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ISOMORPHISM THEOREMS FOR GYROGROUPS AND L-SUBGYROGROUPS

TEERAPONG SUKSUMRAN AND KENG WIBOONTON

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Abstract. We extend well-known results in group theory to gyrogroups, especially the isomorphism theorems. We prove that an arbitrary gyrogroup G induces the gyrogroup structure on the symmetric group of G so that Cayley's Theorem is obtained. Introducing the notion of L-subgyrogroups, we show that an L-subgyrogroup partitions G into left cosets. Consequently, if H is an L-subgyrogroup of a finite gyrogroup G, then the order of H divides the order of G.

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1. Introduction

Let c be a positive constant representing the speed of light in vacuum and let $\mathbb{R}^3_c$ denote the c-ball of relativistically admissible velocities, $\mathbb{R}^3_c = \{\mathbf{v} \in \mathbb{R}^3 ; \|\mathbf{v}\| < c\}$. In [13], Einstein velocity addition $\oplus_E$ in the c-ball is given by the equation

$$\mathbf{u} \oplus_E \mathbf{v} = \frac{1}{1 + \frac{\langle \mathbf{u},\mathbf{v}\rangle}{c^2}} \left\{ \mathbf{u} + \frac{1}{\gamma_{\mathbf{u}}}\mathbf{v} + \frac{1}{c^2}\frac{\gamma_{\mathbf{u}}}{1+\gamma_{\mathbf{u}}}\langle \mathbf{u},\mathbf{v}\rangle \mathbf{u} \right\}$$

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