} + \frac{\kappa(2-\kappa)}{4t^2}\right)$$

where $\kappa = n + \sigma$ and $\sigma \in \mathbb{R}$ is an arbitrary parameter. The generalised harmonic analysis for $L_{\sigma,t}$ gives rise to the (σ,t)-translation, the (σ,t)-convolution, the (σ,t)-spherical Fourier transform, the (σ,t)-Poisson transform, the (σ,t)-Helgason Fourier transform, its inverse transform and Plancherel's Theorem. In the limit of large t, $t \to +\infty$, the resulting hyperbolic harmonic analysis tends to the standard Euclidean harmonic analysis on $\mathbb{R}^n$, thus unifying hyperbolic and Euclidean harmonic analysis.

Contents

doi: 10.7546/jgsp-35-2014-21-60